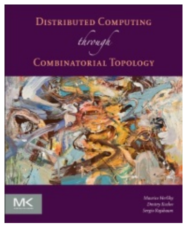
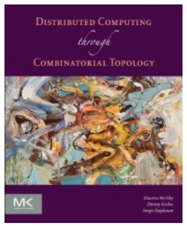
Colorless Tasks: Solvability in Different Models

MITRO207, P4, 2019



Administrivia

- Exam **June 25, B310**
  - Written, 1h30 (13h30-15h00)
  - Annals: check the exercises (and the solutions)
  - Closed books: you can bring two double-side A4 pages with handwritten notes



Road Map

Overview of Models

t-resilient layered snapshot models

Layered Snapshots with *k*-set agreement

Adversaries

Message-Passing Systems

Decidability

Road Map

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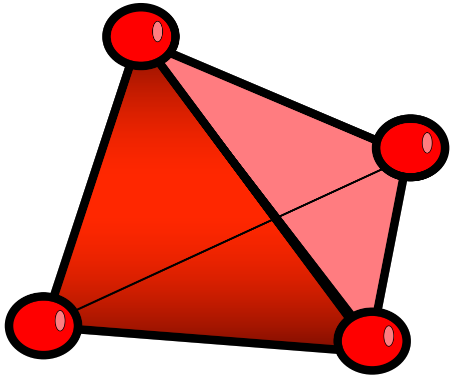
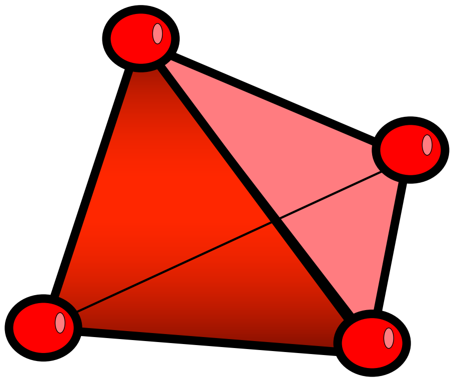
Skeleton



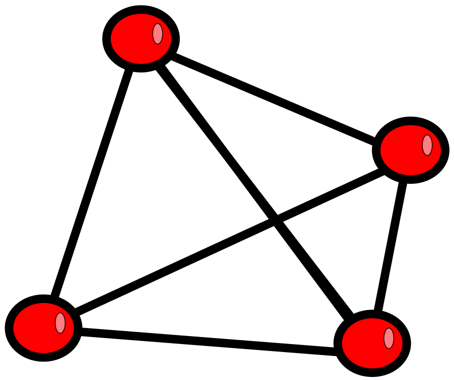
$\mathcal{C}$

(solid tetrahedron)

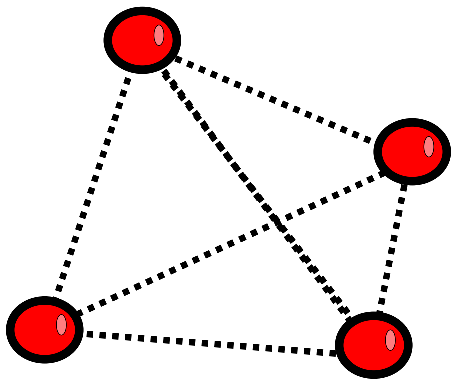


skel^2 $\mathcal{C}$

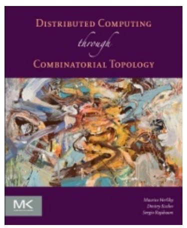
(hollow tetrahedron)



skel^1 $\mathcal{C}$



skel^0 $\mathcal{C}$



Parameter p

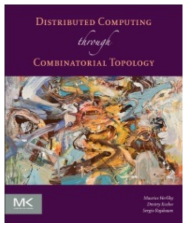
Model characterized by some parameter p, $0 \leq p \leq n$

$(\mathcal{I},\mathcal{O},\Delta)$ has a wait-free protocol iff

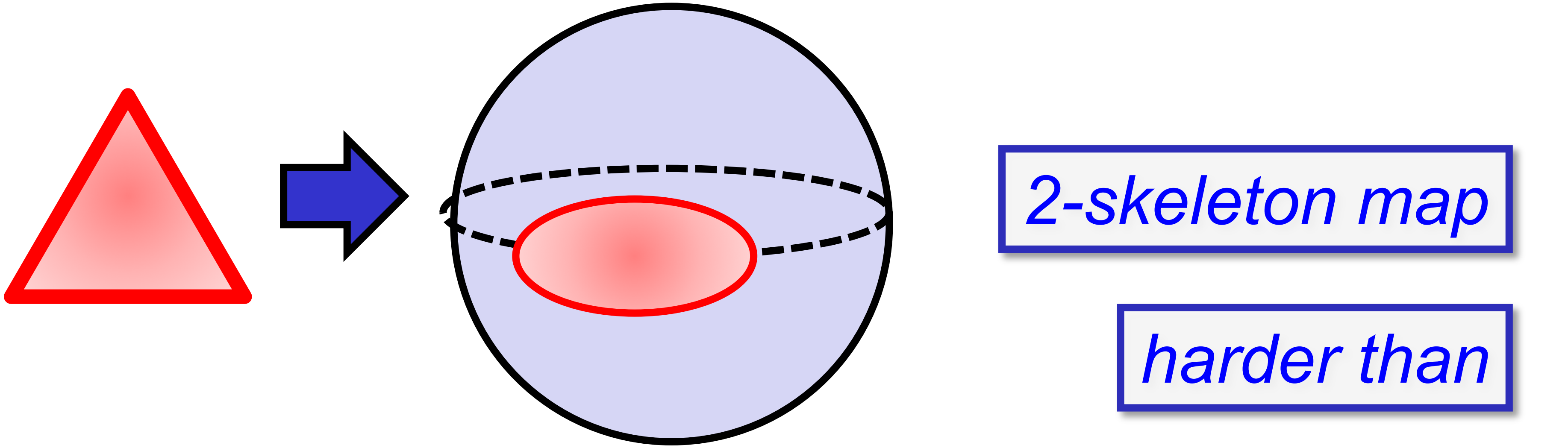
there is a continuous map

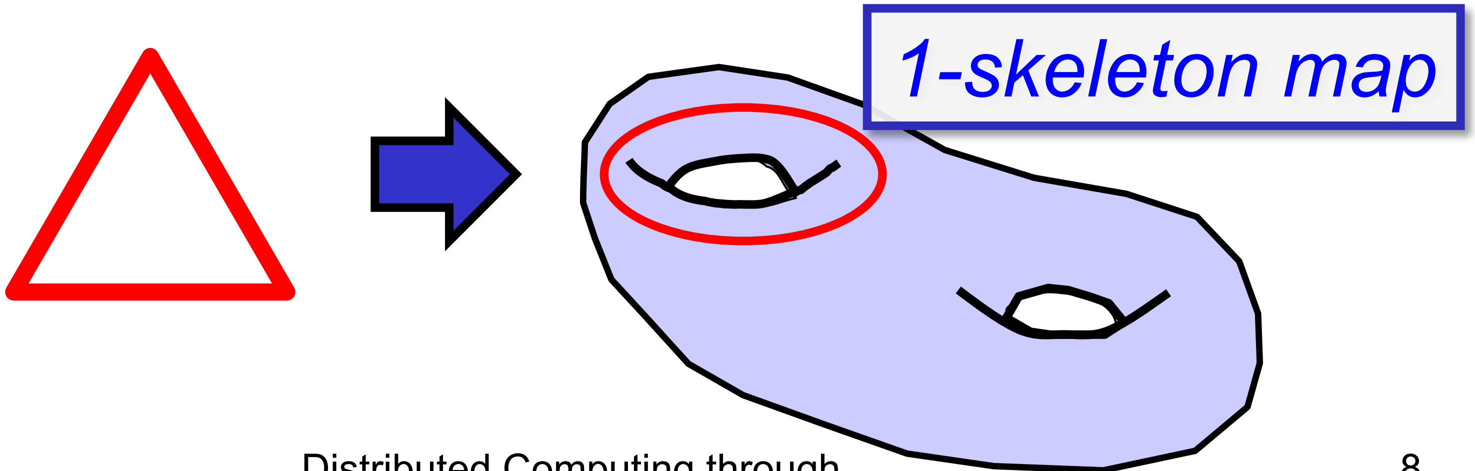
$f\colon |\mathrm{skel}^p\, \mathcal{I}| \to |\mathcal{O}|$

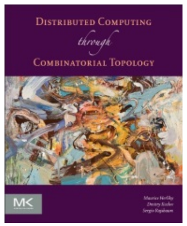
carried by Δ.



Dimension of Skeleton map vs Computational Power







Wait-Free Layered Immediate Snapshots

Up to n out of n+1 can crash

Just can't wait (to be king)

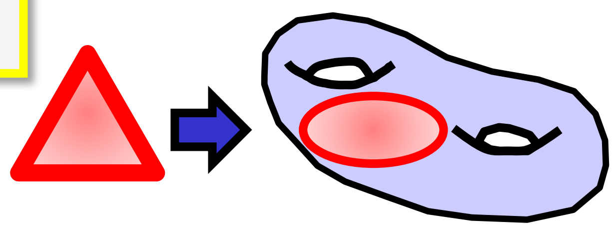
$(\mathcal{I},\mathcal{O},\Delta)$ has a wait-free protocol …

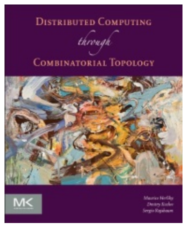
if and only if …

there is a continuous map

$f\colon |\mathrm{skel}^n\, \mathcal{I}| \to |\mathcal{O}|$

carried by Δ.





t-resilient Layered Immediate Snapshots

Up to t out of n+1 can crash

OK to wait for n-t+1

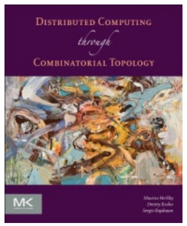
$(\mathcal{I},\mathcal{O},\Delta)$ has a t-resilient protocol …

if and only if …

there is a continuous map

f: $|\text{skel}^t\, \mathcal{I}| \to |\mathcal{O}|$

carried by Δ.



Wait-Free Layered Immediate Snapshot with *k*-set Agreement

shared black boxes that solve *k*-set agreement

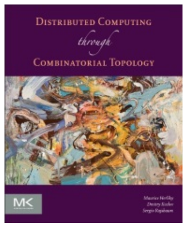
$(\mathcal{I},\mathcal{O},\Delta)$ has a wait-free protocol …

if and only if …

there is a continuous map

$f\colon |\mathrm{skel}^{k-1}\,\mathcal{I}| \to |\mathcal{O}|$

carried by Δ.

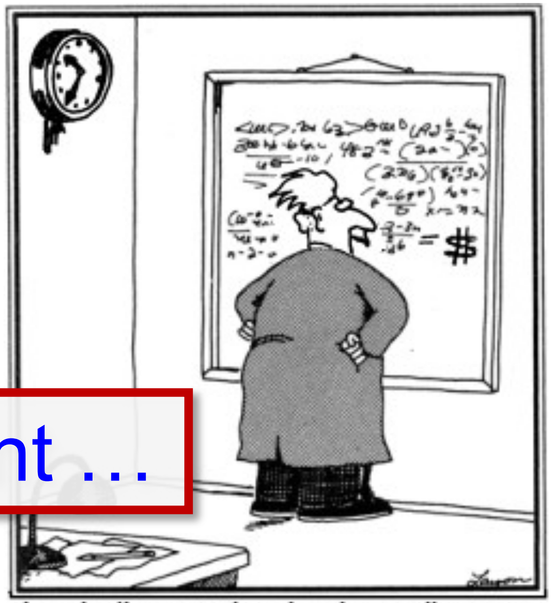


Equivalent Models

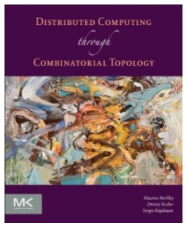
t-resilient model …

wait-free with (*t*+1)-set agreement …

have *identical* computational power!



Einstein discovers that time is actually money.

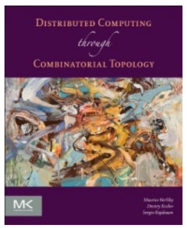


Decidability

Is it *decidable* whether a task has a protocol in a model characterized by:

$f\colon |\mathrm{skel}^p\,\mathcal{I}| \to |\mathcal{O}|$?

decidable if and only if $p \leq 1$!



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t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for ℓ:= 0 to N-1 do
    do
       immediate
         mem[ℓ][i] := view;
         snap := snapshot(mem[ℓ][*])
       until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
    do
    
    until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

	P_0	P_1	…	P_n
Layer 0				
Layer 1				
…				
Layer N-1				

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
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     immediate
        mem[l][i] := view;
        snap := snapshot(mem[l][*])
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

initial view is input value

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for ℓ := 0 to N-1 do
    do
      immediate
        view;
        snap := snapshot(mem[l][*])
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

run for *N* layers

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
```

layer ℓ : immediate write & snapshot of row ℓ

```
for ℓ := 0 to N-1 do
    do
      immediate
        mem[ℓ][i] := view;
        snap := snapshot(mem[ℓ][*])
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
    do
      immediate
        mem[l][i] := view;
        snap := snapshot(mem[l][*])
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

wait to hear from n+1-t processes

why is this live?

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
    do
      immediate
        mem[l][i] := view;
        snap := snapshot(mem[l][*])
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

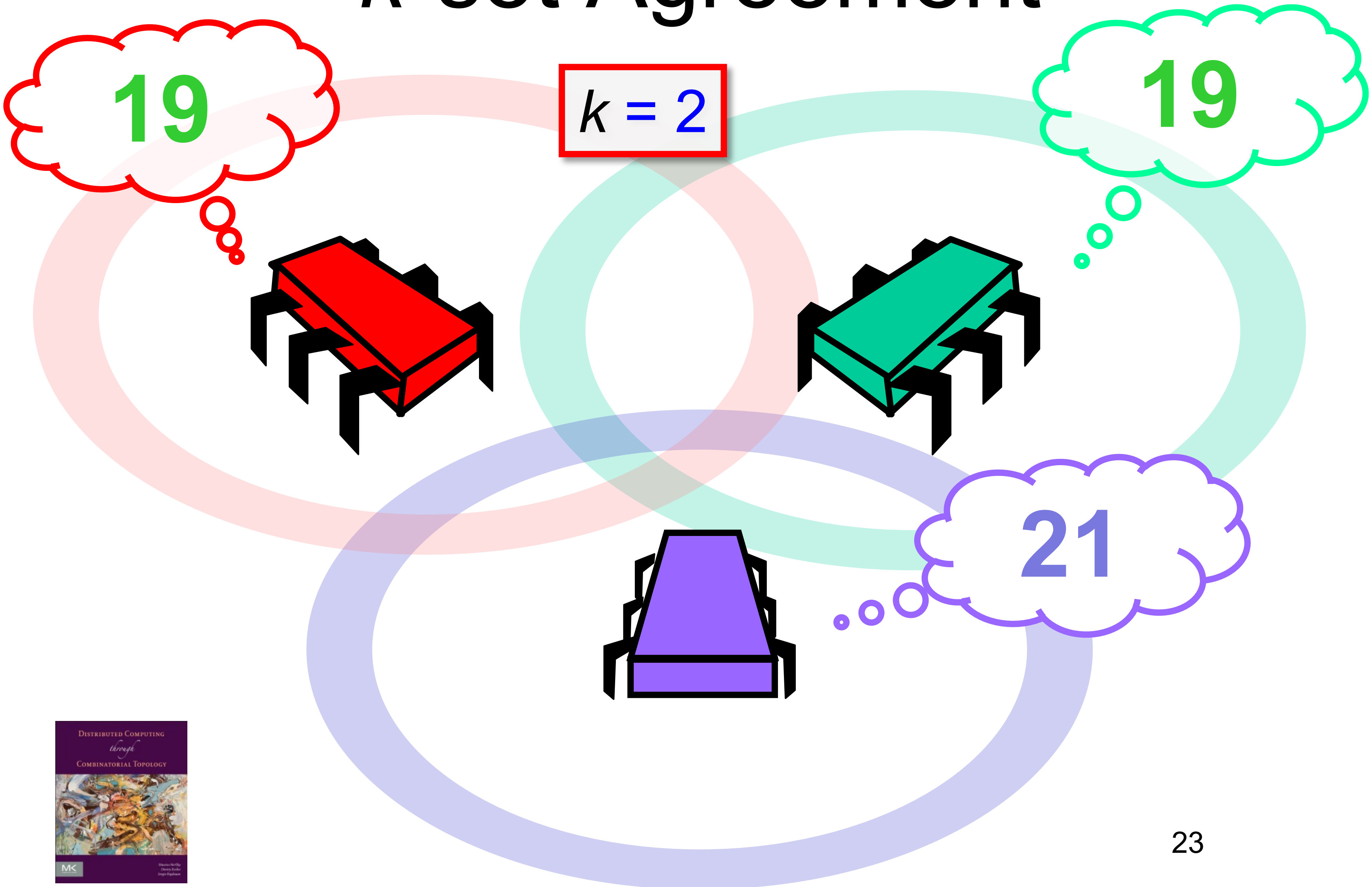
new view is set of values seen

t-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
    do
      immediate
        mem[l][i] := view;
      until |names(snap)| >= n+1-t
    view := values(snap)
return δ(view)
```

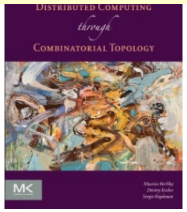
finally apply decision map **δ** to final view

k-set Agreement



(*t*+1)-Set Agreement

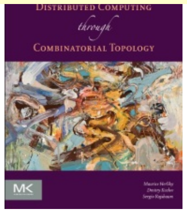
```
view := input
snap: array of Value = ∅
do
   immediate
    mem[0][i] := view;
    snap := snapshot(mem[0][*])
until |names(snap)| >= n+1-t
view := values(snap)
return min(values(view))
```



(*t*+1)-Set Agreement

