Proceedings of the 37th Annual Conference of the Cognitive Science Society,
2069-2074. Austin, TX: Cognitive Science Society.

Some Probability Judgments may Rely on Complexity Assessments

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Abstract

Human beings do assess probabilities. Their judgments are however sometimes at odds with probability theory. One possibility is that human cognition is imperfect or flawed in the probability domain, showing biases and errors. Another possibility, that we explore here, is that human probability judgments do not rely on a weak version of probability calculus, but rather on complexity computations. This hypothesis is worth exploring, not only because it predicts some of the probability 'biases', but also because it explains human judgments of uncertainty in cases where probability calculus cannot be applied. We designed such a case in which the use of complexity when judging uncertainty is almost transparent.

Keywords: Probability, Kolmogorov complexity, simplicity, unexpectedness.

Introduction

Human beings have a natural intuition of 'probability'. They use it, not only to anticipate risks, but much more often when they get the point of narratives based on unexpected events (Dessalles, 2008a). For instance, people are very good at noticing all factors that control unexpectedness in coincidences (Griffiths & Tenenbaum, 2001 ; Dessalles, 2008b) and in near-miss experiences (Teigen, 2005; Dessalles, 2010). Even if unexpectedness does not always match the presence of low probability (Teigen & Keren, 2003; Maguire & Maguire, 2009), the two notions are strongly linked, in a way that will be explored here.

This definite and consistent ability to assess probability appears quite mysterious in the light of the many apparent 'biases' that have been revealed in the past decades. For instance, people wrongly assign low probability to non-representative sequences of similar events (Kahneman & Tversky, 1972; Tenenbaum & Griffiths, 2001). Let's mention some other errors of judgments: the gambler's fallacy (Terrell, 1994), the base-rate fallacy (Bar-Hillel, 1980), the conjunction fallacy (Tversky & Kahneman, 1983) or the simplicity bias in causal explanations (Lombrozo, 2007). What kind of computation can be so wrong that it fails on basic tests and yet is so precise when it comes to judging uncertainty for everyday purposes?

This issue, quite surprisingly, has not been considered a priority. In many psychology experiments, for instance in decision theory, probabilities are provided as input to participants, with the tacit hypothesis that they are able to process them directly. Could it be that judgments of uncertainty are spontaneously achieved through a fundamentally different form of computation? Could it be that probability is no more than a mathematical notion with no cognitive counterpart? The purpose of this paper is not to solve these issues, but to show that *complexity* should not be ruled out as a candidate to account for uncertainty judgments, and that contrary to a common opinion, it is cognitively plausible, perhaps no less than probability itself.

We will first consider the notion of complexity and show how it can be turned into a cognitive notion. Then we will see how notions like condition, independence and subjective probability are reformulated in the framework of *Simplicity Theory*. We will use an example, a story of plagiarism, to show that individuals are sensitive to complexity. Lastly, a small experiment based on this example will be presented. Its purpose is to test some predictions of the theory.

Cognitive Complexity

The mathematical notion of complexity, known as Kolmogorov complexity, emerged in the last fifty years to deal with issues such as randomness, induction in learning and computability. The complexity of a situation is the size of its shortest summary. Or, in other words, its size when it has been maximally compressed. This definition can be made formal by coding situations as binary strings and by finding computer programs that generate them. The complexity $C(s)$ of s is the length of the shortest program that outputs s.

The transposition to cognitive science seems straightforward (Chater, 1999). It is indeed known since Gestalt Theory that human individuals are sensitive to simplicity. The use of complexity in cognitive science has however been hindered by an obvious objection: it is not computable. It is easy to prove that no program can output $C(s)$ when s is given as input. Ideal compression is well-defined, but cannot be computed. This observation led to the conclusion that human minds have no access to complexity. Some authors decided to abandon it altogether in favour of statistical inference (Griffiths & Tenenbaum, 2003), while others attempted to consider computable alternative measures of complexity, such as pattern complexity or Boolean complexity (Simon, 1972; Feldman, 2004). We just need, however, to consider a bounded-resource version of complexity (Chater, 1999). Human beings do have computational power that allows them to detect, for instance, pattern repetition. They are therefore able to perform some compression on perceived situations. For instance, anyone who knows about numbers can detect a pattern in the series 122333444455555, namely "n repeated n times", which leads to significant compression.[1] More generally, any detection of

