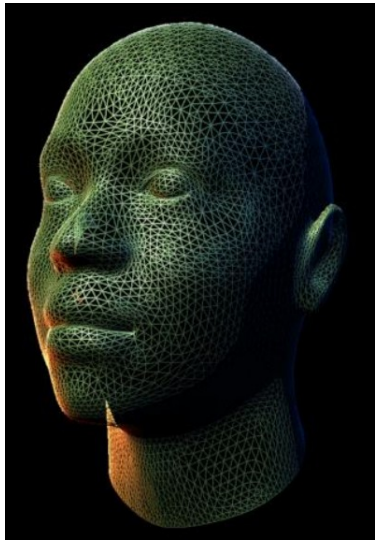
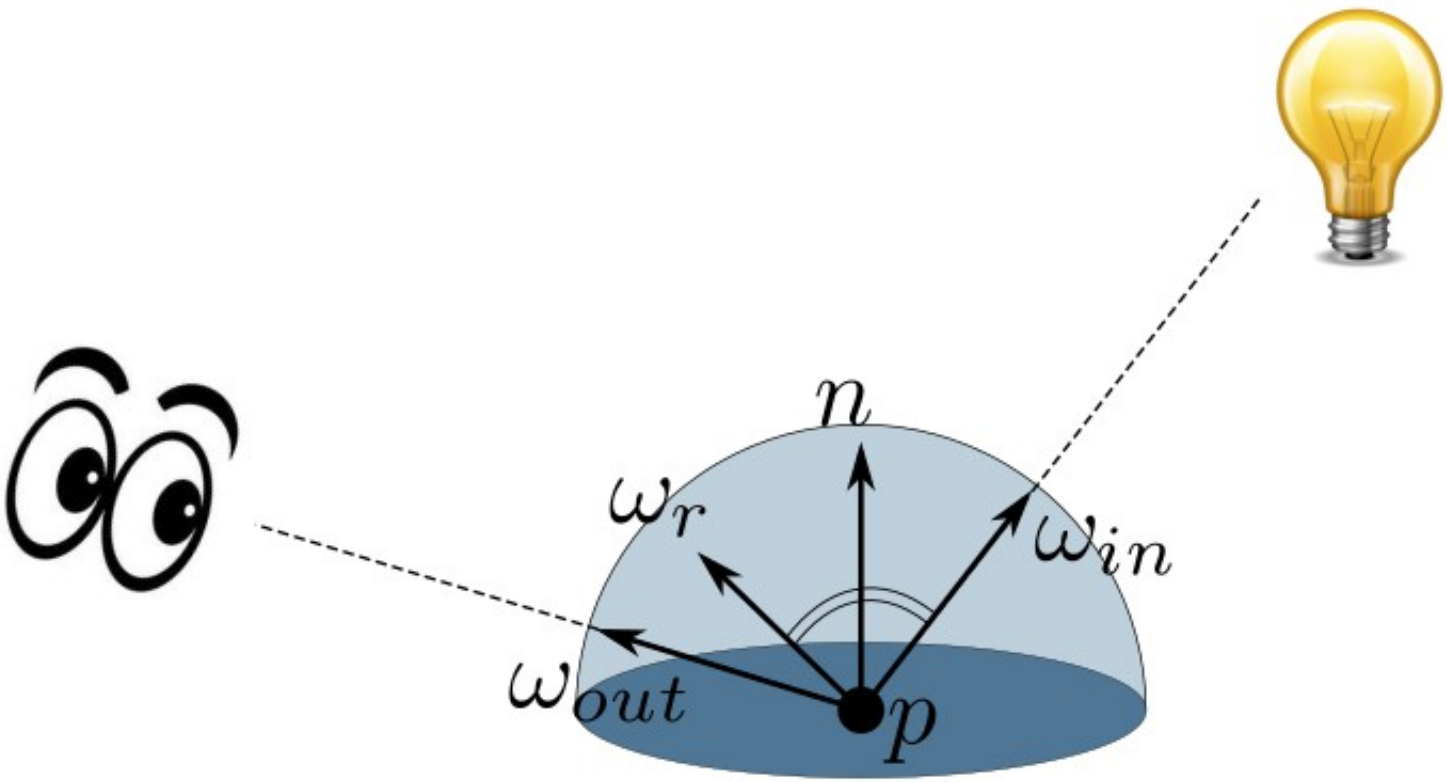
IGR202



Today

- ***Rendering***
  - Real-time (What's the best you can do with low-computation budget?)
  - Materials and Lighting
  - Offline (How fast can you compute as-realistic-as-desired images?)
- ***Geometry & Simulation***
  - Understand meshes & discrete representations
  - Understand « physics »

Ray tracing basic principle



$$L(p,\omega_{out}) = L_e(p,\omega_{out}) + \int_{\omega_{in}\in\mathcal{H}(p,n)} L(p,\omega_{in}) * brdf(p,\omega_{in},\omega_{out}).\ \underbrace{(n\cdot\omega_{in})}_{\text{produit scalaire}}\ .d\omega_{in}$$

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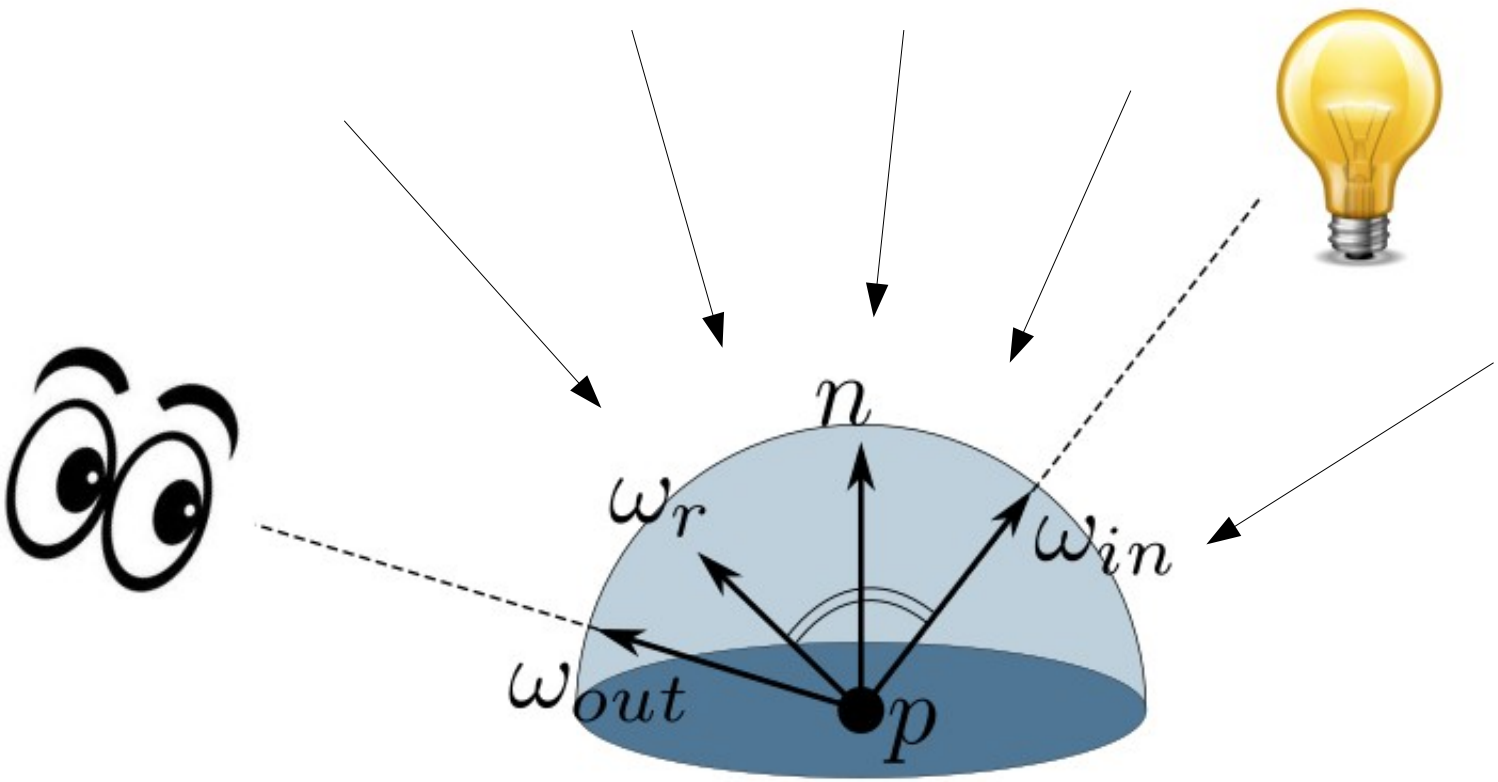

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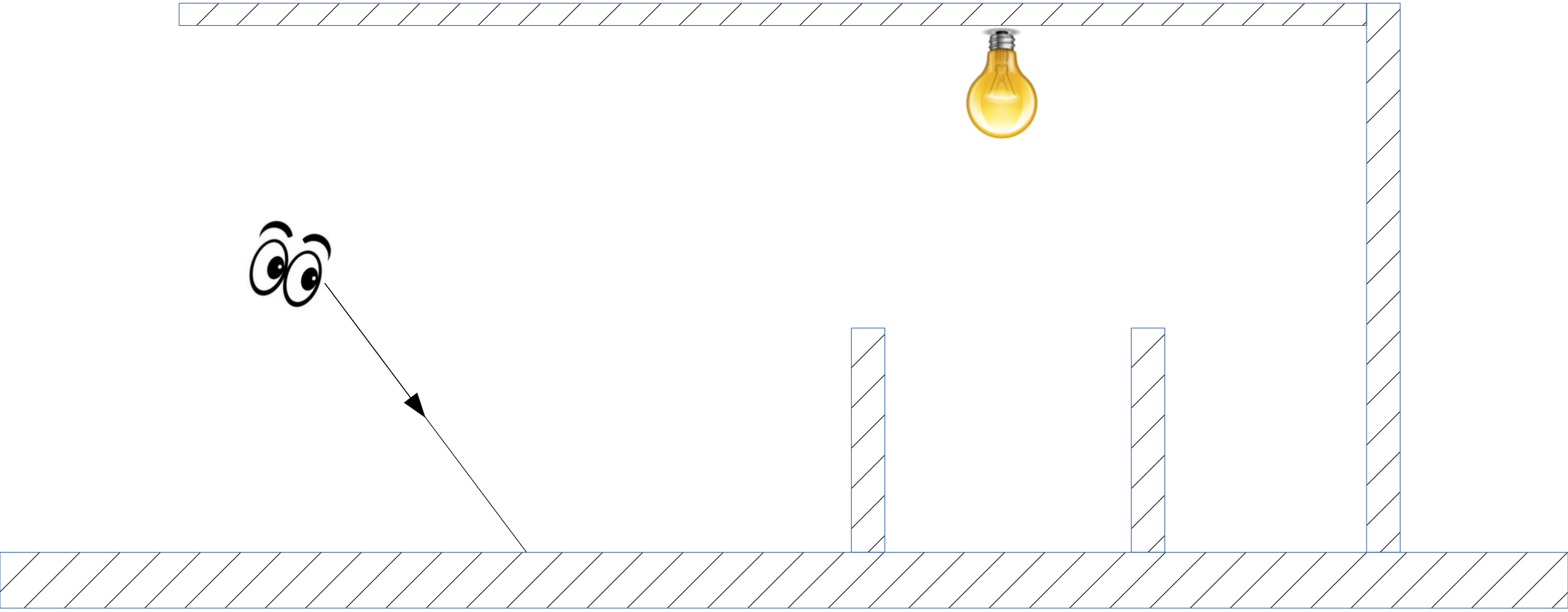

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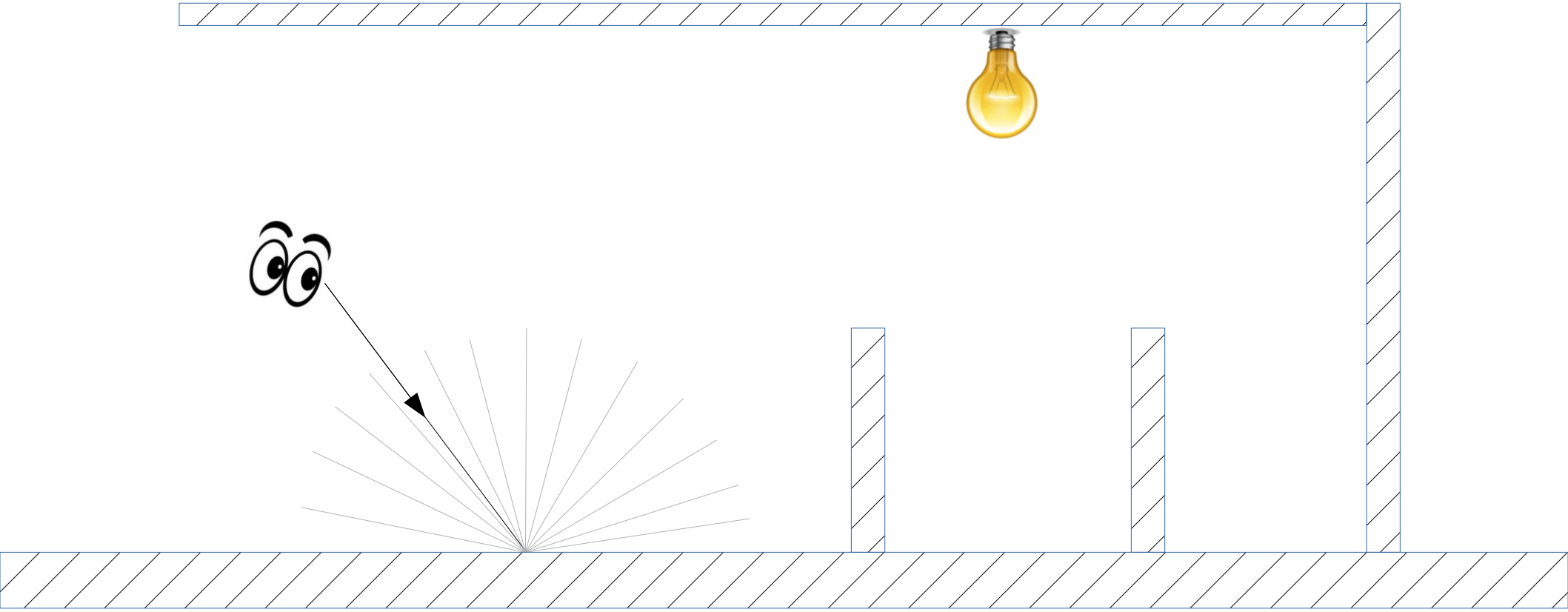