```
view := input
snap: array
do
  immediate
    mem[0][i] := view;
    snap := snapshot(mem[0][*])
until |names(snap)| >= n+1-t
view := values(snap)
return min(values(view))
```

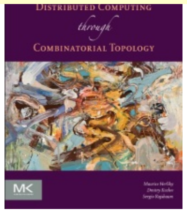
write input and take snapshot



(t+1)-Set Agreement

```
view := input
snap: array of Value = ∅
do
   immediate
      mem[0][i] := view;
      snap := snapshot(mem[0][*])
until |names(snap)| >= n+1-t
view := values(snap)
return min(values(view))
```

wait to hear from n+1-t processes



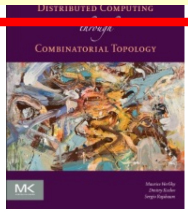
(t+1)-Set Agreement

```
view := input
snap: array of Value = ∅
do
  immediate
    mem[0][i] := view;
    snap := snapshot(mem[0][*])
until |names(snap)| >= n+1-t
view := values(snap)
return min(view)
```

return least value in view

can miss at most t lesser values

most t+1 values returned



Informal Skeleton Lemma

If

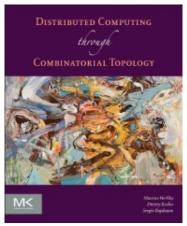
We have a protocol for a task ...

And

A protocol for *k*-set agreement …

Then

WLOG, we can "pre-process" with *k*-set agreement.



Skeleton Lemma

If

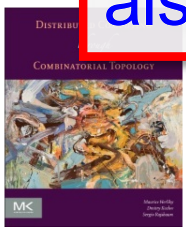
protocol $(\mathcal{I}, \mathcal{P}, \Xi)$ solves task $(\mathcal{I},\mathcal{O},\Delta)$

And

There is a k-set agreement protocol for $\mathcal{I}$

Then

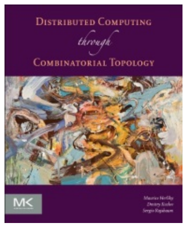
The composition of
k-set agreement with $(\mathcal{I},\mathcal{P},\Xi)$
also solves $(\mathcal{I},\mathcal{O},\Delta)$.



Informal Protocol Complex Lemma

WLOG

We can assume that any protocol complex is a barycentric subdivision of (the skeleton of) the input complex.



Protocol Complex Lemma

If

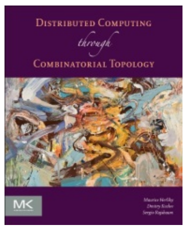
There is a t-resilient layered protocol for $(\mathcal{I},\mathcal{O},\Delta)$ ...

Then

Then there is a protocol $(\mathcal{I},\mathcal{P},\Xi)$ for $(\mathcal{I},\mathcal{O},\Delta)$ such that …

$\mathcal{P} = \mathrm{Bary}^N(\mathrm{skel}^t\, \mathcal{I})$

$\Xi(\sigma) = \mathrm{Bary}^N \bullet \mathrm{skel}^t(\sigma)$.



Theorem

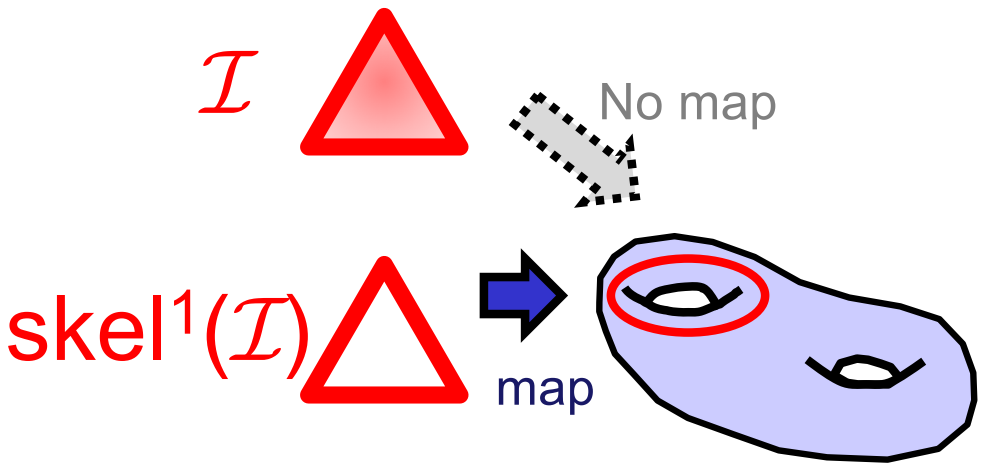
The colorless task $(\mathcal{I},\mathcal{O},\Delta)$ has a t-resilient layered snapshot protocol …

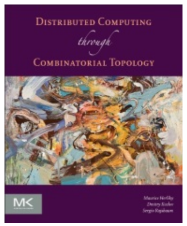
if and only if …

there is a continuous map

f: $|\text{skel}^t\, \mathcal{I}| \to |\mathcal{O}|$

carried by Δ.





Protocol Implies Map

May assume protocol complex is $\mathcal{P} = \text{Bary}^N \text{skel}^t \mathcal{I}$.

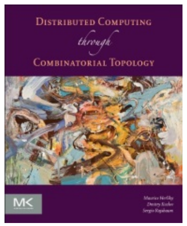
decision map

$\delta: \text{Bary}^N \text{skel}^t \mathcal{I} \to \mathcal{O}$

$|\delta|: |\text{Bary}^N \text{skel}^t \mathcal{I}| \to |\mathcal{O}|$

$|\delta|: |\text{skel}^t \mathcal{I}| \to |\mathcal{O}|$

carried by Δ.



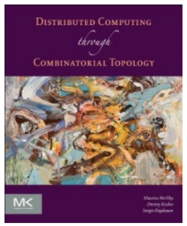
Simplicial Approximation Theorem

- Given a continuous map

$$f\colon |\mathcal{A}| \to |\mathcal{B}|$$

- there is an N such that f has a simplicial approximation

$$\phi\colon \mathrm{Bary}^N \mathcal{A} \to \mathcal{B}$$



Map Implies Protocol

$f\colon |\mathrm{skel}^t\, \mathcal{I}| \to |\mathcal{O}|$

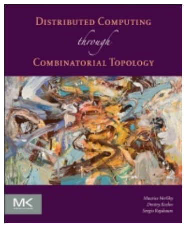
$\phi\colon \mathrm{Bary}^N\, \mathrm{skel}^t\, \mathcal{I} \to \mathcal{O}$

carried by Δ.

Solve using …

barycentric agreement

$(t+1)$-set agreement



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Motivation

Today …

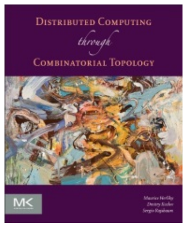
Practically all modern multiprocessors provide synchronization more powerful than read-write …

Like …

test-and-set, compare-and-swap, ….

Here …

we consider protocols constructed by *composing* layered snapshot protocols with *k*-set agreement protocols.



Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of SetAgreement
view := input
for ℓ:= 0 to N-1 do
  view := SA[ℓ].decide(view)
  immediate
    mem[ℓ][i] := view;
    snap := snapshot(mem[ℓ][*])
  view := values(snap)
return δ(view)
```

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of SetAgree
view := input
for l :=
  view
  immec
    mer
    snap := snapshot(mem[l][*])
  view := values(snap)
return δ(view)
```

	P_0	P_1	…	P_n
Layer 0				
Layer 1				
…				
Layer N-1				

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of k-SetAgree
view := input
for l := 0 to N-1 do
  view := SA[l].decide(view)
  immediate
    mem[l][i
    snap :=
  view := va
return δ(view)
```

per-level *k*-set agreement object

Layer 0	agreementObject_0
Layer 1	agreementObject_1
…	…
Layer N-1	$\text{agreementObject}_{N\text{-}1}$

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of k-SetAgree
view := input
for l := 0 to N-1 do
  view: View := SA[l].decide(view)
  immediate
    mem[l][i] := view;
    snap := snapshot(mem[l][*])
  view := values(snap)
return δ(view)
```

initial view is input value

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of k-SetAgree
view := input
for ℓ := 0 to N-1 do
  view:= SA[ℓ].decide(view)
  immediate
    mem[ℓ][i] := view;
    snap := snapshot(mem[ℓ][*])
  view := values(snap)
return δ(view)
```

do *k*-set agreement with others at this level

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of k SetAgree
view := input
for l := 0 to N-1 do
  view:= SA[l].decide(view)
  immediate
    mem[ℓ][i] := view;
    snap := snapshot(mem[ℓ][*])
  view := values(snap)
return δ(view)
```

then do immediate snapshot

Wait-Free Layered Set Agreement Protocol

