Distributed Algorithms

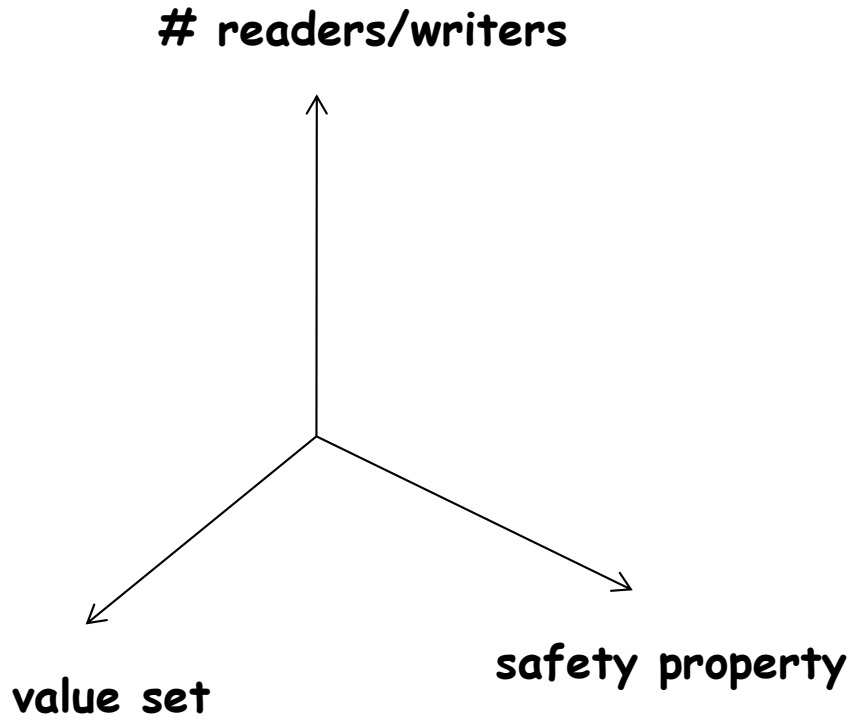
Atomic snapshots and ABD

EFREI, 2019

M1 Big Data

The space of registers

- Nb of writers and readers: from 1W1R to NWNR
- Size of the value set: from binary to multi-valued
- Safety properties: safe, regular, atomic



All registers are (computationally) equivalent!

Transformations

From 1W1R binary safe to 1WNR multi-valued atomic

I. From safe to regular (1W1R)
II. From one-reader to multiple-reader (regular binary or multi-valued)
III. From binary to multi-valued (1WNR regular)
IV. From regular to atomic (1W1R)
V. From 1W1R to 1WNR (multi-valued atomic)
VI. From 1WNR to NWNR (multi-valued atomic)
VII. From safe bit to atomic bit (optimal, coming later)

This class

- Atomic snapshot: reading multiple locations atomically
  - ✓Write to one, read *all*
- Immediate snapshot
  - Write and “immediately” read
- ABD: from message-passing to shared memory

Atomic snapshot: sequential specification

- Each process p_i is provided with operations:
  - ✓ $update_i(v)$, returns ok
  - ✓ $snapshot_i()$, returns $[v_1,\ldots,v_N]$

- In a sequential execution:

  For each $[v_1,\ldots,v_N]$ returned by $snapshot_i()$,
  v_j (j=1,…,N) is the argument of the last $update_j(.)$

  (or the initial value if no such update)

Snapshot for free?

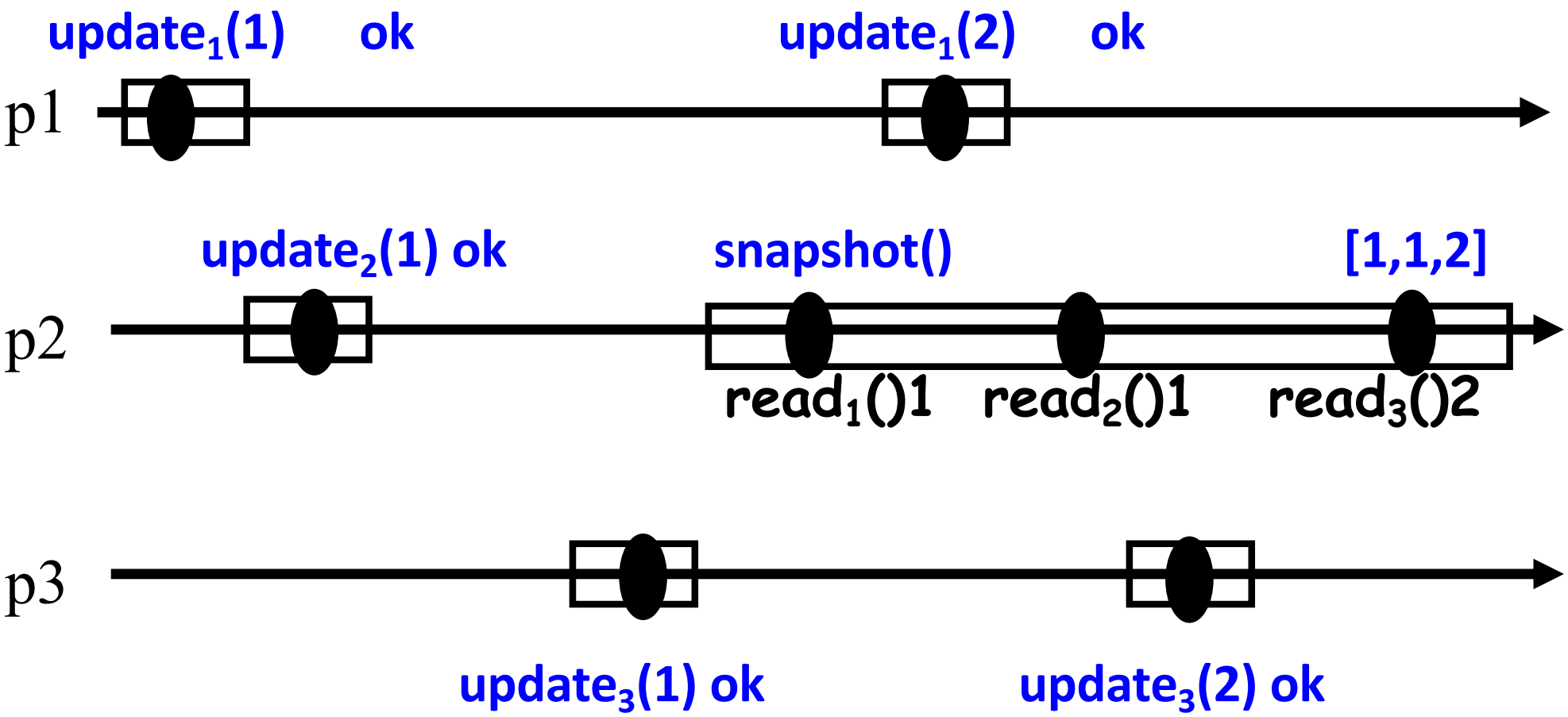
Code for process p_i:

