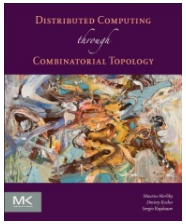
Read-Write Protocols for General Tasks

MITRO207, P4, 2019

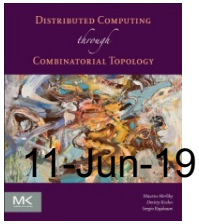


General Tasks

So far, focused on *colorless* tasks …

only *sets* of input and output value matter

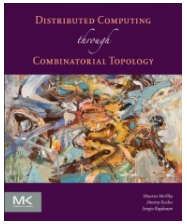
Not who is assigned which values ….



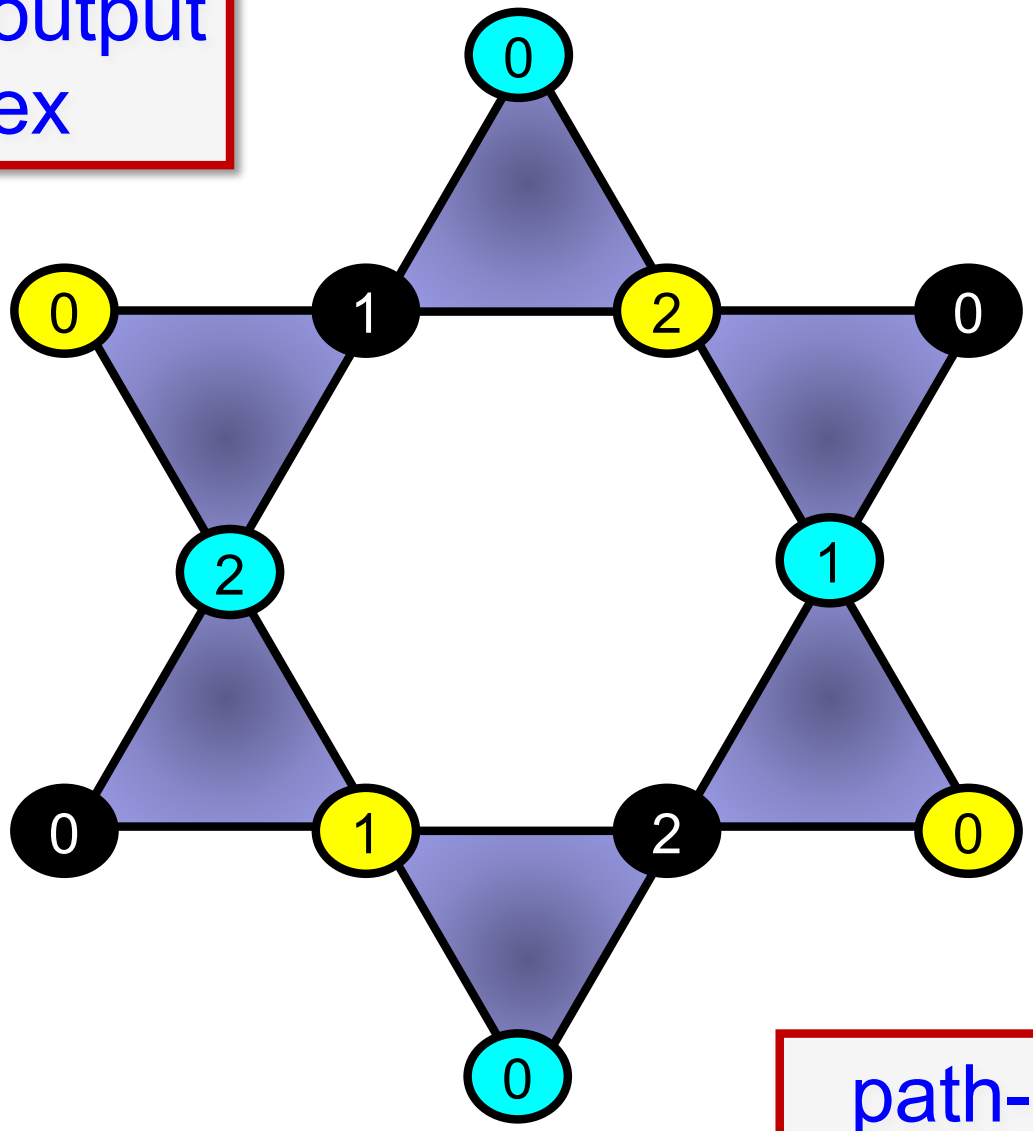
FetchAndIncrement

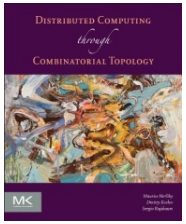
```
shared int v // shared counter

fetchAndIncrement(): int
  atomic      // next 2 steps are atomic
    prior: int := v
    v := v + 1
  return prior
```