```
shared mem array 0..N-1,0..n of Value
shared SA array 0..N-1 of k-SetAgree
view := input
for l := 0 to N-1 do
  view: View := SA[l].decide(view)
  immediate
    mem[l][i] := view;
    snap := snapshot(mem[l][*])
  view := values(snap)
return δ(view)
```

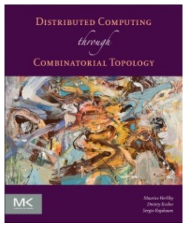
new view is set of values seen

Protocol Complex Lemma

If $(\mathcal{I},\mathcal{P},\Xi)$ is a k-set layered snapshot protocol …

then $\mathcal{P}$ is equal to $\mathrm{Bary}^N\ \mathrm{skel}^{k-1}\,\mathcal{I}$, …

for some $N \geq 0$.



Theorem

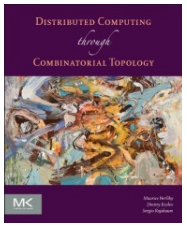
The colorless task $(\mathcal{I},\mathcal{O},\Delta)$ has a wait-free k-set layered snapshot protocol …

if and only if …

there is a continuous map

$f\colon |\mathrm{skel}^{k-1}\,\mathcal{I}| \to |\mathcal{O}|$

carried by Δ.



Theorem

The colorless task $(\mathcal{I},\mathcal{O},\Delta)$ has a wait-free k-set layered snapshot protocol …

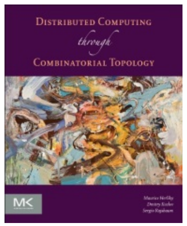
if and only if …

there is a continuous map

$$f\colon |\mathrm{skel}^{k-1}\,\mathcal{I}| \to |\mathcal{O}|$$

carried by Δ.

k-1 skeleton, not t-skeleton!



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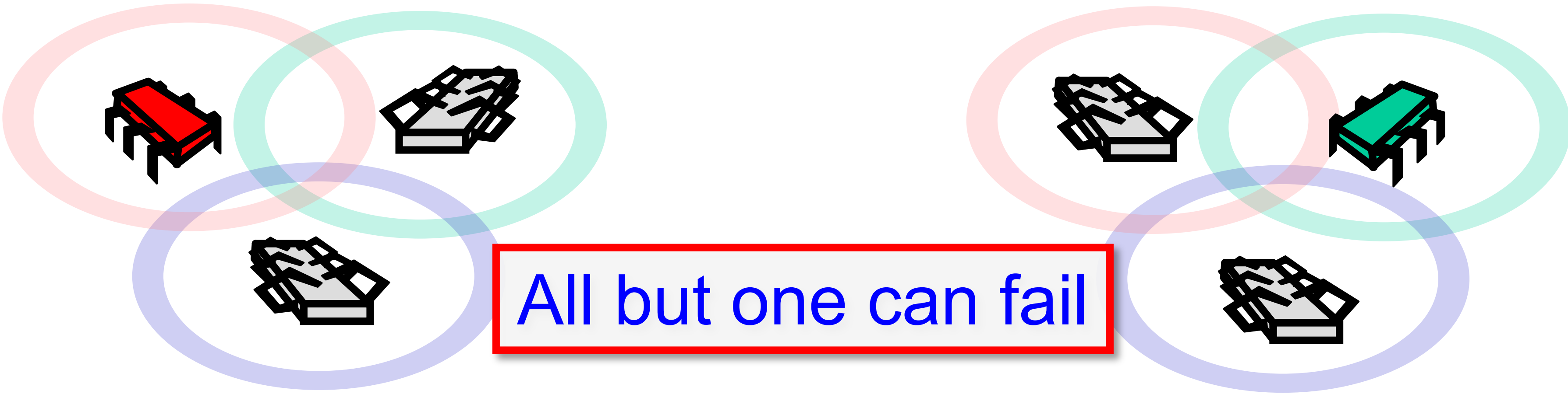
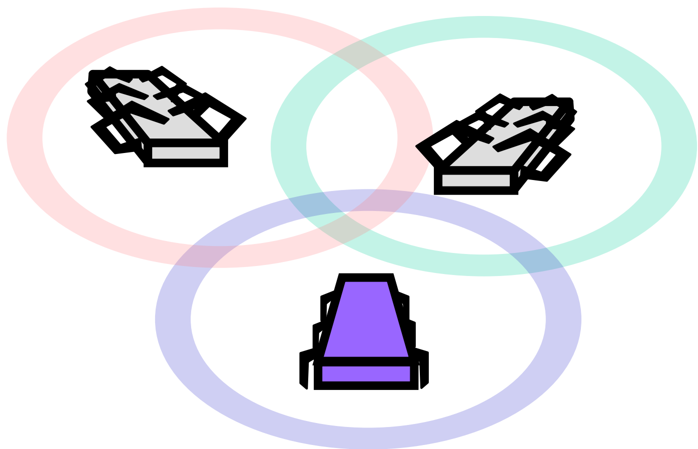
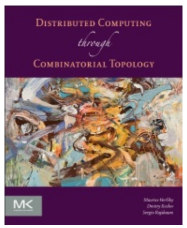
Decidability

Wait-Free

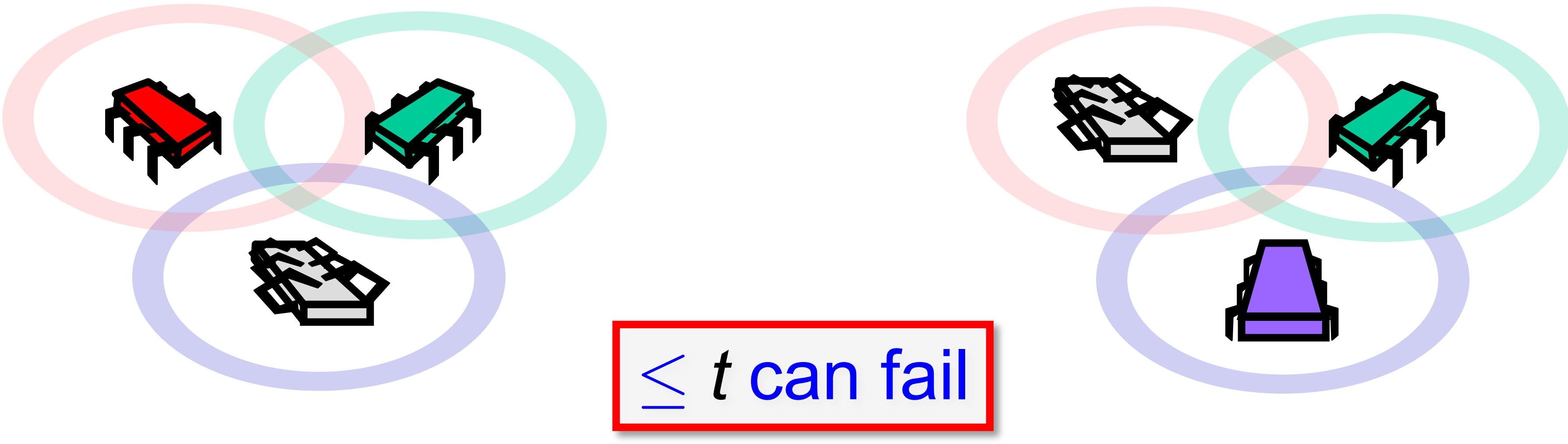




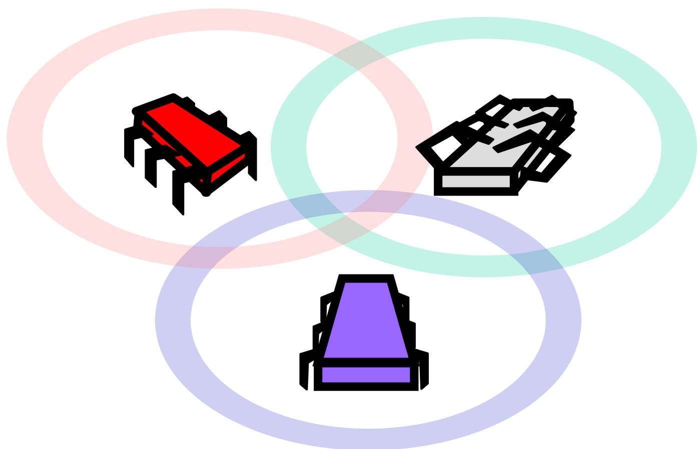


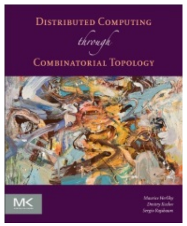
Distributed Computing through Combinatorial Topology