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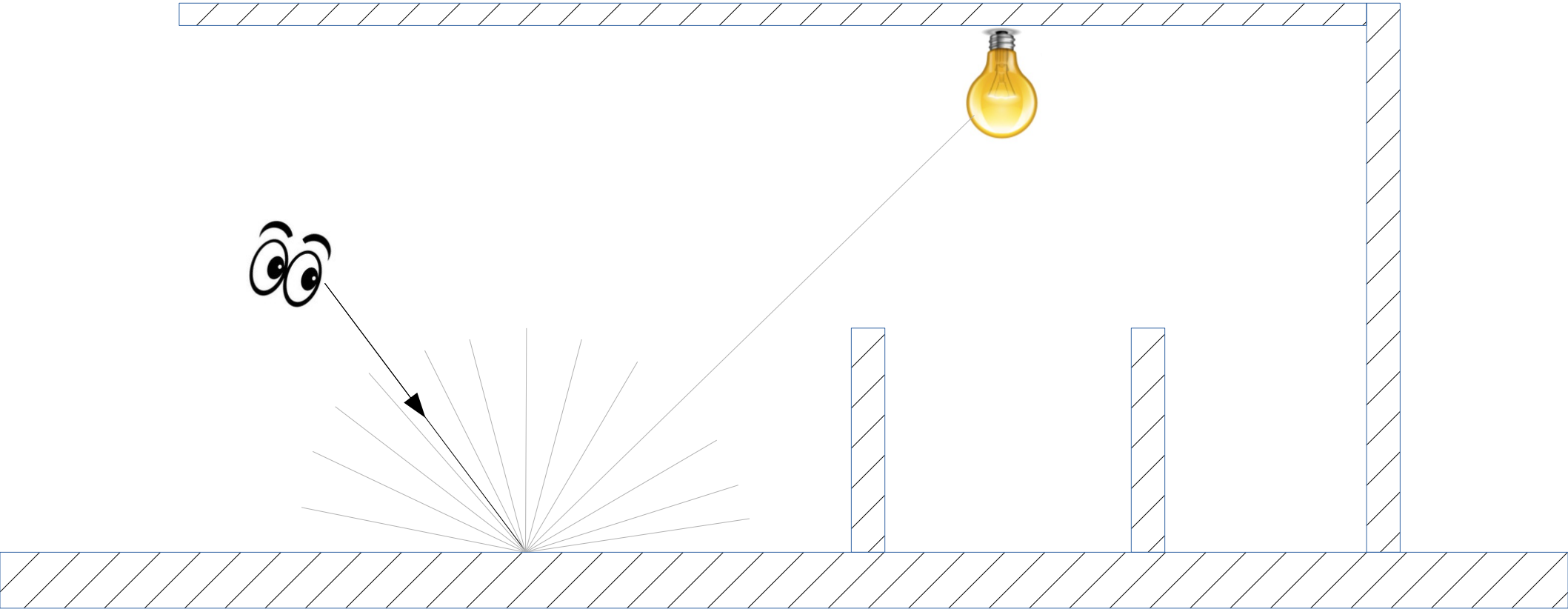

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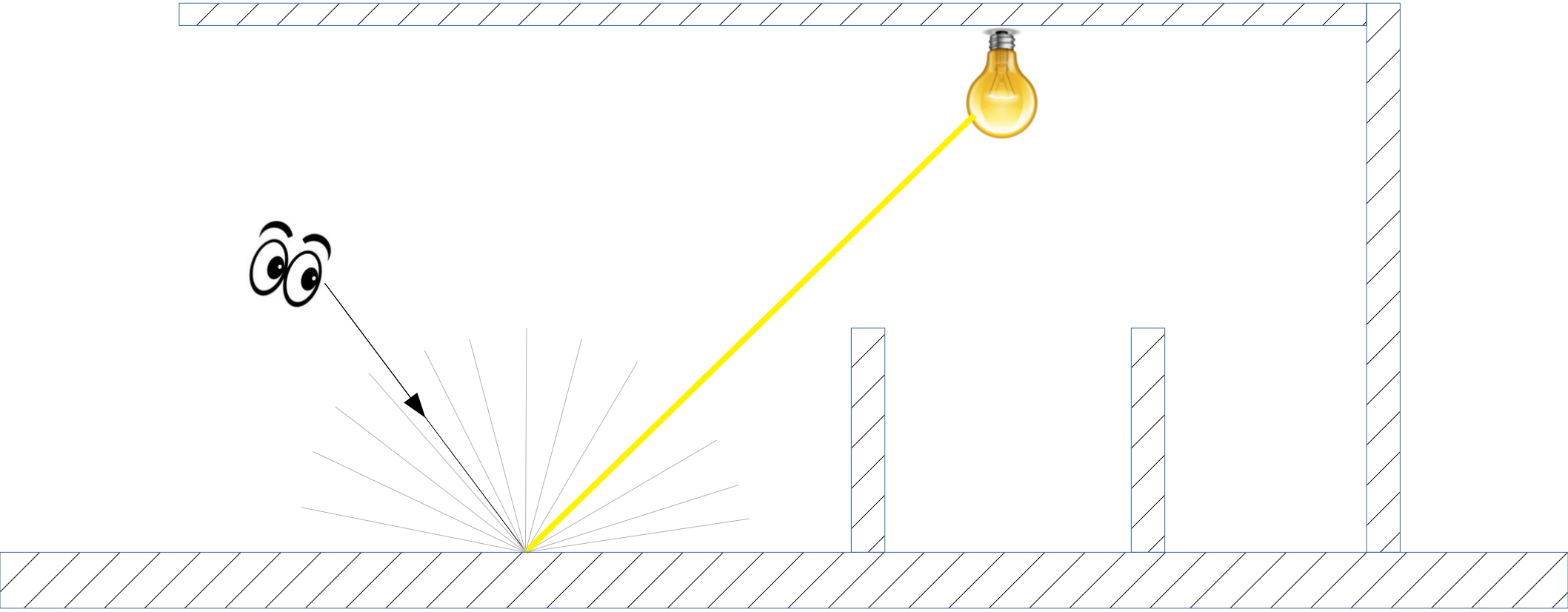

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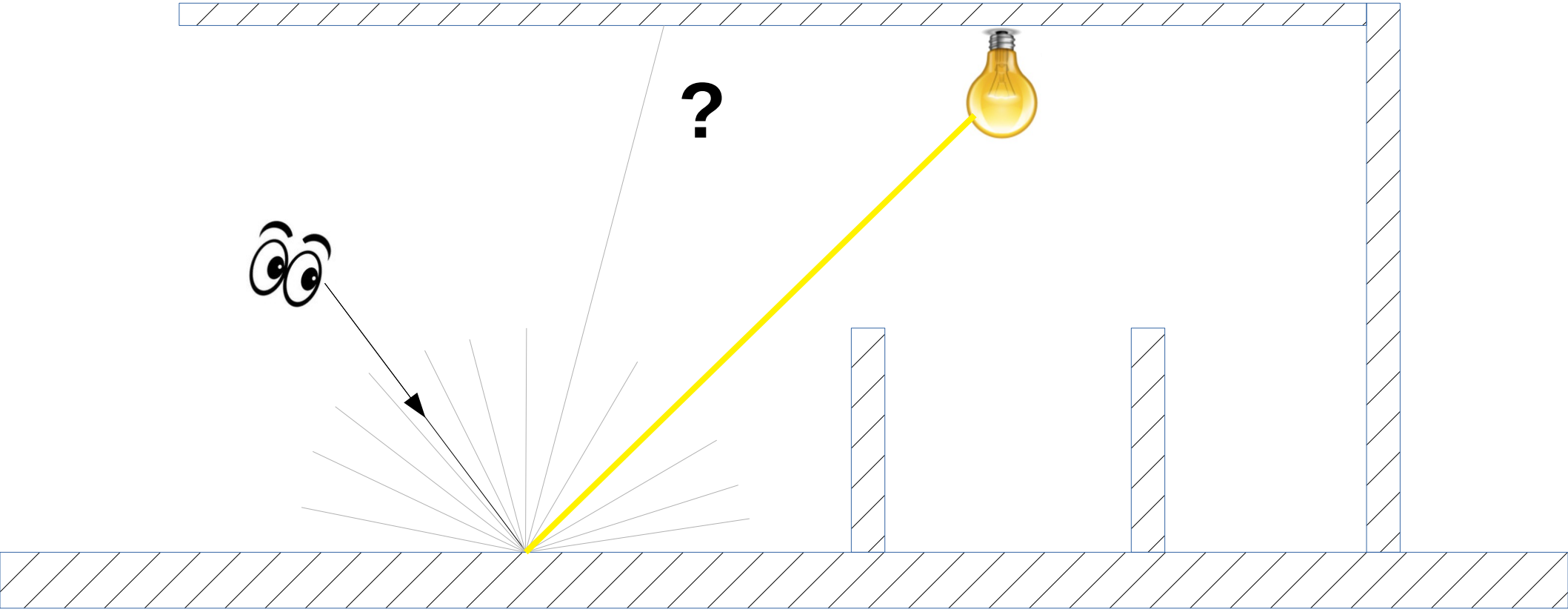

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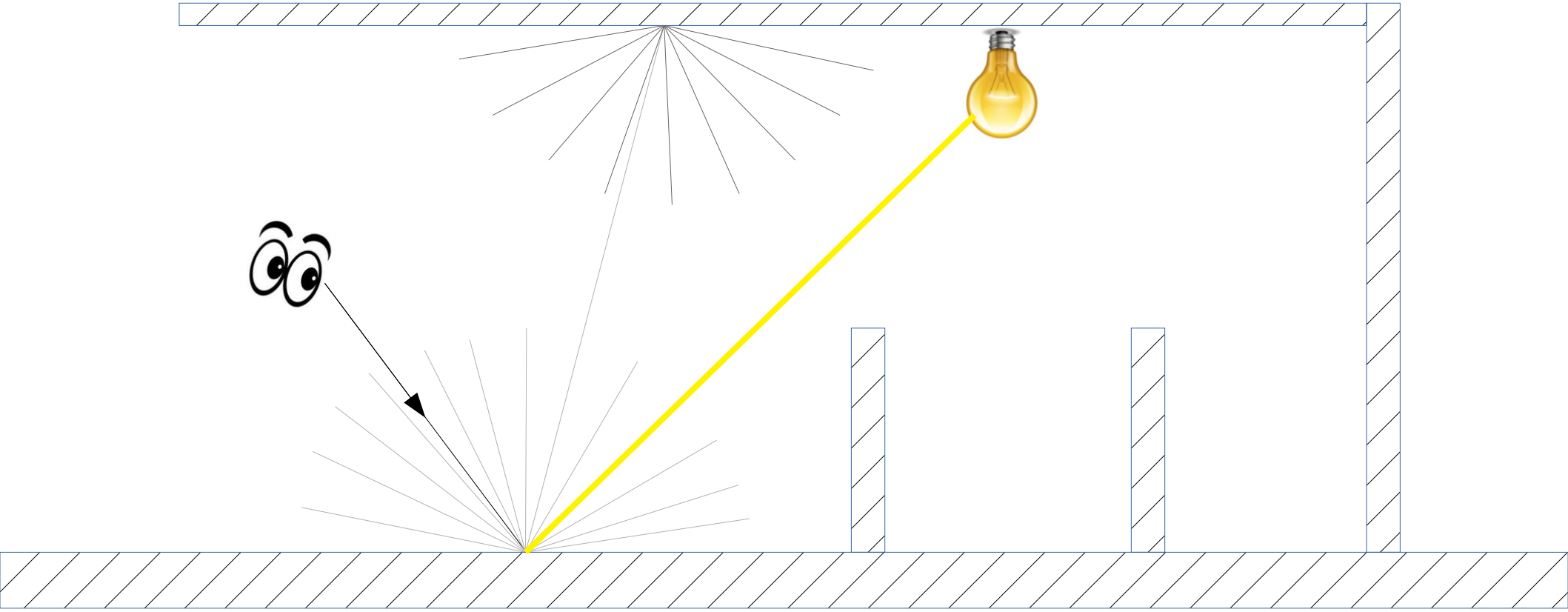

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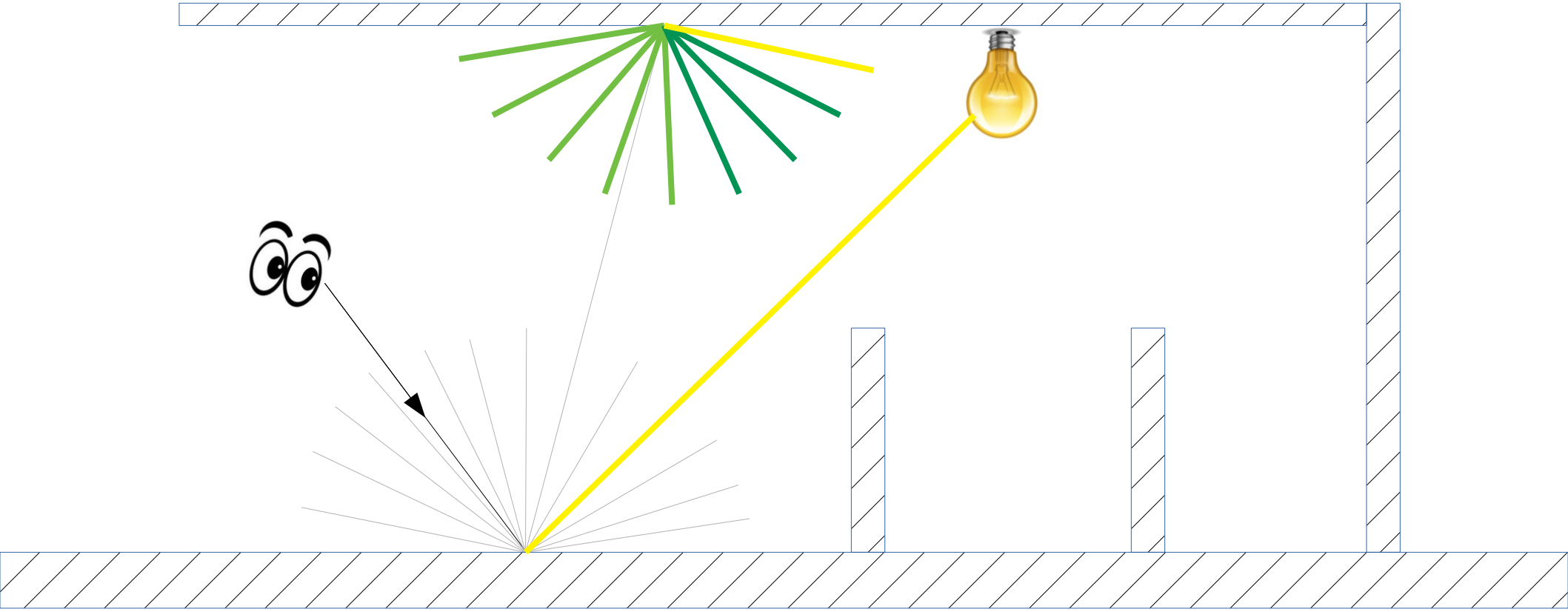

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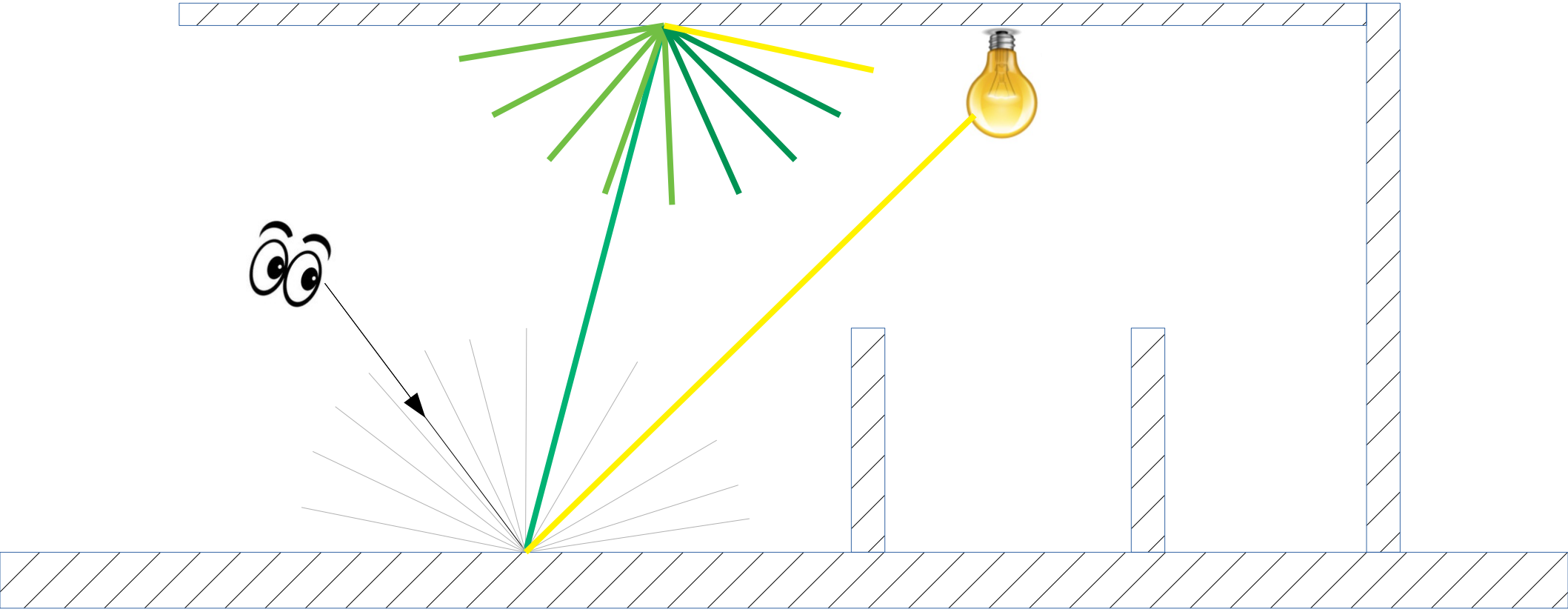