initially:
shared 1WNR *atomic* register $R_i := 0$

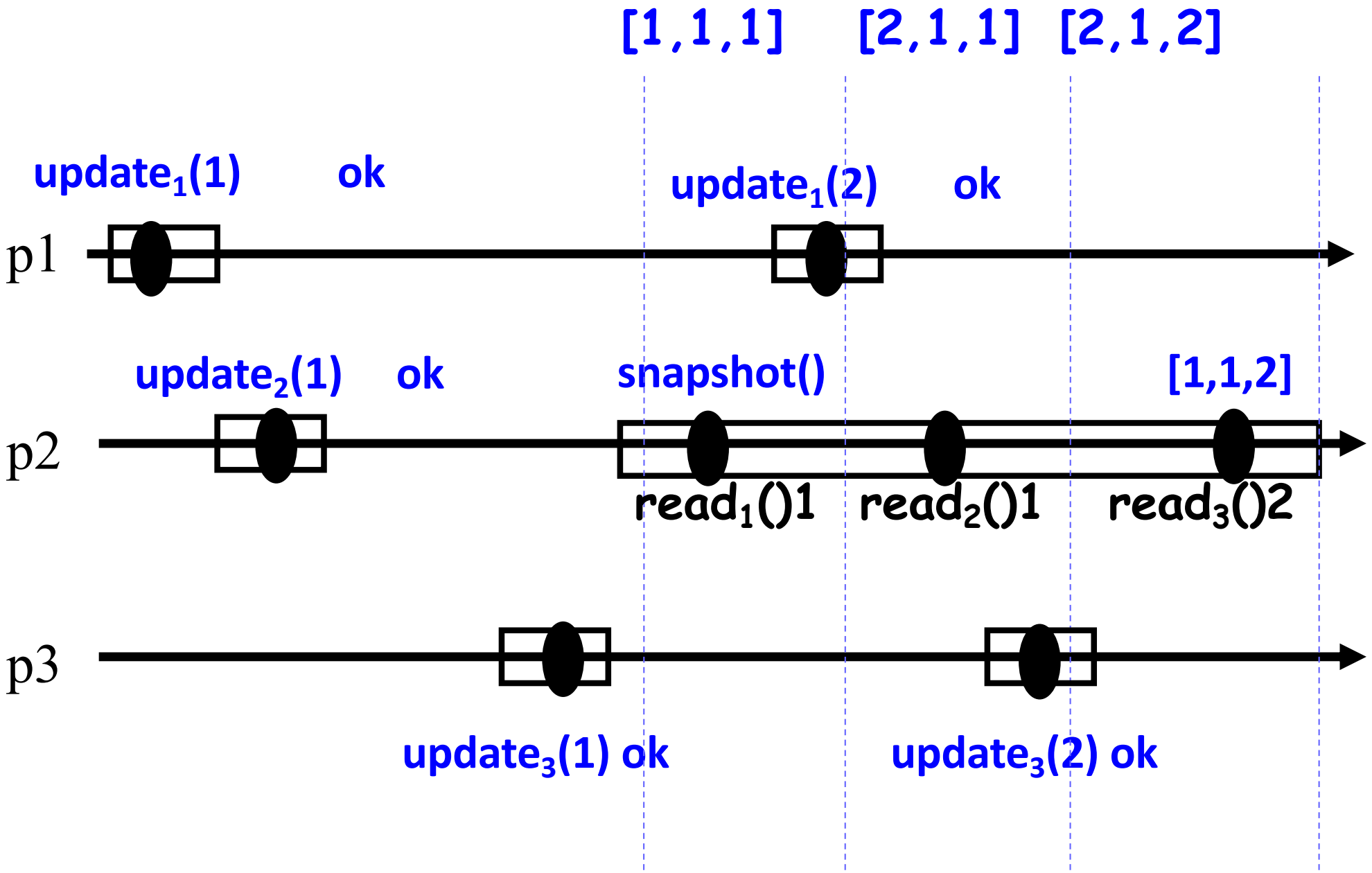
upon snapshot()
$[x_1,\ldots,x_N] := \text{scan}(R_1,\ldots,R_N)$ /*read $R_1,\ldots R_N$*/
return $[x_1,\ldots,x_N]$

upon update$_i$(v)
R_i.write(v)

Snapshot for free?



Snapshot for free?



- What about 2 processes?

- What about lock-free snapshots?
  - ✓At least one correct process makes progress (completes infinitely many operations)

Lock-free snapshot

Code for process p_i (all written values, including the initial one, are unique, e.g., equipped with a sequence number)

Initially:

shared 1W1R atomic register $R_i := 0$

upon snapshot()

$[x_1,\ldots,x_N]$:= scan($R_1,\ldots,R_N$)

repeat

$[y_1,\ldots,y_N] := [x_1,\ldots,x_N]$

$[x_1,\ldots,x_N]$:= scan($R_1,\ldots,R_N$)

until $[y_1,\ldots,y_N] = [x_1,\ldots,x_N]$

return $[x_1,\ldots,x_N]$

upon update$_i$(v)

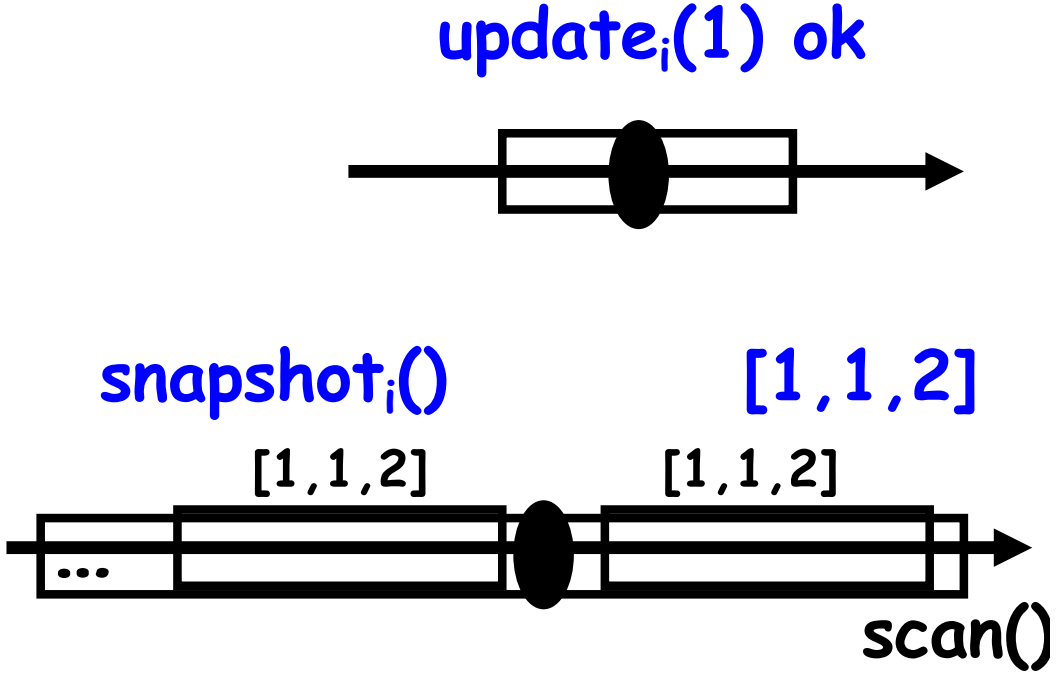
R_i.write(v)

Linearization

Assign a linearization point to each operation

- $update_i(v)$
  - ✓R_i.write(v) if present
  - ✓Otherwise remove the op
- $snapshot_i()$
  - ✓if complete – any point between identical scans
  - ✓Otherwise remove the op

Build a sequential history S in the order of linearization points

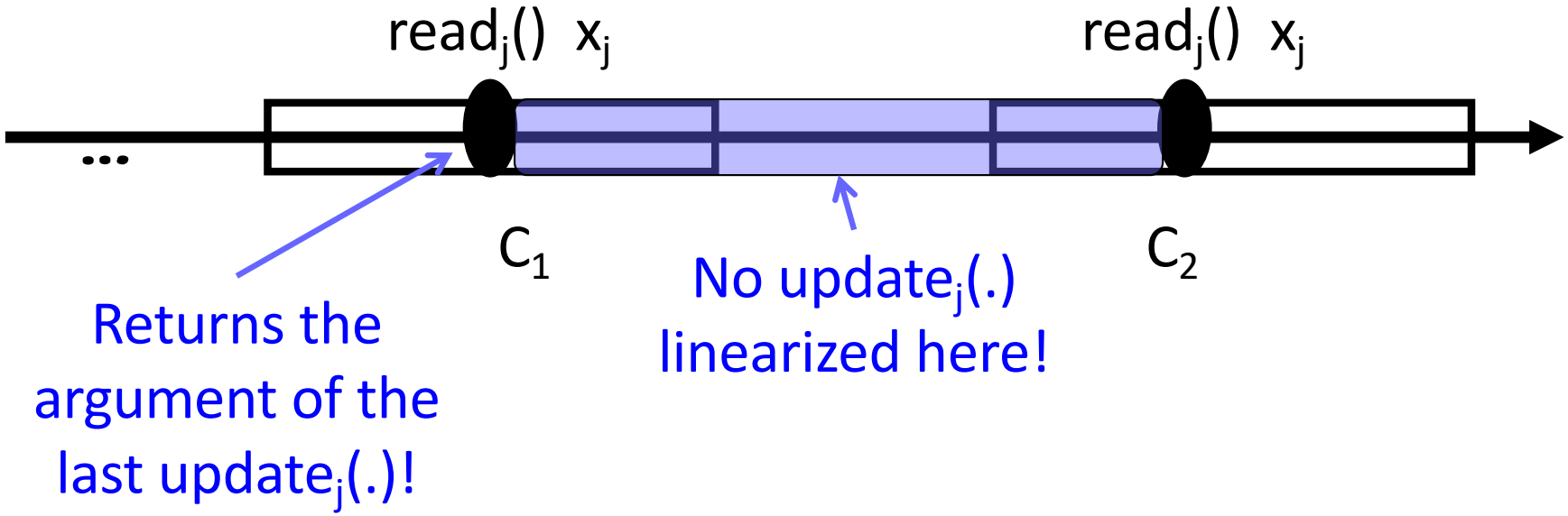


Correctness: linearizability

S is legal: every $snapshot_i()$ returns the last written value for every p_j

Suppose not: $snapshot_i()$ returns $[x_1,\ldots,x_N]$ and some x_j is not the the argument of the last $update_j(v)$ in S preceding $snapshot_i()$

Let C_1 and C_2 be two scans that returned $[x_1,\ldots,x_N]$



Correctness: lock-freedom

An $update_i()$ operation is wait-free (returns in a finite number of steps)

Suppose process p_i executing $snapshot_i()$ eventually runs in isolation (no process takes steps concurrently)

- All scans received by p_i are distinct
- At least one process performs an update between
- There are only finitely many processes => at least one process executes infinitely many updates

What if base registers are regular?