t-resilient



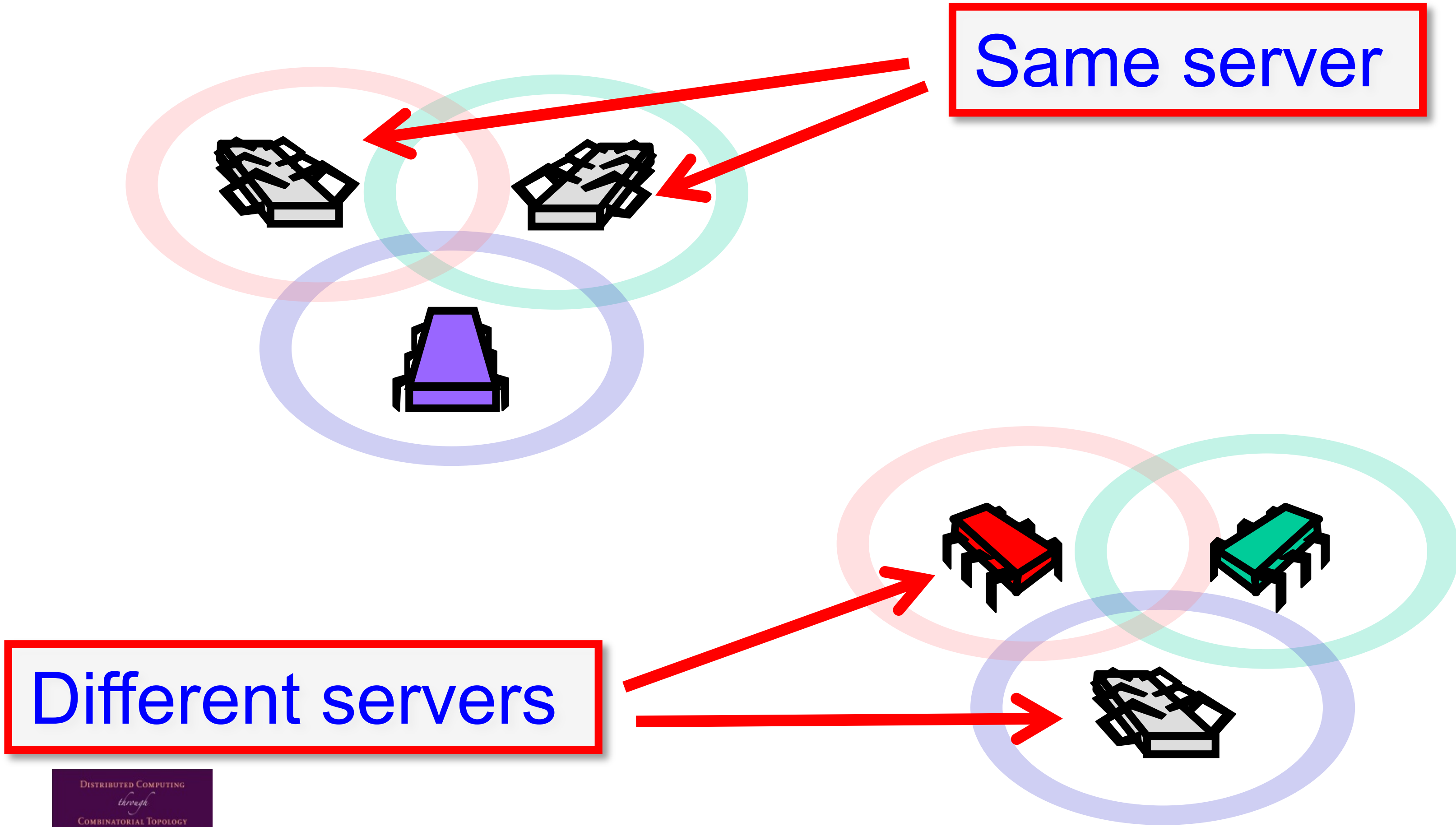
$\leq$ *t* can fail

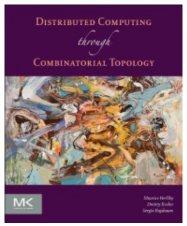




Distributed Computing through Combinatorial Topology

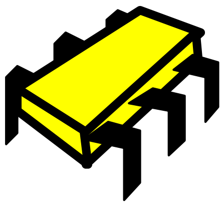
Irregular Failures

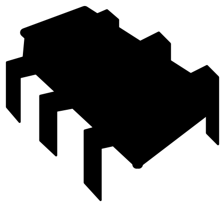


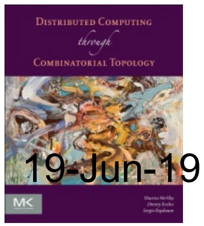


Adversaries

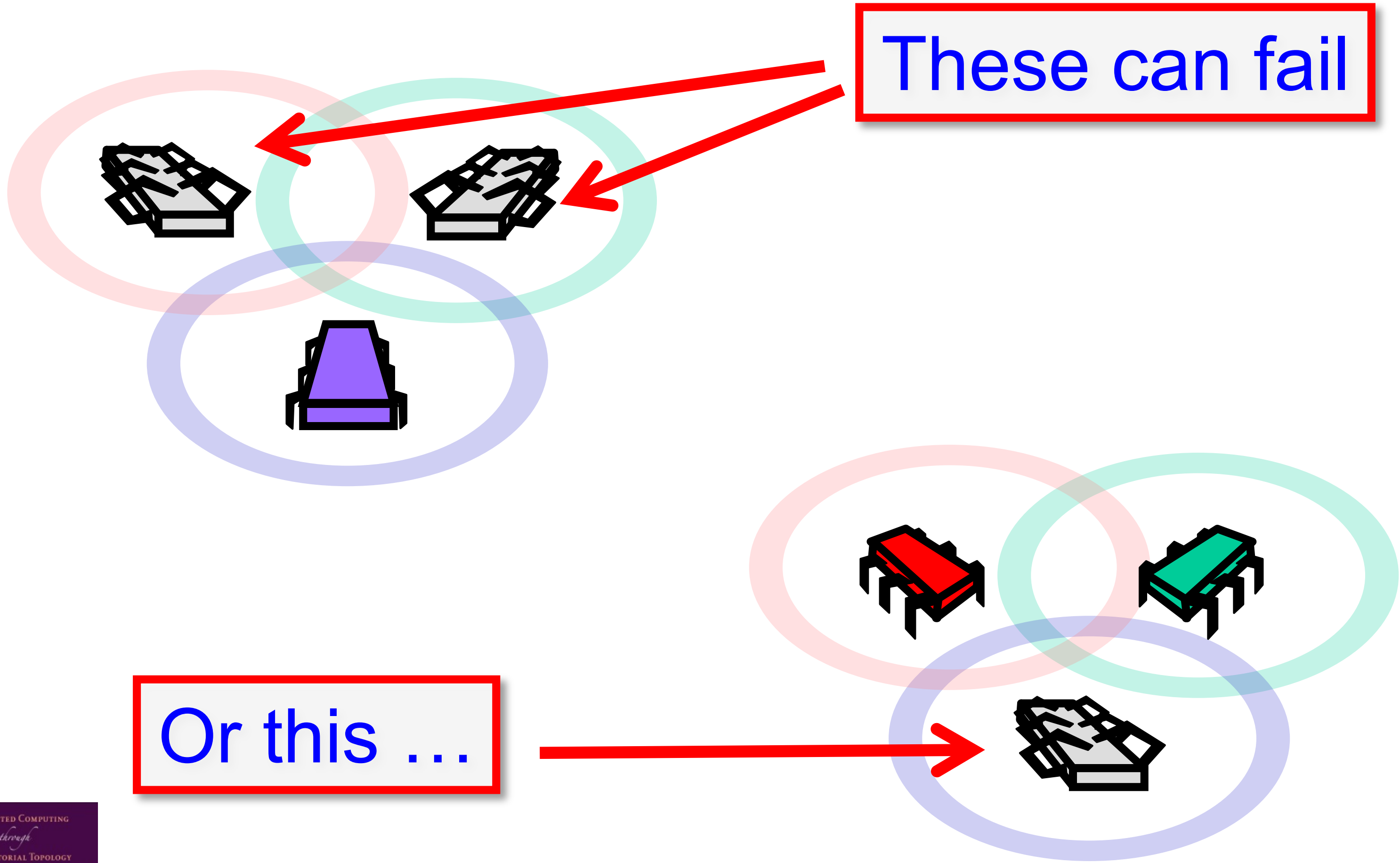

http://pixabay.com/en/chess-figures-game-play-strategy-145184/



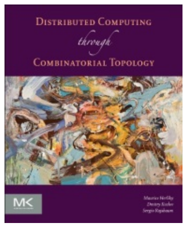




Faulty Sets

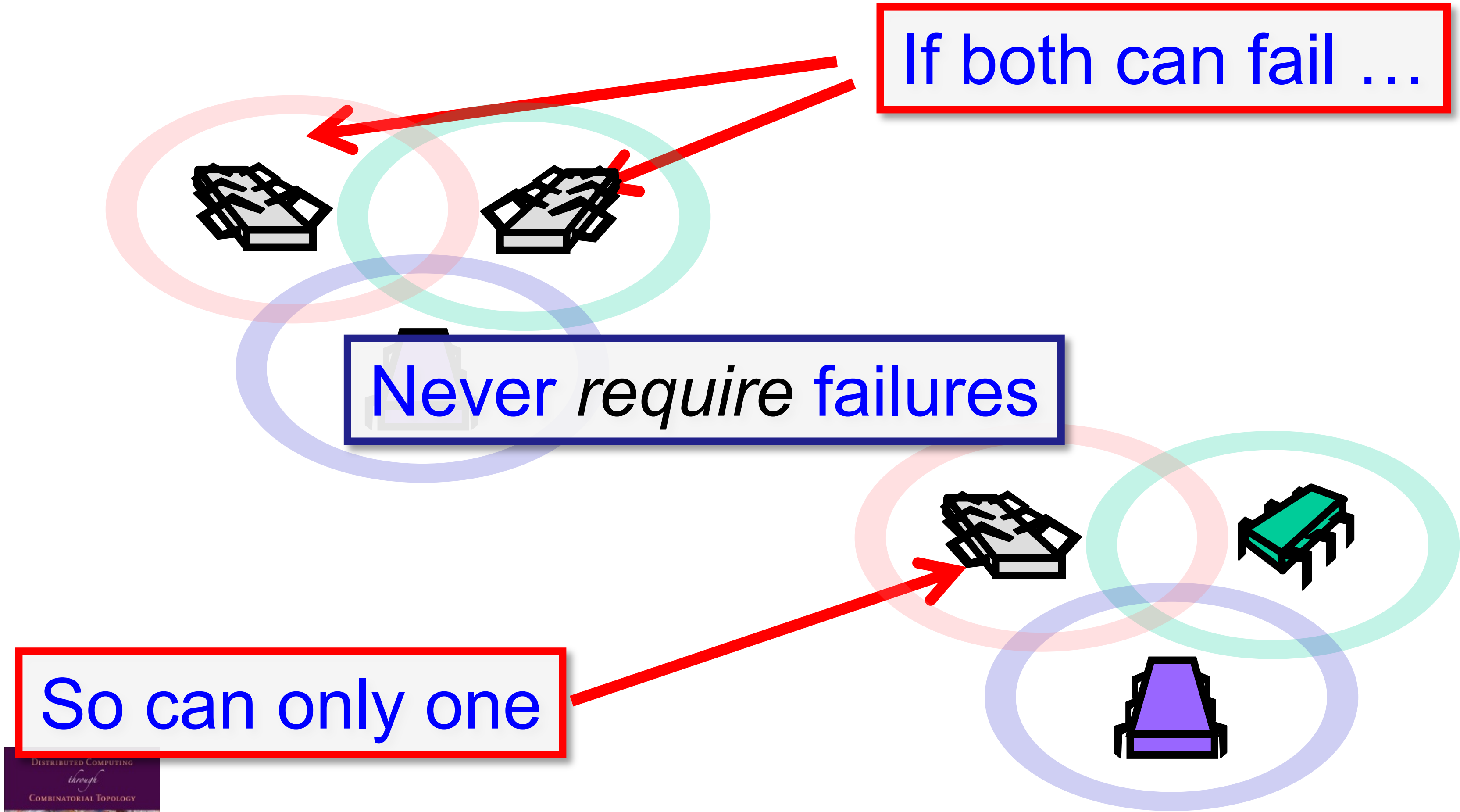


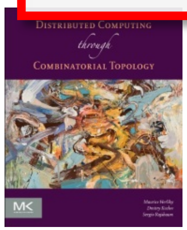
These can fail
Or this …



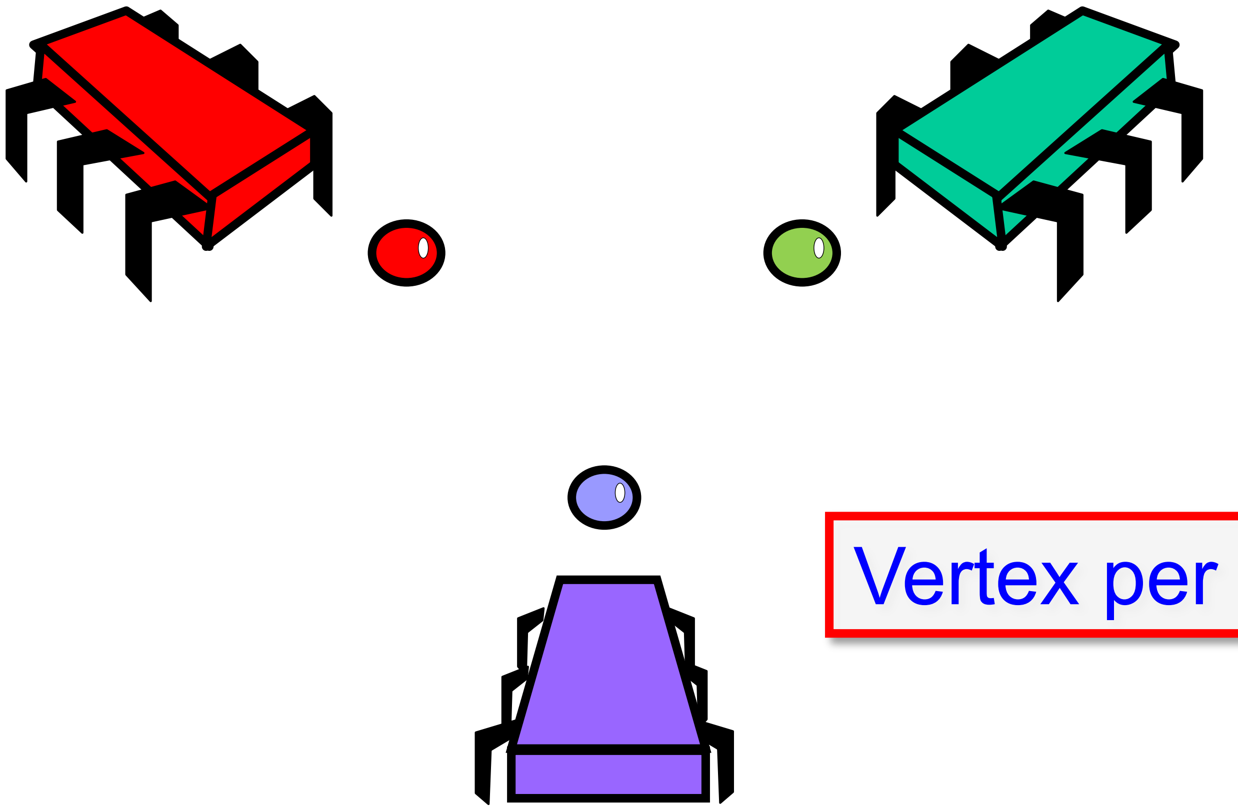
Distributed Computing through Combinatorial Topology

Faulty Sets Closed under Containment

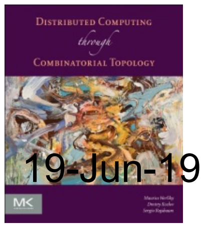




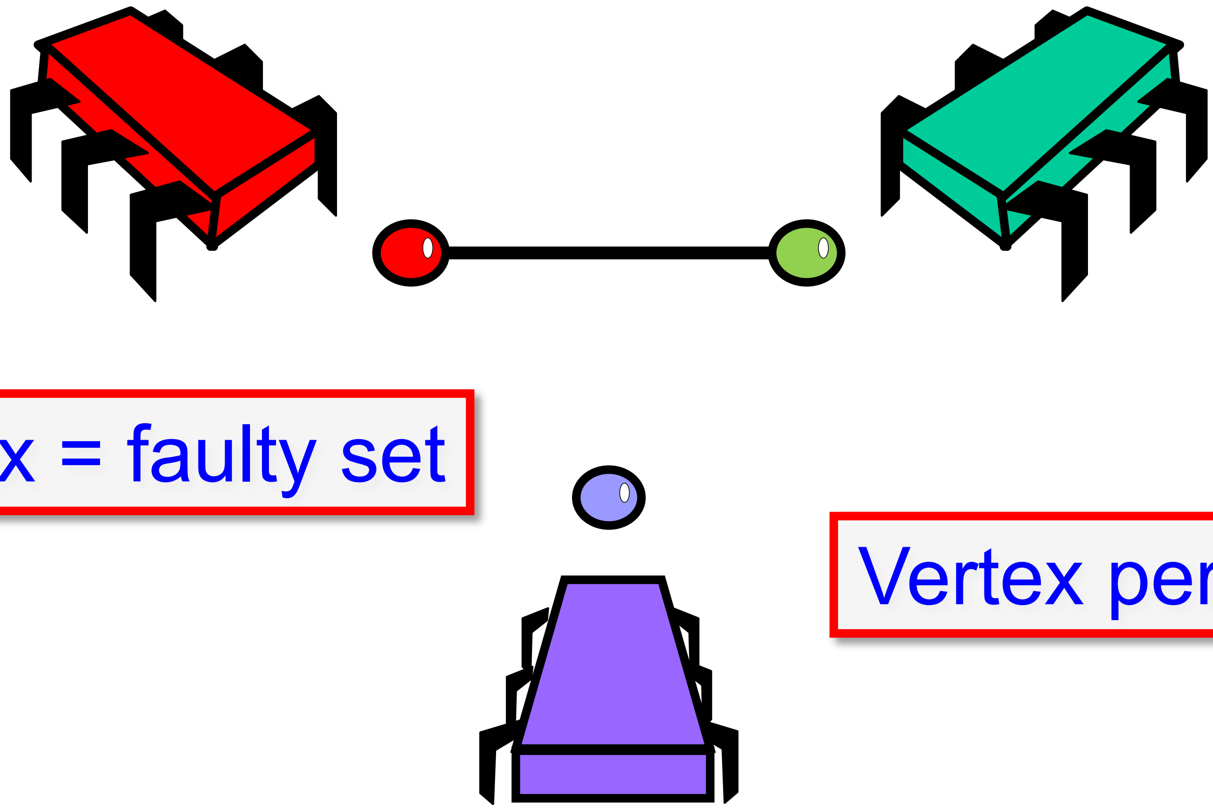
Failure Complex



Vertex per process

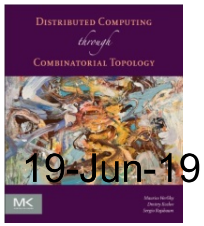


Failure Complex

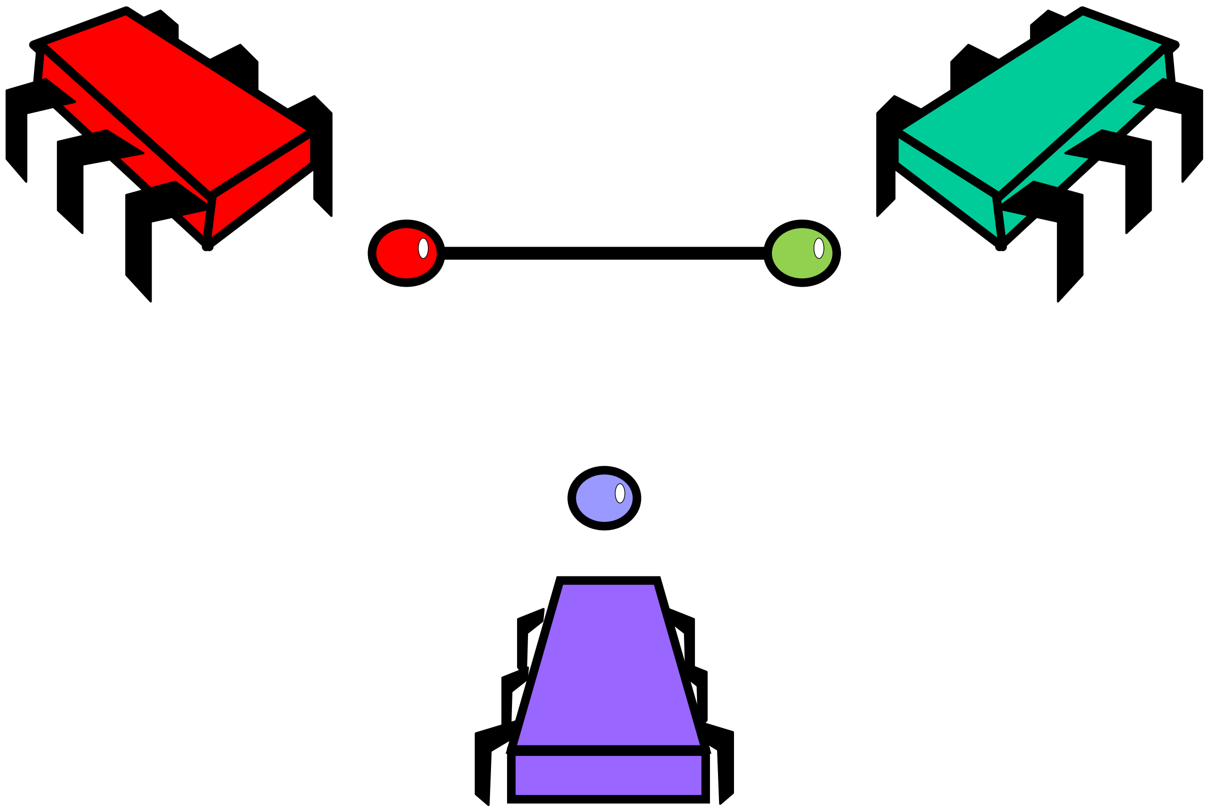


Simplex = faulty set

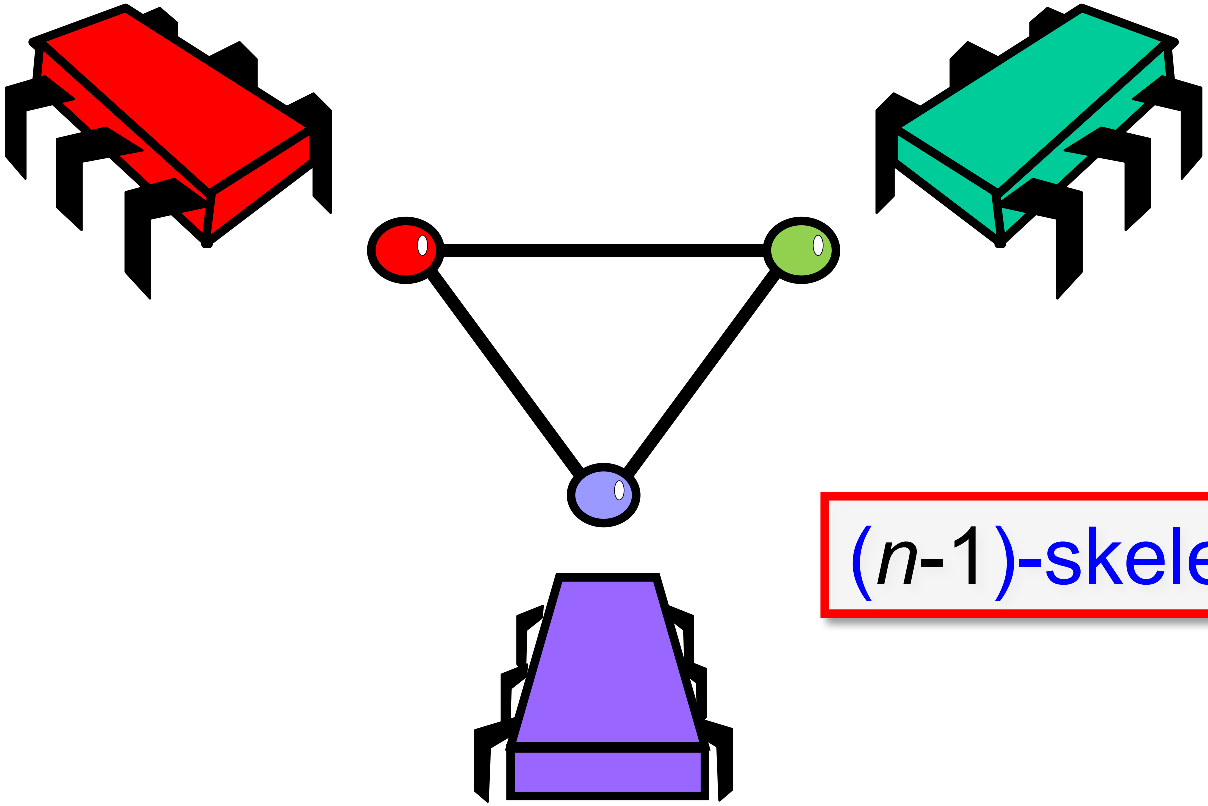
Vertex per process



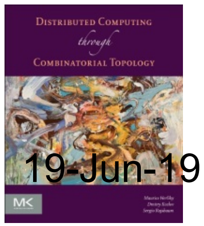
Irregular Failure Complex



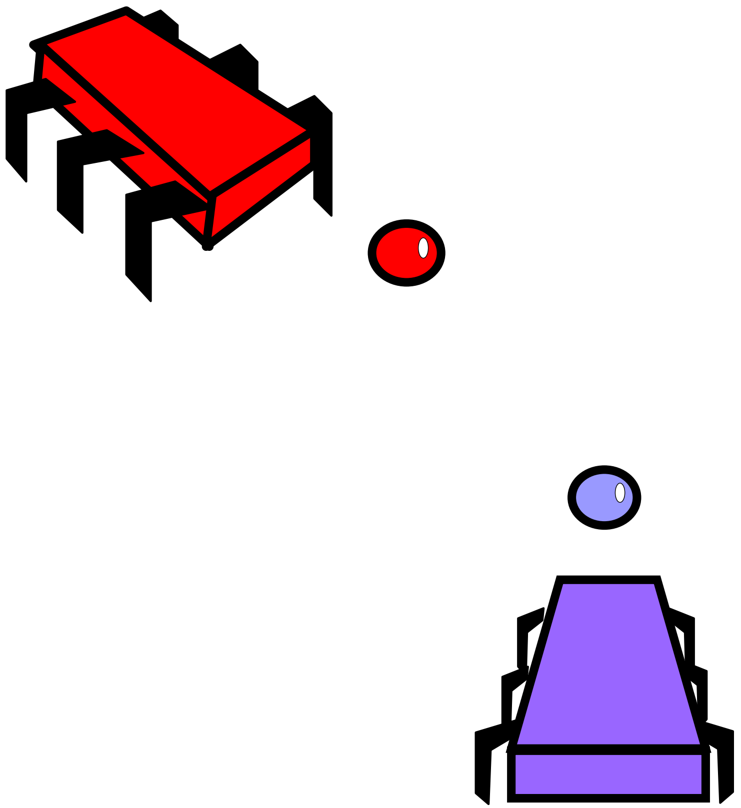
Wait-Free Failure Complex



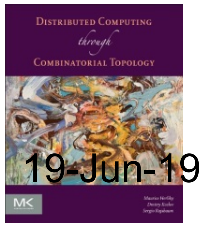
(n-1)-skeleton



t-resilient Failure Complex



(*t*-1)-skeleton

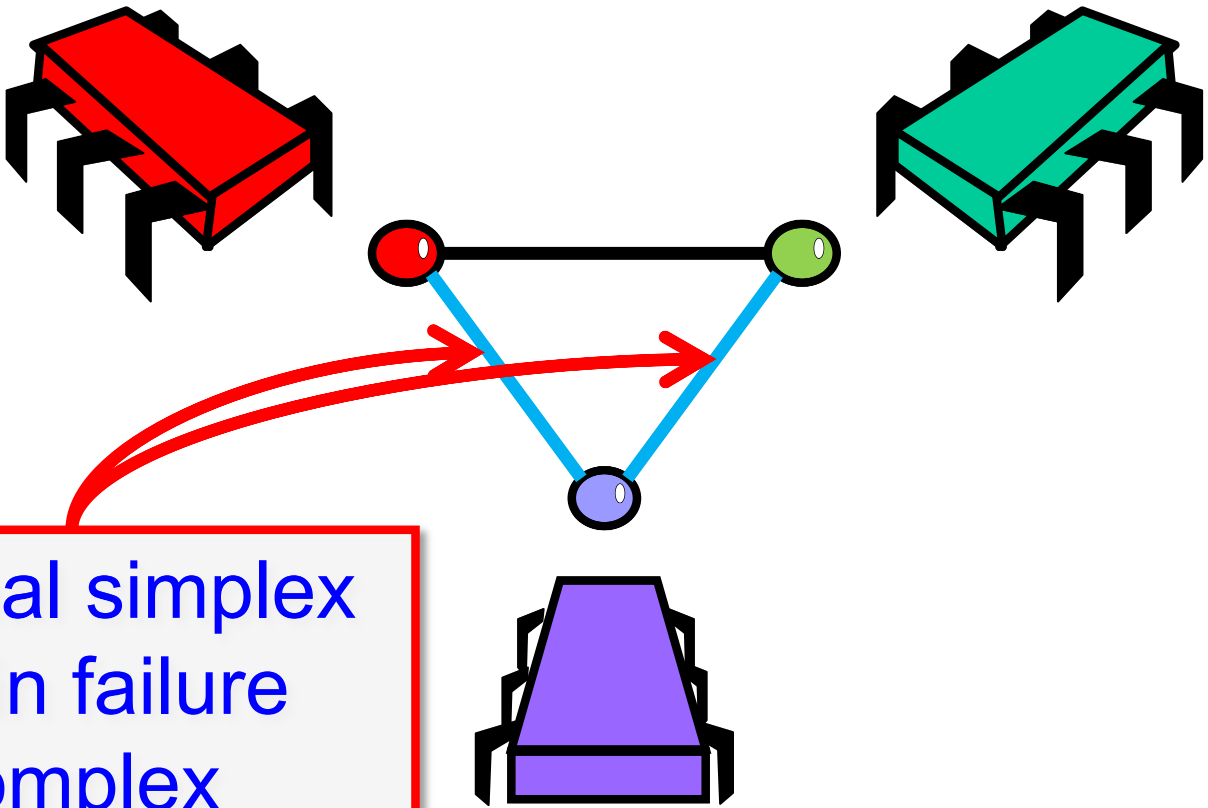


Cores

Minimal set of processes that cannot *all* fail

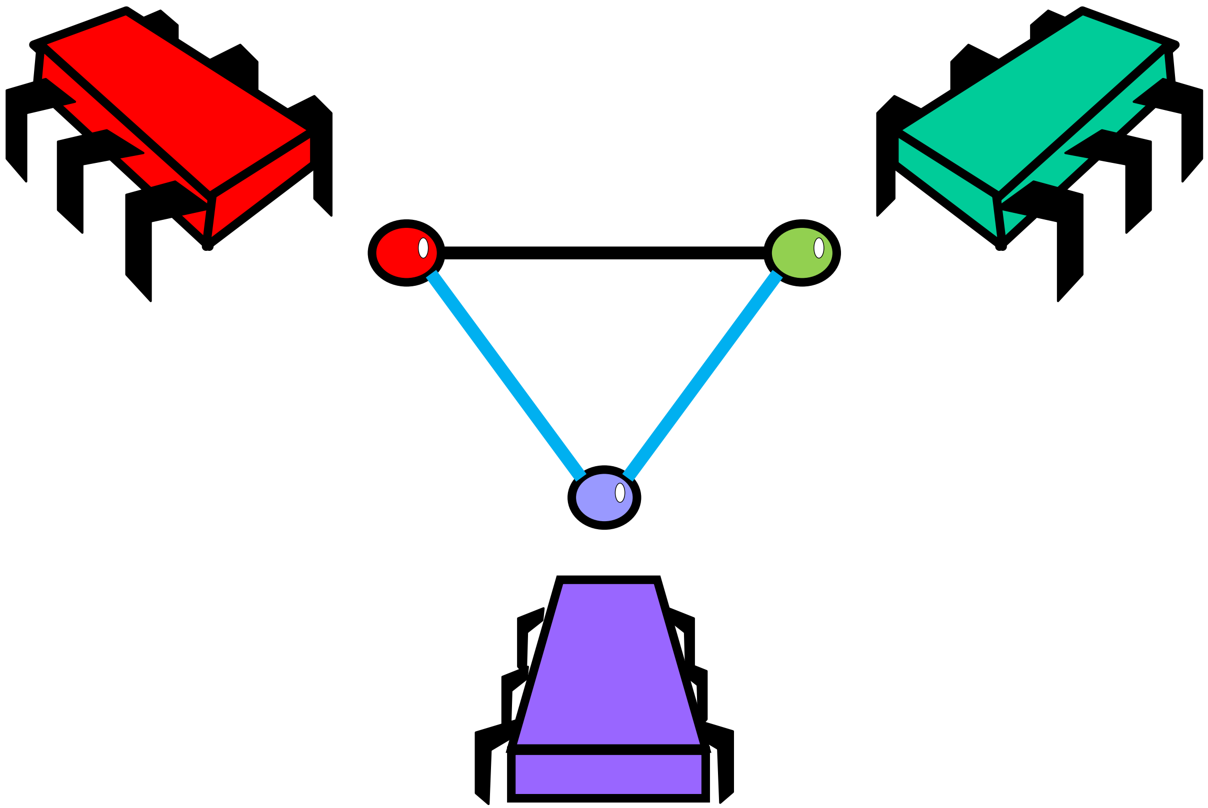