Not colorless because outputs are *unique*



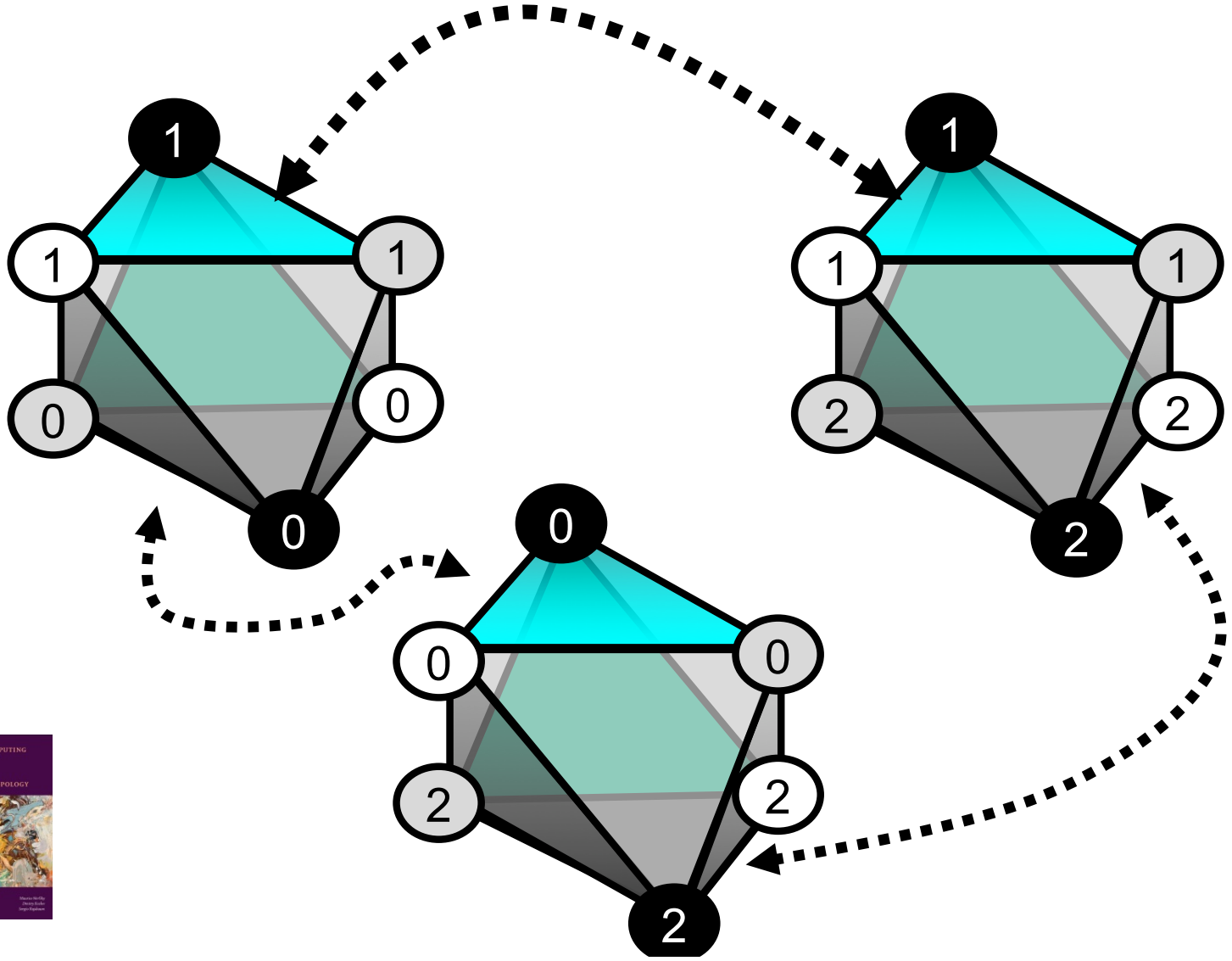
fetch&inc output complex

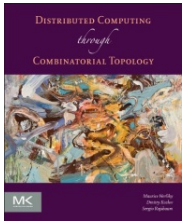




path-connected but has “holes”

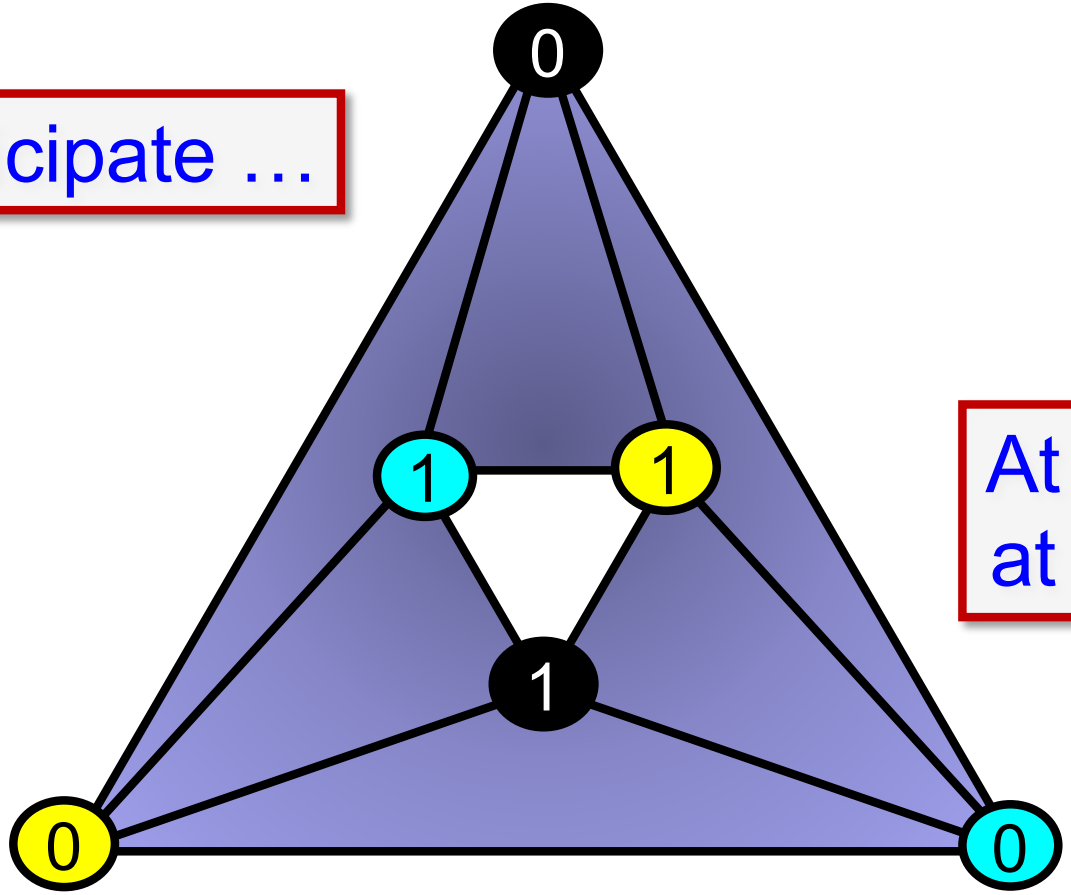
Output: (3,2)-Set Agreement



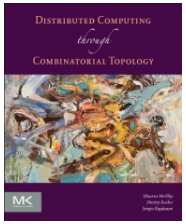


Weak Symmetry-Breaking

If all participate …



At least one 0,
at least one 1.