General case: helping?

What if an update interferes with a snapshot?

- Make the update do the work!

upon snapshot()

$[x_1,\ldots,x_N]$:= scan($R_1,\ldots, R_N$)
$[y_1,\ldots,y_N]$:= scan($R_1,\ldots,R_N$)
if $[y_1,\ldots,y_N] = [x_1,\ldots,x_N]$ then
    return $[x_1,\ldots,x_N]$
else
    let j be such that
        $x_j \neq y_j$ and $y_j=(u,U)$
        return U

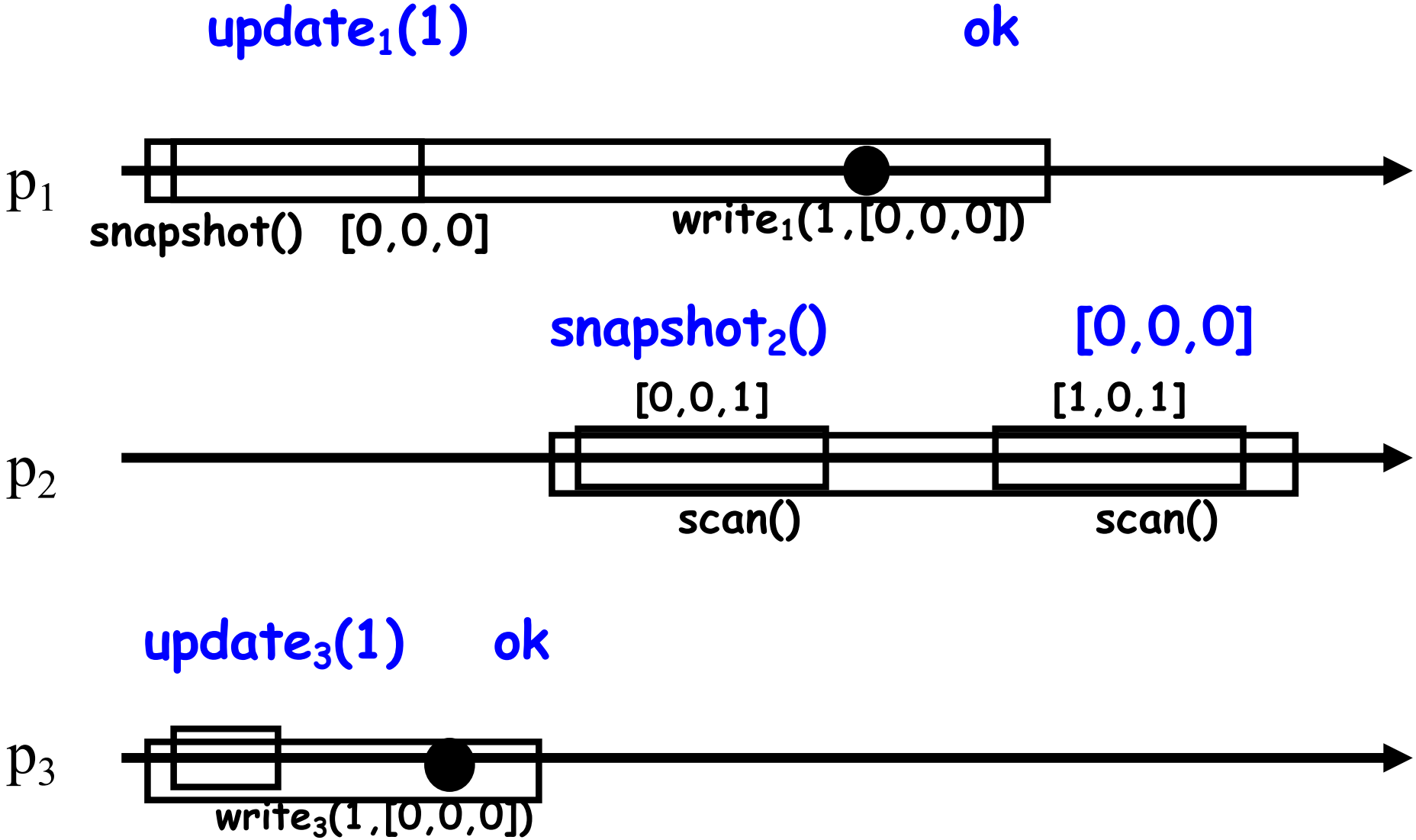
upon update$_i$(v)

S := snapshot()
R_i.write(v,S)

If two scans differ – some update succeeded!

Would this work?

Not that easy!



General case: wait-free atomic snapshot

upon snapshot()

$[x_1,\ldots,x_N]$:= scan($R_1,\ldots,R_N$)

while true do

  $[y_1,\ldots,y_N]$:= $[x_1,\ldots,x_N]$

  $[x_1,\ldots,x_N]$:= scan($R_1,\ldots,R_N$)

  if $[y_1,\ldots,y_N] = [x_1,\ldots,x_N]$ then

    return $[x_1,\ldots,x_N]$

  else if $moved_j$ and $x_j \neq y_j$ then

    let $x_j = (u,U)$

    return U

  for each j: $moved_j := moved_j \vee x_j \neq y_j$

upon update$_i$(v)

  S := snapshot()

  R_i.write(v,S)

If a process moved twice: its last snapshot is valid!

Correctness: wait-freedom

Claim 1 Every operation (update or snapshot) returns in $O(N^2)$ steps (bounded wait-freedom)

snapshot: does not return after a scan if a concurrent process moved and no process moved twice

- At most N-1 concurrent processes, thus (pigeonhole), after N scans:
  - ✓Either at least two consecutive identical scans
  - ✓Or some process moved twice!

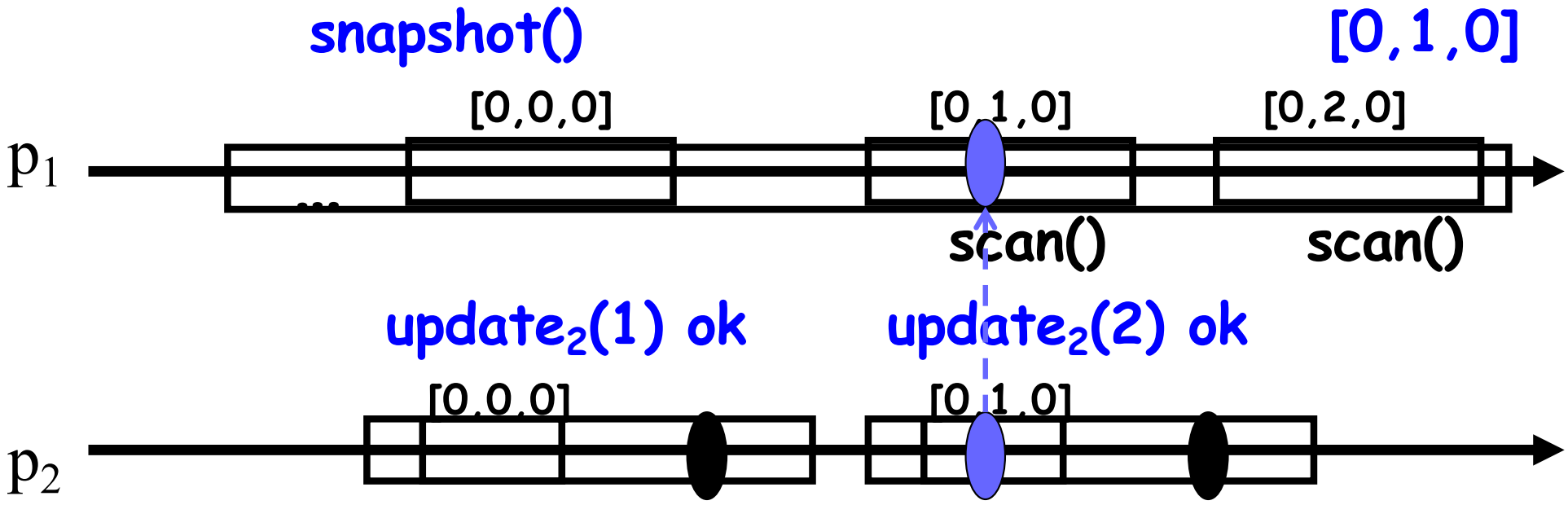
update: snapshot() + one more step

Correctness: linearization points

update$_i$(v): linearize at the R_i.write(v,S)

complete **snapshot()**

- If two identical scans: between the scans
- Otherwise, if returned U of p_j: at the linearization point of p_j's snapshot