[1] Using concatenation, the series can be written Πn^n, $n \leq 5$. This is much more compact than the independent specification of 15 numbers: two instructions (loop and repeat) and an upper limit on the one hand, $15 \times \log_2(10) = 50$ bits on the other hand. If we spare three bits to designate each instruction in the number sequence

structure achieves a compression. Let's call $C_{i,t}(s)$ the size of the best compression that an individual i has been able to produce within time t. This notion is, by definition, computable as soon as a computable cognitive model is available. In what follows, $C(s)$ will be used to designate $C_{i,t}(s)$. In this sense, $C(s)$ is computable.

Description *vs.* Generation

Links between complexity and probability have been noticed from the outset (Solomonoff, 1964). The basic idea is that simpler patterns are more probable. Algorithmic Information Theory (AIT) offers several definitions of algorithmic probability, including $p(s) = 2^{C(s)}$, which amounts to converting each complexity bit into the flip of a fair coin.[2] This definition matches the subjective uncertainty attached to explanatory scenarios: complicated explanations involving many choice points are perceived as less likely.

This definition cannot be the answer, however, as subjective probability sometimes functions the other way around. Lottery draws such as 1-2-3-4-5-6 or 5-10-15-20-25-30 are intuitively felt as much more improbable than 17-19-24-35-38-43, not because the former are more complex, but on the contrary because they are *less* complex (Dessalles, 2006; Maguire et al., 2013). AIT accounts for this effect by introducing a new notion, *randomness deficiency* (Li & Vitányi, 1994). Within the framework of *Simplicity Theory* (ST), randomness deficiency is a special case of complexity *drop* (Dessalles, 2008a).

Why is improbability sometimes attached to complexity (as for causal explanation) and sometimes to simplicity (as for lottery draws)? According to ST, unexpectedness does not correspond to one measure of complexity, but to the difference between two measures of complexity: *generation complexity* and *description complexity*. The latter matches the usual definition of $C(s)$. Note that each individual is regarded as a different computing 'machine': if a lottery draw matches an individual's telephone number, it will be very simple for her, but not for other people.

Generation complexity, on the other hand, is defined as the simplest causal scenario that the individual can figure out to explain a situation. In a lottery, all numbers are believed to be generated by equally complex causal processes. Generation complexity $C_w(s)$ can be measured by the number of choice points and the number of options in the minimal scenario that generates s. For instance, most individuals consider that the presence of a famous actor in their kitchen would require a complex causal scenario.[3] Note that this definition of generation complexity provides a simple notion of *independence*. Two situations s_1 and s_2 are independent iff $C_w(s_1\&s_2) = C_w(s_1) + C_w(s_2)$.

ST defines *unexpectedness* $U(s)$ as the difference between generation and description complexity.

$$U(s) = C_w(s) - C(s). \quad (1)$$

This definition is congruent with Teigen and Keren's observation that surprise corresponds to contrasts between actual outcomes and expectations (Teigen & Keren, 2003; Saillenfest & Dessalles, 2014). Here, expectations correspond to $C_w(s)$ and outcomes to $C(s)$. The above definition of unexpectedness aims at capturing exactly what people regard as surprising, as unlikely, as 'improbable' (in the naïve sense). The correspondence with probability is explored now.

Simplicity Theory and Probability 'Biases'

The main hypothesis explored in this paper is that human beings, in many judgments about uncertainty, rely on unexpectedness rather that on probability. Let's consider the above mentioned 'fallacies' in turn to see if they are compatible with this hypothesis.

