Potential and actual infinite in cognitive models of time

L'infini potentiel et l'infini actuel dans les modèles cognitifs du temps

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Abstract

Representations of time in formal or computational models of cognition must cope with the problem of granularity. A non-zero grain puts an absolute limit on precision, while a zero-grain requires using an uncountable infinity of such grains to represent any non-zero continuous duration. To escape from this dilemma, we propose to replace the notion of absolute time structure by a recursive procedure of event location.

Résumé

Les représentations du temps dans les modèles formels ou computationnels de la cognition doivent faire face au dilemme de la granularité. Un grain non nul impose une limite absolue à la précision, alors qu'un grain nul impose de représenter toute durée non nulle à l'aide d'une infinité non dénombrable de tels grains. Pour échapper à ce dilemme, nous proposons de remplacer la notion absolue de structure temporelle par une procédure récursive de localisation d'événement.

Rapport technique ENST 2001-D-004

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Introduction

Most formal approaches to time rely on a linear continuous representation borrowed from physics. Any non-zero duration is an interval on this line, made of an infinity of instants. This notion of time proves to be quite satisfactory for calculus. It may also be relevant to cognitive modeling: if we compare the mind-brain with a dynamical system (van Gelder, 1998), then it is normal to use the usual time line of physics to compute derivatives and predict state evolution. If, however, we want to account for the cognitive ability to deal explicitly with time, as in planning or language understanding, then representing time as an infinite set of punctual instants seems absurd: memory, in brains as in computers, is limited to finite quantities.

Infinity has always been a delicate issue for philosophers and cognitive scientists. Our mind is unable to grasp infinitely large or infinitely tiny quantities directly. Children understand quite soon that by repeatedly adding *1*, they will reach infinity. This procedural conception of infinity is known, since Aristotle, as ‘potential infinity’, to be contrasted with the ‘actual infinity’ of infinite sets given extensively. The former is obviously accessible to computers and minds, the latter obviously not.

The question addressed in this paper is to know whether we are able to offer cognitive representations of time that avoid any use of actual infinity. As we will see, the answer is far from obvious. In what follows, we first show that in classical models of time offered by logic or Artificial Intelligence, the only way to escape from using actual infinity is to put a limit on precision by means of non-zero grain representing an atom of duration. Then, we will use Zeno’s paradox, not only to show that both punctual instants and atoms are untenable as models of our ability to reason about time, but also as a way of revealing aspects of this ability. Lastly, we will suggest an alternative way of representing time conceptualization: we will abandon the idea of a static structure in favor of a mental procedure that allows us to conceive of temporal relations.

Limits of Classical Models of Time

Formal representations of time offered by logic and used in Artificial Intelligence make use of actual infinite. There have been several attempts to cope with time in logic, but most of these theoretical frameworks rely on the mathematical idealization of a temporal line as a set of punctual instants. Statements, in classical logic, are a-temporal. The main problem encountered by logicians was to account for the non-permanence of truth or falsity through time. One obvious solution was to add time as an argument in most predicates. It may appear quite unsatisfactory, though, that a statement like ‘Socrates is dead’ be instantiated in different, non-correlated statements at each instant t of the time line. An elegant solution, introduced by Prior (Prior, 1957; 1967), was to incorporate the time dimension into the framework of Kripke’s modal logic. The time parameter becomes a semantic modality, it appears as subscript in models and needs not appear as an explicit argument in predicates. In other words, predicates do not necessarily depend on time, but their evaluation does.

Time structure, in modal logic, appears only indirectly in the properties of the accessibility relation, which links each ‘world’ to its futures. Though many structures can be considered (*e.g.* discrete or even circular time), structures of practical interest, especially for natural language modeling, presuppose ‘natural’ properties like linearity (because instants of our lives seem to be ordered), density (one can think of other instants between any pair of instants) and continuity (one feels no gaps in the flow of time). Natural time, in this context, looks thus very much like the real line of mathematics.

One may hope that the use of modal operators like P may allow avoiding any explicit reference to punctual instants. $P\alpha$ means that α was true at some instant in the past, without precise reference to that instant. An axiom like $P\alpha \supset PP\alpha$ may express density without referring explicitly to instants: if α was true at some time in the past, then at some intermediary time (between that time and present), α could also be said to have been true. If this process is iterated ($PPP...P\alpha$ is also true), we need an infinity of instants within a bounded duration. This infinity, however, now appears as potential and not necessarily actual. By avoiding any mention of punctual instants, operators seem to escape from the difficulty of a zero-grained conception of duration. This advantage is however paid by a gross loss in expressive power.