The linearization is:

- Legal: every snapshot operation returns the most recent value for each process
- Consistent with the real-time order: each linearization point is within the operation's interval
- Equivalent to the run (locally indistinguishable)

(Full proof in the lecture notes, Chapter 6)

Quiz 1: atomic snapshots

1. Prove that one-shot atomic snapshot satisfies self-inclusion and containment:
   - ✓Self-inclusion: for all i: v_i is in S_i
   - ✓Containment: for all i and j: S_i is subset of S_j or S_j is subset of S_i
2. Show that the atomic snapshot is subject to the ABA problem (affecting correctness) in case the written values are not unique

One-shot atomic snapshot (AS)

Each process p_i:

update$_i$(v_i)

S_i := snapshot()

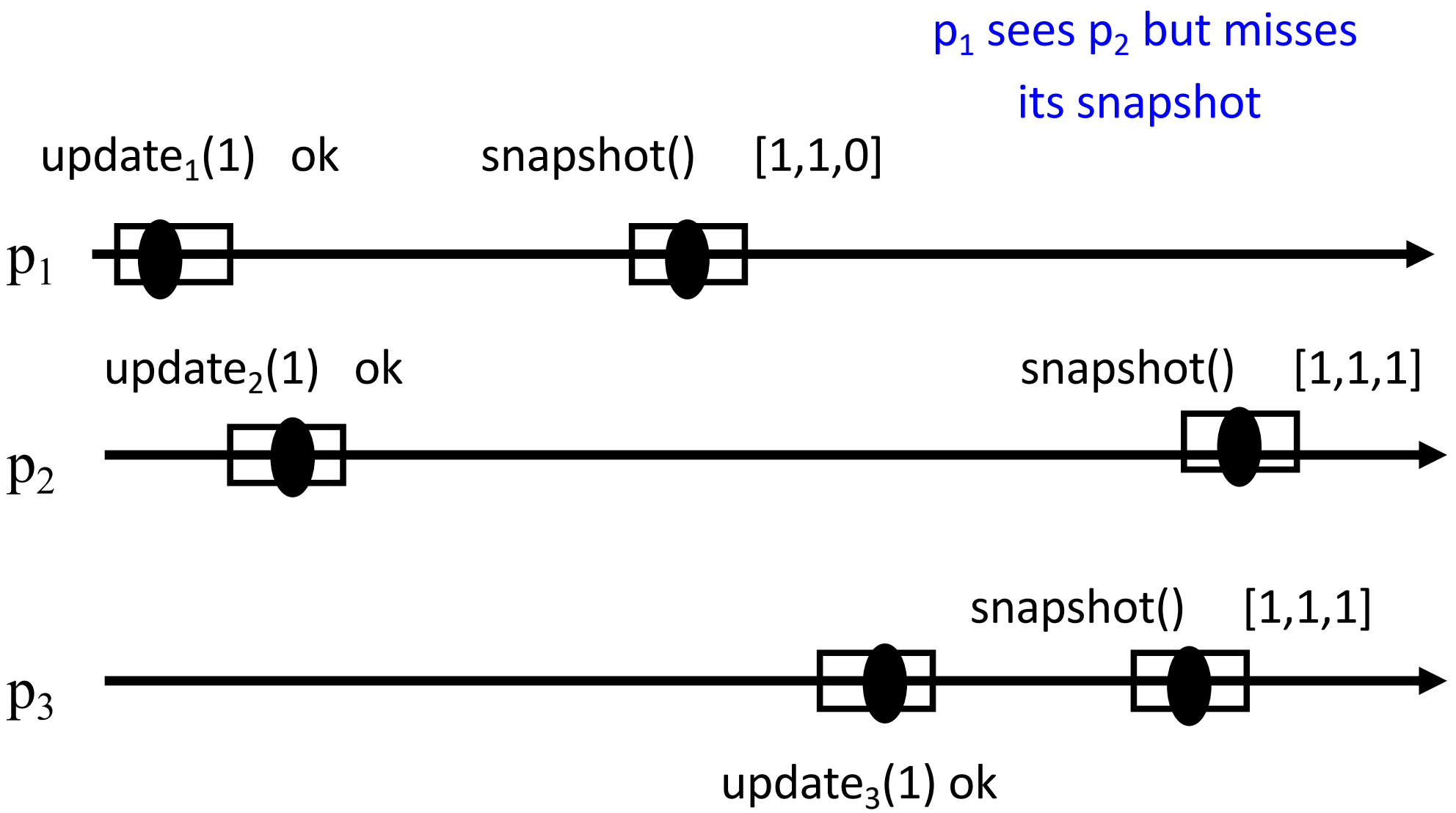
$S_i = S_i[1],\ldots,S_i[N]$

(one position per process)

Vectors S_i satisfy:

- Self-inclusion: for all i: v_i is in S_i
- Containment: for all i and j: S_i is subset of S_j or S_j is subset of S_i

“Unbalanced” snapshots



Enumerating possible runs: two processes

Each process p_i (i=1,2):

$update_i(v_i)$

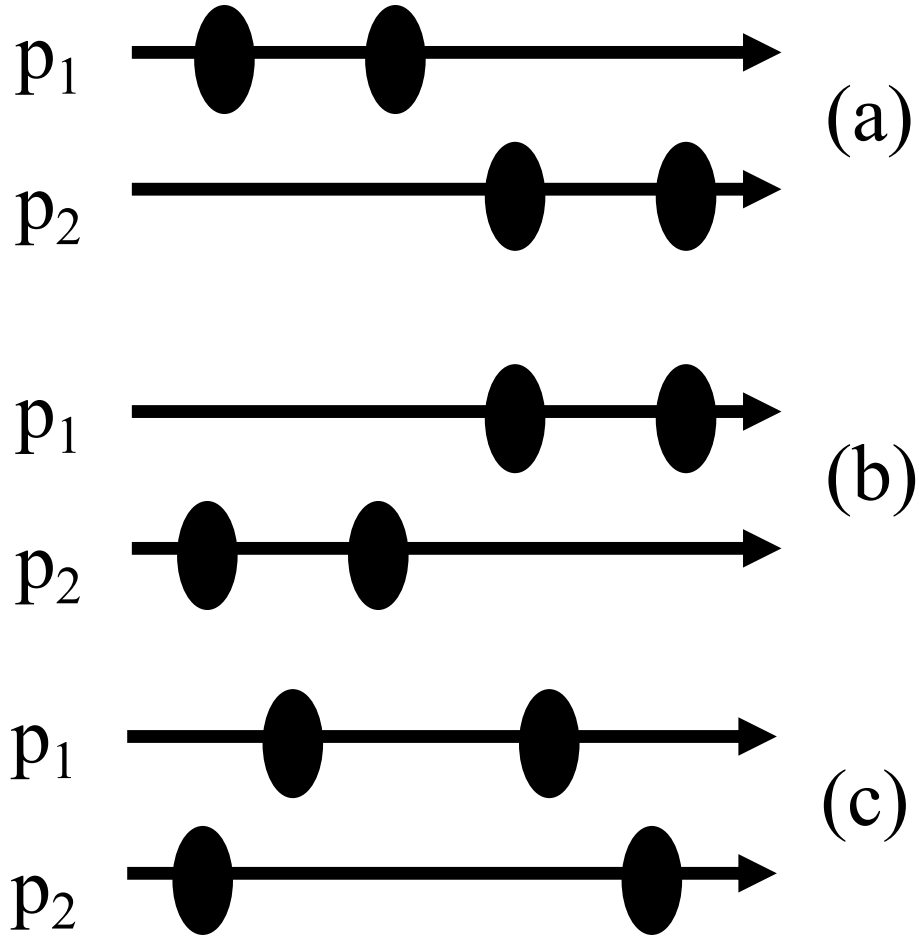
S_i := snapshot()