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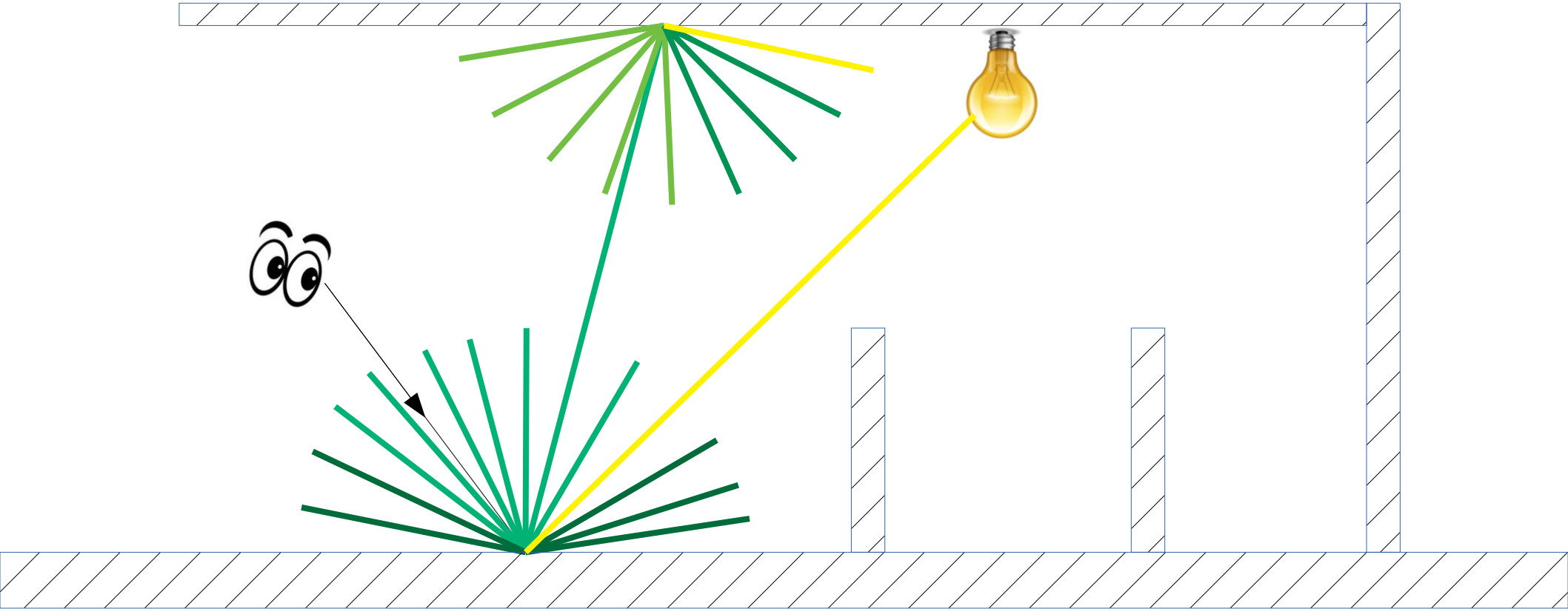

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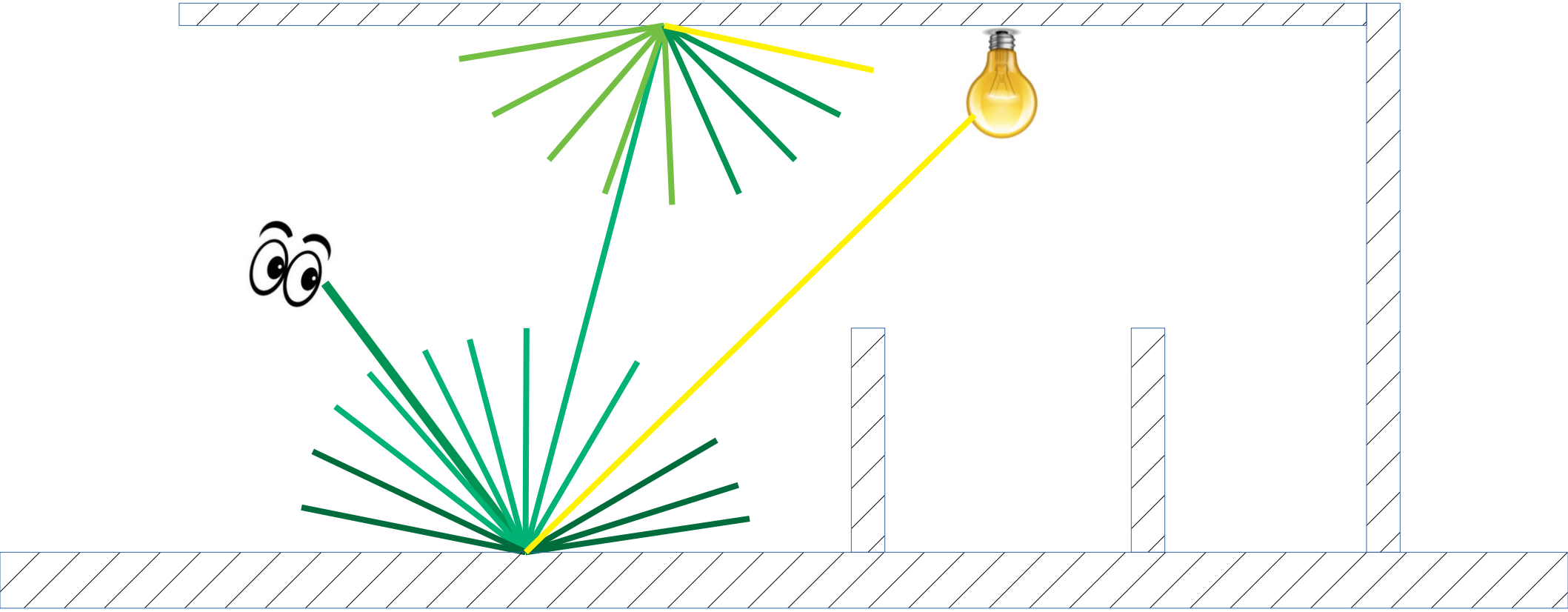

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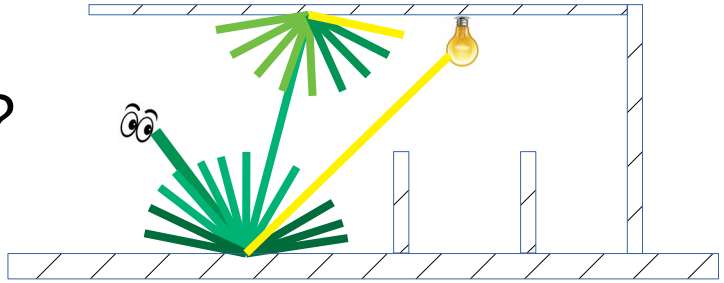
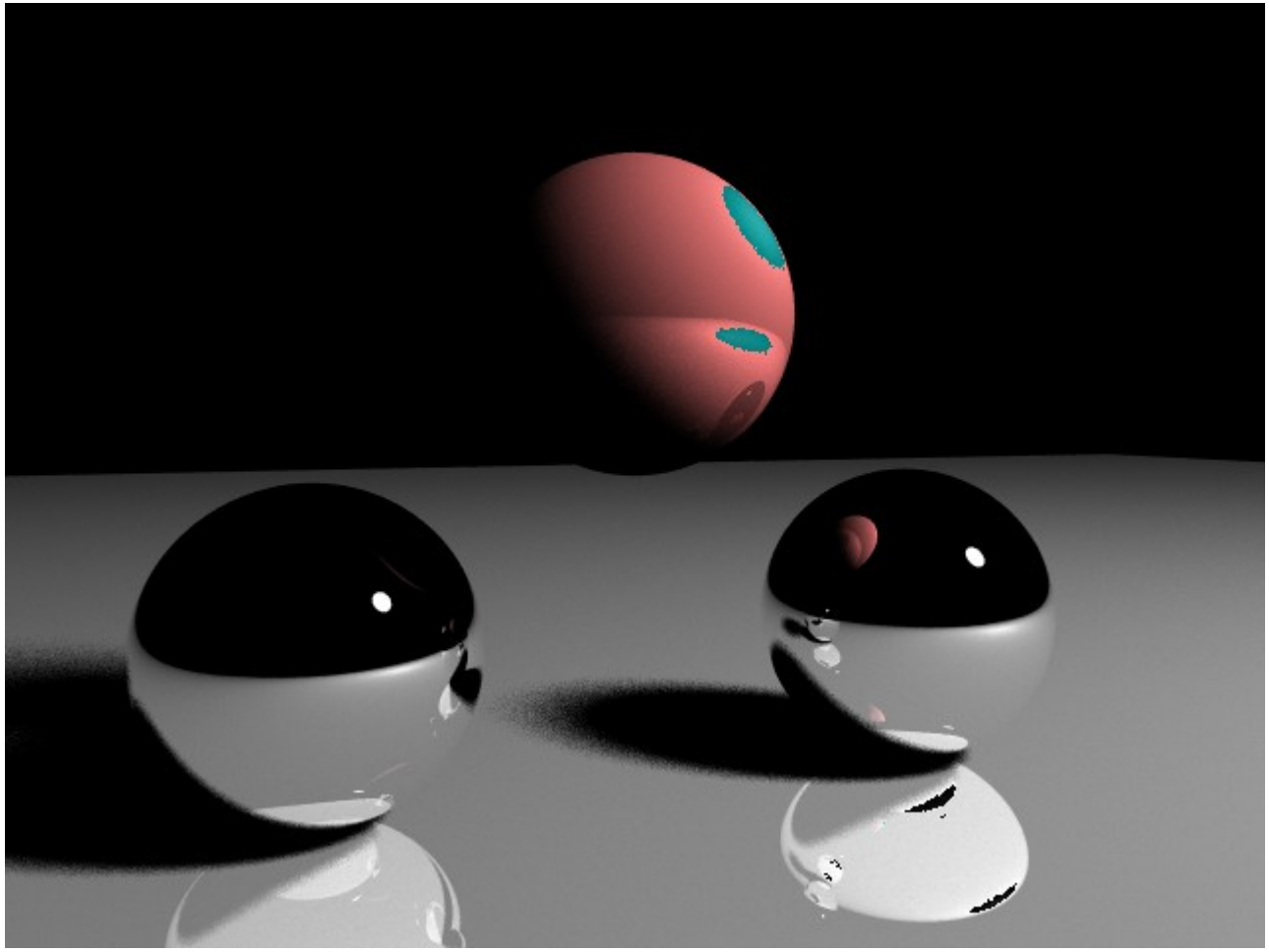
Ray tracing basic principle

- ***Rebounds***
  - How many rays at each intersection ?
  - When do we stop ?



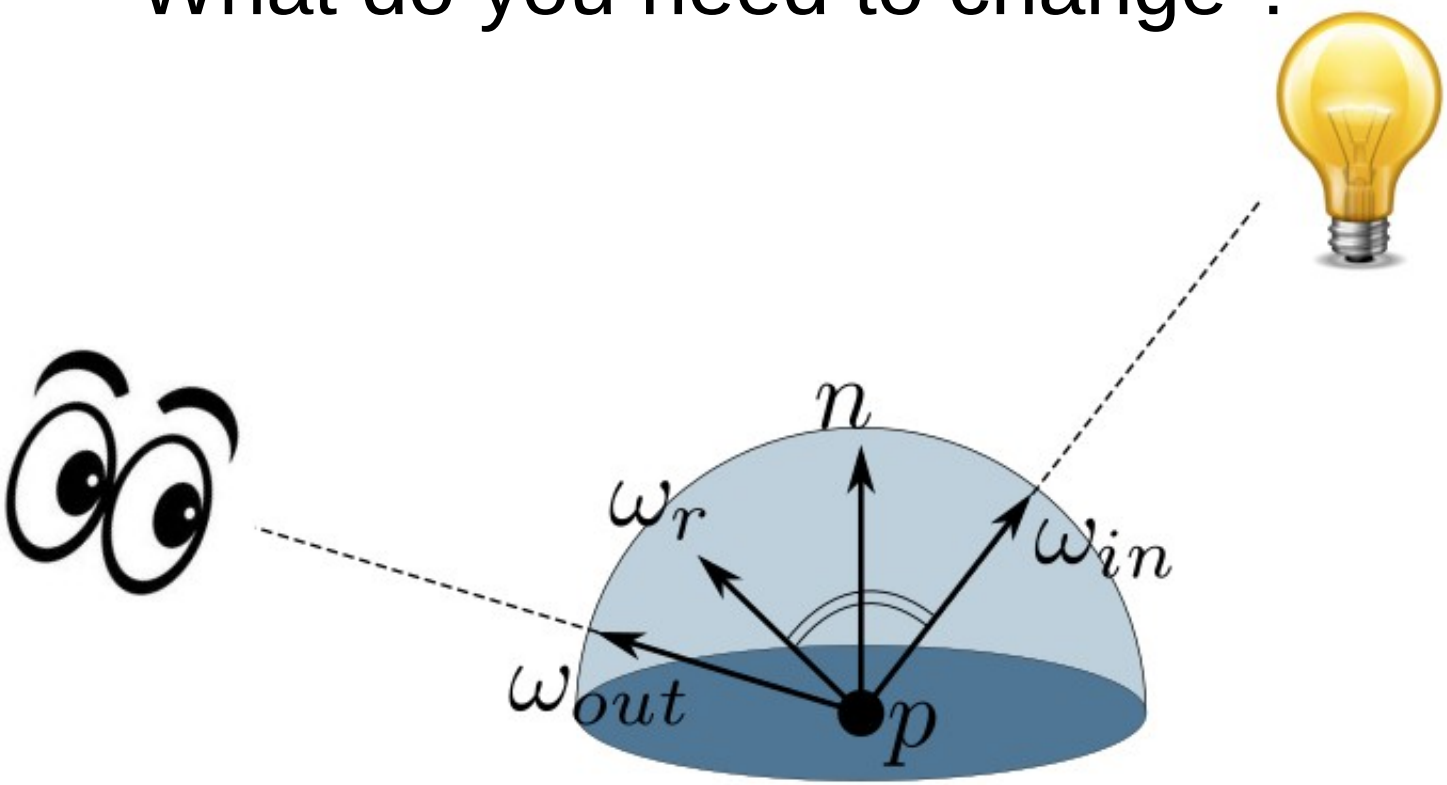
- ***Implementation***
  - Recursive vs path-based
  - Ray/Scene intersection
- ***Simple effects « for free »***
  - Mirrors/Reflections
  - Glass/Refractions , Caustics
  - ...

Reflections



Reflections

What do you need to change ?