The limits of modal logic operators are well known. For instance, operators like *P* and *F* do not allow to express intervals of time. The introduction of more powerful operators like *S* and *U* still leaves us with a variety of problems concerning temporal anaphora, indexicality and present (Kamp & Reyle, 1993). For instance, we cannot easily express 'She said he has no money', since once situated in the past through the operator *P*, we have no simple means to refer back to present. The reason for these limitations is that all modal operators are, by essence, amnesic: they can only operate on the time of their operand, without being sensitive to time references hidden deeper in the operand.

There seems to be another solution to avoid using punctual instants: one can adopt a time ontology based on intervals (Kamp, 1979; van Benthem, 1979). The idea has old philosophical roots. For some authors, time would not be a primitive notion, but would be derived from the concept of event (Whitehead, 1929; Davidson, 1980). Unfortunately, punctual instants are not absent from interval-based formalization. They can be constructed by means of basic relations between intervals, precedence and overlapping. The interval representing a fact like 'Mary came home' may be compared with two other intervals, representing two successive states of affairs: 'Mary was not at home' and 'Mary was at home'. If we want to avoid contradiction, these two intervals cannot overlap. If we want to preserve the principle of excluded middle, there cannot be any gap between them. As a consequence, both intervals must be contiguous. They must be connected by a single, punctual instant representing the infinitely small duration of Mary's arrival. The use of intervals may prove to be technically sound and useful in many situations of practical interest (Allen, 1984), when the use of atomic intervals is not problematic. As cognitive model, though, a temporal representation based on intervals cannot save us from actual infinity.

Another idea to avoid using punctual instants representing infinitely small duration is to introduce uncertainty or fuzziness in time location. The idea seems particularly relevant from a cognitive perspective, since time reference in ordinary language is rarely precise: even when we say 'at two o'clock', we rarely mean at 2:00:00.00. An interesting attempt to formalize time in this way is offered by De Glas' locology (De Glas, 1992) applied to Desclés' model of time (Desclés, 1990). These authors adopt a temporal ontology based on different kinds of intervals: closed intervals for events, open intervals for states. One further original idea of their approach is to make interval boundaries 'thick'. Each subset of the time line has a 'heart' and a 'shadow' which replace the interior and adherence of conventional topology. The difference between shadow and heart, for an interval, is not reduced to a pair of isolated instants. This kind of model presents multiple advantages (De Glas & Desclés, 1996). It does not, however, escape from the time line ontology in which any non-zero duration is implemented by an infinite set of instants. Since boundaries are thick, they possess, themselves, an interior, their 'heart', which itself has thick boundaries, and so on until we reach the point as limit. Even if boundaries are blurred, punctual instants are thus constitutive of any time period in such models.

The primary purpose of most formalizations of time is theoretical or technical, it is not to offer a plausible picture of how the mind performs temporal reasoning. We must observe, however, that none of these formalizations is adequate as cognitive model, because all of them fall into what we may call Zeno's dilemma: choose between actual infinity and an atomic conception of time.

What Zeno's Paradox Reveals About our Conceptualization of Time

Instantaneous Duration

Any cognitive model of time representation must face Zeno's paradox of dichotomy. We can always think of a duration which is one half of any given duration. Both ways out of this paradox seem wrong. One is to forbid dichotomy beyond some point. There would be some atoms of time, below which our mind would not be able to grasp time duration conceptually. We have no candidate for such an atom. The alternative option is that our mind can grasp zero-length duration. By accepting this, one would admit that our mind is able to conceptualize actual infinity. In the dichotomy procedure, indeed, zero-length duration is obtained after an infinite number of divisions. Moreover, the mere possibility of zero-length duration, though mathematically sound, appears as cognitively puzzling, as Lakoff and Núñez have nicely shown (Lakoff & Núñez, 2000). For instance, two of such zero-length periods of time, if distinct, cannot be consecutive. Deleting a zero-length period from some non-null duration should have no effect, but it has one, since it creates a gap in the non-null duration which becomes discontinuous. A juxtaposition of any finite number of zero-length time periods, even a huge one, is unable to form non-zero duration. One needs an infinity of such zero-length periods to do so. Similarly, the dichotomy procedure cannot be reversed, since multiplying again and again a zero-length time period by *2* does not bring to anything else.

Zero-length duration and non-zero duration have so different behaviors that we may suspect the two notions to be of different cognitive nature. In Lakoff and Núñez's terms, they belong to different metaphors. It seems that instantaneous duration cannot be conceptualized as such, but rather as a non-zero tiny duration which is smaller than any other given non-zero duration. The idea is not to go back to

Leibniz's notion of infinitesimal[1], but rather to follow Aristotle's idea when he limits the power of our mind to the conception of potential infinity. In other words, our mind can only conceive of non-zero duration. When forced to imagine a zero-length time period, it just imagines some tiny duration yet smaller than all other duration that were previously mentioned.