Renaming

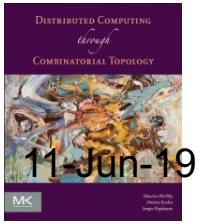
Input

Unique names from $\{0 \ldots N\}$, $N \gg n$

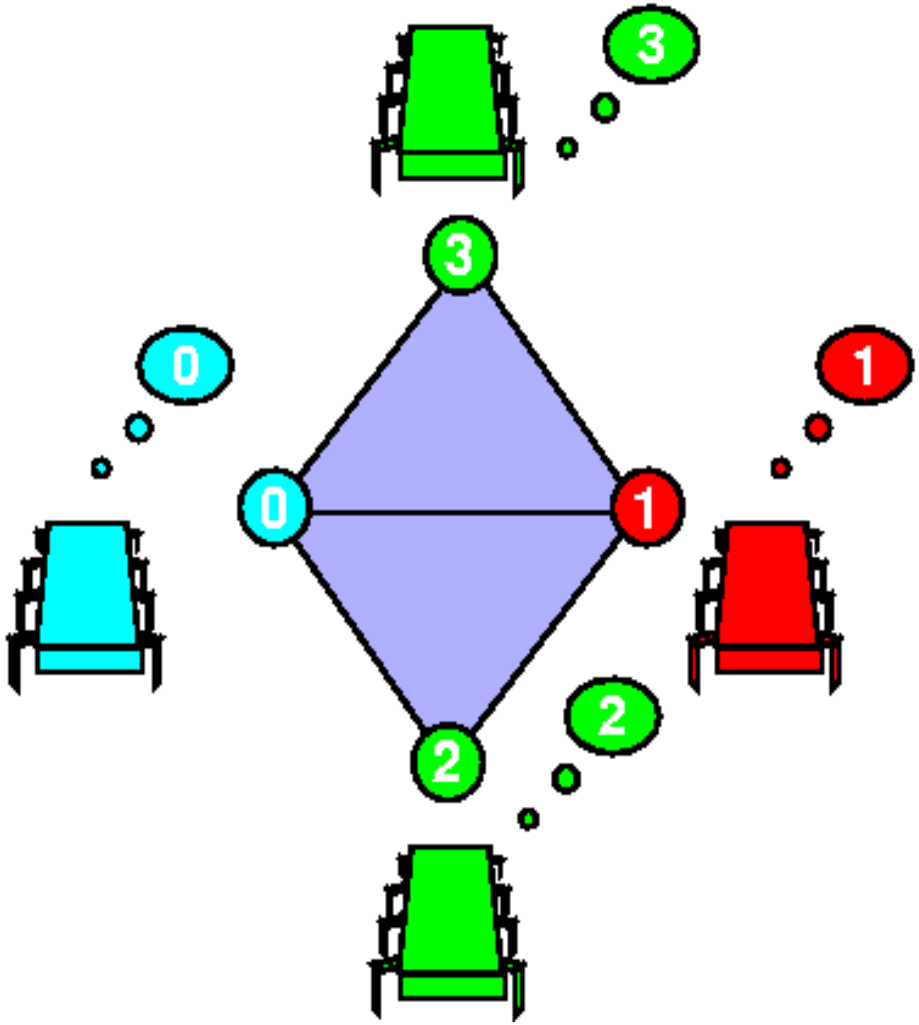
Output

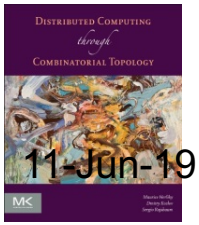
Unique names from $\{0 \ldots M\}$, $N \gg M$

Output complex called *rook complex*



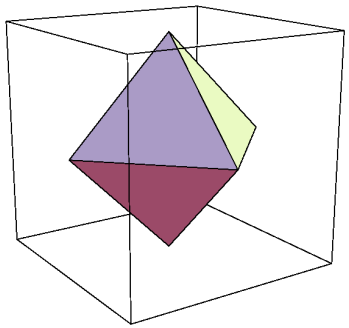
Colored Complexes



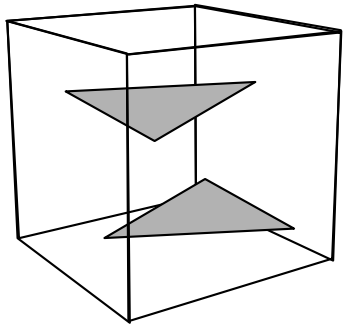


Review: Task Specification

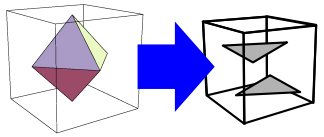
$(\mathcal{I}, \mathcal{O}, \Delta)$



Input complex

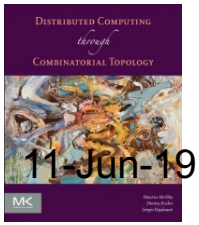


Output complex



Carrier map

$\Delta: \mathcal{I} \rightarrow 2^{\mathcal{O}}$

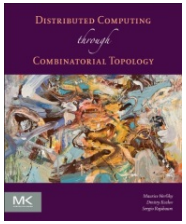


Immediate Snapshot Executions

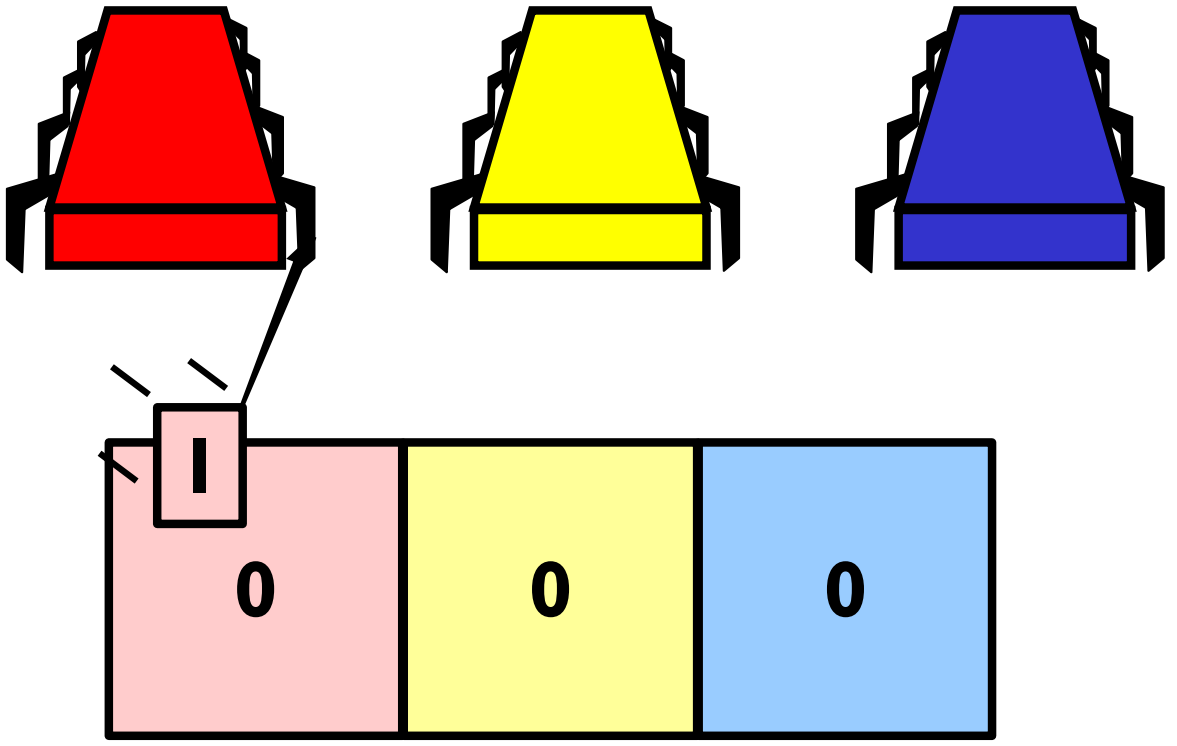
Restricted form of Read-Write memory

(Equivalent to regular R-W memory)

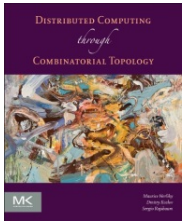
But we will not prove it yet.