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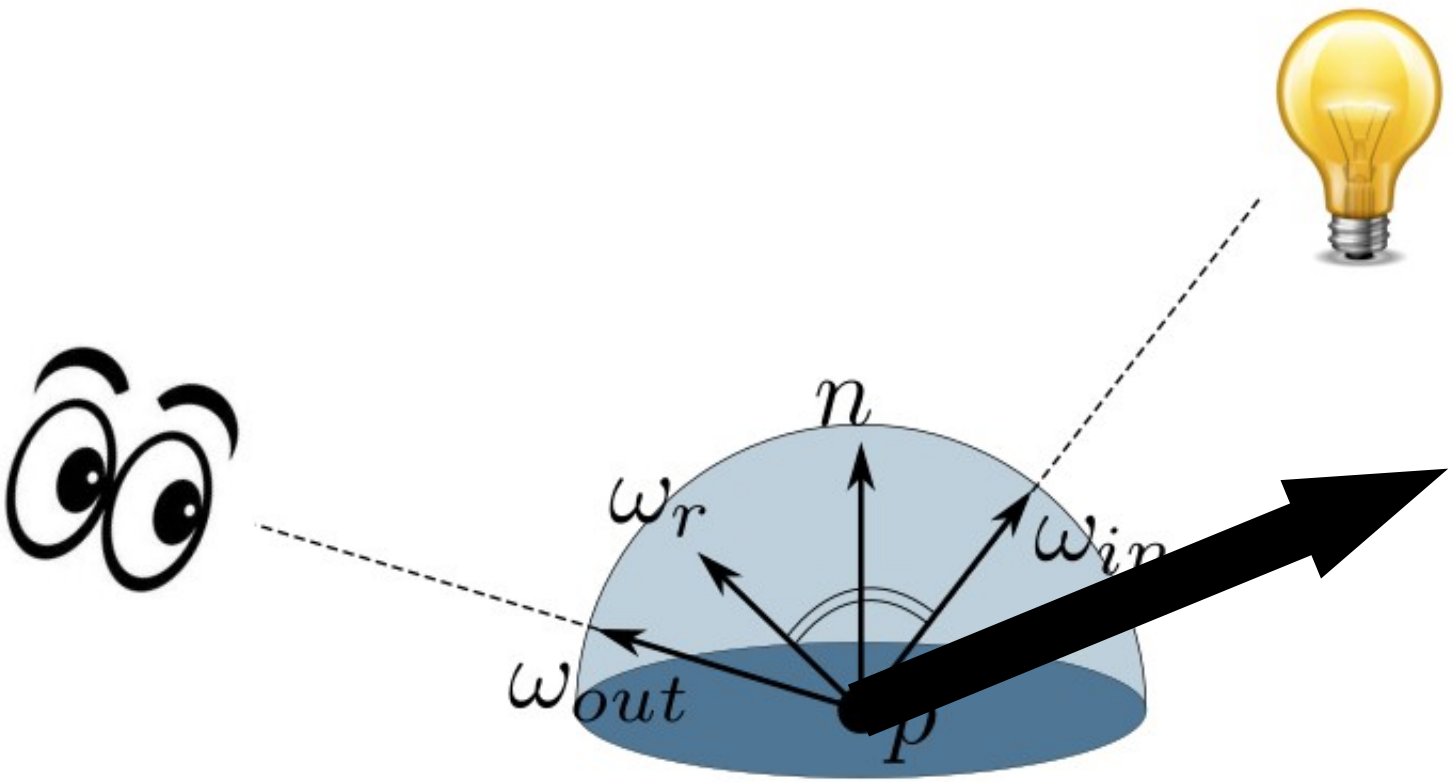

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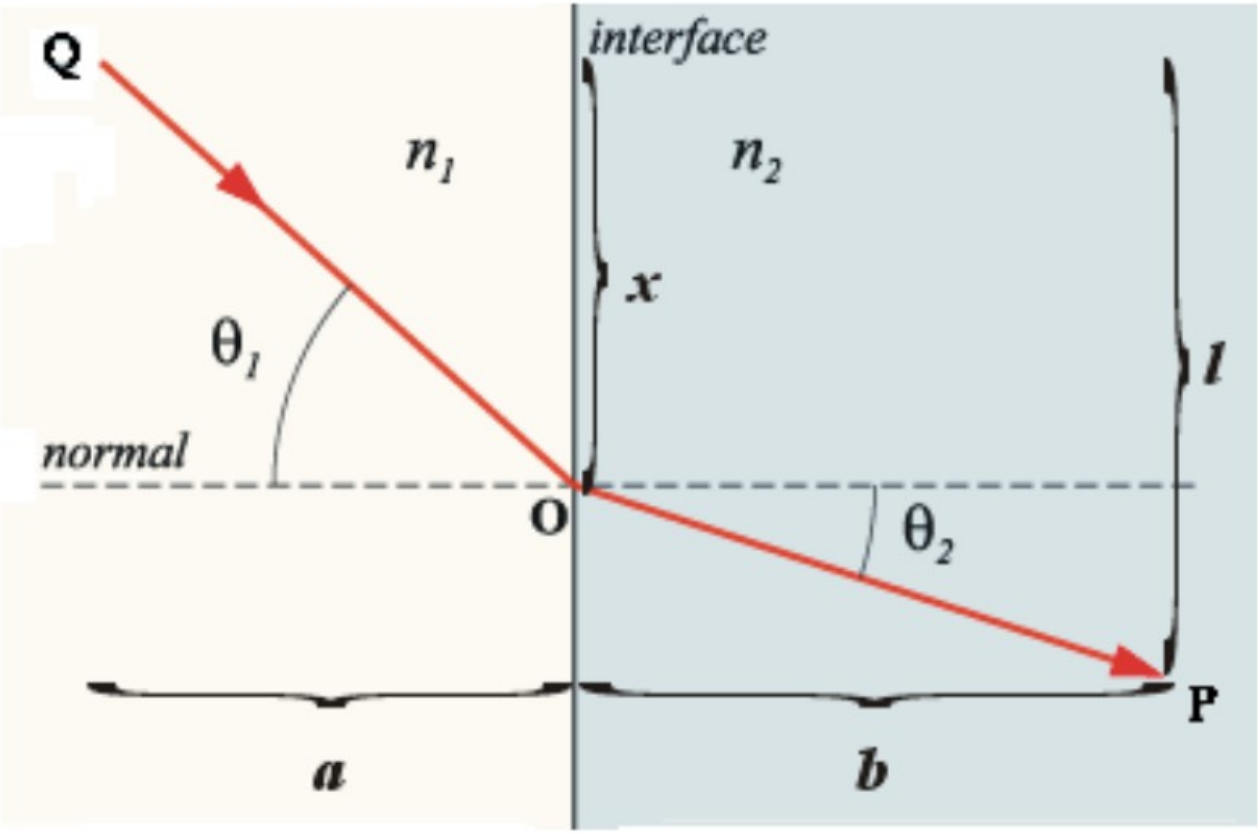
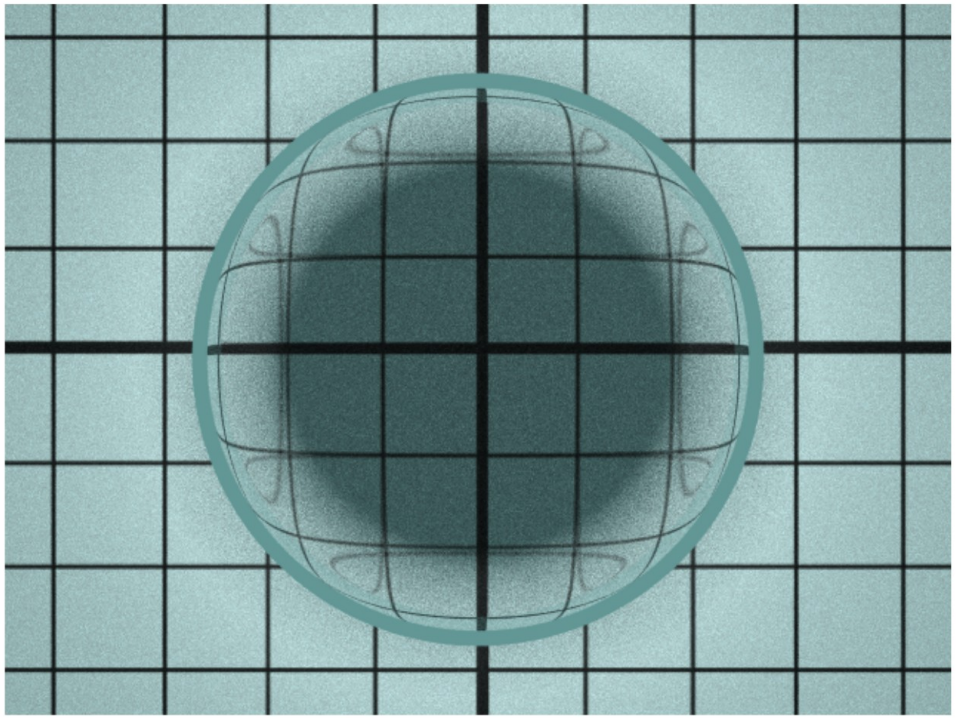
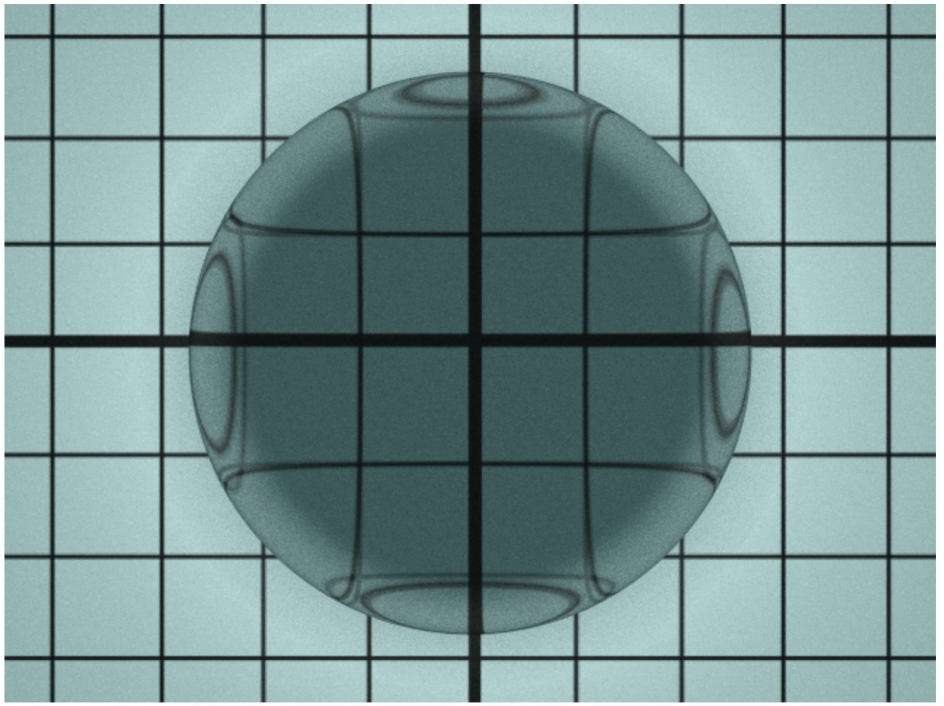


Law of sines

Refractions





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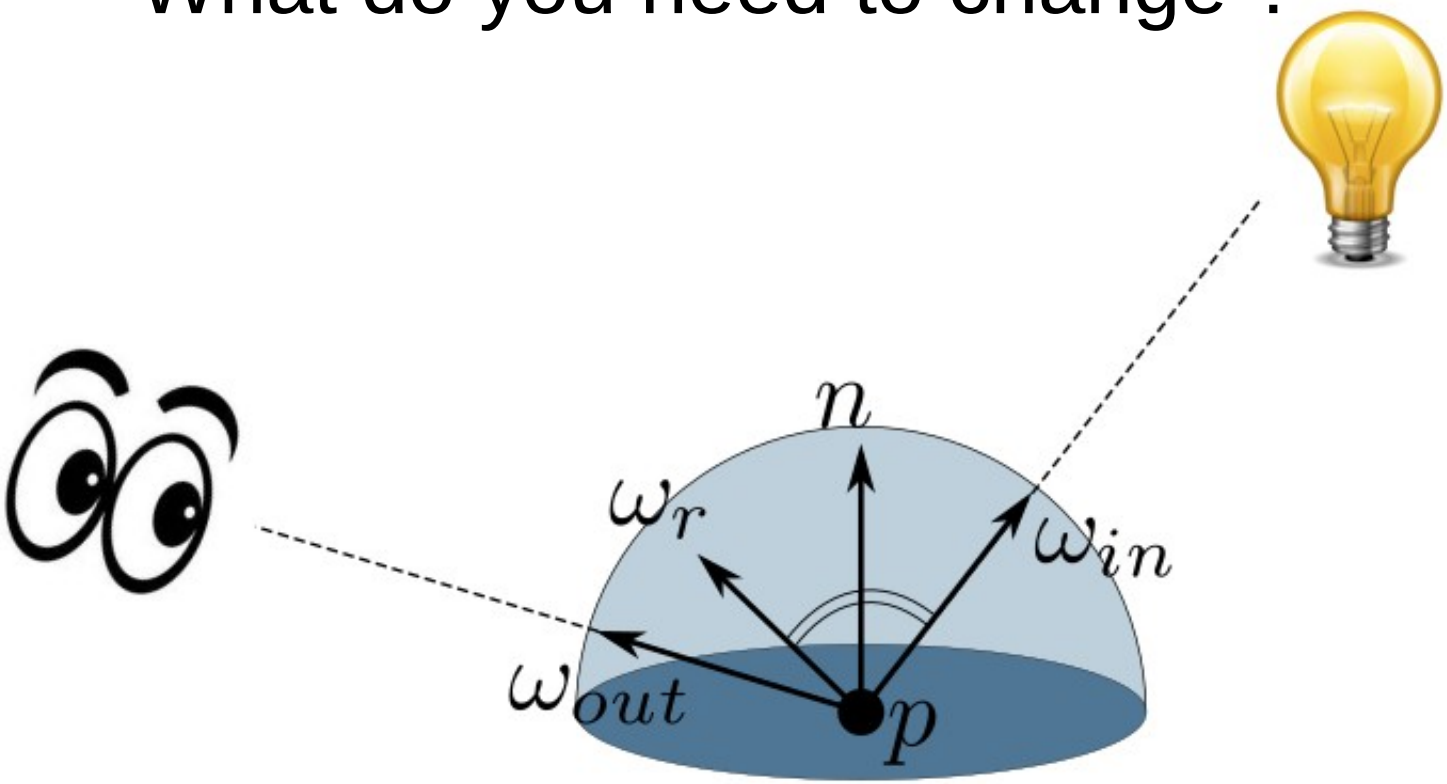