Three cases to consider:

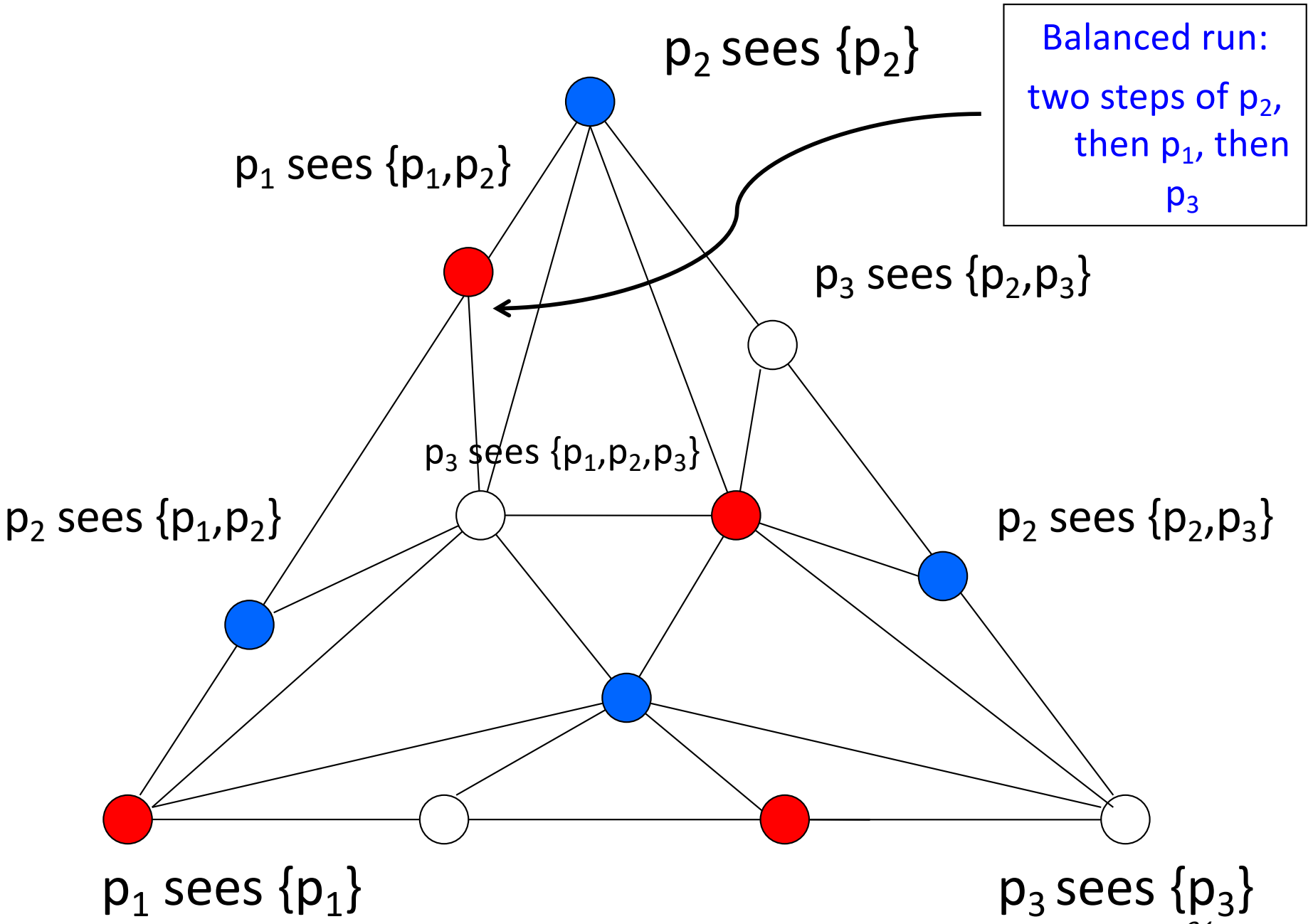
(a) p_1 reads before p_2 writes

(b) p_2 reads before p_1 writes

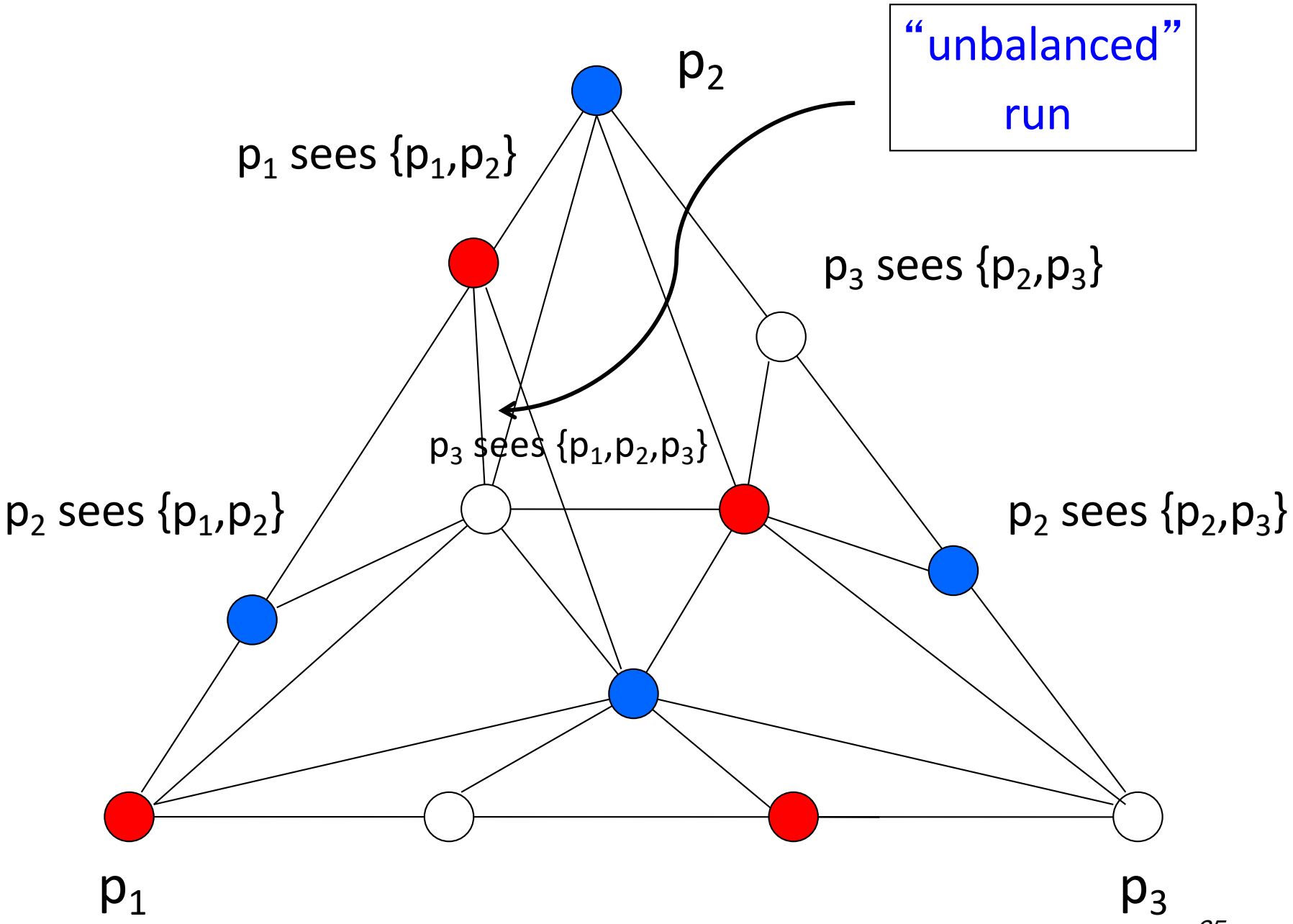
(c) p_1 and p_2 go "lock-step": first both write, then both read



Topological representation: one-shot AS



Topological representation: one-shot AS



One-shot *immediate* snapshot (IS)

One operation:
WriteRead(v)