Safe to wait for at least one member of a particular core to show up

Cores & Failure Complex

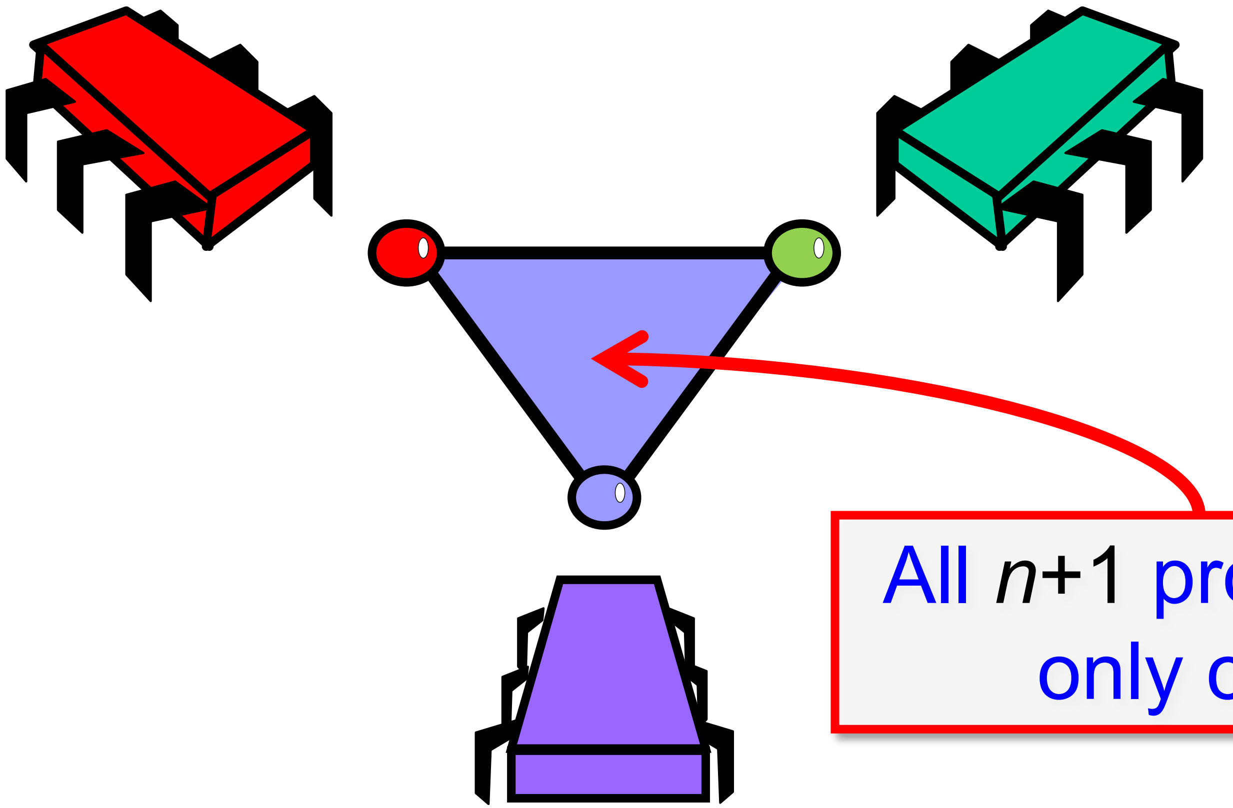


Minimal simplex not in failure complex

Irregular Failure Complex

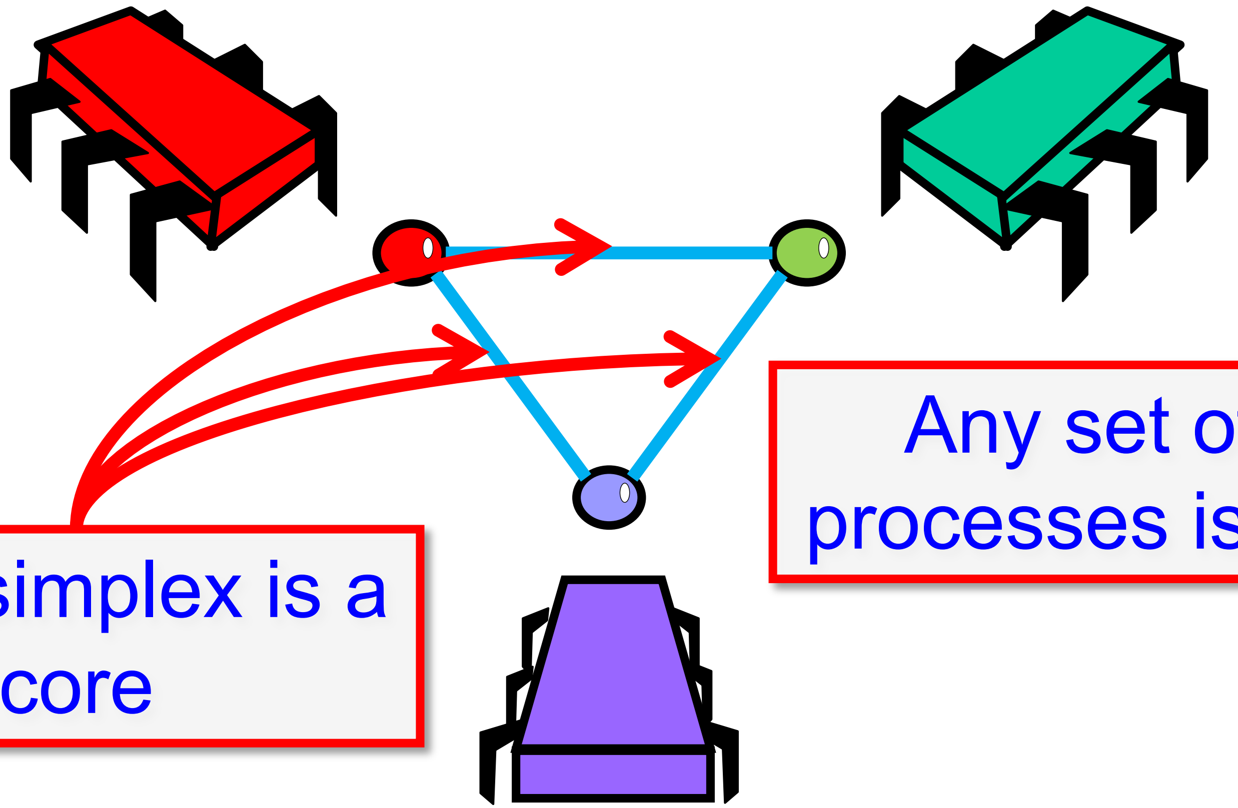


Wait-Free Failure Complex



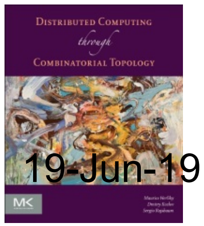
All n+1 processes only core

t-resilient Failure Complex



Any *t*-simplex is a core

Any set of *t*+1 processes is a core



Cores

For many models,

minimum core size…

Completely determines adversary's power to solve *any* colorless task!

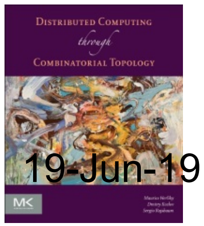
So adversaries with same min core size solve the same colorless tasks

Survivor Sets

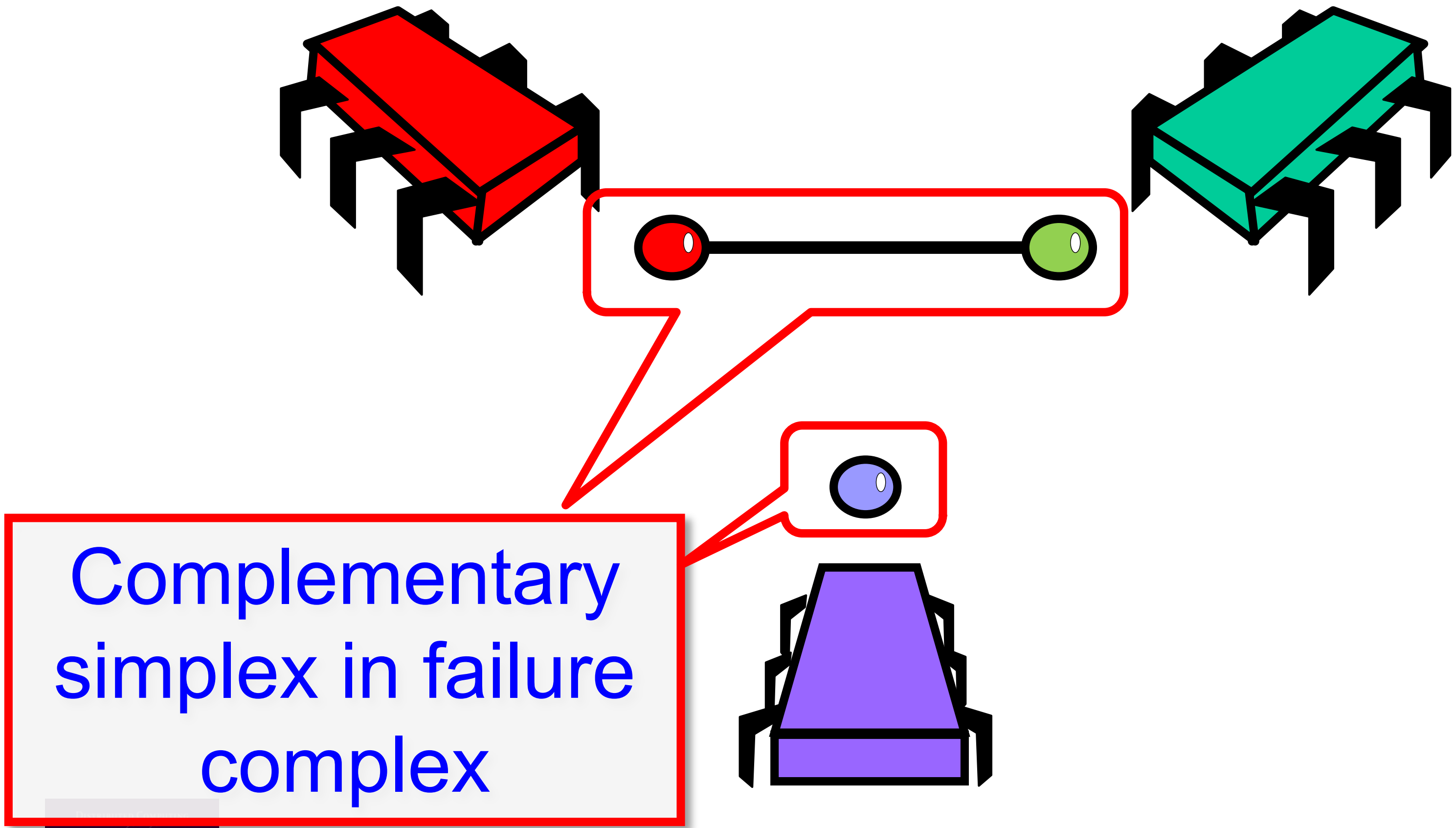
Minimal set of processes that might *all* survive

Safe to wait for all members of some survivor set to show up

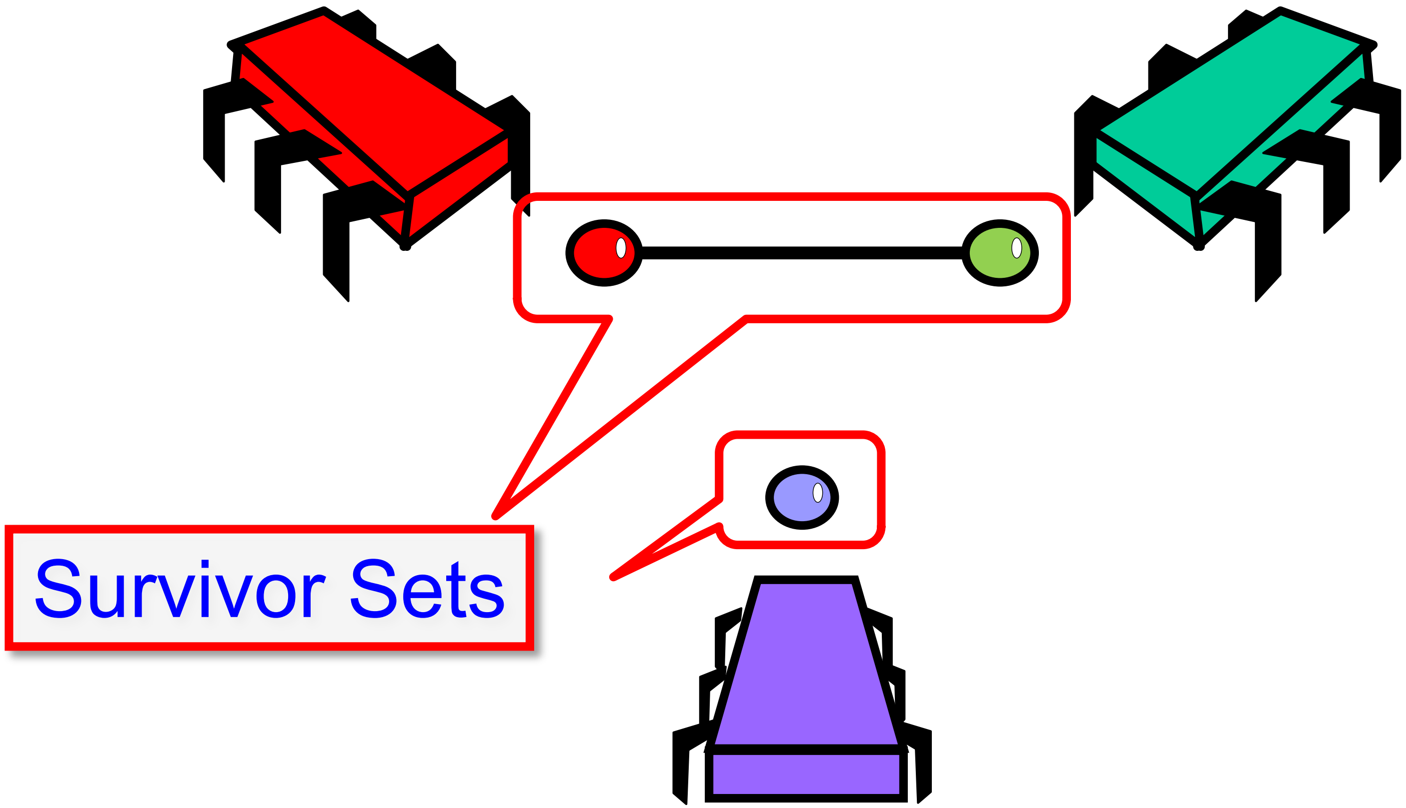
Dual to cores: each one determines the other



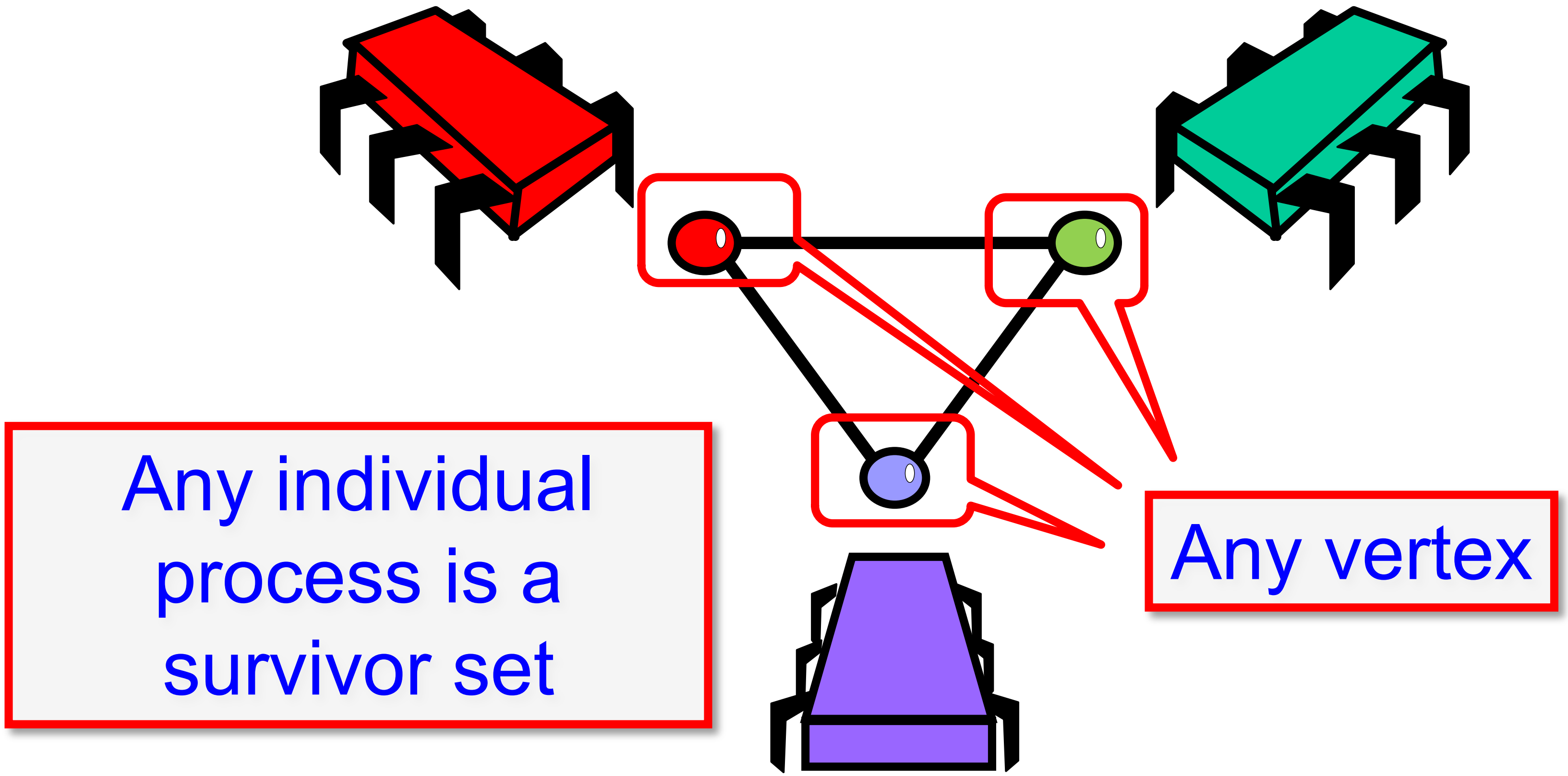
Survivor Sets in Failure Complex

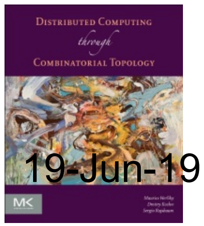


Irregular Failure Complex

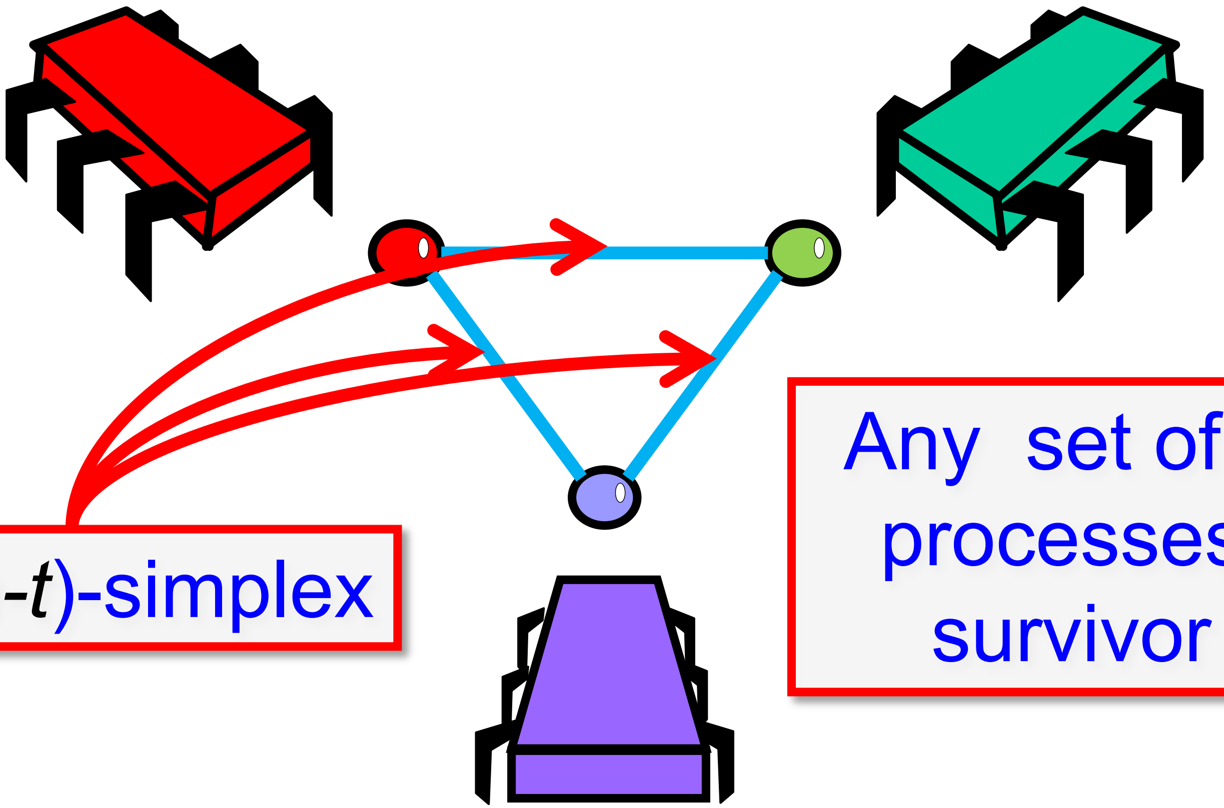


Wait-Free Failure Complex



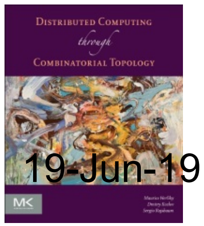


t-resilient Failure Complex



Any (n-t)-simplex

Any set of n-t+1 processes is a survivor set



$\mathbb{A}$-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for ℓ := 0 to N-1 do
  do
    immediate
      mem[ℓ][i] := view;
      snap := snapshot(mem[ℓ][*])
    until survivor set ⊆ names(snap)
  view := values(snap)
return δ(view)
```