In the gambler's fallacy (Terrell, 1994), people are reluctant to bet on recently drawn numbers. This behavior is deviant in the eyes of Probability Theory (PT). Why would a memory-less lottery avoid recent numbers? If 571 was drawn four weeks ago in a weekly lottery, people behave as if they considered the probability that 571 be drawn, not twice at a four week distance, but twice *within* a four week interval. PT explains neither the phenomenon nor the 'within' hypothesis required to account for its fading with time. According to ST, gamblers bet on the least unexpected outcome. If 571 was recently drawn, it is much simpler to describe than $\log_2(1000) \approx 10$, which is the number of bits required to distinguish among the 1000 options in the lottery studied by Terrell. The simplest description of 571 now amounts to 2 bits, as it consists in giving its rank in the list of past winning numbers. The gambler's fallacy results from a decrease in $C(s)$, while $C_w(s)$ remains constant. This makes recent winning numbers too unexpected to be bet on. The effect lasts as long as the complexity of locating 571 in the winning list remains small enough.

As observed by H. Simon (1972), simplicity accounts for biases of representativeness as well. People would consider a series of eight births like GGGGBBBB, where four girls and then four boys are born in the family, as less 'probable' than a more complex pattern like GGBGBBGB. Here also, $C_w(s)$ is kept constant while $C(s)$ varies. The effect is a mystery for PT, but it is again predicted by ST if we suppose that individuals are sensitive to unexpectedness. As (1) shows, simple structures make $U(s)$ larger, hence the feeling of improbability. This simplicity effect can be quantified and matches experimental results (Simon, 1972). For the same reason, remarkable lottery draws like 1-2-3-4-5-6 are regarded as virtually impossible by most people

context, the first code would need only $2\times3+\log_2(5) < 9$ bits.

[2] This definition is sometimes regarded as problematic. If we code situations s as numbers n_s, then for most situations, $C(s) \approx \log_2(n_s)$, and $\Sigma p(s) = \infty$ instead of 1. This problem is avoided, either by considering prefix-free codes, or by regarding numbers like 297 and 2971 as non exclusive (as the latter contains the former).

[3] See www.simplicitytheory.org for further details. The site answers some frequently asked questions about ST, including why a situation that is the most complex in its class turns out to be simple for that reason; or the converse: why a standard object, like a common window, turns out to be complex, as it requires a lengthy description to be distinguished from all other windows.

(Dessalles, 2006). Note that contrary to PT, ST does not invoke here any *ad hoc* notion such as representativeness.

In the conjunction fallacy (Tversky & Kahneman, 1983), people find it less probable that Paul, a former Green activist, would drive a big SUV rather than if he drived a big SUV functioning with LPG (SUV are known to waste more energy than standard cars and LPG is known to be less polluting than gasoline). Within the set-theoretic framework of PT, this seems absurd, as the set of SUV drivers includes the set of LPG-SUV drivers. How does ST account for this phenomenon? By noticing that the causal generation of the SUV case is more complex than for the LPG-SUV case.

The two situations: Paul driving a SUV (s_1) and Paul driving a LPG-SUV (s_2), differ both by their description and their generation. Assuming that Paul is already in the context, situation s_1 can be described using the concept of green activist (G) and the concept of SUV (f_1).

$$C(s_1) = C(G) + C(f_1|G).$$

The vertical bar in $C(a|b)$ denotes conditionality. It means that b is available to describe a. For instance, $C(a|a) = 0$. s_2 requires an additional feature f_2 = 'LPG' to be described.

$$C(s_2) = C(G) + C(f_1|G) + C(f_2|G, f_1).$$

(we ignored other features, such as 'drive', that are common to s_1 and s_2). Here, concepts[4] are prototypical situations evoked by words. Considering prototypes instead of sets is presented as a human flaw by Tversky and Kahneman (1983). But the hypothesis that words be associated with sets rather than prototypes is a constraint imposed by PT and has little cognitive support.[5]