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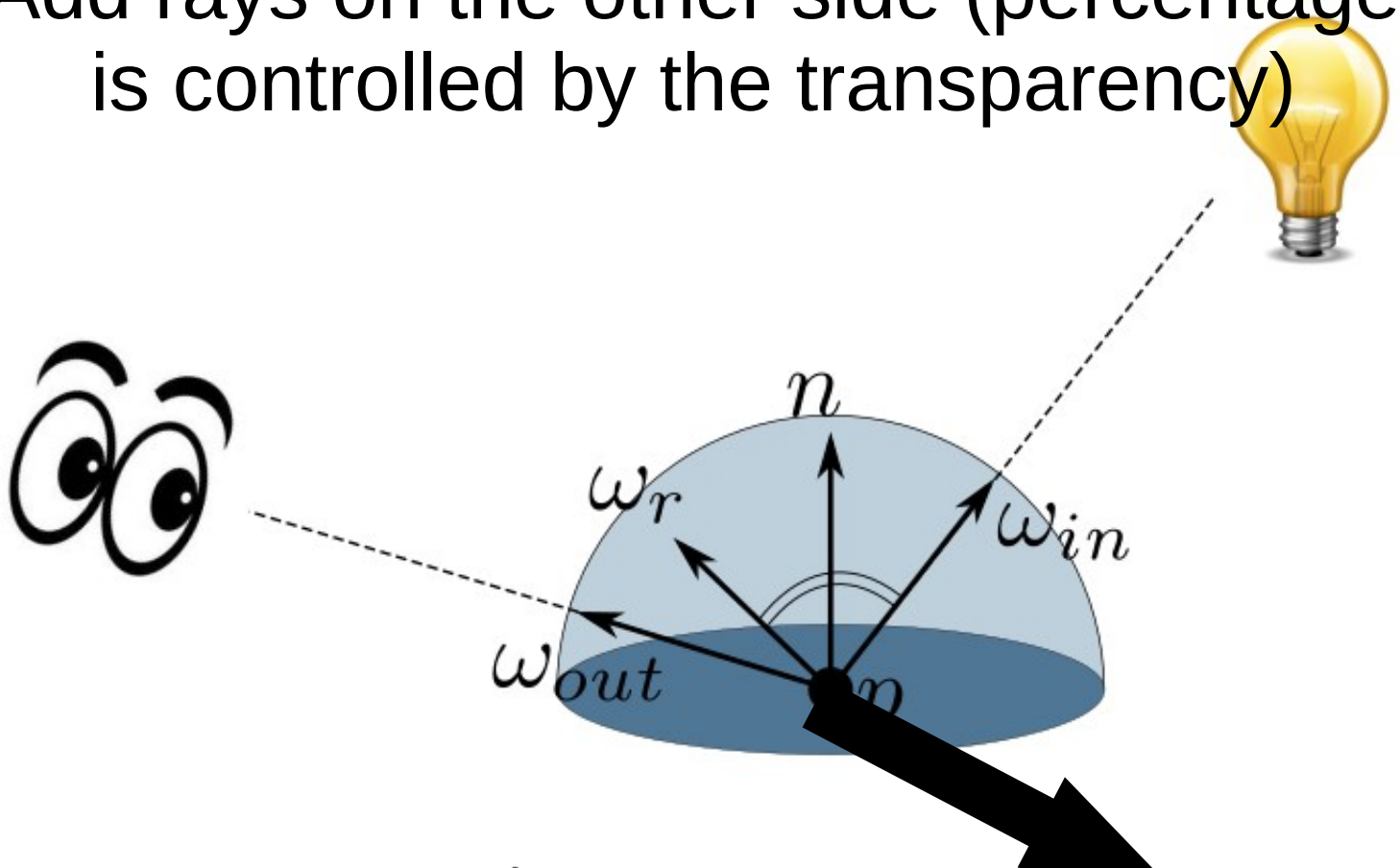
Refractions

Add rays on the other side (percentage is controlled by the transparency)



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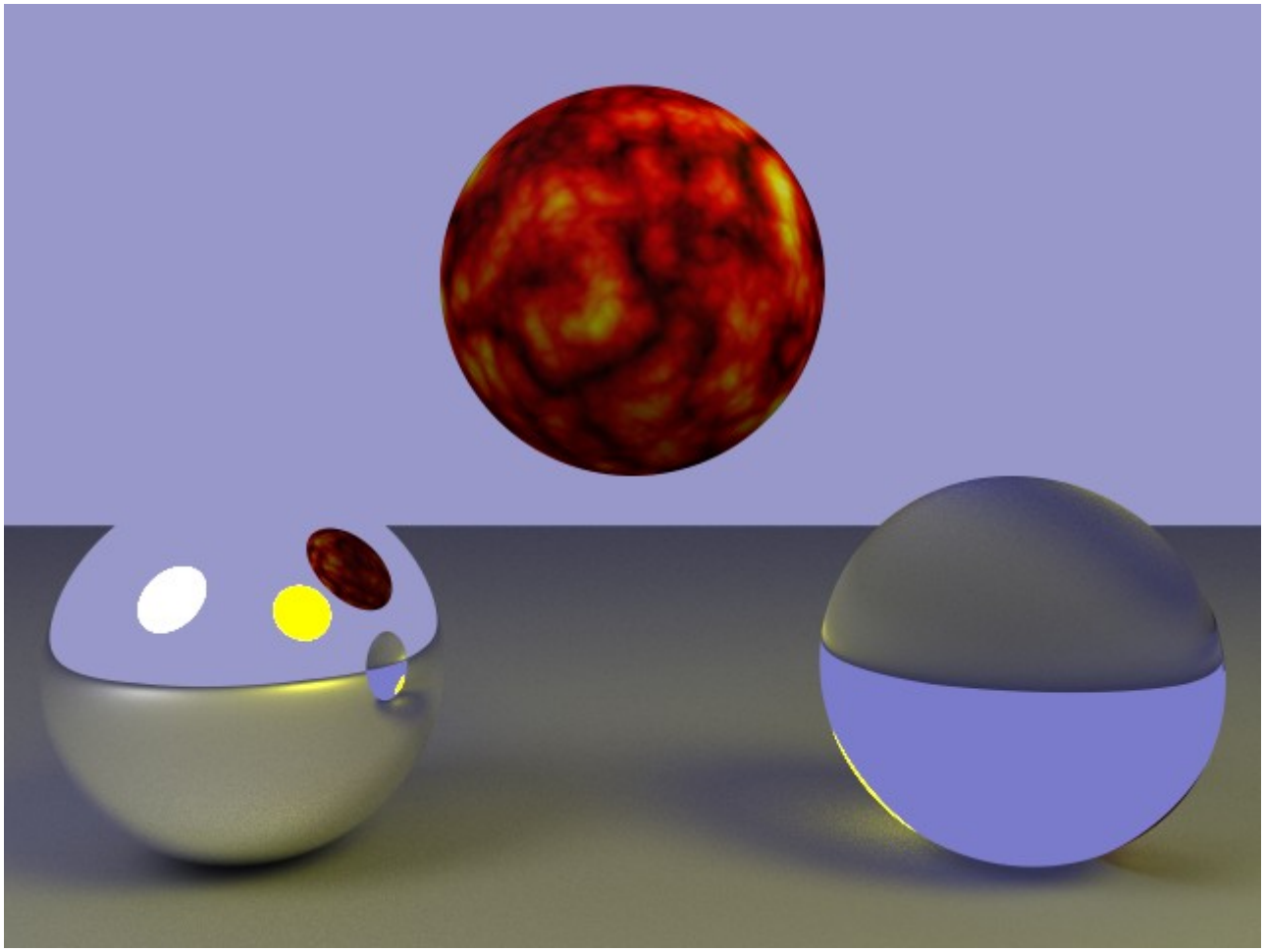
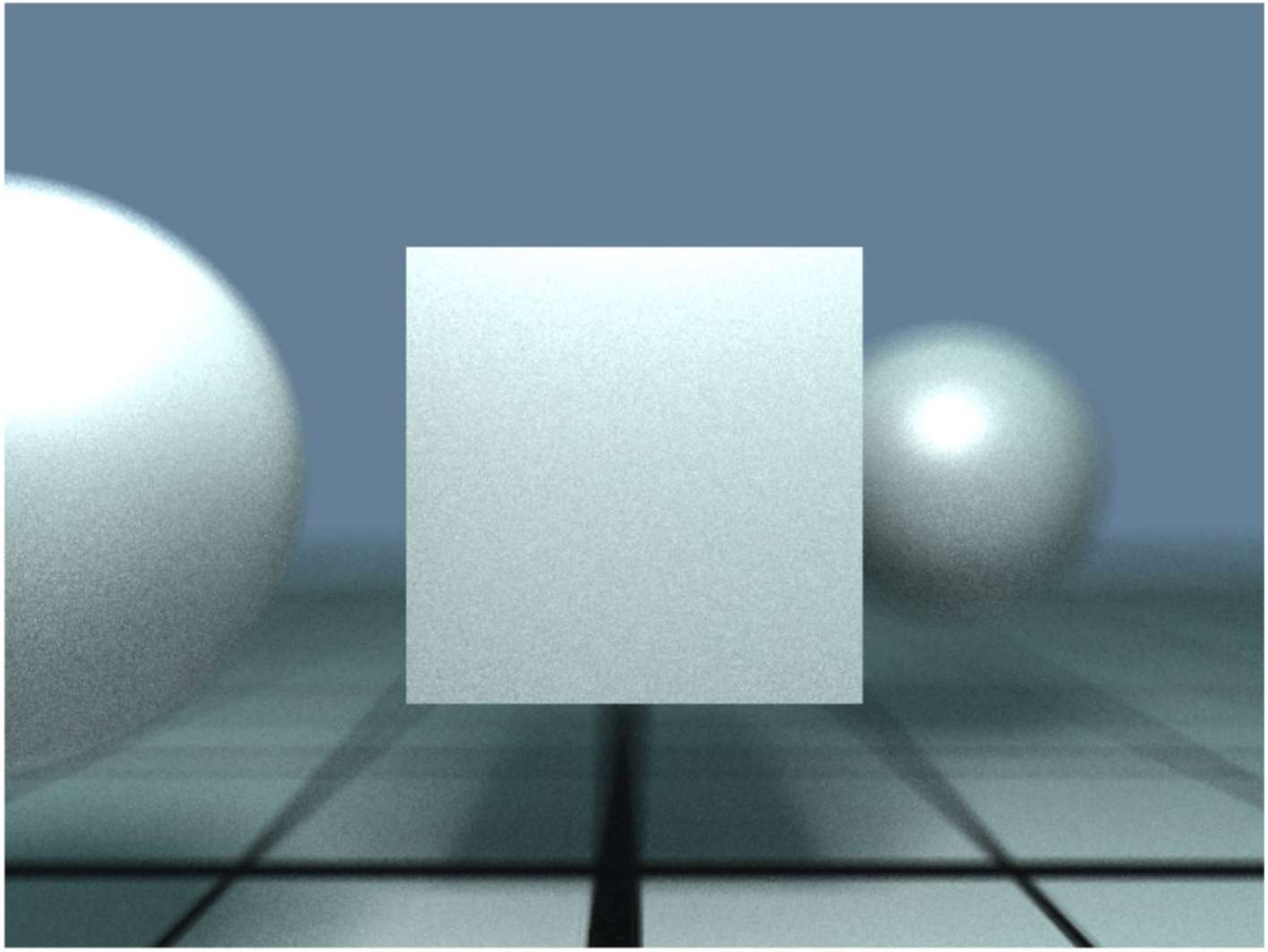
Refractions

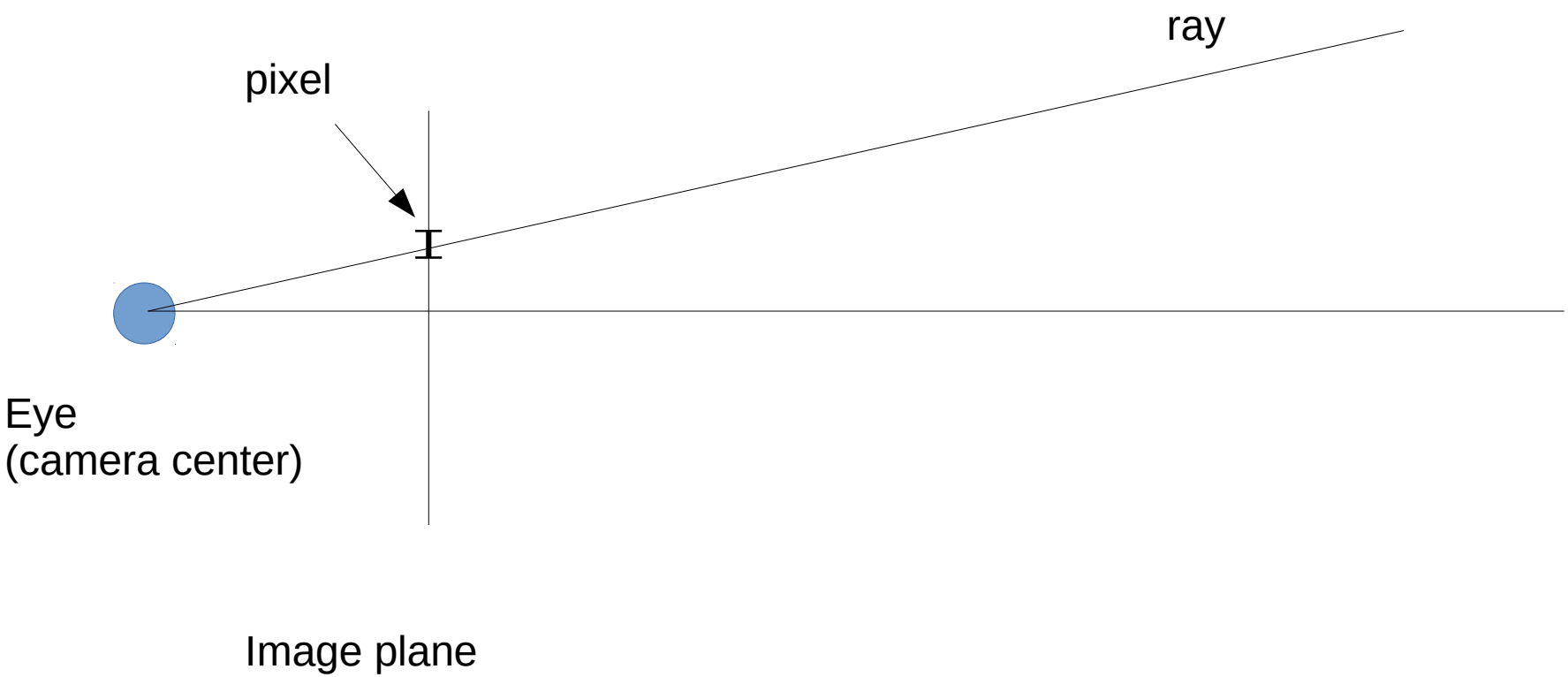


Create caustics

Depth of field



What to change ?