Aristotle's distinction between actual and potential infinity seems a good way to explain our incapacity to conceive duration as a set of punctual instants. There is, however, a problem. It seems that potential infinity is not a valid notion, since it generates the kind of absurdities that Zeno's revealed with his paradoxes.

Zeno's Paradox

Since Zeno of Elea formulated his famous paradoxes, scholars attempted to solve them. It is commonly accepted that the discovery of the notion of limit put an end to the puzzle created by Parmenides' disciple. Some authors, though, still look for original 'solutions'. For instance, McLaughlin suggests that Zeno's arrow may be moving, unnoticed, during those unobservable instants postulated by non-standard number theory (McLaughlin, 1994).

Zeno's most famous paradox states that Achilles never reaches the tortoise, since each time he reaches the tortoise location, the animal has moved forward by a certain distance, which represents a new target for Achilles, and so on endlessly. It is important to notice that this paradox was never absolute. Aristote, for instance, had a clear vision of where the 'error' originated from: time, like distance, is indefinitely divisible. Thus, we may think that Achilles reaches his moving target in a finite amount of time, even if our mind imagines an infinite number of successive phases. The mathematical notion of limit tells us little more. It confirms Aristotle's statement that the sum of an infinity of time intervals may be finite and allows to compute the limit. In the case of Achilles and the tortoise, the mathematical solution brings however only little satisfaction. One does not need to compute infinite series to estimate the time when Achilles reaches the tortoise: just divide the distance by the speed difference. Zeno's lesson is quite different: if we follow his line of reasoning, our mind becomes unable to conceptualize the *possibility* that Achilles reaches the tortoise.

The test is easy to perform, even with individuals fully aware of the mathematical notion of limit. If individuals accept to consider a phase just before Achilles reaches the tortoise, then they must acknowledge that this phase cannot be the last one, since once Achilles reaches the target location, the target has moved forward. Thus there is no last phase in the reasoning. When presented with the puzzle, children adopt an atomic view of how things proceed: Achilles passes the tortoise in one step (Núñez, 1994). Adults may escape the puzzle by refusing to conceptualize the next phase, backing up to the cinematic scene where Achilles obviously passes the slowly moving animal. In any case, Zeno's trap functions for anyone who accepts to follow his reasoning, which consists in visualizing repeatedly the scene 'just before' Achilles succeeds.

Such paradoxes seem to prove that limiting the power of our mind to potential infinite is unsound. If we have no conceptual access to actual infinite, then we cannot conceive that Zeno's loop can come to an end and we wrongly conclude that Achilles remains behind the tortoise.

Accepting Zeno's Lesson

On important hypothesis of the present paper is that Zeno's paradoxes are not based on ill-formed reasoning. According to this hypothesis, there is no point trying to 'solve' them. They are a consequence of some fundamental property of the mind and we would rather explore the consequences. The situation imagined by Zeno creates a cognitive illusion. In case of perceptive illusions, one part of our mind knows that another part makes a wrong judgement. There is no difference here. Thanks to cinematic reasoning[2], we know that Achilles passes the tortoise, even if our analytic abilities, being unable to grasp the precise moment conceptually, tell us otherwise. If we accept to consider Zeno's paradoxes as cognitive illusions, then we can accept Aristotle's statement about our limitation to potential infinite as well. In the case of time, we are unable to conceptualize infinitely small duration, and we are thus exposed to Zeno's illusion that Achilles remains behind the tortoise.

The fact that mathematics succeeds in characterizing time, by means of the real line, as a set of punctual instants does not contradict our hypothesis. Mathematics offer finite means to describe how infinitely small duration or infinite series behave. It does not follow that we are able to have direct access to such notions. Physics tells us how a photon is supposed to behave, but we must rely on poor and inconsistent metaphors to imagine what photons look like (something like a small sphere or a wave). What Lakoff and Núñez show is that the metaphors we may use to conceptualize points in space or time are grossly misleading: points should have some (tiny) surface, but they don't; a large number of them should occupy some space, but this does not happen; two distinct points should be able to touch each other, but they aren't; and so on. We imagine spatial points as something like small spheres, and instants as tiny duration. This is wrong, but we do not seem to have cognitive alternatives.

If we accept Zeno's lesson[3], actual infinite remains out of reach of the analytic part of our mind. The same is true for digital computers. We must nevertheless account for the fact

[1] Infinitesimals are also very strange cognitive entities, as Lakoff and Núñez demonstrate in their book.