If we abandon PT's extensional constraint, we can compute how s_1 and s_2 differ on the generation side. s_1 evokes a *typical* situation, *i.e.* a gasoline SUV. Since LPG is supposed to be more Green-friendly than standard gasoline, the contradiction with Paul's past as Green activist is less flagrant in s_2 than in s_1. This means that the minimal causal scenario explaining s_2 is less complex than the minimal causal scenario leading to s_1. In other words: $C_w(s_2) < C_w(s_1)$. We get:

$$U(s_1) - U(s_2) = C_w(s_1) - C_w(s_2) + C(f_2|G, f_1) > 0. \qquad (2)$$

ST correctly predicts that s_1 will appear more unexpected, and therefore less probable, than s_2. This prediction matches the so-called 'conjunction fallacy'. Note that this account, derived from ST, is not unrelated to Maguire *et al.*'s (2013) explanation. These authors introduce different 'models', which correspond to the causal scenarios underlying $C_w(s_1)$ and $C_w(s_2)$. Though adopting a complexity-based approach, their description is expressed in terms of probability distributions. (2) shows that the phenomenon can be parsimoniously analyzed in terms of complexity exclusively. The detour through probability is unnecessary.

If individuals judge uncertainty based on complexity rather than on probability, several other 'fallacies' are no longer problematic. Simplicity bias in causal explanations (Lombrozo, 2007) and base-rate neglect (Bar-Hillel, 1980) rely on experiments in which probabilities are provided as numbers (percentages) to participants. While educated individuals may be able to translate unexpectedness into probability estimates, it is a too strong assumption to suppose that the converse might be true.

Unexpectedness and Subjective Probability

ST has been developed to account for the human ability to assess the unexpectedness of events after they have occurred. *Ex post* probability[6] is defined as:

$$p(s) = 2^{-U(s)}. \qquad (3)$$

Formula (3) explains why a simple sequence like 1-2-3-4-5-6 is felt as much more improbable than a complex one like 17-19-24-35-38-43. It also explains why events that are rare, unique or extreme according to a *simple* criterion are perceived as improbable when they occur; it explains why rare events (like a fire) are regarded as less probable when they occur in the vicinity; it explains recency effects in the news; it also explains why coincidences are exaggeratedly perceived as improbable (Falk, 1983) (see simplicitytheory.org for a review). In all these examples, probability judgments are performed *ex-post*, after the fact.

In all the above mentioned classical studies on probability bias, individuals were asked whether a situation was more 'probable' than another. This corresponds to an *ex-ante* judgment. Ideally, from the *ex-ante* perspective, s is already determined: $C(s) = 0$, and *ex-ante* unexpectedness is $U_a(s) = C_w(s)$. The central thesis of this paper is that people translate the word 'probable' by considering both $U_a(s)$ and $U(s)$. In our lottery examples, $U_a(s)$ is constant and only $U(s)$ varies. In the SUV example, $U_a(s)$ and $U(s)$ vary in the same direction. But only for a relevant feature like LPG.

ST predicts that individuals' behavior will be different if f_2 is a neutral feature such as 'the SUV was red'. Suppose there are 16 possible SUV colors. One needs $C(f_2|f_1) = 4$ bits to designate the actual color. On the generation side, 4 additional bits are also required in a causal scenario to orient the choice among the 16 colors: $C_w(s_2) = C_w(s_1) + 4$. We get: $U(s_2) = U(s_1)$, and s_2 will be judged no more unexpected than s_1. In a narrative like: "Remember Paul, the former Green activist?", the two mentions "I saw him driving a big SUV" and "I saw him driving a big red SUV" offer exactly the same unexpectedness. The detail about the color is irrelevant,[7] if *relevance* is defined as: 'contributing to

[4] For any concept x, $C(x)$ can be approximated as $C(x) \approx \log_2(r)$, where r is the rank of a word expressing x in a list of words sorted by frequency of occurrence in a corpus. Assuming Zipf's law, r can also be the frequency itself, or the relative number of hits on a Web search engine (Cilibrasi & Vitányi, 2007). A specific corpus can be used to refine estimates for given individuals.

[5] The set of all SUVs is a mathematical abstraction which is not computable, either objectively or cognitively.