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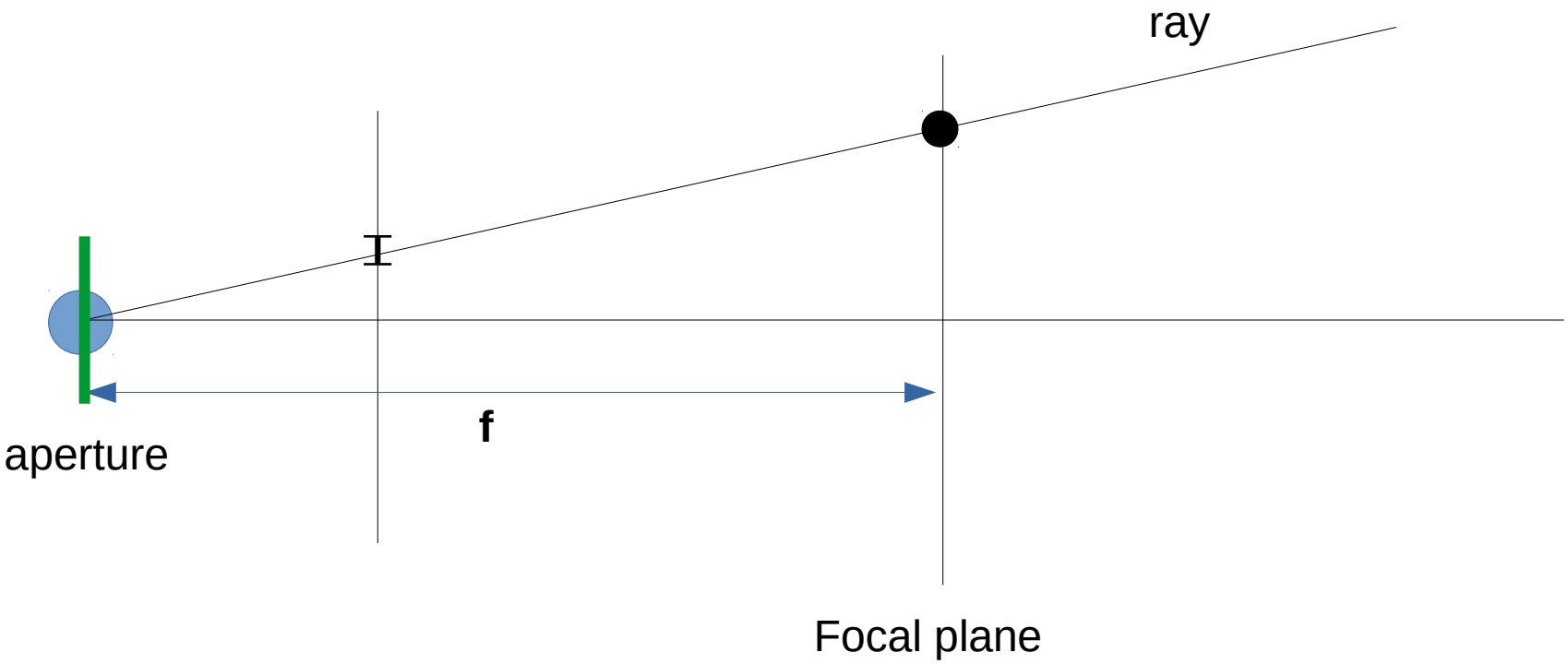
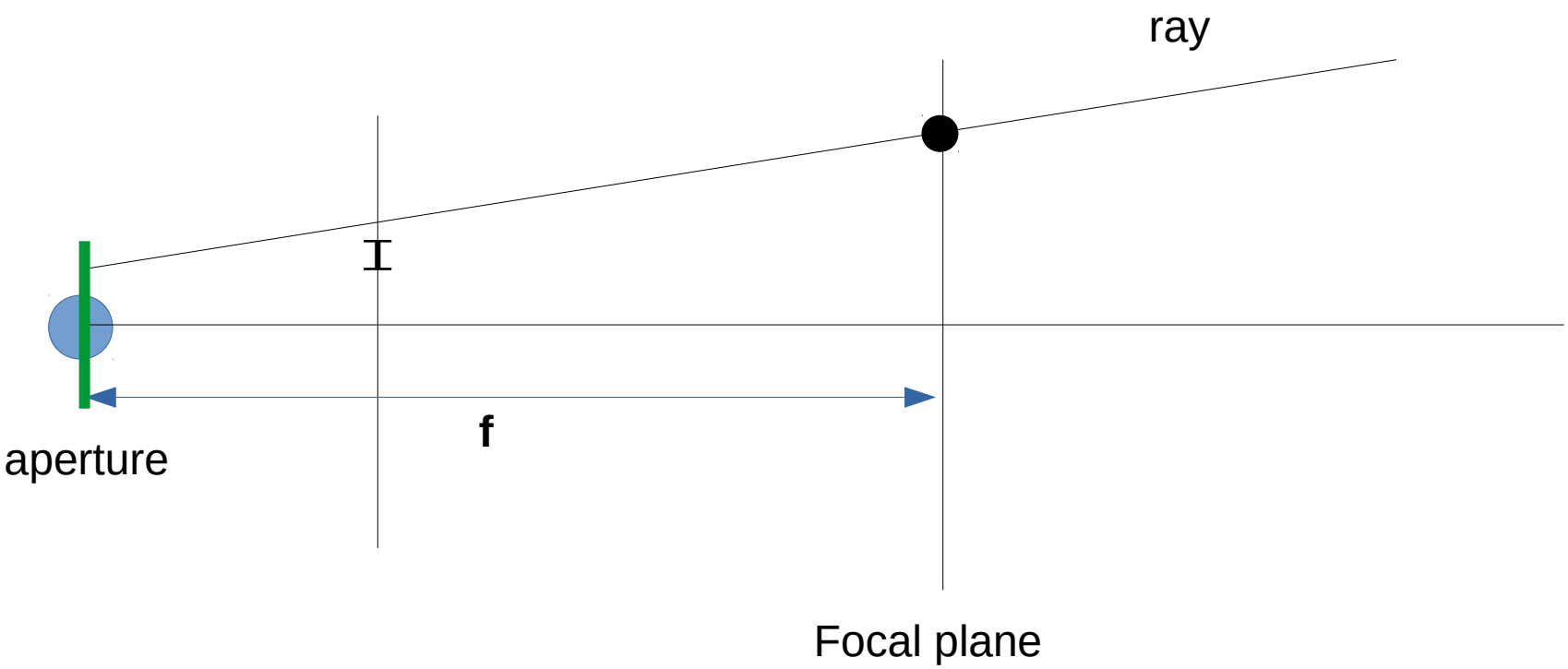


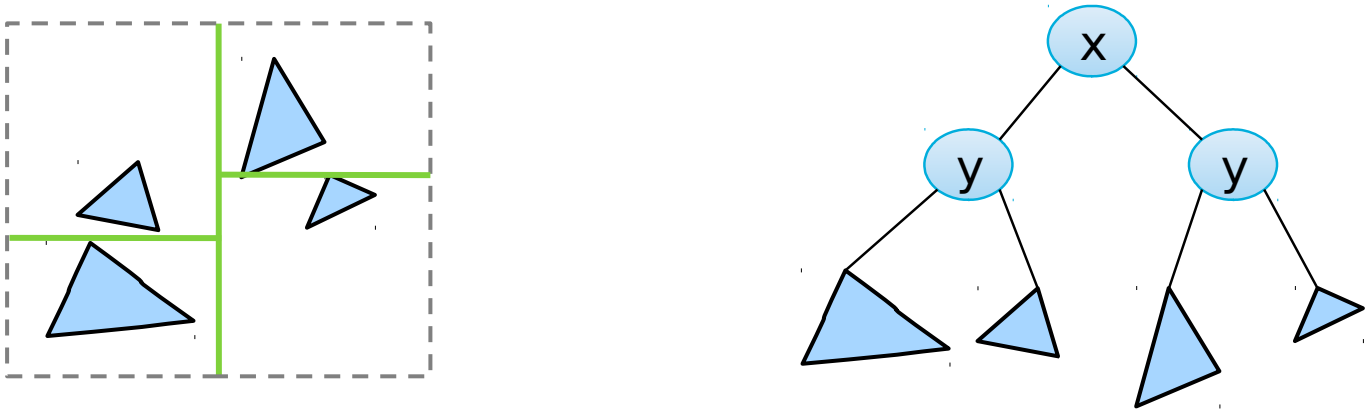
New ray from random point in aperture



Careful ! You still add the contribution of the ray to the original pixel !

Spatial hierarchies: kd-trees

- Binary tree of space subdivisions
  - Each is axis-aligned plane



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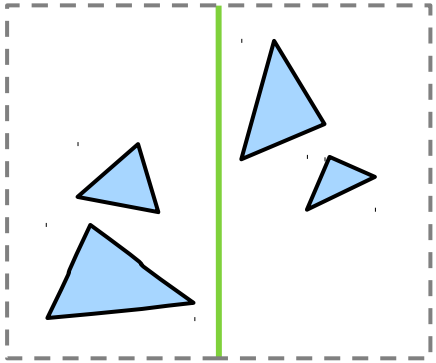

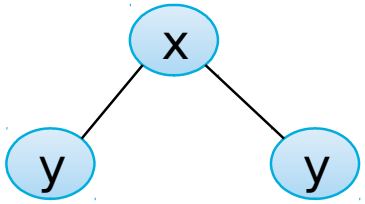

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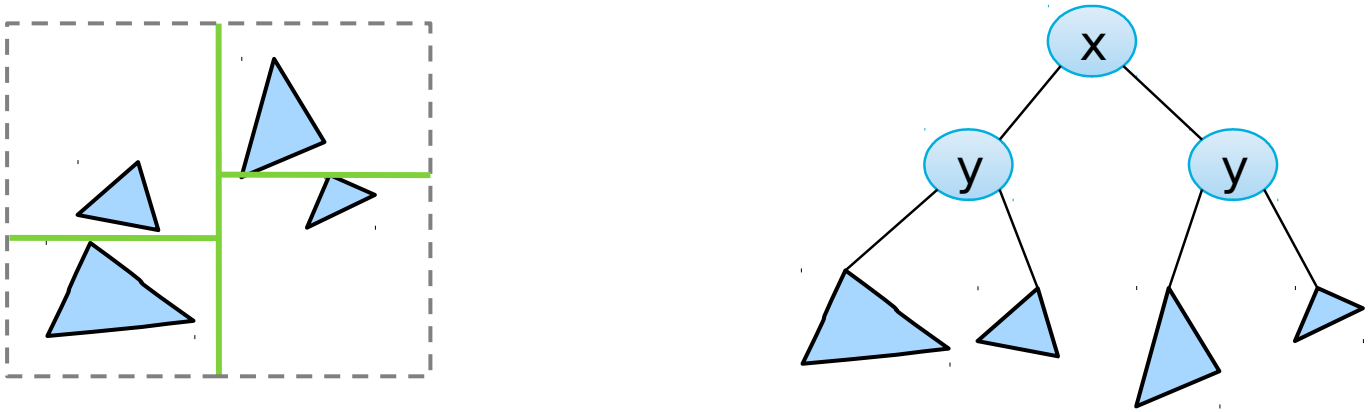

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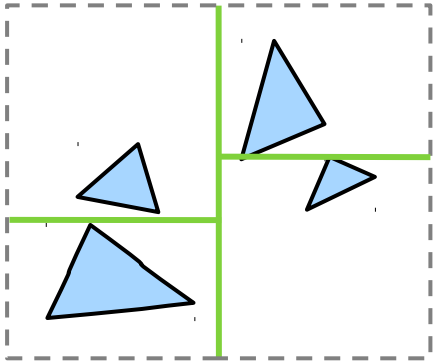
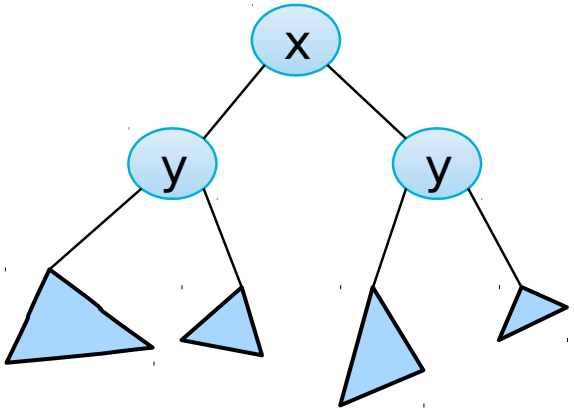
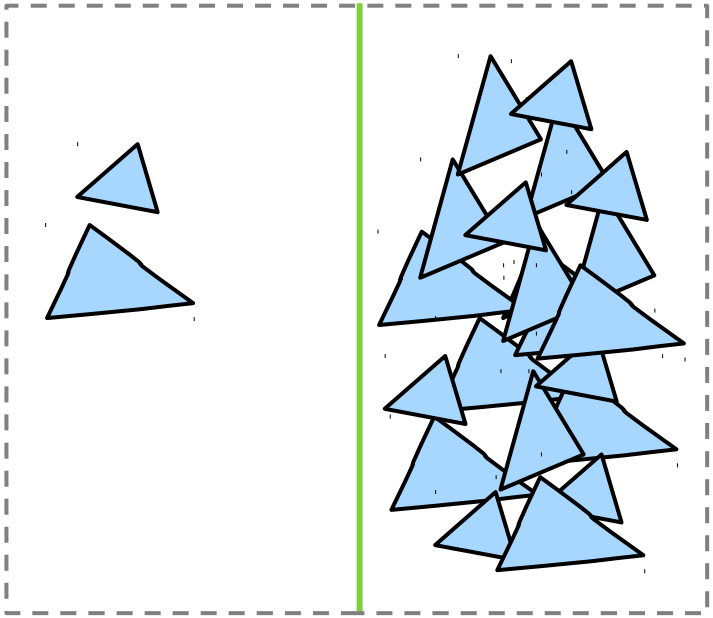


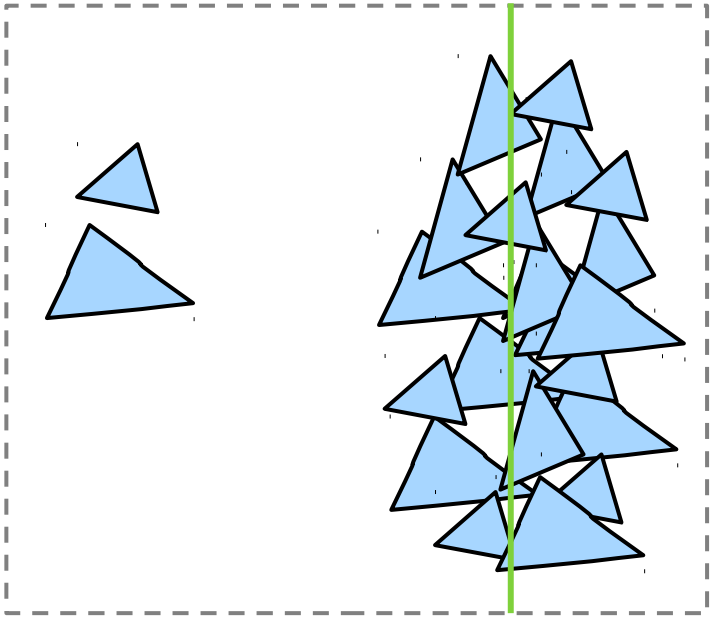


Question :
How to decide where to cut ?

Spatial hierarchies: kd-trees

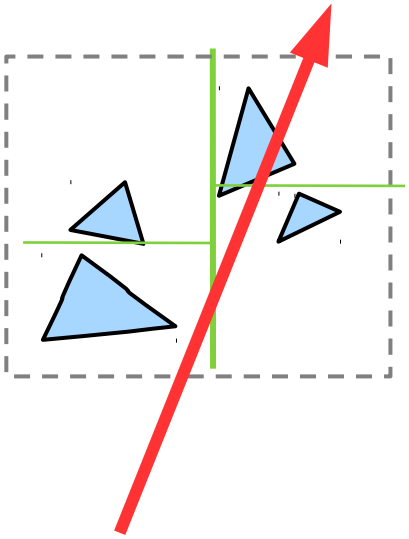
- Prefer median to mean value





Question :
In which node do we put a triangle that intersects the cutting plane ?

Ray – kdtree intersection



- Traversing a kd-tree: recursive
  - Start at root node
  - For current node:
    - If inner node:
      - Find intersection of ray with plane
      - If ray intersects both children, recurse on near side, then far side
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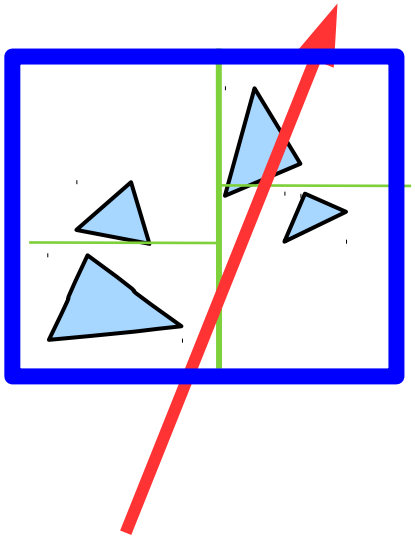
Ray – kdtree intersection



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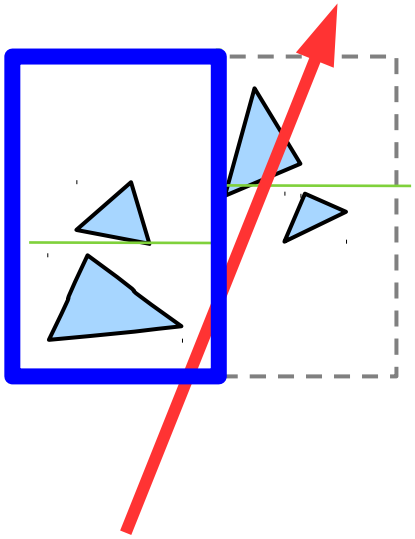

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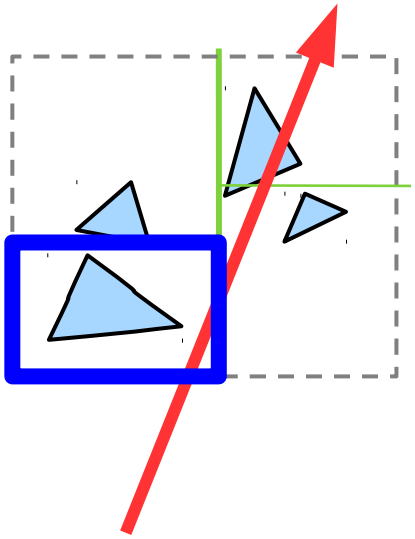