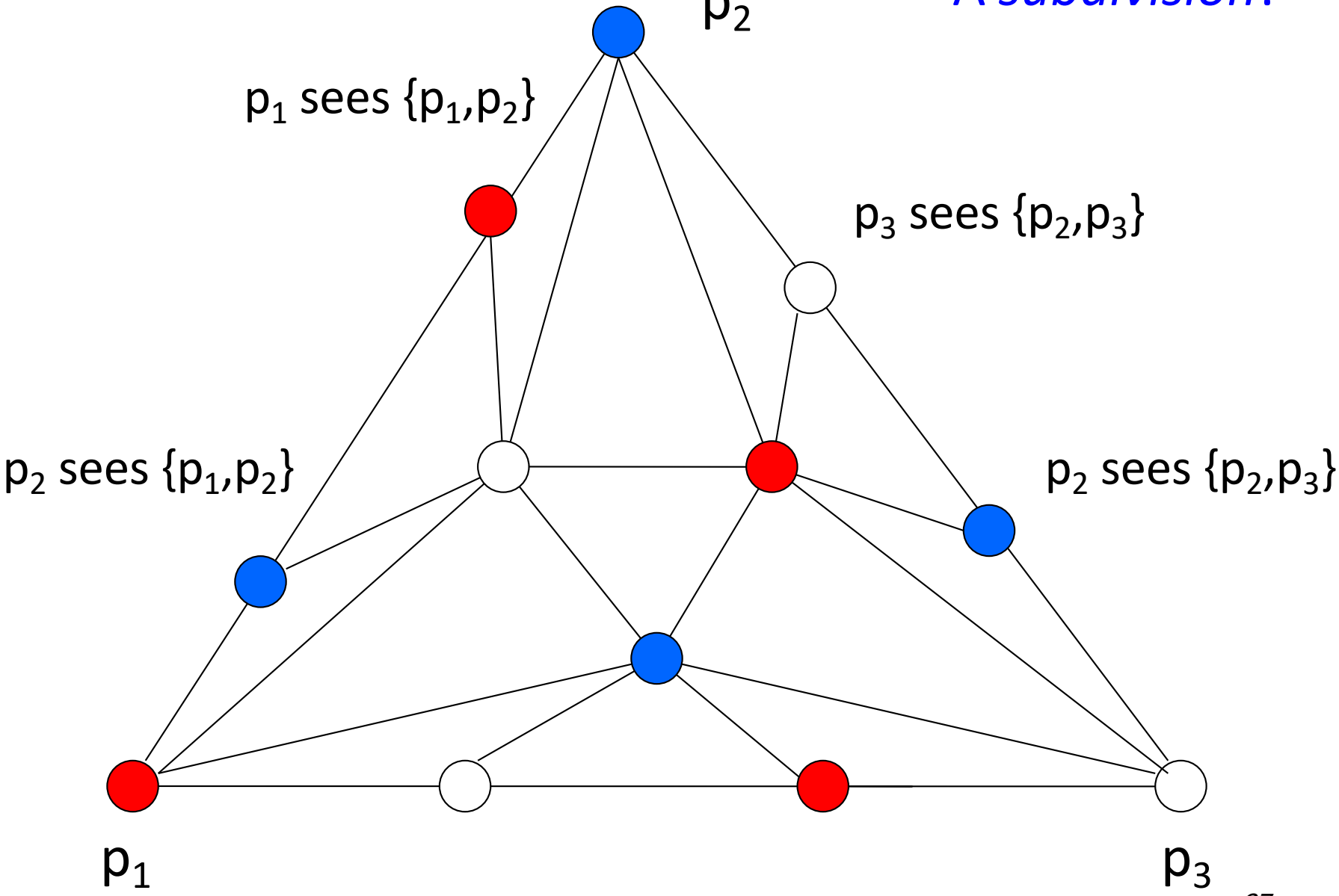
Each process p_i:
S_i := WriteRead$_i$(v_i)

Vectors $S_1,\ldots,S_N$ satisfy:

- Self-inclusion: for all i: v_i is in S_i
- Containment: for all i and j: S_i is subset of S_j or S_j is subset of S_i
- Immediacy: for all i and j: if v_i is in S_j, then is S_i is a subset of S_j

Topological representation: one-shot IS

A *subdivision*!



IS is equivalent to AS (one-shot)

- IS is a restriction of one-shot AS => IS is stronger than one-shot AS
  - ✓Every run of IS is a run of one-shot AS

- Show that a few (one-shot) AS objects can be used to implements IS
  - ✓One-shot ReadWrite() can be implemented using a series of update and snapshot operations

IS from AS

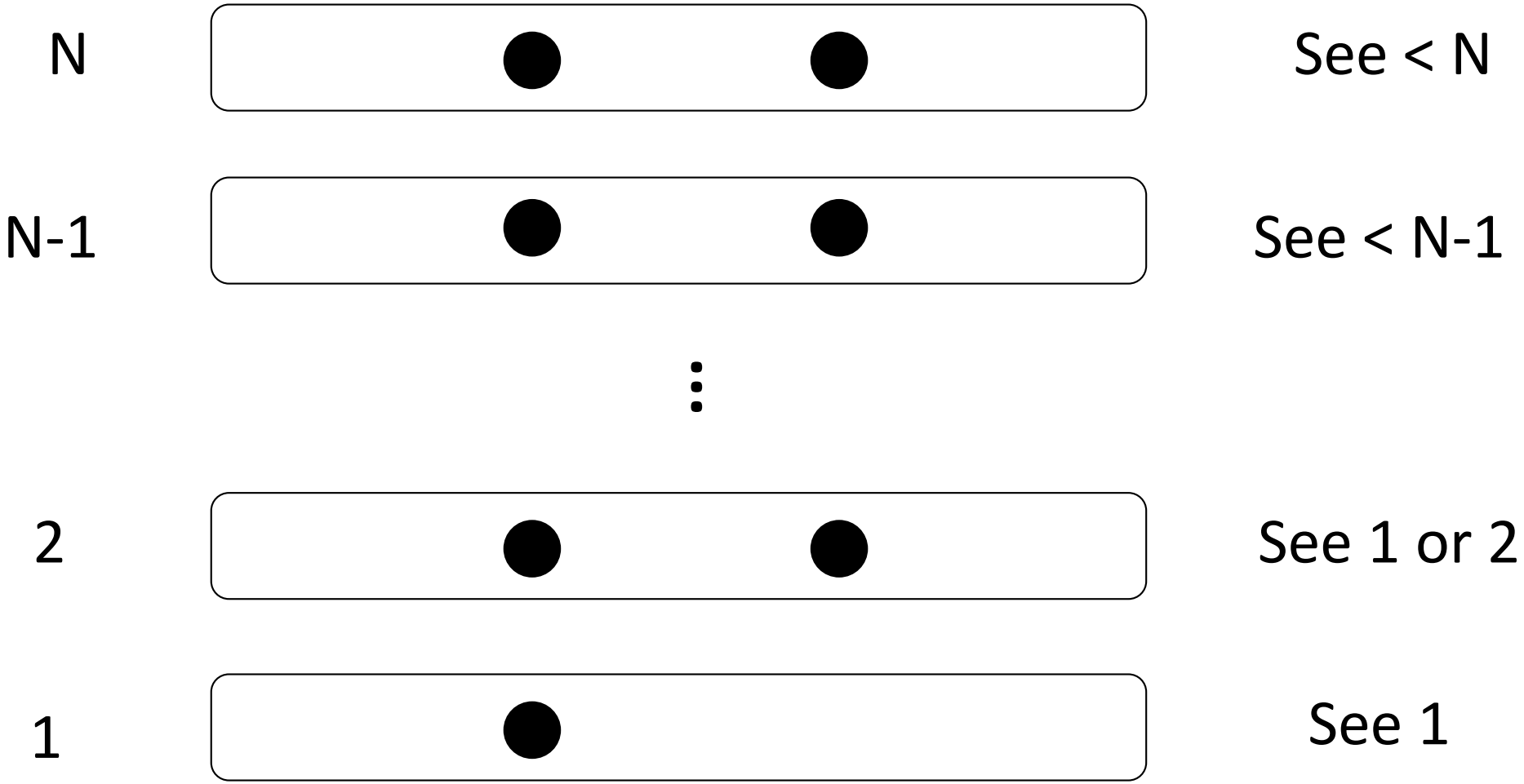
shared variables:

$A_1,\ldots,A_N$ – atomic snapshot objects, initially [T,…,T]

Upon WriteRead$_i$(v_i)

```
r := N+1
while true do
        r := r-1              // drop to the lower level
        A_r.update_i(v_i)
        S :=  A_r.snapshot()
        if |S|=r then         // |S| is the number of non-T values in S
            return S
```

Drop levels: two processes, N>3



Correctness

The outcome of the algorithm satisfies Self-Inclusion, Snapshot, and Immediacy

- By induction on N: for all N>1, if the algorithm is correct for N-1, then it is correct for N
- Base case N=1: trivial

Correctness, contd.