[6] Technically, *ex-post* probability $p(s)$ as defined by (3) corresponds to $Prob(happens(s) \mid s)$. It is not itself a probability measure: the sum of $p(s_i)$ for different events s_i may exceed 1.

[7] Unless it is used to emphasize that the story is true.

unexpectedness' (Dessalles, 2013). However, $U_a(s_2)$ is larger than $U_a(s_1)$ by 4 bits. This may lead most people to regard 'red-SUV' as less probable than mere 'SUV', thus respecting the conjunction axiom this time. Note that PT is unable to take the relevance of the feature into account.

The Plagiarism Story

To make the case of complexity even stronger, we searched for a situation in which individuals make a definite judgment of uncertainty that cannot be explained by probability calculus. Consider the following story.

> Story 1: *Ms S. is accusing Mr D. of having stolen her manuscript and of having published it under his name. Fortunately, she hid her name in the book. Her name is (option n_1: Sami); (option n_2: Schildget). It can be retrieved by taking (option r_1: the first letter of each chapter); (option r_2: the first or the second letter of each chapter, depending on the chapter's parity).*

We could not find anyone, in informal inquiries among students, who chose options other than n_2 and r_1 when asked to maximize Ms S.'s chances to win her case. Why do the more complex name and the simpler retrieving algorithm make plagiarism so obviously more likely in this story?

PT would merely predict that a shorter name is more likely to 'occur' by chance in the book, without any precision about what 'occur' means. It is unable to account for the role of the algorithm used to retrieve the name.

ST's explanation is straightforward: plagiarism is probable if the co-occurrence *by chance* of Ms S.'s name (N) and Mr D.'s book (B) is highly unexpected, *i.e.* if $U(N \,\&\, B)$ is large. 'By chance' here means that N and B are supposed to be independent. By definition of independence, $C_w(N \,\&\, B) = C_w(N) + C_w(B)$. On the description side, $C(N \,\&\, B) \leq C(B) + C(N|B)$. The algorithm A provided by Ms S. to retrieve her name in the book gives an upper bound of $C(N|B)$: $C(N|B) \leq C(A)$. If we assume that neither B nor N is unexpected by itself, we get by applying (1):

$$U(N \,\&\, B) \geq C(N) - C(A). \tag{4}$$

ST thus explains why a complex (*i.e.* long) name and a simple algorithm make plagiarism more probable in story 1. The complexity of the retrieving algorithm, A, is directly understood to play a crucial role. A probability-based model could not account for this effect without many *ad hoc* assumptions. It is more parsimonious to consider that individuals have direct access to complexity assessments.

We designed a small experiment as a first attempt to explore the Plagiarism story in more details and try to test finer grain phenomena. This time, we introduce new variables, such as the size of the book. Here is the second story.

> Story 2: *Ms Schmidt is accusing Mr Durand of having plagiarized in his book B_2 a passage T_2 of size S that is almost identical to a passage T_1 from her book B_1. The sizes of the two books are S_1 and S_2. T_1 is located at page p_1* of *B_1 and T_2 at page p_2 in B_2. T_2 is found in one piece (option 1) / in three pieces distributed over three paragraphs in p_2 (option 2).*

Can we predict how parameters S, S_1, S_2, p_1, p_2 and the two options influence plagiarism probability? PT has something to say about this. It will predict that the probability of T_1 appearing in B_2 by chance would decrease with S and increase with S_1 and S_2. But in the absence of any specific knowledge about the borrowing mechanism, it would assume uniform probability for p_1 and p_2, and their value would be irrelevant. Comparing options 1 and 2 would be somewhat tedious. One would need to imagine all ways of splitting T_1 into several pieces to determine the probability that T_1 would end up in three, instead of one, two or more than three pieces.