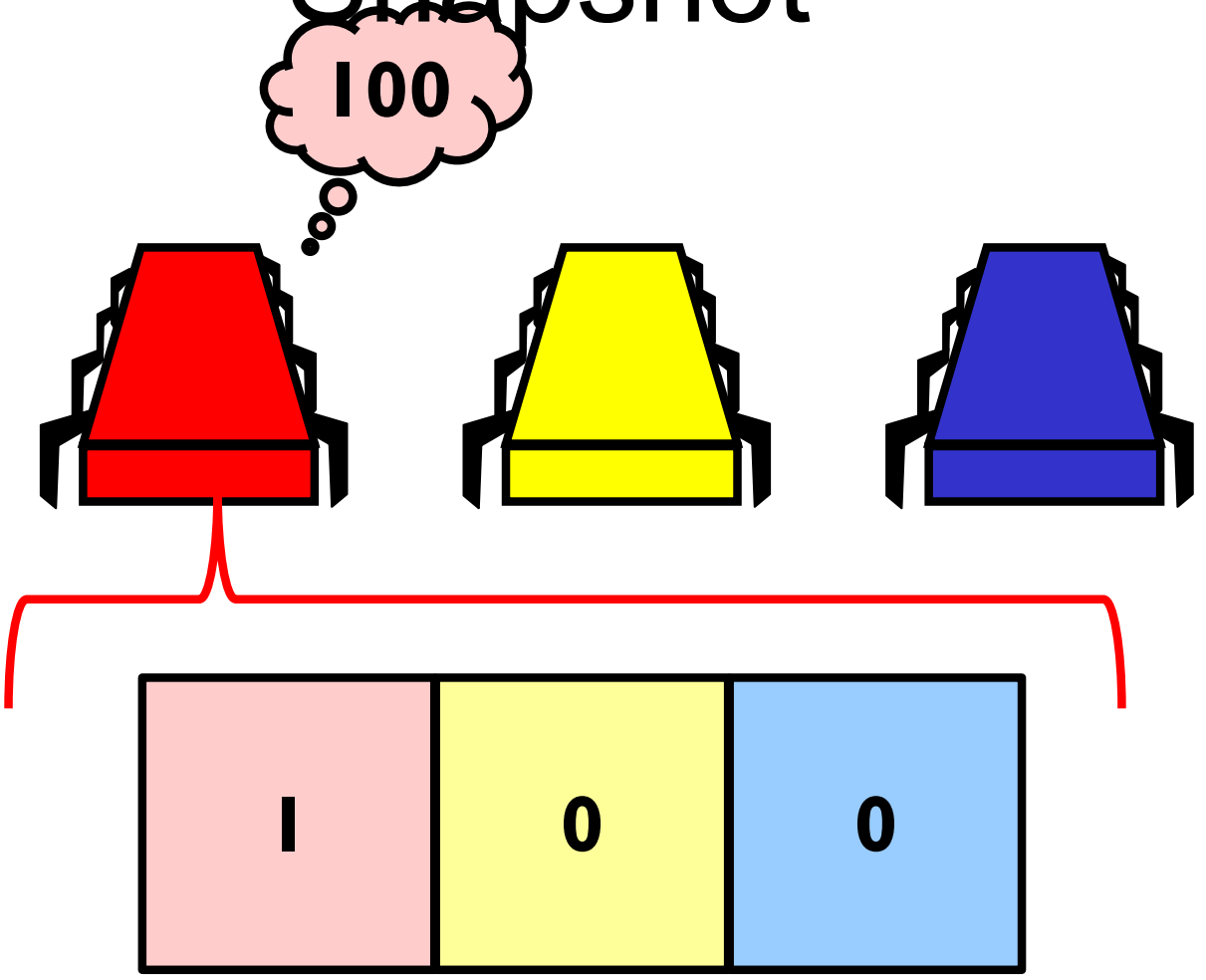
Write



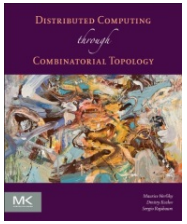
Single-writer, multi-reader variables



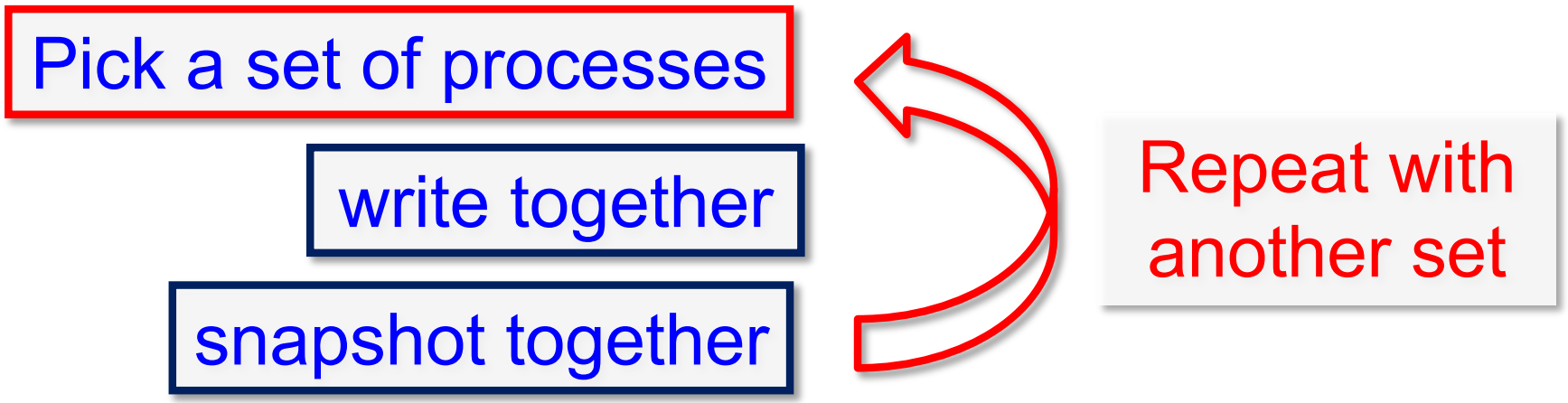
Snapshot

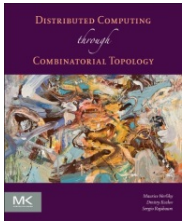


Single-writer, multi-reader variables

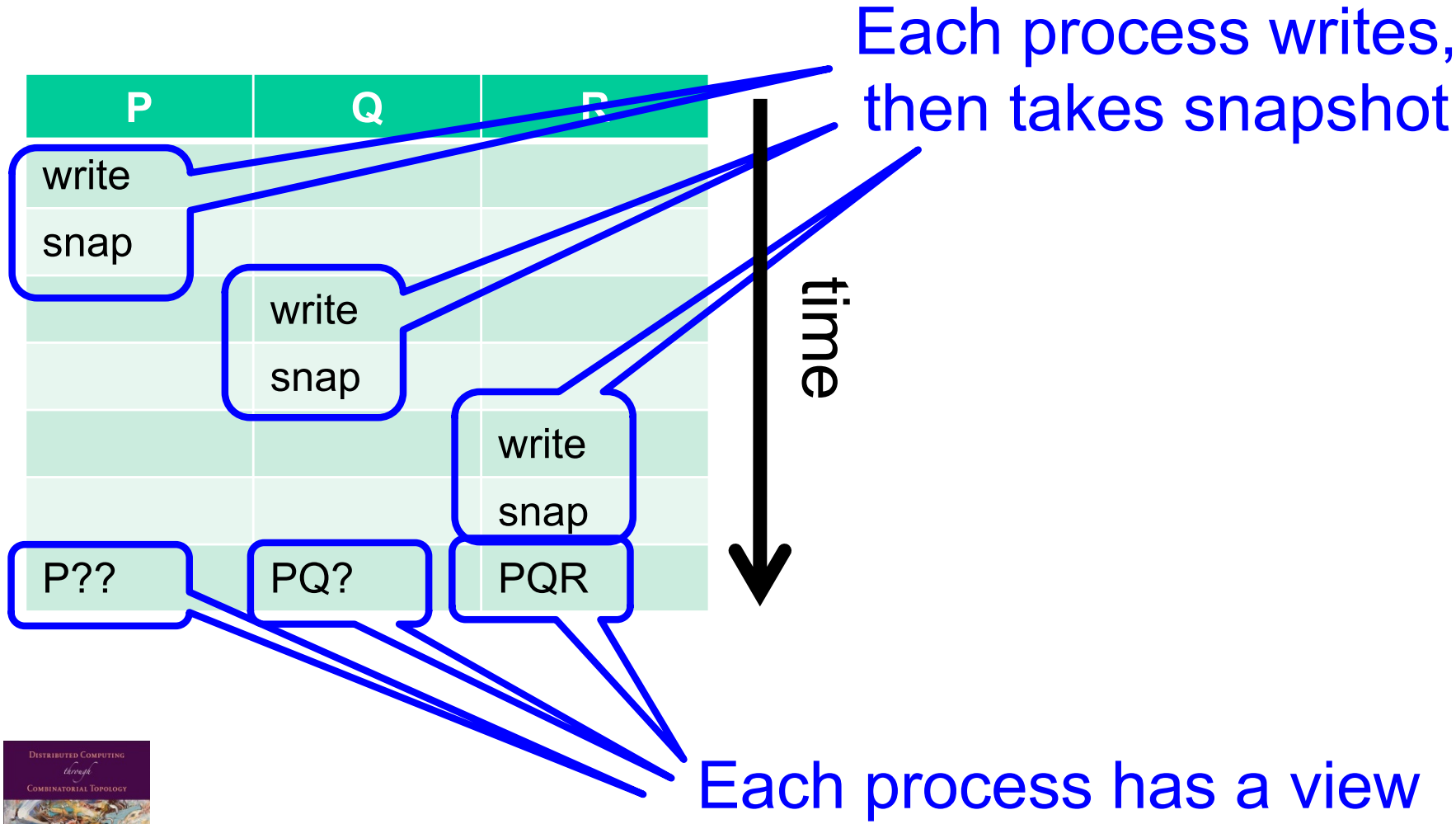


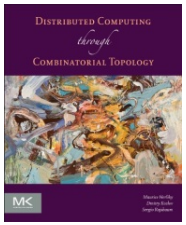
Immediate Snapshot Executions





Example Executions