$\mathbb{A}$-Resilient Layered Immediate Snapshot Protocol

```
shared mem array 0..N-1,0..n of Value
view := input
for l := 0 to N-1 do
  do
    immediate
      mem[l][i] := view;
      snap := snapshot(mem[l][*])
  until survivor set⊆ names(snap)
view := values(snap)
return δ(view)
```

wait to hear from a survivor set

why is this live?

Road Map

Overview of Models

t-resilient layered snapshot models

Layered Snapshots with *k*-set agreement

Adversaries

Message-Passing Systems

Decidability

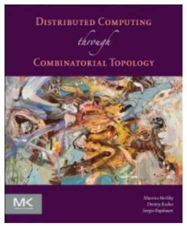
Message Passing

There are *n*+1 asynchronous processes …

that send and receive messages …

via a *fully-connected* communication network.

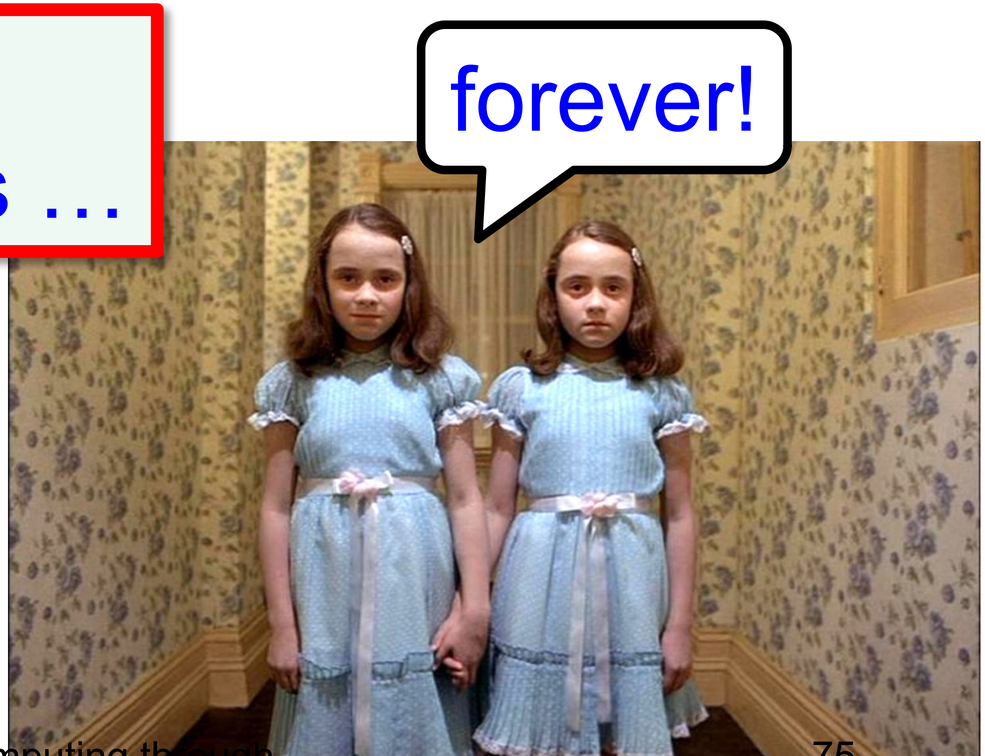
Message delivery is *reliable* and *FIFO*

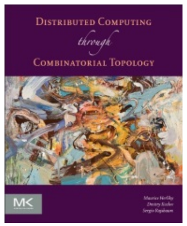


Message-Passing Protocols

decide after finite # steps

but protocol
forwards messages …



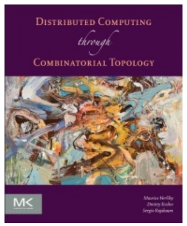


Communication Syntax

send(P, v_0, …, v_ℓ) **to** Q

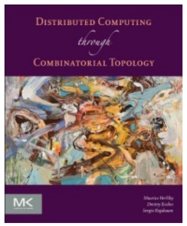
send(P, v_0, …, v_ℓ) **to all**

upon receive(P, v_0, …, v_ℓ) **do**
    ... // handle message



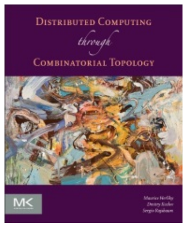
Forwarding

```
background // forward messages forever
    upon receive(P_j,v) do
      send(P_i,v) to all
```



Get Values from *n*+1-*t* Processes

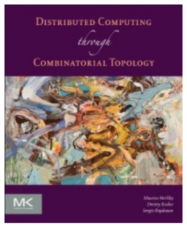
```
getQuorum(): Set of Value
  V: Set of Value := ∅
  q: int := 0
  do
    upon receive(Q,v) do
      V := V ∪ {v}
      q := q + 1
  until q = n+1-t
  return V
```



Get Values from *n*+1-*t* Processes

```
getQuorum(): Set of Value
  V: Set of Value := ∅
  q: int := 0
  do
    upon receive(Q,v) do
      V := V ∪ {v}
      q := q + 1
  until q = n+1-t
  return V
```

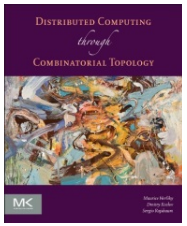
Initially, nothing.



Get Values from *n*+1-*t* Processes

remember values and count
(only the first value per sender counts)

```
q: int := 0
do
  upon receive(Q,v) do
    V := V ∪ {v}
    q := q + 1
until q = n+1-t
return V
```

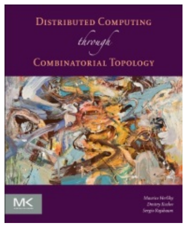


Get Values from n+1-t Processes

```
getQuorum(): Set of Value
  V: Set of Value := ∅
  q: int := 0
  do
```

safe to wait for n+1-t values

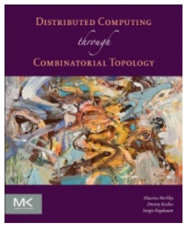
```
      V := V ∪ {v}
      q := q + 1
  until q = n+1-t
  return V
```



Get Values from *n*+1-*t* Processes

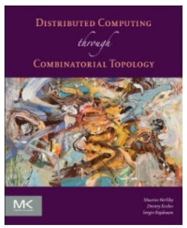
```
getQuorum(): Set of Value
  V: Set of Value := ∅
  q: int := 0
  do
      V := V ∪ {v}
      q := q + 1
  until q = n+1-t
  return V
```

return values when enough received



Protocol for (*t*+1)-Set Agreement

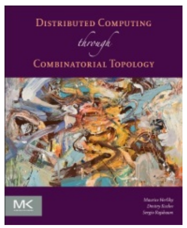
```
SetAgree(v_i) : value
  send(P, v_i) to all
  V: Set of Value := getQuorum()
  return min(V)
```



Protocol for (*t*+1)-Set Agreement

```
SetAgree(v): value
  send(P, v) to all
  V: Set of Value := getQuorum()
  return min(V)
```

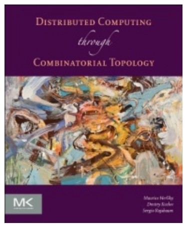
broadcast my value



Protocol for (*t*+1)-Set Agreement

```
SetAgree(v) : value
  send(P, v) to all
  V: Set of Value := getQuorum()
  return min(V)
```

get values from all but *t*

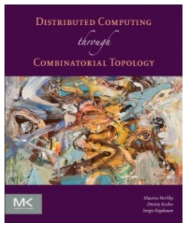


Protocol for (*t*+1)-Set Agreement

```
SetAgree(v) : value
   send(P, v) to all
   V: Set of Value := getQuorum()
   return min(V)
```