In the ST framework, plagiarism is blatant when the coincidence between the content of B_1 and of B_2 is too unexpected. There is coincidence if these contents are supposed to be independent:

$$C_w(B_1\&B_2) = C_w(B_1) + C_w(B_2). \tag{5}$$

On the description side:

$$C(B_1\&B_2) \leq C(B_1) + C(B_2|B_1). \tag{6}$$

Following (1), the unexpectedness of the coincidence, $U(B_1\&B_2)$, corresponds to the complexity drop between generation (5) and description (6). Assuming that neither B_1 nor B_2, as sequence of words, is unexpected by itself[8] ($C_w(B_i) = C(B_i)$), we get:

$$U(B_1\&B_2) \geq C(B_2) - C(B_2|B_1). \tag{7}$$

The right-hand side of (7) is the compression of B_2 allowed by the knowledge of B_1. This compression is due to the resemblance between T_1 and T_2. Ideally, $C(T_2)$ could be spared in the description of B_2. There is a tax to pay, however, which is the complexity of the procedure needed to get T_2 from B_1. To compute a lower bound of the compression, we may compute the complexity of the following procedure: locate T_1 in B_1; use algorithm A to transform T_1 into T_2; determine target location in B_2; insert T_2. From (7), we get:

$$U(B_1\&B_2) \geq C(T_2) - (C(l_1) + C(A) + C(l_2)). \tag{8}$$

l_1 and l_2 designate the precise locations of T_1 and T_2 in B_1 and B_2. If there are about n words in a page, then:

$$U(B_1\&B_2) \geq C(T_2) - (C(p_1) + C(A) + C(p_2) + 2 \log_2(n)). \tag{9}$$

In option 1, if texts are strictly identical, then A_1 is a mere copy and $C(A_1) = 0$. In option 2, $C(A_2)$ has a definite value, as it requires at least four numbers: two cut points in T_1 to make the three pieces, and the size of two gaps to specify how to insert the pieces in the target text. To make things concrete, we may say that $C(A_2) \sim 4 \log_2(n)$. Formula (9) allows us to draw the following predictions. Plagiarism is more likely if:

[1] T_2 is a long excerpt, making $C(T_2)$ large
[2] A is simple
[3] Sizes S_i are small, as $C(p_i) \leq \log_2(S_i)$, while n is not too large.

[8] One way to be unexpected in this way would be to be more compressible than exepected, as Georges Perec's *Grand palindrome*.

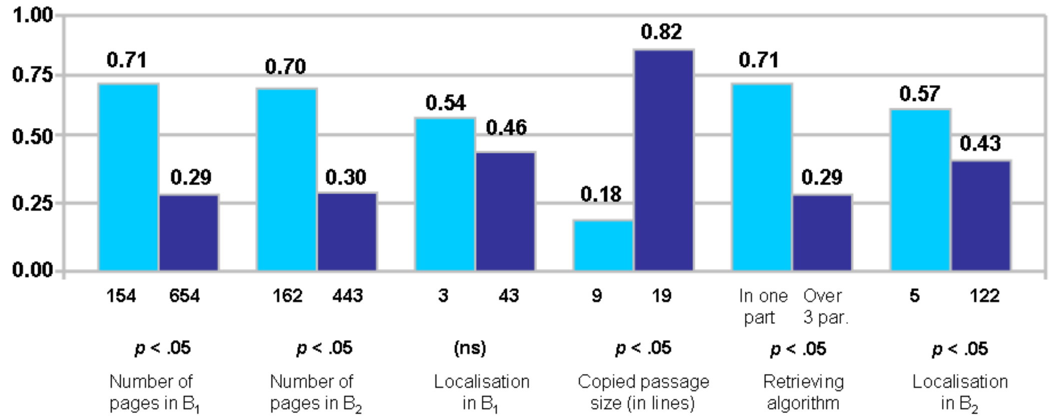

Figure 1: Answers for each alternative of the 6 propositions in Story 2

[4] p_i is small (while n is not too large), as in this case $C(p_i) \sim \log_2(p_i) < \log_2(S_i)$.

If B_1 is not known in advance, $C(B_1)$ is augmented by the determination of Ms Schmidt and by the determination of B_1 in her works. We could then add the two following predictions:

[5] Ms Schmidt is a famous author
[6] She has written few books.