- Suppose the algorithm is correct for N-1 processes
- N processes come to level N
  - ✓At most N-1 go to level N-1 or lower
  - ✓(At least one process returns in level N)
  - ✓Why?
- Self-inclusion, Containment and Immediacy hold for all processes that return in levels N-1 or lower
- The processes returning at level N return all N values
  - ✓The properties hold for all N processes! Why?

Iterated Immediate Snapshot (IIS)

Shared variables:

IS_1, IS_2, IS_3,… // a series of one-shot IS

Each process p_i with input v_i:

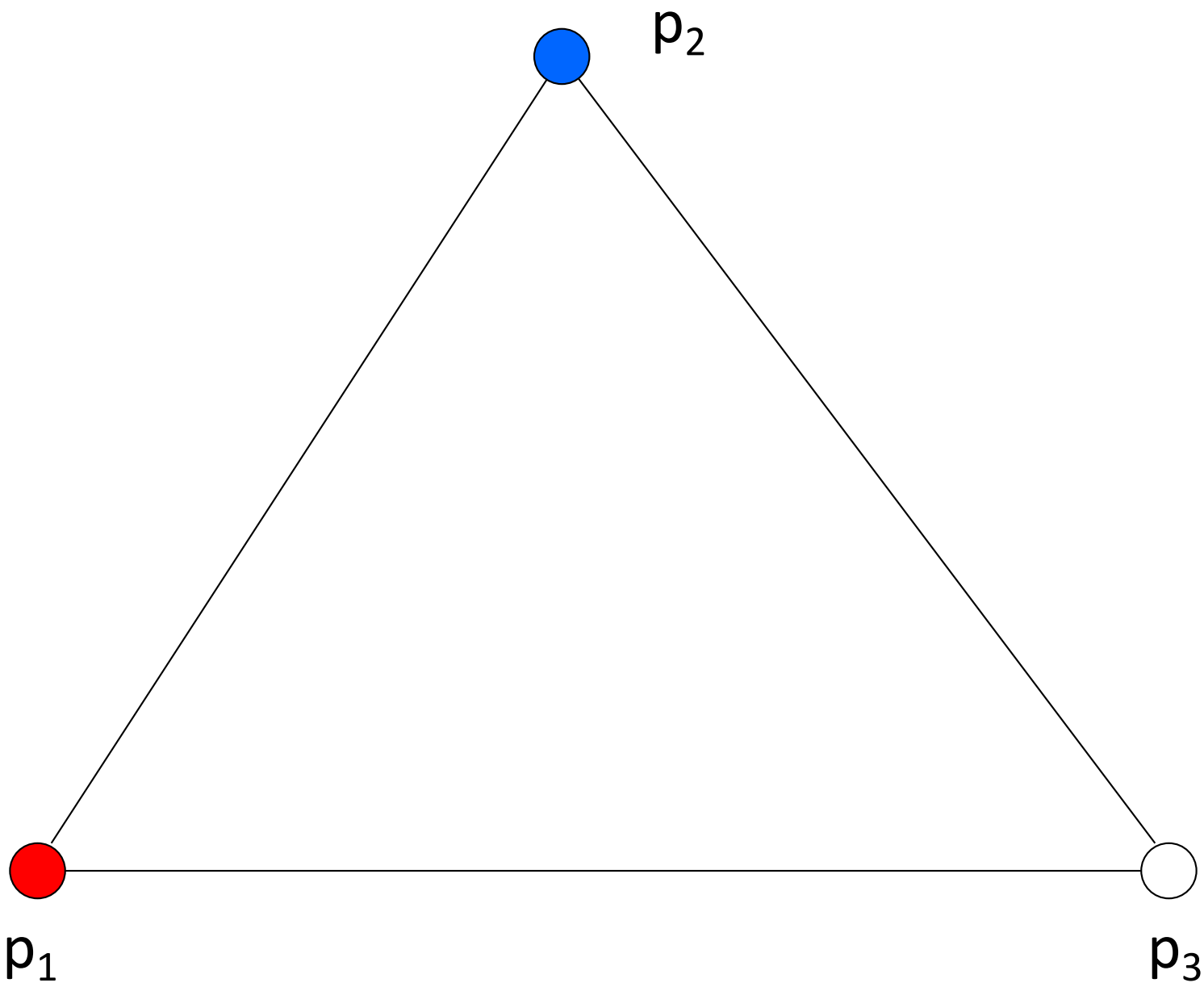
r := 0

while true do

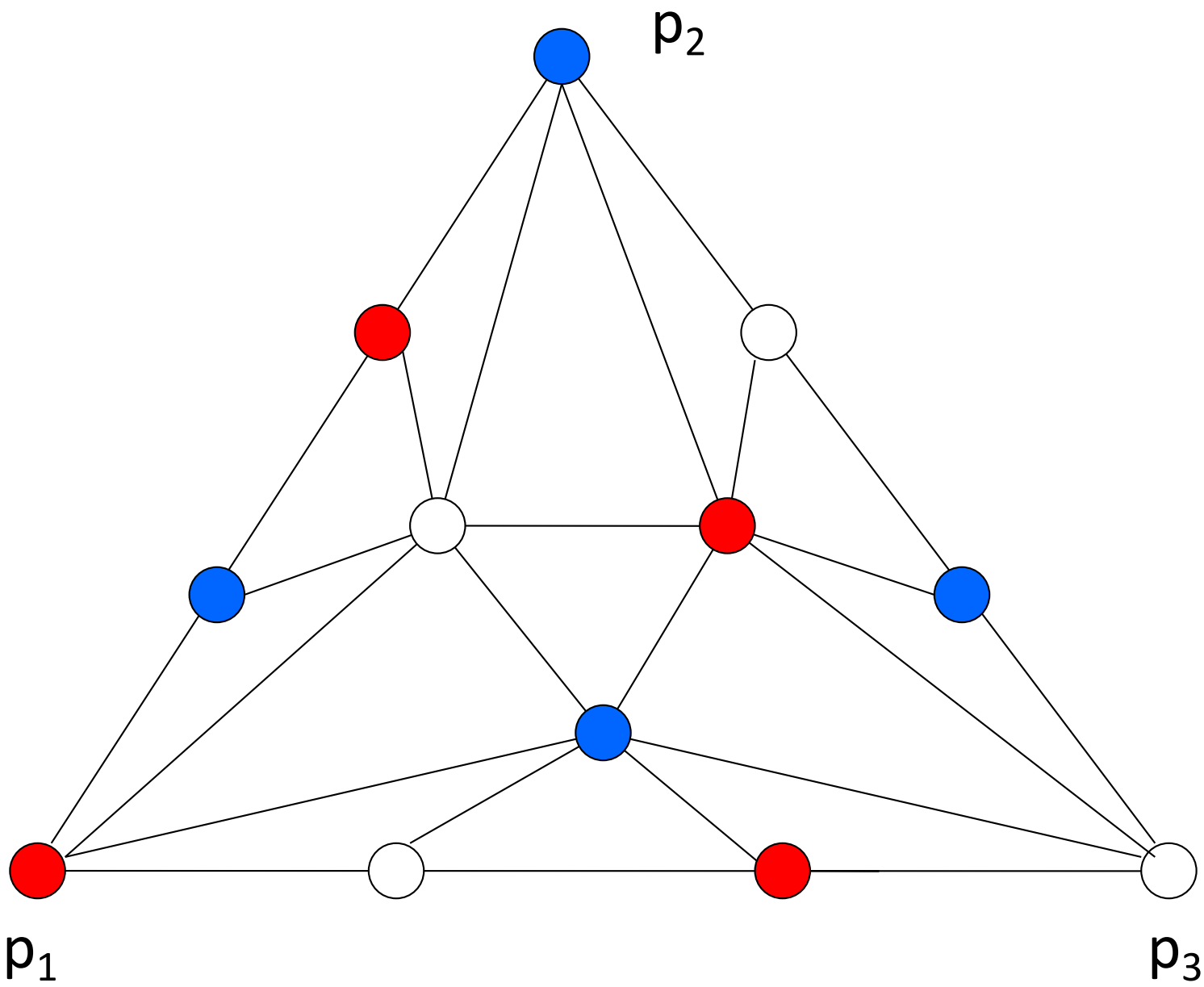
r := r+1

v_i := IS_r.$WriteRead_i(v_i)$

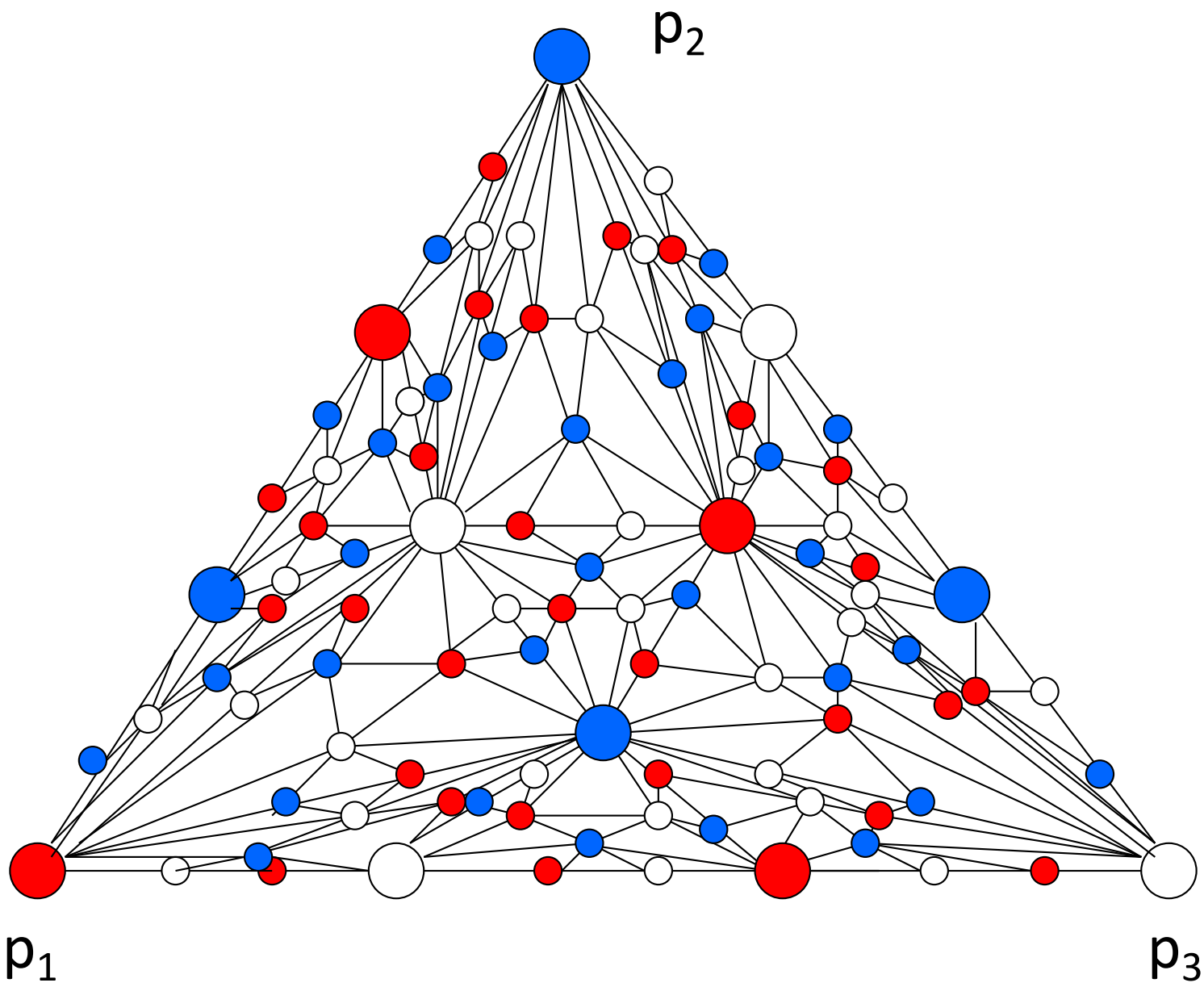
Iterated standard chromatic subdivision (ISDS)



ISDS: one round of IIS



ISDS: two rounds of IIS



IIS is equivalent to (multi-shot) AS

- AS can be used to implement IIS (wait-free)
  - ✓Multiple instances of the construction above (one per iteration)
- IIS can be used to implement (multi-shot) AS in the lock-free manner:
  - ✓At least one correct process performs infinitely many read or write operations
  - ✓Good enough for protocols solving distributed tasks!

Message-passing

- Consider a network where every two processes are connected via a reliable channel
  - ✓no losses, no creation, no duplication

- Which shared-memory results translate into message-passing?
- Implementing a distributed service

Implementing message-passing

Theorem 1 A reliable message-passing channel between two processes can be implemented using two one-writer one-reader (1W1R) read-write registers

Corollary 1 Consensus is impossible to solve in an asynchronous message-passing system if at least one process may crash

ABD algorithm (Attiya, Bar-Noy, Dolev): implementing shared memory