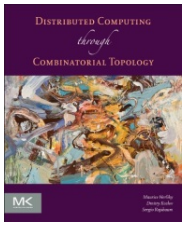


Example Executions

P	Q	R
write		
snap		
	write	
	snap	
		write
		snap
P??	PQ?	PQR

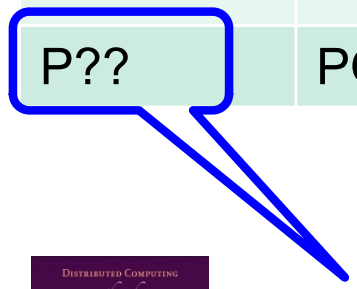
Moving last process one round earlier,

Changes this view from PQ? to PQR

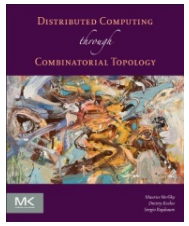


Example Executions

P	Q	R
write		
snap		
	write	write
	snap	snap
		write
		snap
P??	PQR	PQR

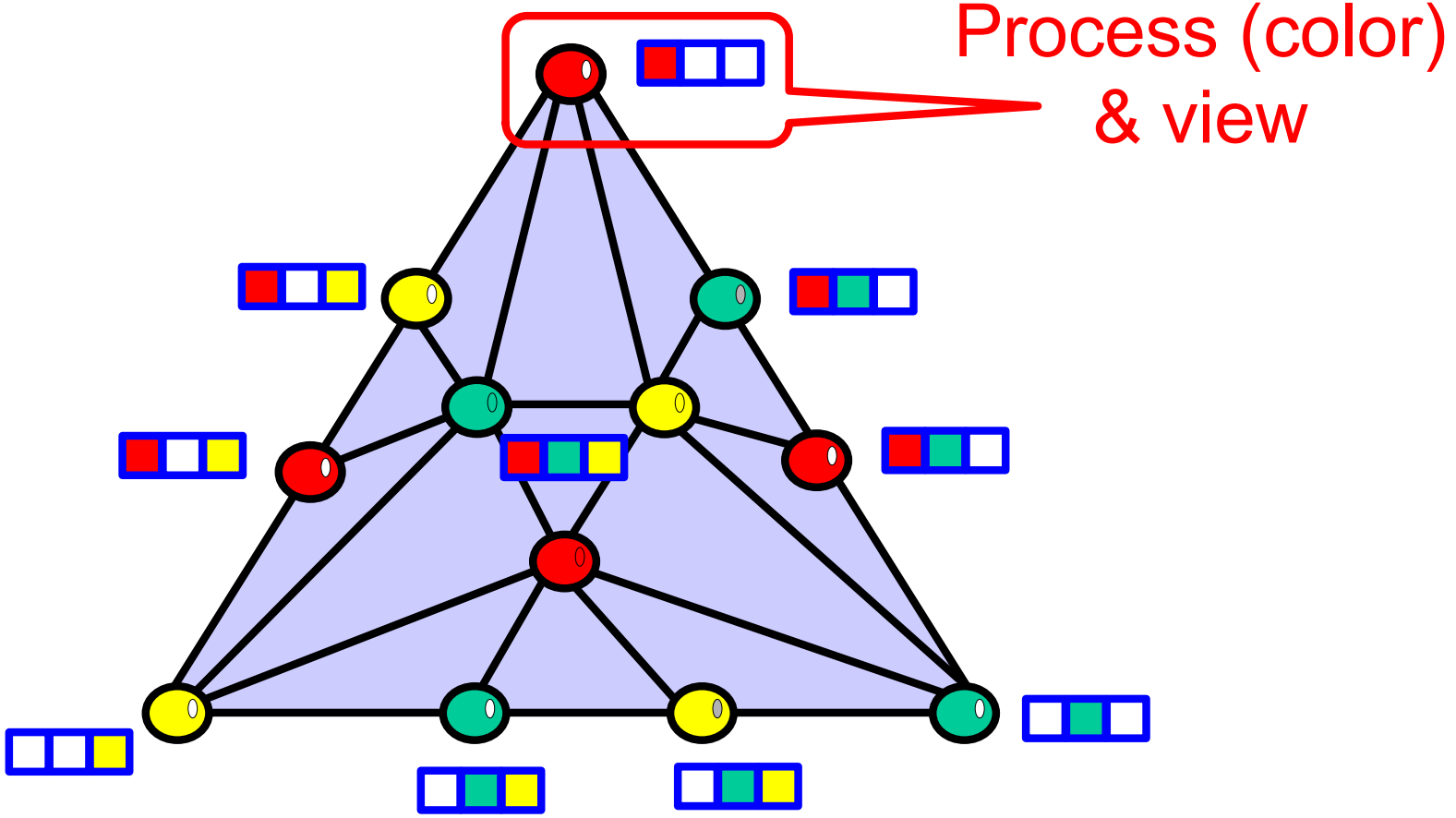


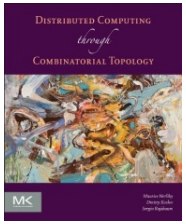
Moving last processes one round earlier,



Changes this view from P?? to PQR

Protocol Complex

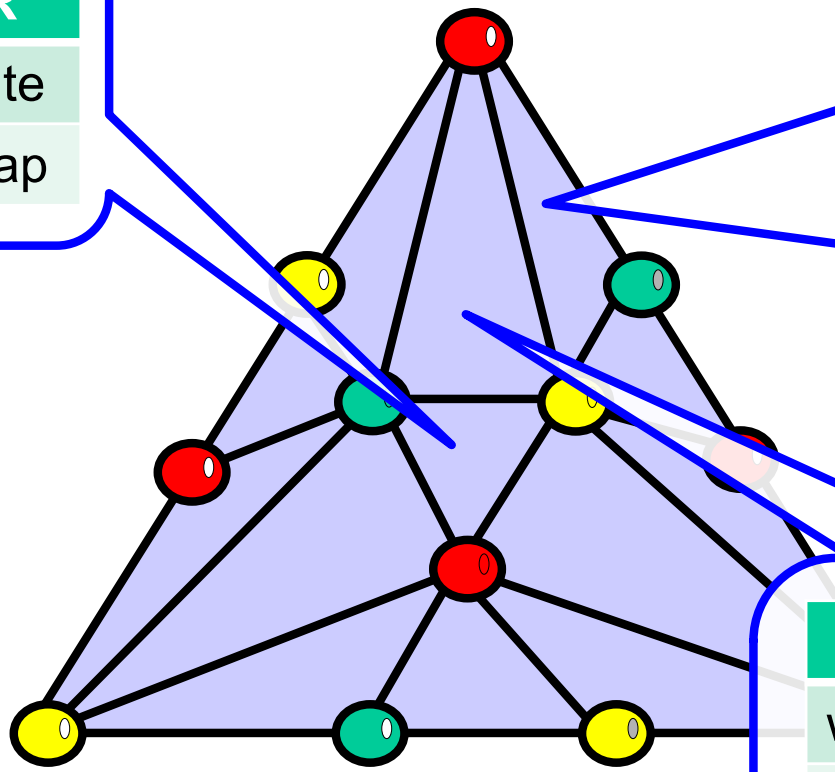




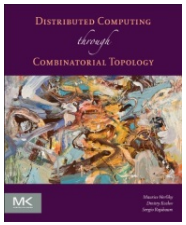
Protocol Complex

P	Q	R
write	write	write
snap	snap	snap

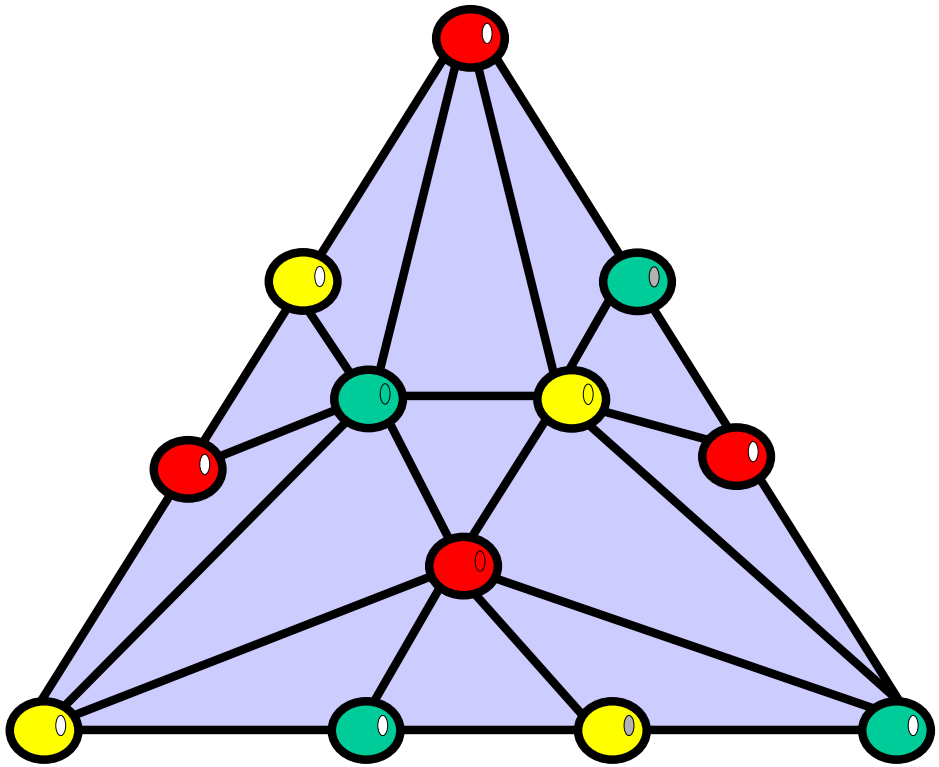
P	Q	R
write		
snap		
	write	
	snap	
		write
		snap