[2] This reasoning may rely on analog devices, as suggested by the literature on mental images and made plausible by dynamical models (van Gelder, 1998).

[3] Zeno's paradoxes were meant as support for Parmenides' theory about the impossibility of any movement. We adopt a milder position here!

that our mind is able to interpret temporal relations of some complexity without being limited in precision.

Representing Time as a Recursive Procedure

Abandoning objective time

We observed that formal or computational models of time structures could not avoid presupposing actual infinite, unless they accept to consider atoms of time. Zeno's repeated dichotomy procedure shows that it can't be otherwise. If we want to avoid both actual infinite and atoms, we must abandon the idea of an objective time structure (Ghadakpour, 1998). This may help solve some related problems encountered in the automated interpretation of natural language. Consider a sentence like "Fifteen billions years ago, during the first three picoseconds of the universe, symmetry got broken". It is not possible to represent the temporal organization of events mentioned in this statement[4] on a fixed temporal structure, for two reasons. First, representing the 10^{30} picoseconds between now and the beginning of the Big Bang would be quite a waste. Second, even if we do so, we will be unable to position the very beginning of the Big Bang with the required precision of less than a picosecond on such a representation. Human beings have no trouble understanding the statement (whatever their opinion about it). How do they perform such a feat?

A first idea to make a consistent model of this performance is to make a distinction between *moments* and *epochs*. Considering this dual ontology avoids the strange situation in which any non-zero interval of time is made of an actual infinity of instants. Epochs are not sets of events. They are interpreted as interpolations between events, while events are located within or outside an epoch. One may object that if moments aren't punctual, there is no difference between a moment and the epoch it covers. As we will see, moments do not cover anything in this model.

A second idea is to make a distinction between *conceptual time* and *phenomenal time*. The former allows us to make distinctions like precedence or to perform temporal reasoning; moments and epochs belong to it. The latter is what provides all concrete information about time: the fact that a falling apple takes about one second to fall down or that it will take one hour to mow the lawn. Phenomenal time makes use of personal memories, but also of factual knowledge: one knows that the Big Bang took place billions year ago, and that a billion years is much more than a picosecond, even if we cannot experience these phenomena directly.

A third idea is to introduce the notion of *temporal map*. A temporal map is a projection from conceptual time onto phenomenal time. It crucially depends on the scale we choose. If we merely refer to the wedding of our friends' parents, twenty years ago, a precision of one year or two is not crucial. If we attempt to locate the wedding by means of some contemporary event, *e.g.* a presidential election, a precision of a few days may be in order. If we refer to the time it took for the bride to say yes during the ceremony, then a precision of one or two seconds is required. So the precision (granularity) in phenomenal time and the scale of the temporal map vary accordingly.

These three principles: moments *vs.* epochs, conceptual time *vs.* phenomenal time, temporal maps of various scales, allow us to interpret the preceding statement relative to the Big Bang asymmetry, what was problematic with a system based on an objective time structure. The interpretation requires two different temporal maps (Figure 1). The first one allows us to locate the moment of the Big Bang as distinct from the present moment. The two moments are separated by an epoch, which projects on phenomenal time at a scale of billions of years[5]. The second map allows us to separate two other moments: the very beginning of the Big Bang and the moment when asymmetry does occur. These two moments are separated by an epoch which projects on phenomenal time at a scale of a few picoseconds (which is known to be a very short time, comparable with elementary operations in electronic circuits). We must now understand how these different maps are successively invoked.

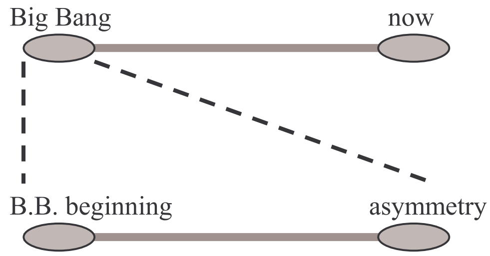

Figure 1: Two successive temporal maps

Recursive event location

The interpretation of a sentence like "In 1912, during the night of April 14, as the Titanic was sinking, a young man made a promise" requires a correct temporal location of events. It is not April 14 of the current year, the promise was not made anywhere in April, and so on. According to our model, this interpretation may require up to five different *temporal maps*[6], corresponding to five different scales (Figure 2).

Part (1) of the process presupposes that we are able to determine the scale of the map. The basic idea is that the scale is just precise enough to allow separation. Since there

[4] The example refers to matter-antimatter asymmetry, but it is not meant to have any scientific validity in physics.