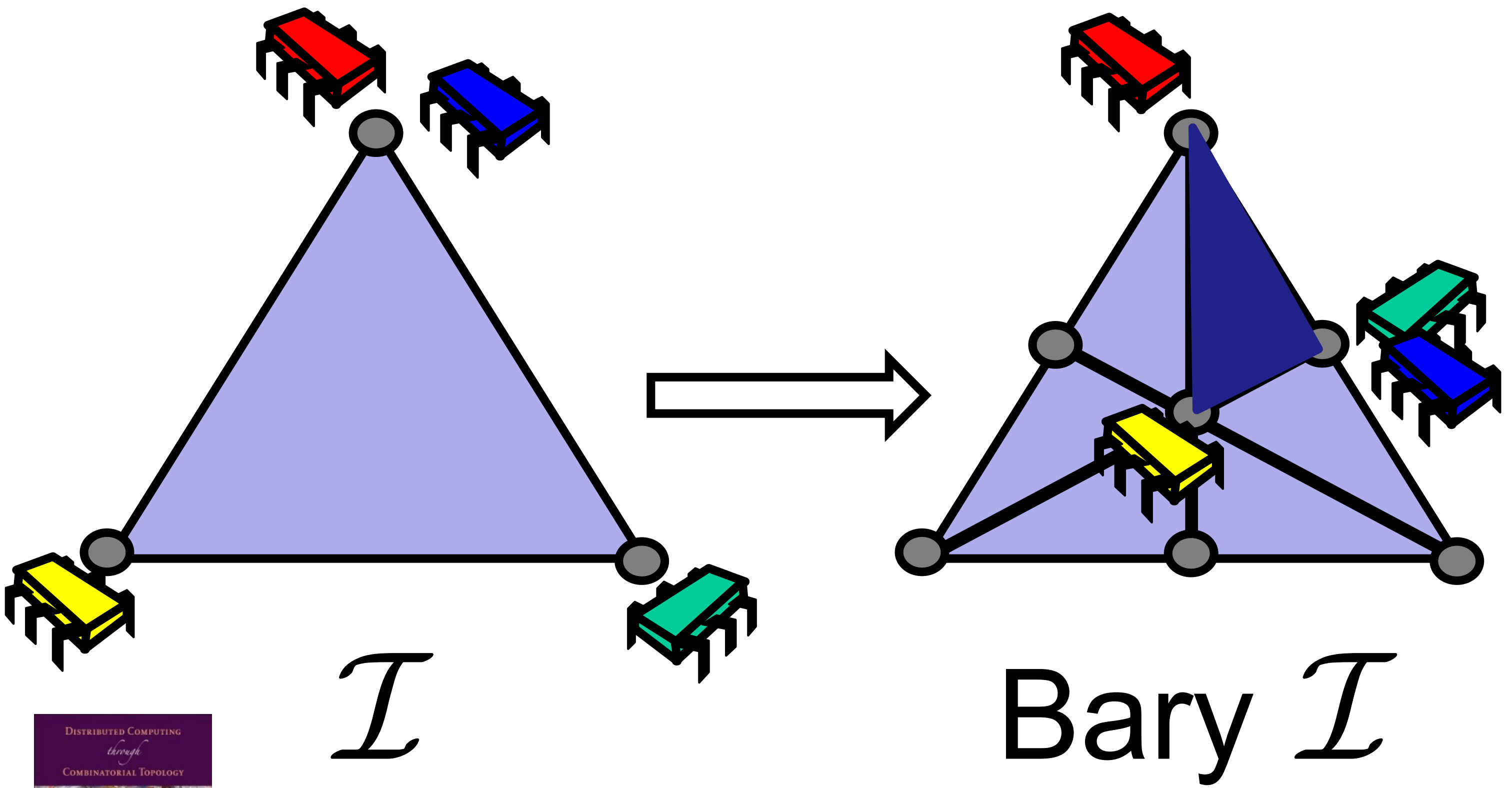
return min value received

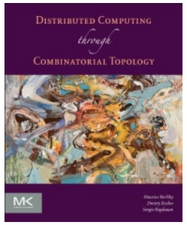
possible to “miss” only *t* lesser values



Barycentric Agreement

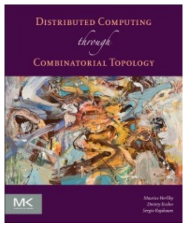
Assuming n+1>2t





Barycentric Agreement Protocol

```
BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := {v_i}
  count: int := 0
  while count < n+1-t do
    send(P_i, V_i) to all
    on receive(P_j, V_j) do
      if V_i = V_j then count := count + 1
                    else if V_j \ V_i ≠ ∅ then
                            V_i := V_i ∪ V_j
                            count := 0
  return V_i
```



Barycentric Agreement Protocol

BaryAgree(v_i: Vertex): set of Vertex

V_i: set of Vertex := $\{v_i\}$

count: int := 0

while count < n+1-t do

send

Set of messages P_i *has received*

on receive(P_j, V_j) do

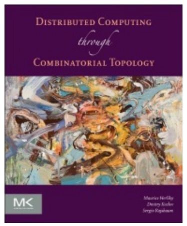
if $V_i = V_j$ then count := count + 1

else if $V_j \setminus V_i \neq \emptyset$ then

$V_i := V_i \cup V_j$

count := 0

return V_i



Barycentric Agreement Protocol

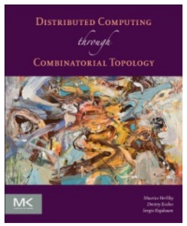
BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := {v_i}

  count: int := 0
  while count < n+1-t do

    send(P_i, V_i) to all

keep track of confirmations received so far

    on receive(P_j, V_j) do
      if $V_i = V_j$ then count := count + 1
        else if $V_j \setminus V_i \neq \emptyset$ then
          $V_i := V_i \cup V_j$
          count := 0
  return V_i



Barycentric Agreement Protocol

BaryAgree(v_i: Vertex): set of Vertex

V_i: set of Vertex := $\{v_i\}$

count: int := 0

while count < n+1-t do

send(P_i, V_i) to all

on receive(P_j, V_j) do

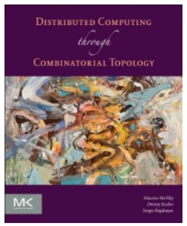
get confirmation from each non-faulty process

else if $V_j \setminus V_i \neq \emptyset$ then

$V_i := V_i \cup V_j$

count := 0

return V_i



Barycentric Agreement Protocol

BaryAgree(v_i: Vertex): set of Vertex

broadcast message set received

count: int := 0

while count < n+1-t do

send(P_i, V_i) to all

on receive(P_j, V_j) do

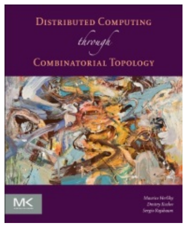
if $V_i = V_j$ then count := count + 1

else if $V_j \setminus V_i \neq \emptyset$ then

$V_i := V_i \cup V_j$

count := 0

return V_i

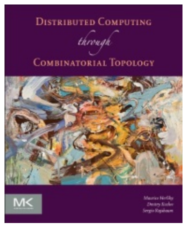


Barycentric Agreement Protocol

BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := $\{v_i\}$
  count: int := 0
  while count < n+1-t do
    send(P_i, V_i) to all

collect responses

on receive(P_j, V_j) do
      if $V_i = V_j$ then count := count + 1
        else if $V_j \setminus V_i \neq \emptyset$ then
          $V_i := V_i \cup V_j$
          count := 0
  return V_i



Barycentric Agreement Protocol

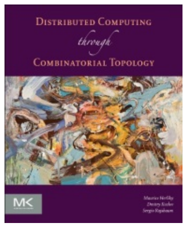
BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := $\{v_i\}$
  count: int := 0

remember if message confirms my view

  send(P_i, V_i) to all
  on receive(P_j, V_j) do

if $V_i = V_j$ then count := count + 1

    else if $V_j \setminus V_i \neq \emptyset$ then
      $V_i := V_i \cup V_j$
      count := 0
  return V_i



Barycentric Agreement Protocol

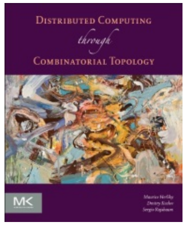
BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := $\{v_i\}$
  count: int := 0

otherwise learned something new, start over

    send(P_i, V_i) to all
    on receive(P_j, V_j) do
      if $V_i = V_j$ then count := count + 1

else if $V_j \setminus V_i \neq \emptyset$ then
  $V_i := V_i \cup V_j$
  count := 0

  return V_i

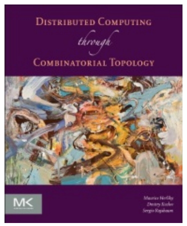


Barycentric Agreement Protocol

BaryAgree(v_i: Vertex): set of Vertex
  V_i: set of Vertex := $\{v_i\}$
  count: int := 0
  while count < n+1-t do
    send(P_i, V_i) to all
    on receive(P_j, V_j) do
      if $V_i = V_j$ then count := count + 1
      then
        $V_i := V_i \cup V_j$
        count := 0

return when enough agree

return V_i



Wait, There's More!

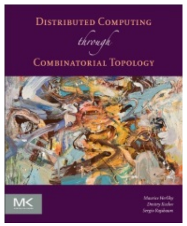
background

upon receive(P_j, V_j) do

$V_i := V_i \cup V_j$

send(P_i, V_i) to all

the operating system runs forever …

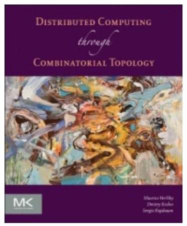


Wait, There's More!

keep forwarding new values

background

upon receive(P_j, V_j) do
    $V_i := V_i \cup V_j$
    send(P_i, V_i) to all



Lemma: Protocol Terminates

Suppose P_i runs forever …

Eventually V_i assumes final value V …

Non-faulty P_j, where $V_j = V'$, receives V

P_j must have sent V_j to P_i

$V_j \subset V$ OR $V_j = V$

P_j will sent V to P_i

P_j has sent V to P_i

return V_i

All V_i, V_j Totally Ordered

BaryAgree(v_i: Vertex): set of Vertex

If P_i broadcasts $V^{(0)}, \ldots, V^{(k)}$, then $V^{(i)} \subseteq V^{(i+1)}$

count: int := 0

To decide …

P_i received V_i from X, $|X| \geq n+1-t$

P_j received V_j from Y, $|Y| \geq n+1-t$

if $V_i = V_j$ then count := count + 1

some $P_k \in X \cap Y$ sent both V_i, V_j

Recall that n+1>2t

so V_i, V_j are ordered.

return V_i

Theorem

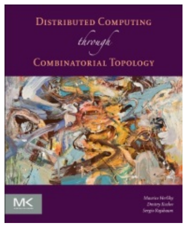
For $2t < n+1$, colorless task $(\mathcal{I},\mathcal{O},\Delta)$ has a t-resilient message-passing protocol …

if and only if …

there is a continuous map

$f\colon |\text{skel}^t\,\mathcal{I}| \rightarrow |\mathcal{O}|$

carried by Δ.



Theorem

For $2t < n+1$, colorless task $(\mathcal{I},\mathcal{O},\Delta)$ has a t-resilient message-passing protocol …

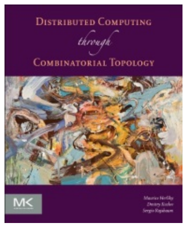
if and only if …

there is a continuous map

$f: |\mathrm{skel}^t\, \mathcal{I}| \to |\mathcal{O}|$

carried by Δ.

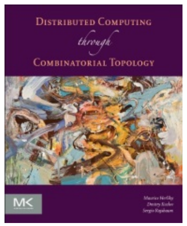
same as snapshot when $2t < n+1$!



Protocol implies map

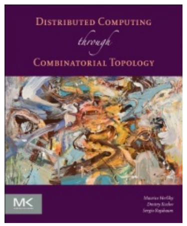
- Any t-resilient message passing protocol implies a t-resilient layered snapshot protocol
  - Snapshots are "stronger" than message-passing (even when 2t$\geq (n+1)$)
- A t-resilient layered snapshot protocol implies a map

$$f\colon |\mathrm{skel}^t\,\mathcal{I}| \to |\mathcal{O}|$$



Map implies protocol

- There exists a simplicial approximation
  ϕ: $\mathrm{Bary}^N \mathcal{I} \to \mathcal{O}$ carried by Δ
- Run t-set agreement for simplex agreement on $\mathrm{skel}^t \mathcal{I}$ (works even when 2t$\geq (n+1)$)
- Run N iteration on Barycentric agreement (for 2t$< (n+1)$) and use ϕ



Road Map

Overview of Models

t-resilient layered snapshot models

Layered Snapshots with *k*-set agreement

Adversaries

Message-Passing Systems

Decidability

Automatic Proofs?

What if we could program a Turing machine to tell whether a task has a protocol?

In wait-free read-write memory?

Or other models?

We could …

automatically generate conference papers

No need for grad students

Alas no

Whether a protocol exists for a task in …

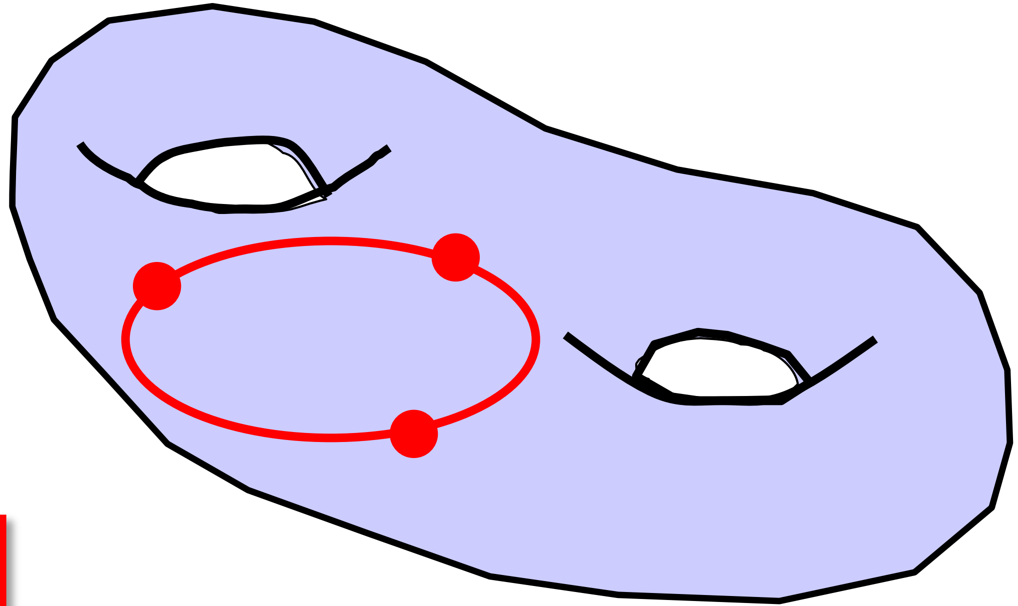
Read-write memory for 3+ processes …

Read-write memory & k-set agreement …

for $k > 2$

Is *undecidable*.