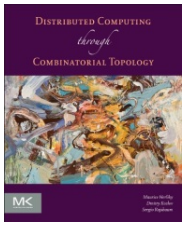
P	Q	R
write		
snap		
	write	write
	snap	snap



Standard Chromatic Subdivision



$\mathrm{Ch}(\sigma)$



Combinatorial Definition (I)

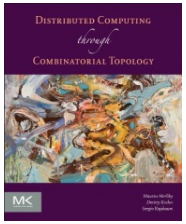
Input simplex σ

Protocol complex $\mathcal{IS}(\sigma)$

Vertex (P_i, σ_i)

Process name

view $\sigma_i \subseteq \sigma$



Combinatorial Definition (II)

Each process's write appears in its own view

$P_i \in \text{names}(\sigma_i)$.

Snapshots are ordered

$\sigma_i \subseteq \sigma_j$ or vice-versa

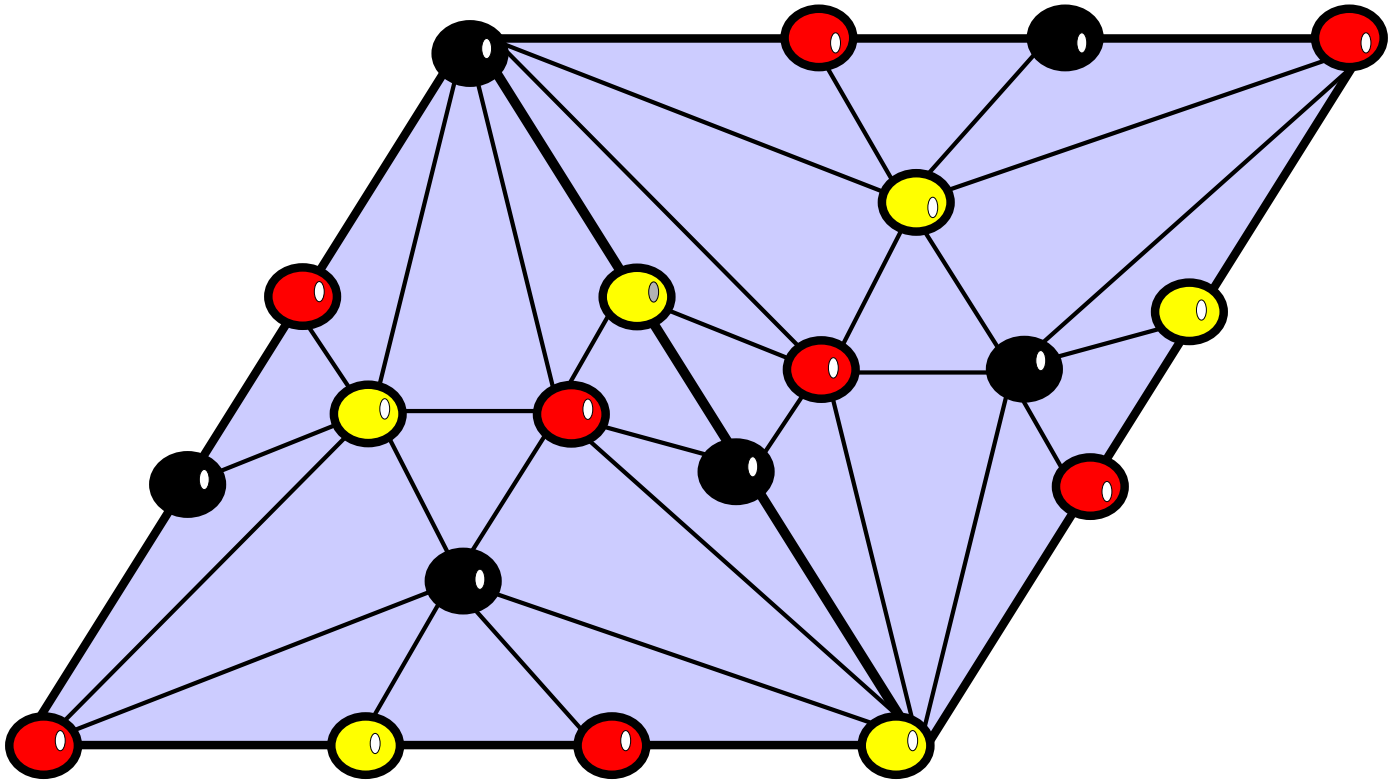
Each snapshot ordered immediately after write

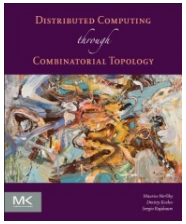
if $P_i \in \text{names}(\sigma_j)$, then $\sigma_i \subseteq \sigma_j$.

If P_j saw P_i write ...

then P_j's snapshot not later than P_i's.