Theorem 2 A 1W1R read-write register can be implemented in a (reliable) message-passing model where a majority of processes are correct

Implementing a 1W1R register

```
Upon write(v)
  t++
  send [v,t] to all
  wait until received [ack,t] from a majority
  return ok

Upon read()
  r++
  send [?,r] to all
  wait until received {(t',v',r)} from a
  majority
  return v' with the highest t'
```

Implementing a 1W1R register, contd.

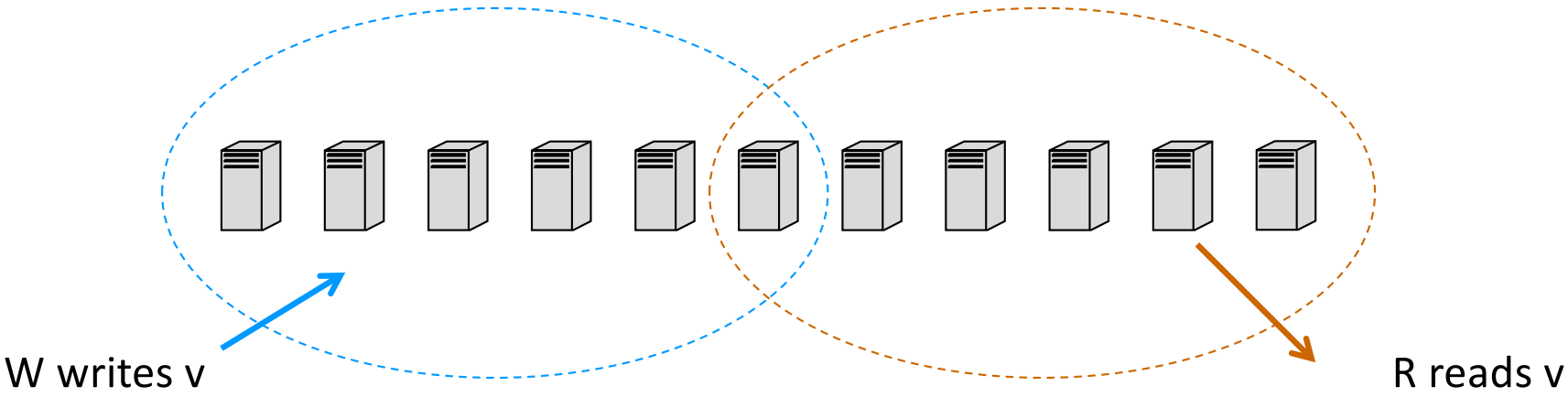
```
Upon receive [v,t]
  if t>t_i then
        v_i := v
        t_i := t
        send [ack,t] to the writer

Upon receive [?,r]
  send [v_i,t_i,r] to the reader
```

A correct majority is necessary

Otherwise, the reader may miss the latest written value

The quorum (set of involved processes) of any write operation must intersect with the quorum of any read operation:

at least a majority of processes must be correct



Quiz 2

- Does the ABD algorithm run by one writer and multiple readers implement an atomic (linearizable) register?
- If not, can it be turned in an atomic one?
- How to support multiple writers?

Simplified key-value store: sequential specification 1

State:

set of key-value pairs (k,v), k in N, v In V (set of values)

Operations:

- add(k,v) – add a new pair to the set, return true iff the key is not in the set
- get(k) - return the value in the set with key k (predefined “bottom” value returned if k is not in the set)

Simplified key-value store: sequential specification 2

State:

set of key-value pairs (k,v), k in N, v In V (set of values)

Operations:

- put(k,v) – set the value with key k to v
- get(k) - return the value in the set with key k (predefined “bottom” value returned if no value with key k is put until now)