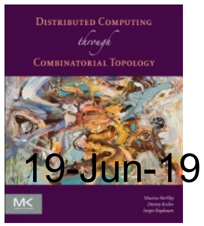
Loop Agreement



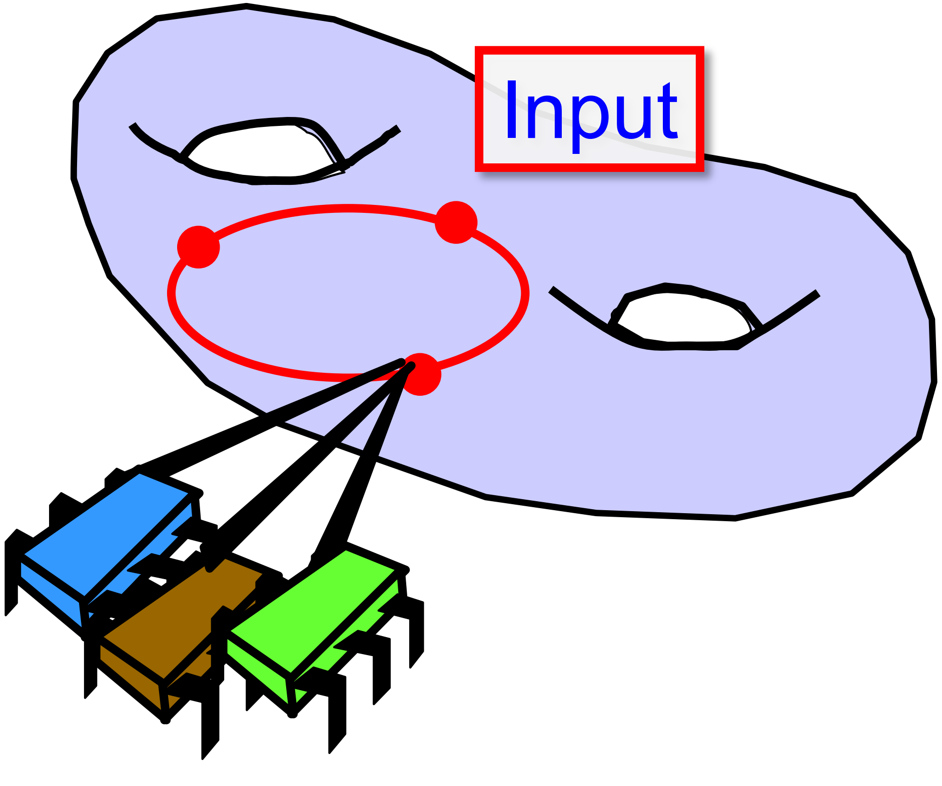
Complex

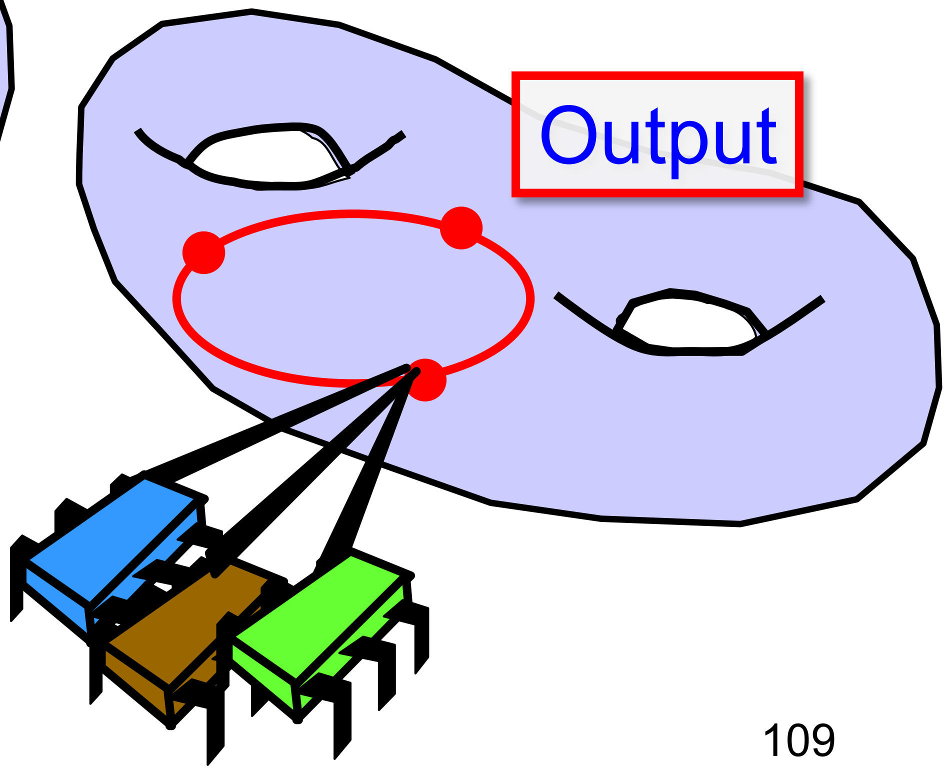
Loop

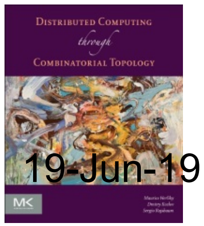
Three *rendez-vous* points



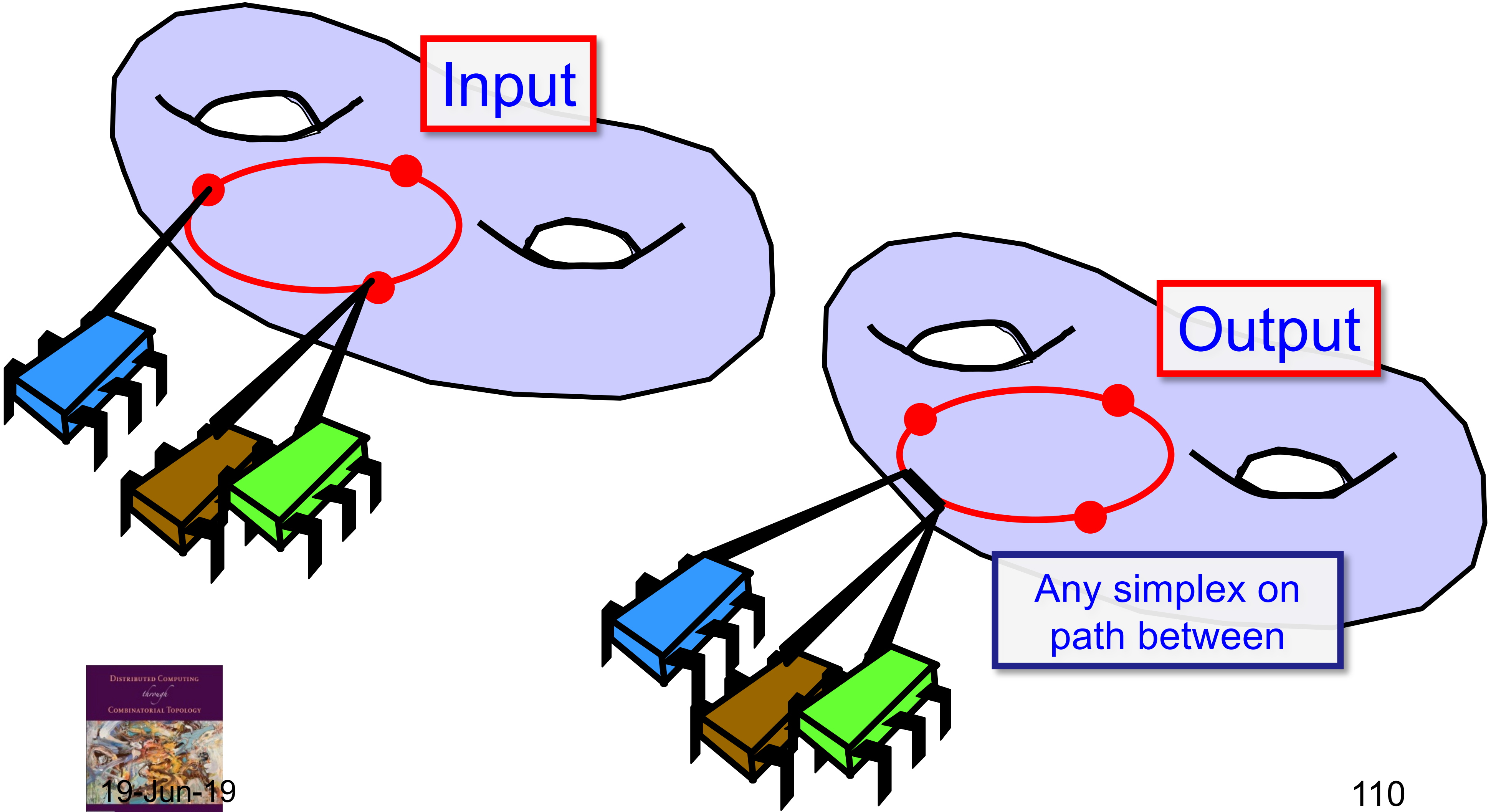
One Rendez-Vous Point



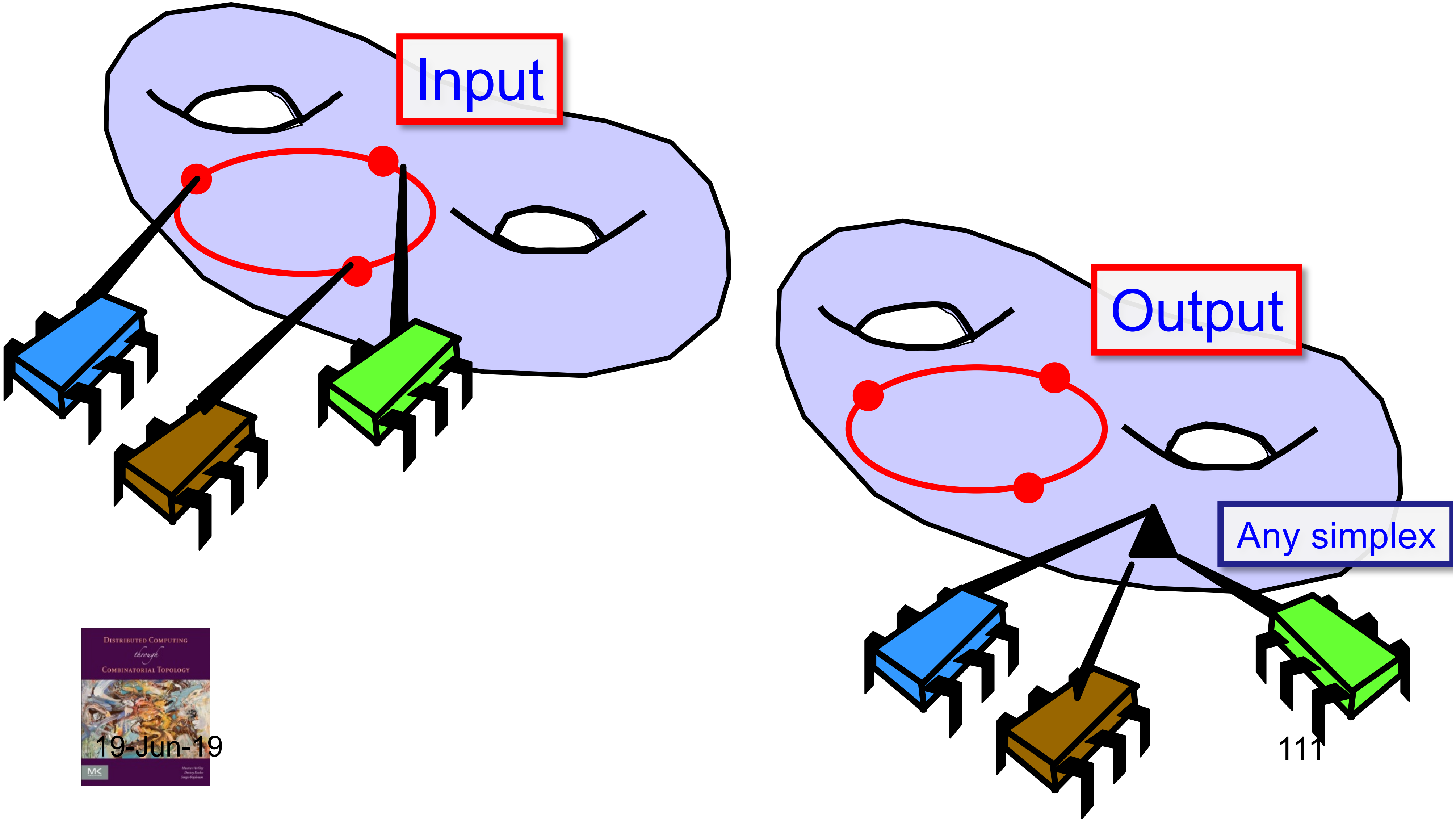




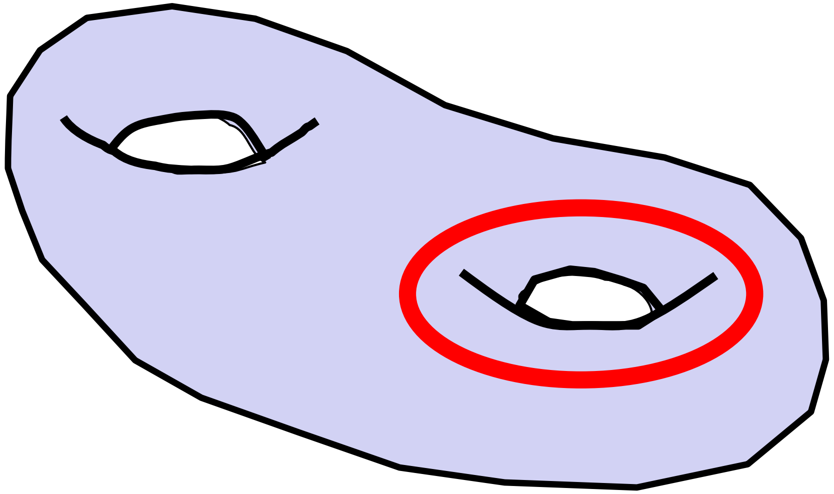
Two Rendez-Vous Points



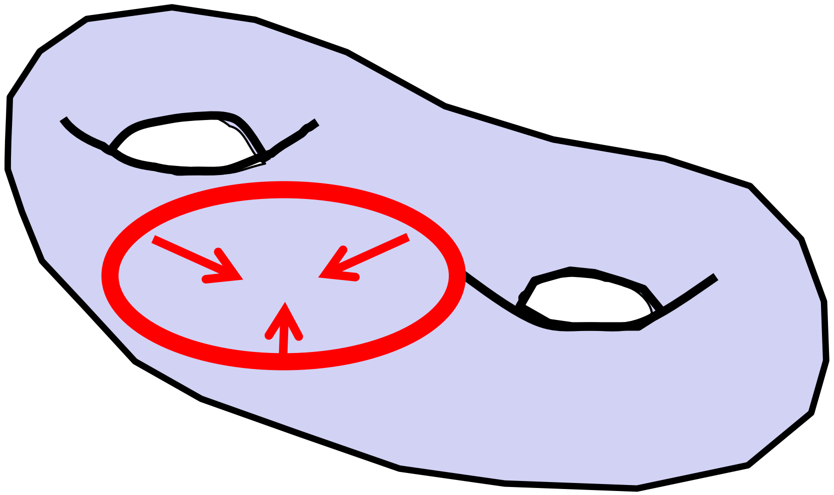
Three Rendez-Vous Points



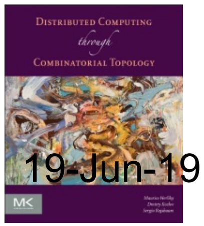
Contractibility



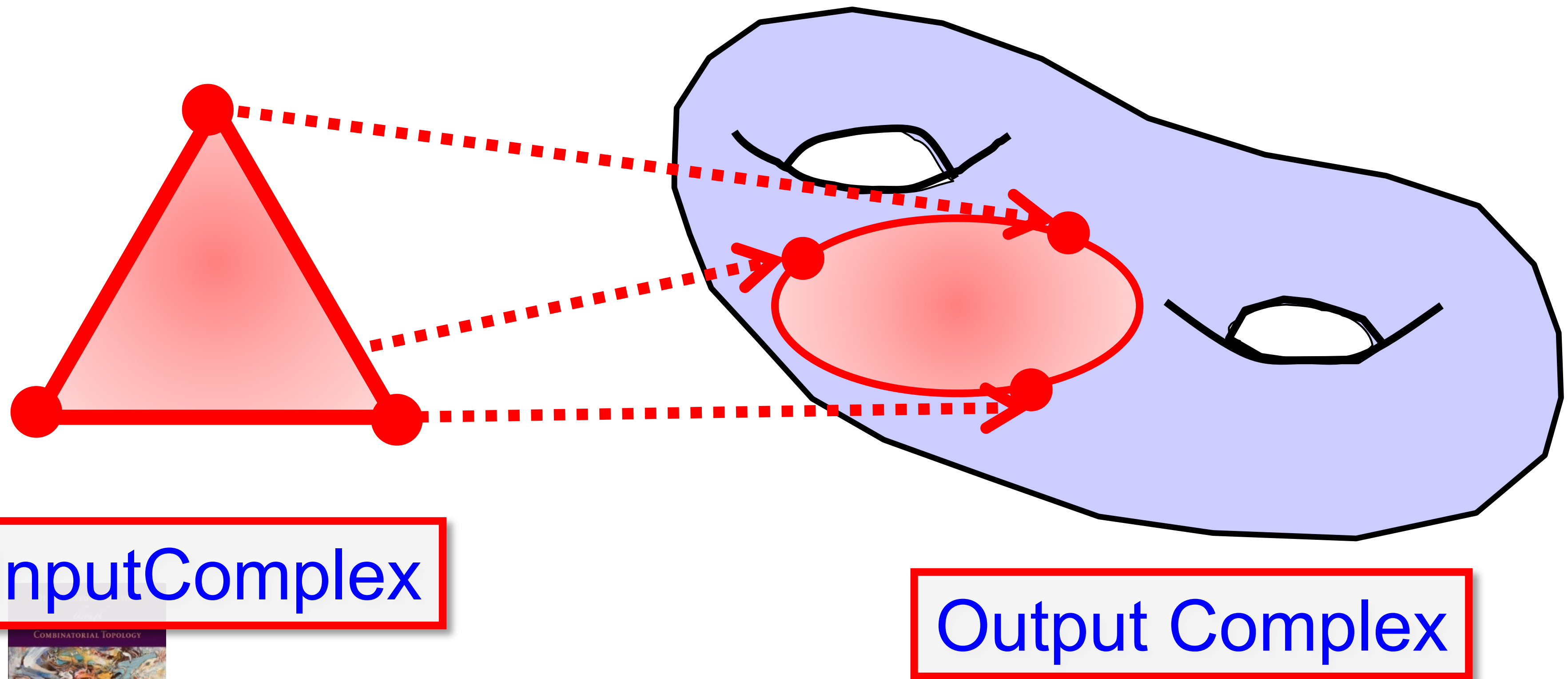
contractible



not contractible



Solvable Iff Loop Contractible



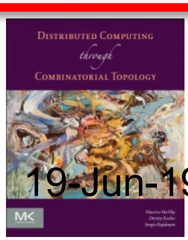
Undecidability

But Contractiblity is *undecidable* …

even for finite complexes!

(reduces to the word problem for finitely-presented groups)

Undecidable whether a task has a protocol in wait-free read-write memory



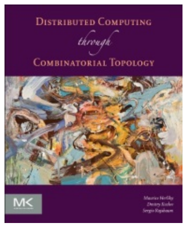
Other Models

Wait-free read-write memory plus *k*-set agreement , for $k > 2$

Solvable iff $f : \mathrm{skel}^{k-1}\, \mathcal{I} \to \mathcal{O}$ exists …

Implies contractible, for $k > 2$

Undecidable whether a task has a protocol in wait-free read-write memory plus *k*-set agreement , for $k > 2$


Distributed Computing
through
Combinatorial Topology
MK