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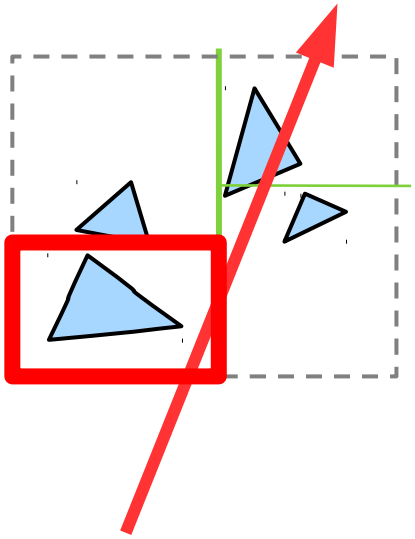

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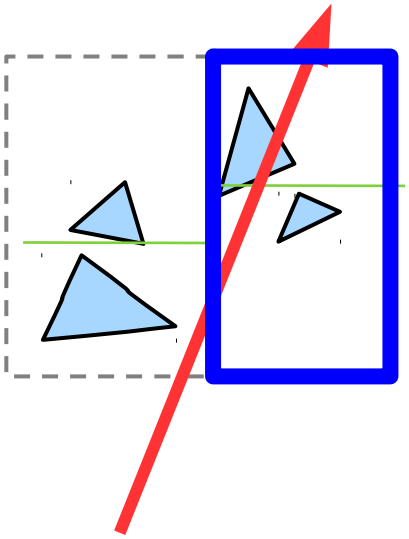

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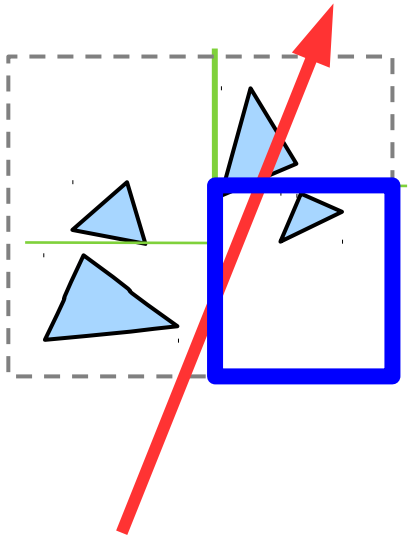

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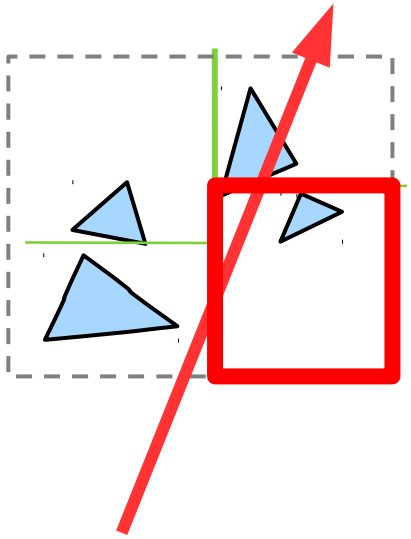
Ray – kdtree intersection



- Traversing a kd-tree: recursive
  - Start at root node
  - For current node:
    - If inner node:
      - Find intersection of ray with plane
      - If ray intersects both children, recurse on near side, then far side
      - Otherwise, recurse on side it intersects
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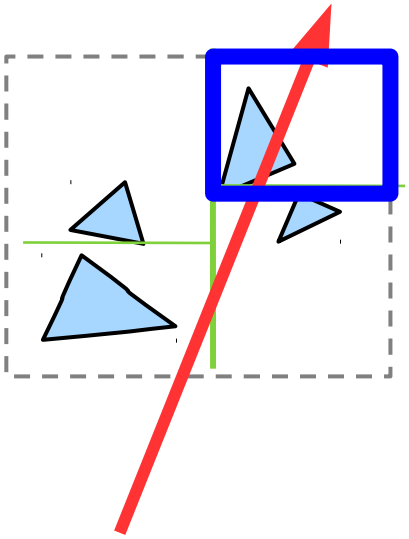

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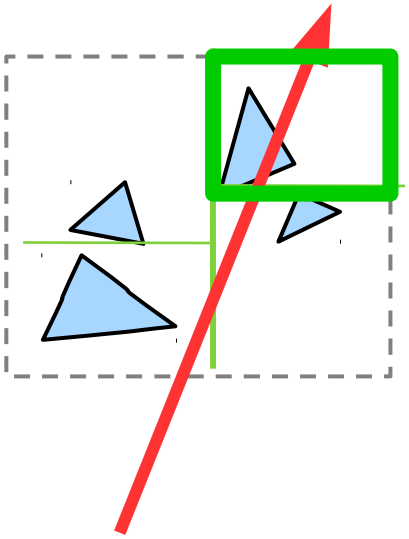

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Kd-tree traversal

- Simple and fast implementation
  - In practice: using stack, not recursion
  - Very quick intersection test (couple FLOPS + tests)
- Overall: logarithmic complexity for each ray