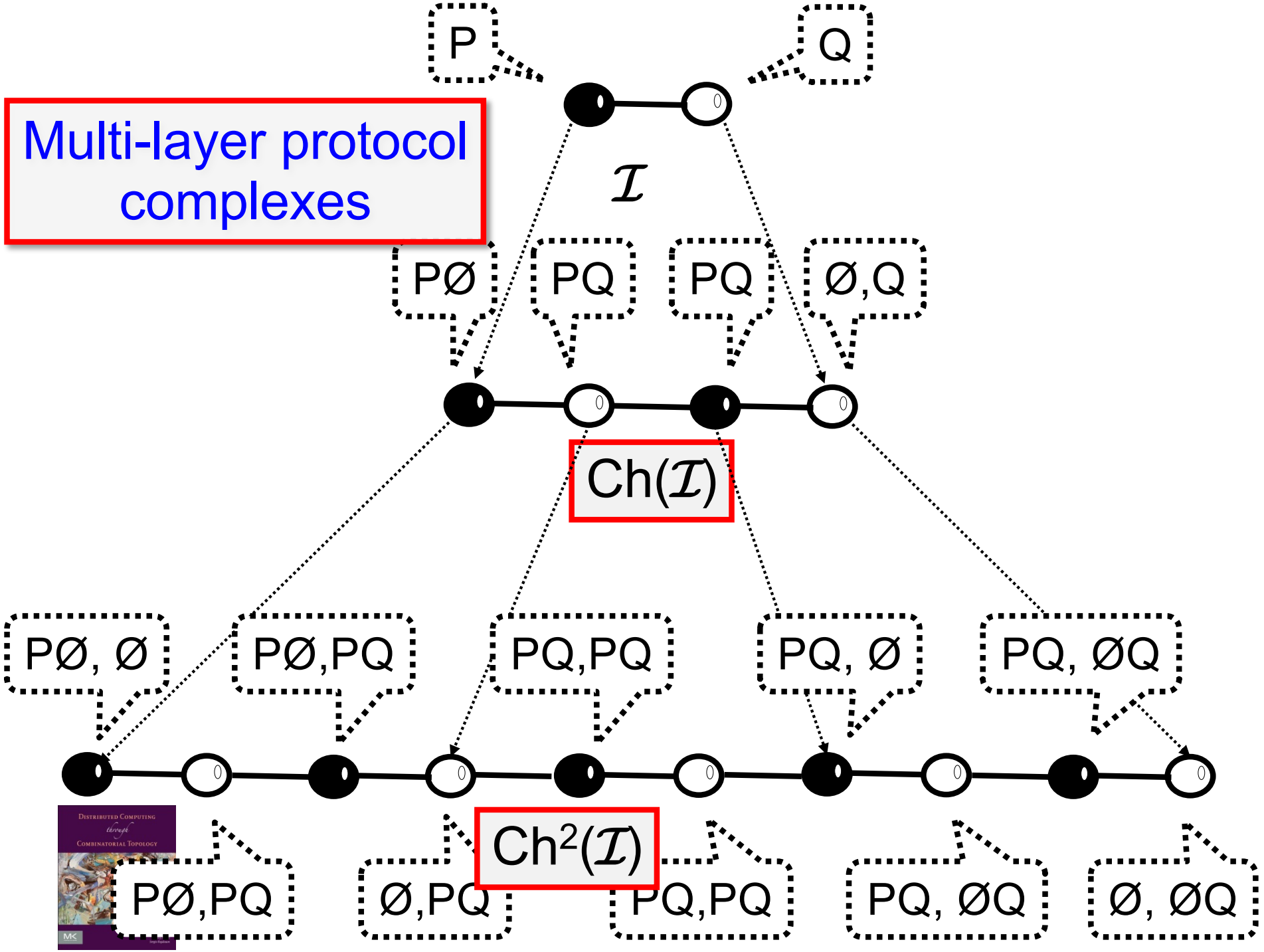
2 Input Simplexes, 1 Round





$$\mathrm{Ch}(\sigma_0) \cup \mathrm{Ch}(\sigma_0)$$

Multi-layer protocol complexes

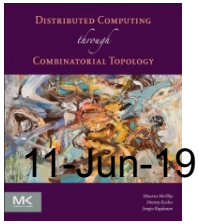


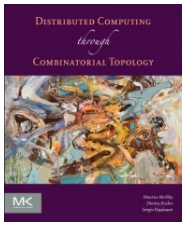
Summary

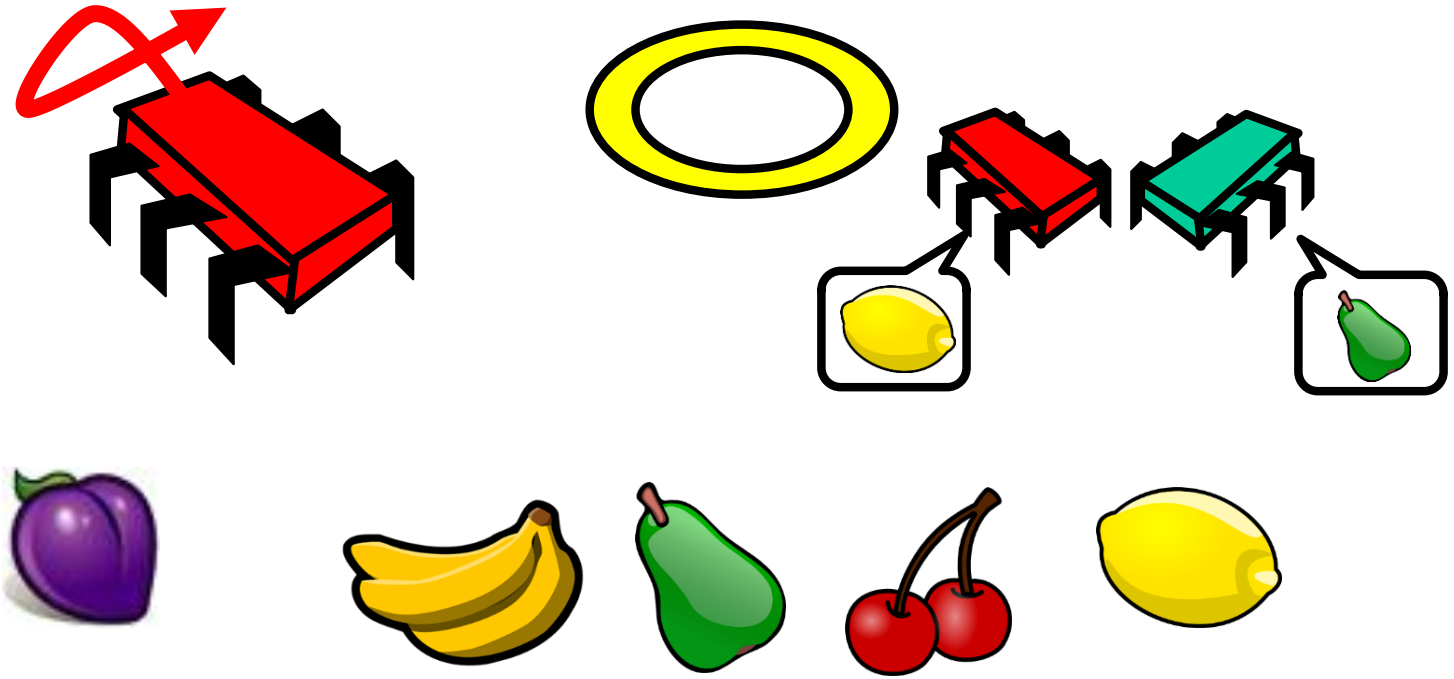
Tasks and protocols described as …

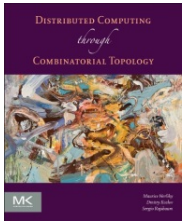
colored complexes.

Protocols are layered immediate snapshot








Distributed Computing
through
Combinatorial Topology