[5] Few people have an accurate perception of the ratio between such quantities and the time of a human life. For most, it's just 'a lot of time'. Our model makes this inaccuracy compatible with the ability to interpret the statement.

[6] Interpretations may take shortcuts. April 14 may be interpreted directly as a given day of 1912. Conversely, we did not locate the night in Figure 2.

are at most three objects on a map (one moment within an epoch, or two moments around one epoch), we need a granularity of less than one third of the total period covered in the phenomenal time. For example, we need a grain which is finer than *5* billion years to separate two events on a period of *15* billion years, but certainly not much finer. Even with such a coarse grain, epochs and moments can occupy a grain each and still remain distinguishable.

This example illustrates the fact that the location of events may require the repeated application of a same process. This process can be sketched as follows:

(1) Locate one or two moments on a temporal map, corresponding to the first events to be located.

(2) When a new event location requires considering the interior of an event, create a new map.

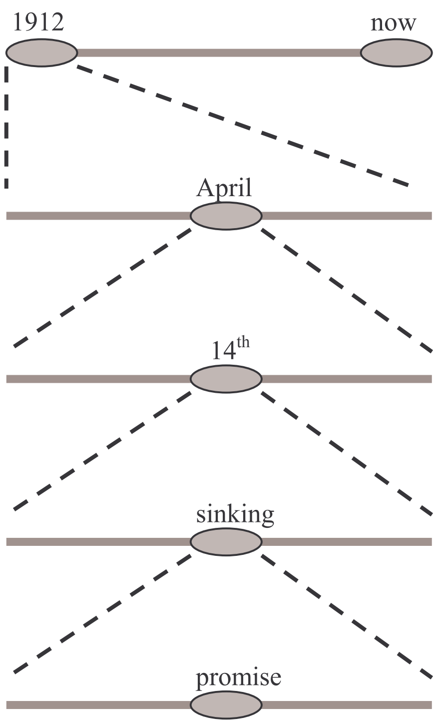

Figure 2: Recursive location of events

Part (2) of the process is what makes the whole operation recursive. It is triggered by the necessity to access the interior of an event, which is inaccessible in the current map. There are a variety of linguistic markers that help to signal the necessity of (2): aspect ('was sinking') as well as grammatical words ('during', 'as'…)[7].

The necessity of phase (2) lies at the core of the model. A temporal map is used to *separate* one event from another, or to *locate* one event within an epoch, *and for nothing else*. Events are represented by moments on a given map. Whenever one wants to convert a moment into an epoch, one has to change the map. By doing so, one changes the scale of the projection onto phenomenal time.

Conclusion: potential infinite precision

The recursive device sketched in the preceding section avoids the major flaw of other systems of time representation, considered as cognitive models. It implements potential infinite, but avoids actual infinite. Thanks to the recursive location of events, we can reach arbitrary accuracy. There is no atom, the system can always 'zoom in' to look inside any period of time. The system is nevertheless immune to infinite regression leading to actual infinite. The reason for this is that it is not based on an objective time structure. Its behavior, when confronted to Zeno's dichotomy procedure, is (perhaps paradoxically) what we should expect from a cognitive model of time: like human beings, it enters a potentially endless loop.

At each step of Zeno's story, our mind creates a temporal map[8]. We distinguish the current moment from the moment when Achilles will reach the tortoise's current position. These two moments are separated by an epoch. But the tortoise is already farther. We are thus asked to distinguish the second moment from a third one, when Achilles reaches the new tortoise's position. This new distinction requires a new map. Unfortunately, this map is identical to the preceding one. It is supposed to project with a more detailed scale on phenomenal time (because the tortoise is slow), but there are no landmarks to ground this scale in mental imagery. All successive maps look alike. This is why Zeno's procedure drives us into a potentially infinite loop. Having no access to actual infinite, and thus to the limit, our analytic mind creates the wrong belief that Achilles is unable to reach the tortoise. As we see, our model accounts for this cognitive illusion.

We are currently working at an implementation of the recursive procedure described in this paper. The mechanism itself is straightforward, but we have to interface it with a database of long term memories and with a natural language processing program, in order to demonstrate it practical usefulness.

The main interest of our model is to replace a fixed time structure, where all remembered events would be represented on a single basic scale, by the ability to locate any given event by time comparison with known events. The point of the paper is to suggest that this ability is achieved by a multi-scale recursive procedure.

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[7] These phenomena have been widely investigated; see (Bach, 1986). Though, there is a tendency to consider these temporal properties as part of lexical semantics, which we do not endorse.

[8] It also creates a spatial map, which bears much resemblance to the temporal one.

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