Recursive (pseudo) implementation

```
Vec3  Scene ::rayTrace( Ray r ) {
    if( intersectLight(r) )
        return lightColor ;
    Vec3 color(0,0,0) ;
    p,n = intersectScene( r ) ;
    for(  int i = 0 ; i < N ; ++i  ){
        Ray r2 = randomRay( p , n ) ;
        color += brdf( p , n , -r , r2 ) * rayTrace( r2 ) * dr2 ;
    }
    return color ;
}
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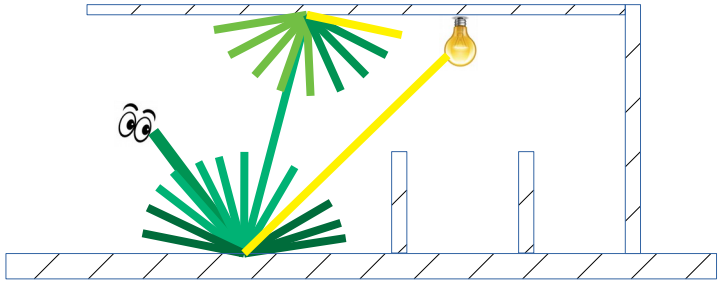

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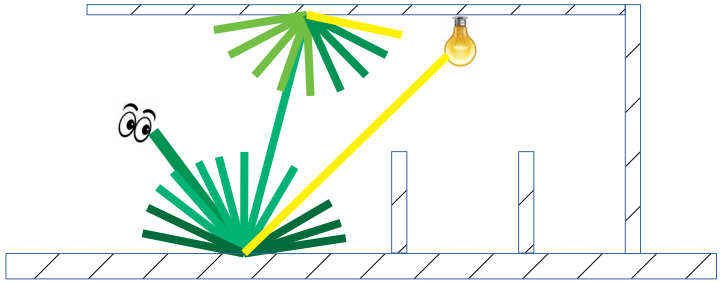

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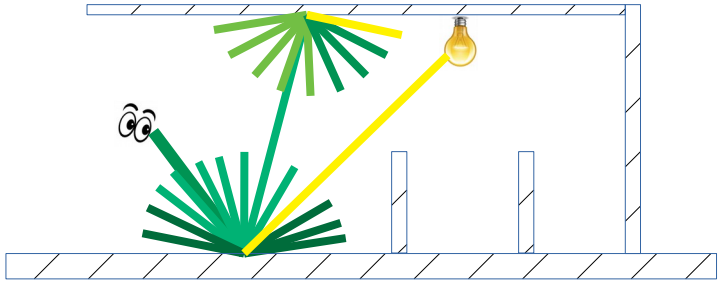

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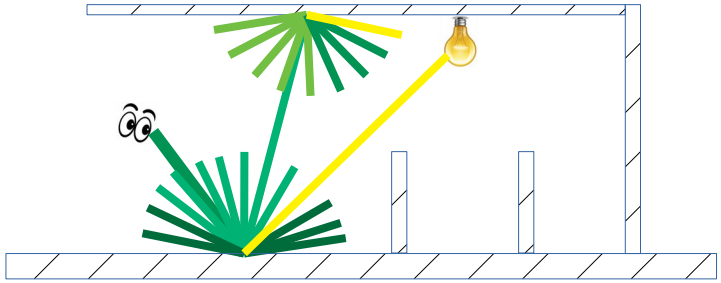

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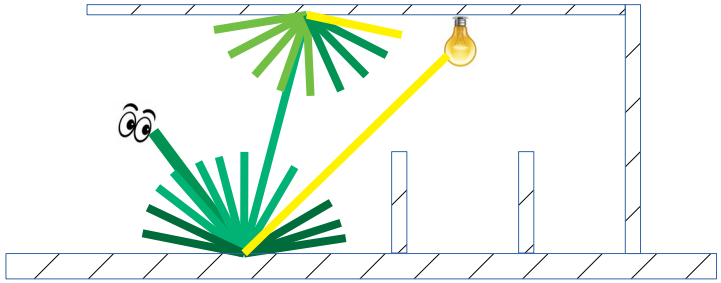

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Non-recursive (pseudo) implementation

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        for(  int j = 0 ; j < p.depth() ; ++j  )
            pathColor *= brdf( p.in(j) , p.out(j) ) * p.material(j) ;
        color += pathColor * p.light() * dp
    }
    return color ;
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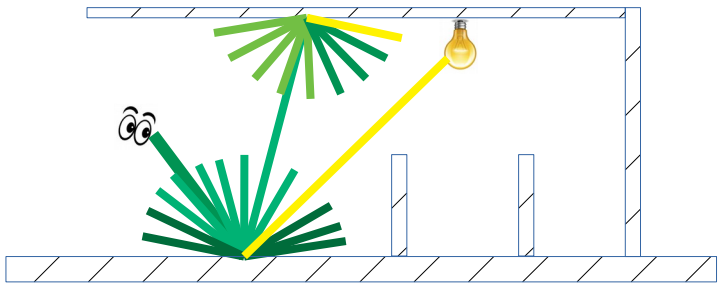

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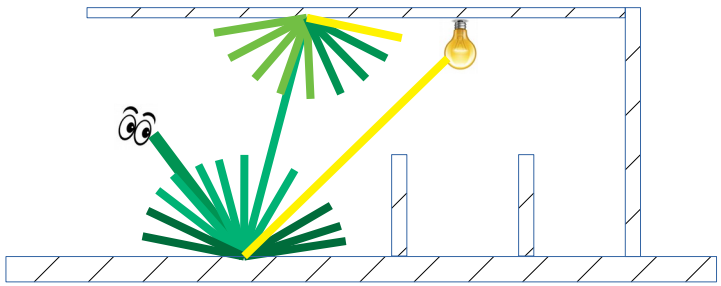

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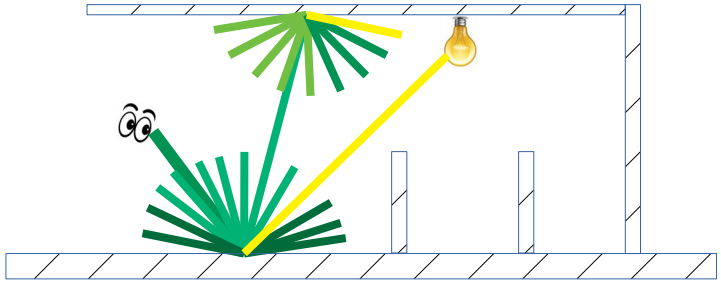

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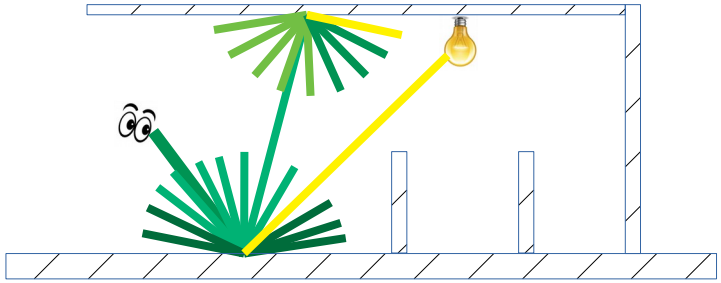

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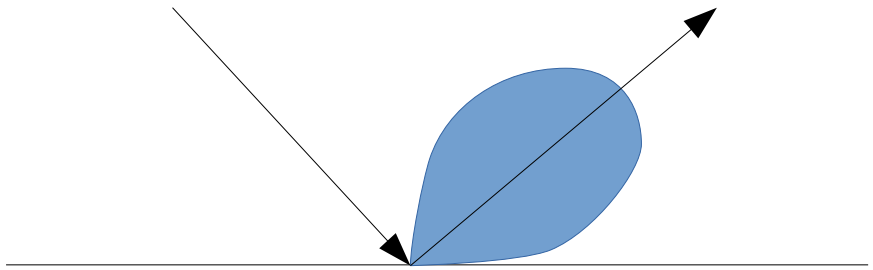
Recursive vs path-based

- ***Recursive :***
  - Easy to implement (see here for example)
  - Requires casting « shadow rays » in practice
- ***Path-based***
  - More complicated
  - Allows for parallelization
  - Allows for useful extensions :
    - Bidirectional
    - Metropolis

Importance sampling



BRDF

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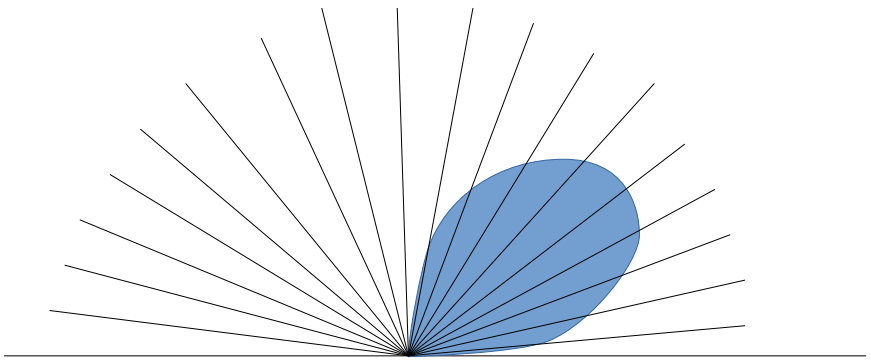

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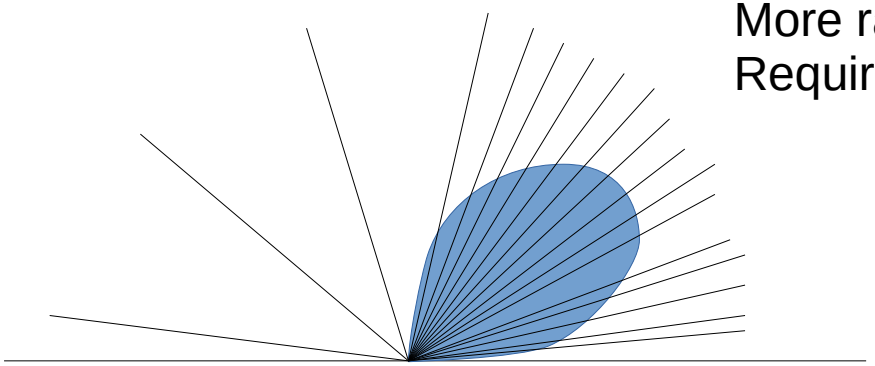

BRDF

More rays here :
Requires to set a smaller step size dw_i

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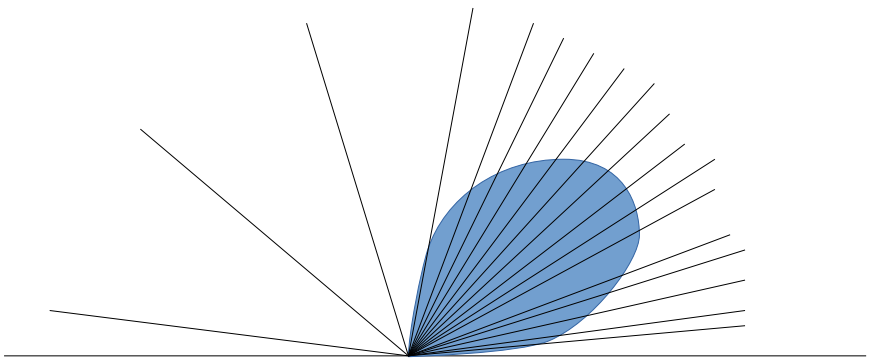

BRDF

$$\int f(x)\,dx = \int \frac{f(x)}{p(x)}\,p(x)\,dx$$

$$\int f(x)\,dx = \frac{1}{N}\sum_i \frac{f(x_i)}{p(x_i)}$$

if x_i follows the distribution p

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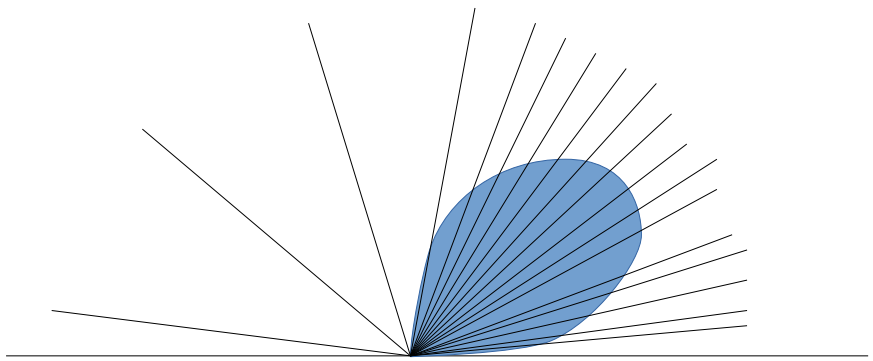

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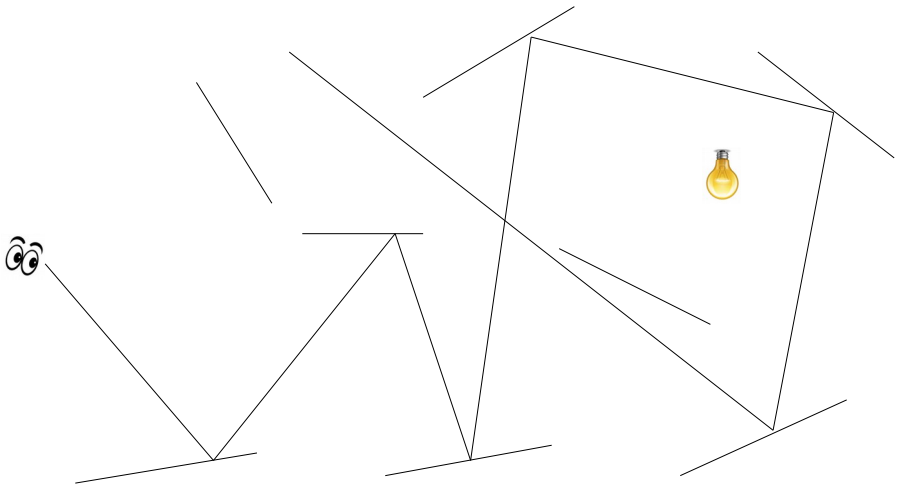
In practice, almost everything is importance-sampled (lights, bounces, …).
Variance is best reduced when p equals f.

$$L(p,\omega_{out}) = L_e(p,\omega_{out}) + \int_{\omega_{in}\in\mathcal{H}(p,n)} L(p,\omega_{in}) * brdf(p,\omega_{in},\omega_{out}).\ \underbrace{(n\cdot\omega_{in})}_{\text{produit scalaire}}\ .d\omega_{in}$$

Bidirectional path tracing principle

- It is sometimes difficult to hit the lights (small lights, occluders)
  → slow convergence



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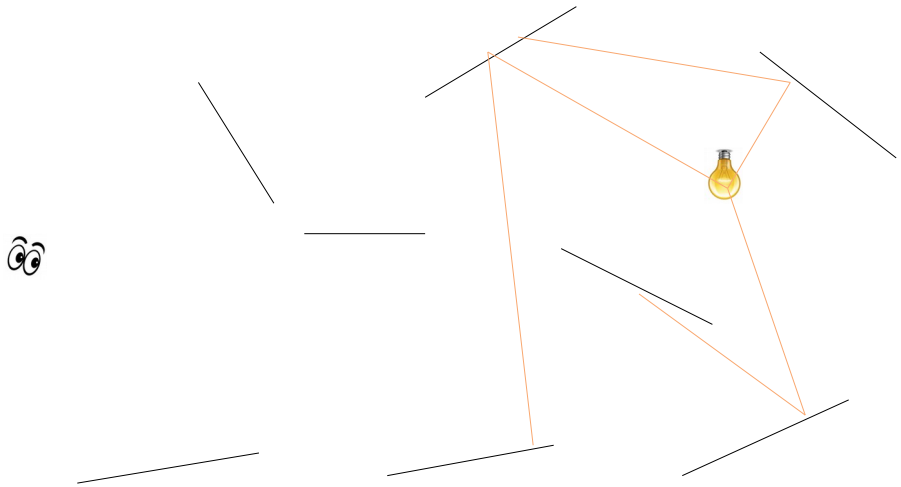
- We can generate sub-paths from the lights



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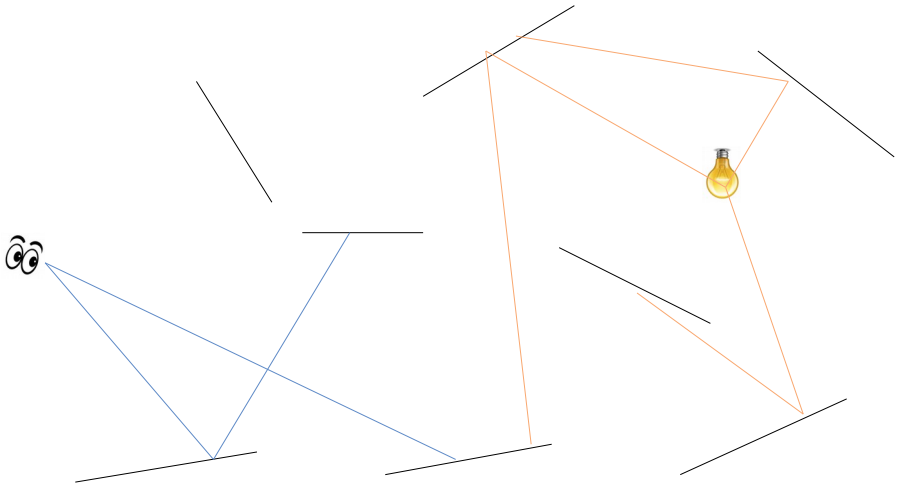
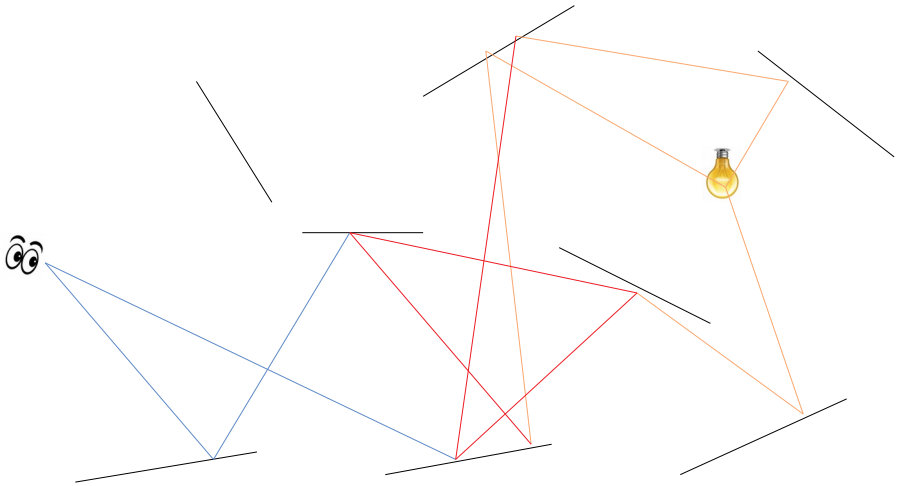


Bidirectional path tracing principle

- It is sometimes difficult to hit the lights (small lights, occluders) → slow convergence
- We can generate sub-paths from the lights, and sub-paths from the camera
- And connect them



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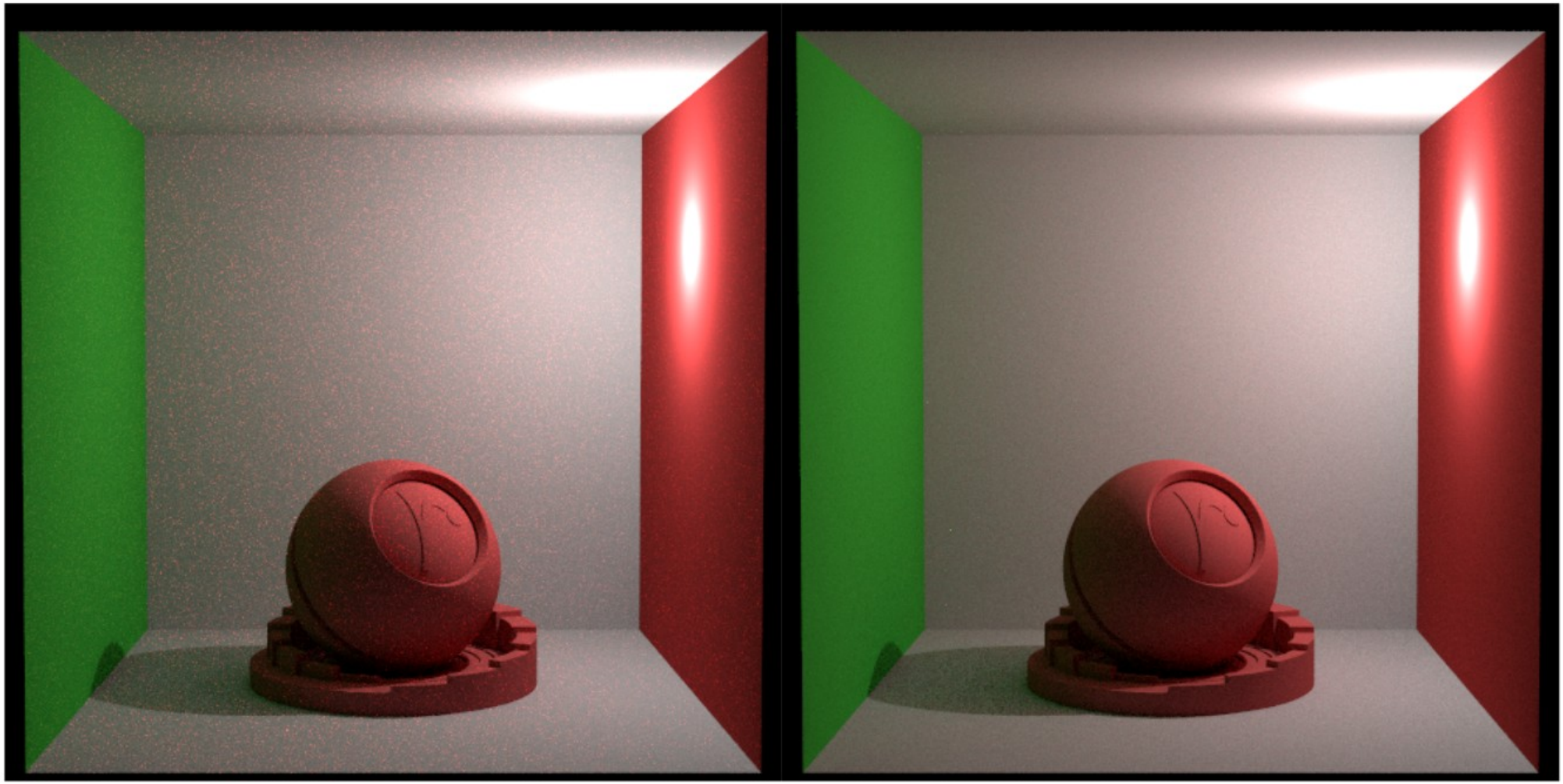

(a) unidirectional (b) bidirectional

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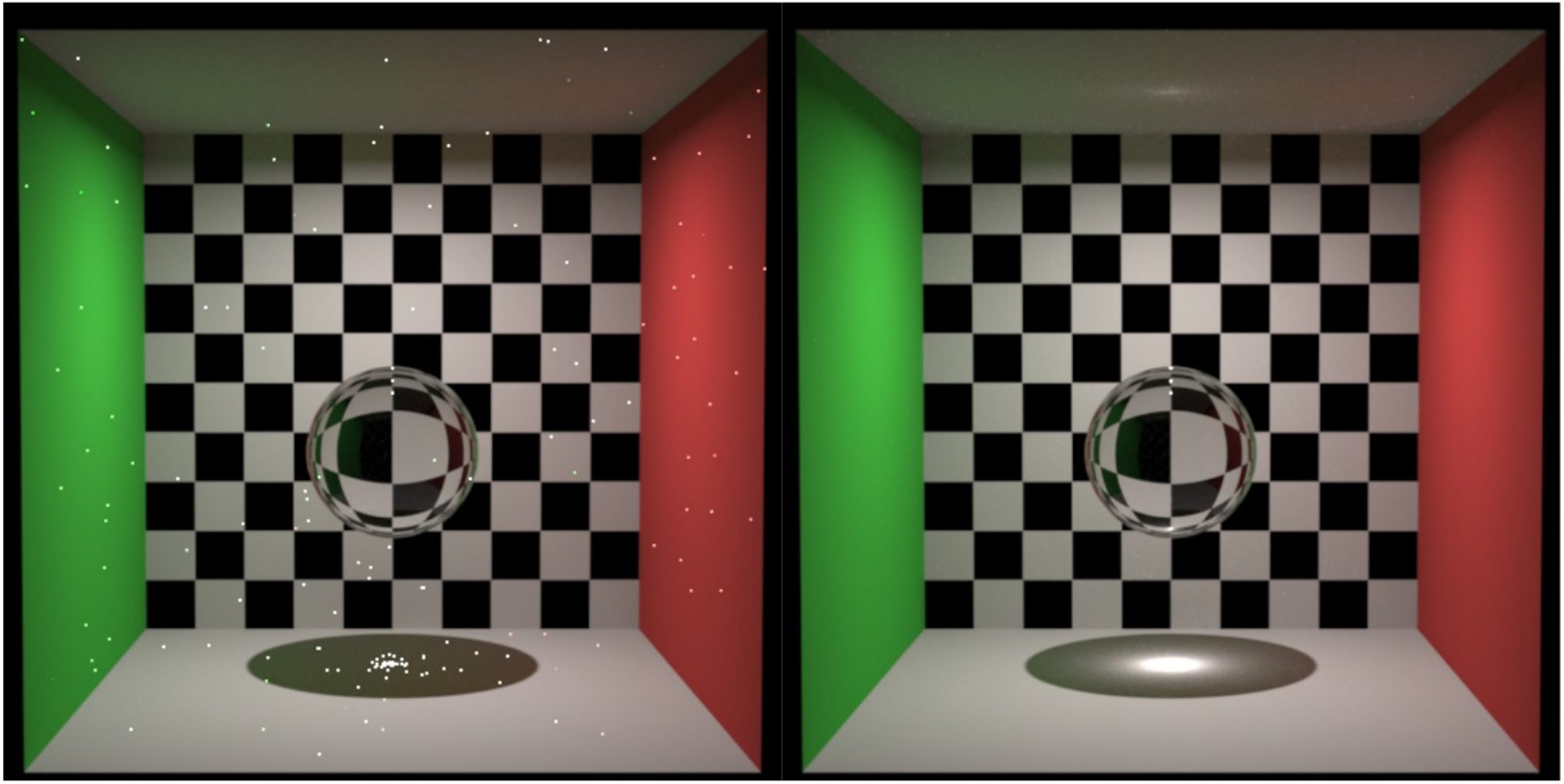

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