Note that prediction [1] has a massive effect, as compared with predictions [2]-[4]. Each word in T_2 contributes by $\log_2(N)$ bits to unexpectedness, where N is the size of the lexicon (if one compares text creation to a uniform lottery among words). In comparison, [4] may spare 6 bits only if $p_1 = 3$ and $S_1 = 250$. The following small experiment is an attempt to put predictions [1]-[4] to the test.

Experiment

Participants were presented Story 2 (in French) with the following options:

- Ms Schmidt's book is a 154/654 -page book.
- Mr Durand's novel is a 162/443 -page book.
- The passage mentioned by Ms Schmidt is located page number 3/43 in her collection of short stories.
- The passage mentioned by Ms Schmidt is 9/19 lines long.
- The passage mentioned by Ms Schmidt can be found in one part / spread over 3 paragraphs in Mr Durand's novel.
- The passage mentioned by Ms Schmidt can be located page number 5 / 122 in Mr Durand's novel book.

A total of 352 individuals (aged from 16 to 63, mean 28.64 (std. dev. 6.66), 276 females, 91 males, 15 unknown gender) participated to the test online. Participants were recruited via social networks and billposting. We manually checked the answer files for individuals who provided incomplete results or whose response time was less than 30 seconds.

Results and discussion

Percentages of answers for each alternative proposed are presented in Figure 1. We tested for significance using binomial tests. Results indicate a significant effect ($p < 0.05$) for the number of pages of the books, the size of the passage, its location in Mr Durand's book and the complexity of the algorithm that leads from one text to another. The location of the passage in Ms Schmidt's book did not lead to a significant effect ($p \approx 0.11$). These results agree with our predictions [1], [2], [3] and [4]. Note that a probabilistic model would predict [1] and [3], perhaps [2], but not [4].

The non-significant result for the localization in Ms Schmidt's book is due in part to its small expected contribution in comparison with [1]. Moreover, an informal inquiry among additional participants suggests that some individuals may have performed second-order reasoning: borrowing a passage from Ms Schmidt's first pages would be too conspicuous and would make plagiarism not rational and therefore less likely. Due to its design that did not anticipate this reaction, Story 2 turns out to be less convincing than Story 1.

In further work, we plan to test predictions [5] and [6], as the contribution is larger ($\sim \log_2(A)$ where A is the number of authors), and also because probability calculus would not naturally take the author's celebrity into account.

Conclusion

The first aim of this paper is to question the human ability to process probability as such. Despite the existence of numerous human 'biases', the fact that uncertainty judgments rely on some form of probability calculus is often taken for granted without questioning its cognitive plausibility.

Our second aim is to put forward the possibility that *complexity* can be directly assessed by individuals. Complexity has not been sufficiently considered as a good candidate to explain judgments of uncertainty, due to prejudices concerning its non-computability, to misconceptions about how it should be computed (see note 3), and to the (wrong) belief that probability itself is always computable (see note 5). Notions which are quite intricate when expressed in probabilistic terms are more intuitively defined in terms of complexity: independence and causal indeterminism are captured by generation complexity; conditionality and simplicity come naturally with description complexity.

We hypothesized that unexpectedness, as defined in Simplicity Theory as the difference between generation and description complexity, is used by individuals to judge about uncertainty. This hypothesis has two advantages. (1) It offers new and parsimonious accounts of the various cognitive 'biases'; (2) It accounts for situations in which probability theory would be partially silent. We designed two versions of the Plagiarism story to make up cases in which judgments of uncertainty seem to rely on complexity rather than on probability.

Simplicity Theory has been designed in an unrelated context: to account for interest and relevance in narratives (Dessalles, 2008; Saillenfest & Dessalles, 2014). Quite remarkably, as we showed here, it can be successfully applied *with no modification* to judgments of uncertainty. No *ad hoc* hypotheses, such as recency avoidance, representativeness or randomness deficiency, have been introduced. This invites us to consider complexity as a plausible dimension of cognitive processing.

Acknowledments

This study is supported by grants from the programme Futur&Ruptures and from the "Chaire Modélisation des Imaginaires, Innovation et Création".

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