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Peculiarities and predictions of rogue waves in mid-infrared quantum cascade lasers under conventional optical feedback

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ABSTRACT

Quantum cascade lasers (QCLs) are optical sources exploiting radiative intersubband transitions within the conduction band of semiconductor heterostructures.¹ Mid-infrared QCLs have been thoroughly considered for applications such as spectroscopy,² free-space communications³ and countermeasure systems.⁴ Under conventional optical feedback, QCLs have been proven to operate in several non-linear dynamic regimes, including deterministic chaos,⁵ entrainment of low-frequency fluctuations⁶ and square wave all-optical modulation.⁷ We extend the understanding of non-linear phenomena in QCLs with the experimental study of rogue waves. Rogue waves represent random isolated events with amplitudes well above that of neighboring ones, occurring more often than expected from the distribution of lower amplitude events.⁸ In the optical domain, rogue waves were first demonstrated experimentally in 2007 in the context of super-continuum generation in optical fibers⁹ and have since been observed in a wide variety of configurations such as semiconductor lasers.¹⁰ In QCLs, the extra power from these sudden bursts can be used in order to improve the efficiency of mid-infrared remote sensing or countermeasure systems. It can also be a helpful tool for neurophotonics clusters aiming to reproduce synaptic transmissions in an all-optical system. As a step toward a reliable control over these rare spikes, we carry out a statistical analysis of the interval between rogue events and show that precursors always occur before these events. The advantage of these precursors is to have a characteristic time longer than that found in other semiconductor lasers exhibiting the same non-linear phenomena. Birth of giant pulses like dragon-kings events are also discussed and analyzed.

Keywords: Quantum cascade lasers, Mid-infrared photonics, Optical rogue waves, Dragon-king events

1. INTRODUCTION

Rogue waves have long been considered a maritime legend among others like Krakens or Scylla and Charybdis. But following pictures taken by sailors in the 20th century, they are now closely monitored because not only they exist but they can be harmful even for the largest tankers¹¹ and for deep-sea oil platforms.¹² The careful study of these maritime events started in the middle of the 20th century with the first records of the heights of the swell.^{13,14} Thorough efforts have then allowed to draw a global picture of the rogue wave phenomenon in the ocean, with recent papers still debating about their origin and occurrence rate.^{15,16} Before 2007 and the first observation of optical rogue waves, this term was exclusively utilized to describe water waves. Following the discovery of events with similar properties but sometimes different mechanisms, the term 'rogue waves' has now been extended to a wide range of domains such as economics,¹⁷ acoustics¹⁸ and astrophysics.¹⁹ The similarity

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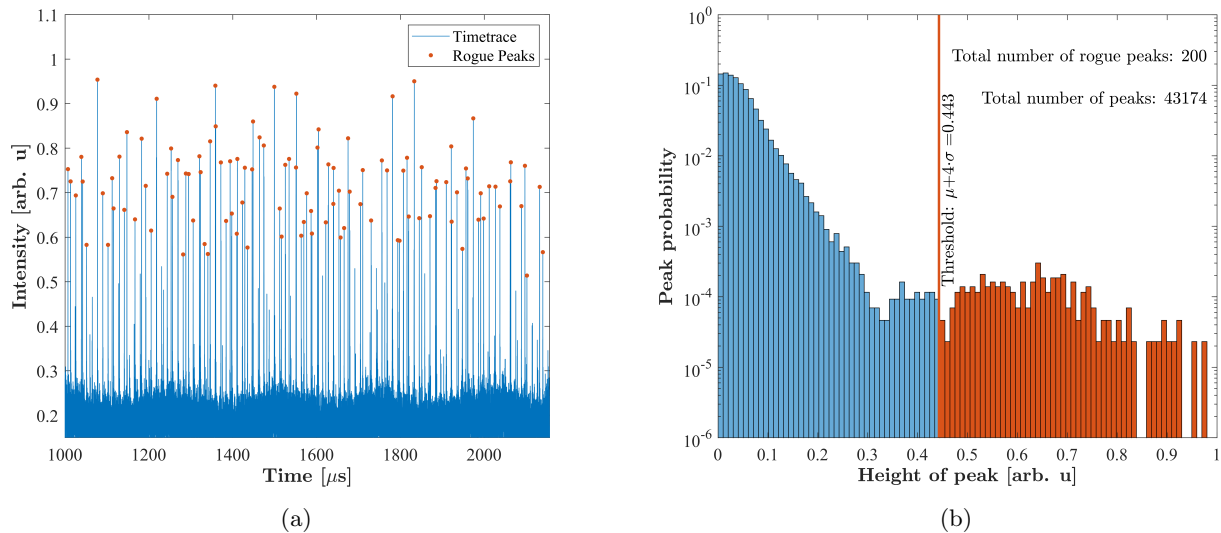


Figure 1: (a) Experimental time trace with 200 events (out of 43174) meeting the rogue wave criterion. For the sake of clarity, only half of the time trace is displayed; (b) Histogram of the retrieved maxima intensity and the related threshold for the criterion $\mu + 4 \times \sigma$.

with the oceanographic concept resides in the threshold criterion to take into account in order to consider an event as a rogue wave. In order to detect rogue waves from experimental data, the probability density function (PDF) of wave heights is commonly used.⁸ Once extracted from the data, heights are compared to a threshold value above which events can be considered as rogue. Various threshold values can be found in the literature and the strictest one, leading to the identification of the lowest number of extreme events, should be used to account rogue waves. This term is applied for single waves that are extremely unlikely as judged by the Rayleigh distribution of wave heights.²⁰ In practice this means that when one studies wave dataset of a finite length, a wave is considered to be a rogue wave if the wave height is higher than 2 times the significant height H_s of the dataset,²¹ or sometimes 2.2 times H_s ,²² which represents the average of the higher third of heights. Another threshold widely used relies on the standard deviation (σ) and the mean value (μ) of the data and an event is considered as extreme if it exceeds $\mu + 8 \times \sigma$.¹⁰ However, a criterion with a lower threshold (down to $\mu + 4 \times \sigma$) is also commonly used without loss of generality.^{23,24} This threshold alone is however not sufficient to identify rare events and it is necessary to verify that the PDFs represent long-tailed distributions. We have already reported experimental rogue wave results in QCLs when taking into account the most restrictive threshold ($\mu + 8 \times \sigma$) to ensure that the extreme pulses we observed can be considered rogue.²⁵ In this paper, we focus on a lower threshold, namely $\mu + 4 \times \sigma$, in order to compute relevant statistics with more events. As we will see hereafter, these events still comply with the PDF distortion and are not events belonging to the Gaussian distribution.

2. ROGUE WAVES IN QCLS

Following the efforts about water rogue waves, the concept of optical rogue waves was first introduced in the context of super-continuum generation in optical fibers at near-infrared wavelength.⁹ The experiments allowed observing pulses with extreme intensities and narrow timescales occurring rarely, yet much more frequently than expected based on the relatively narrow distribution of typical events. The statistical distribution of the events was indeed characterized by a L-shape showing a distortion of the regular Gaussian distribution. Experimental findings were backed by numerical simulations relying on the generalized Non-Linear Schrödinger Equation. The model was a powerful tool to confirm the soliton behaviour of the observed extreme events and it paved the way for the study of rogue events, not only in optical systems, but also in several physical systems because it was the first demonstration of such phenomenon apart from hydrodynamics. The concept of rogue waves has then been extended to several optical domain, such as nonlinear driven cavities,²⁶ Raman fiber amplifiers and

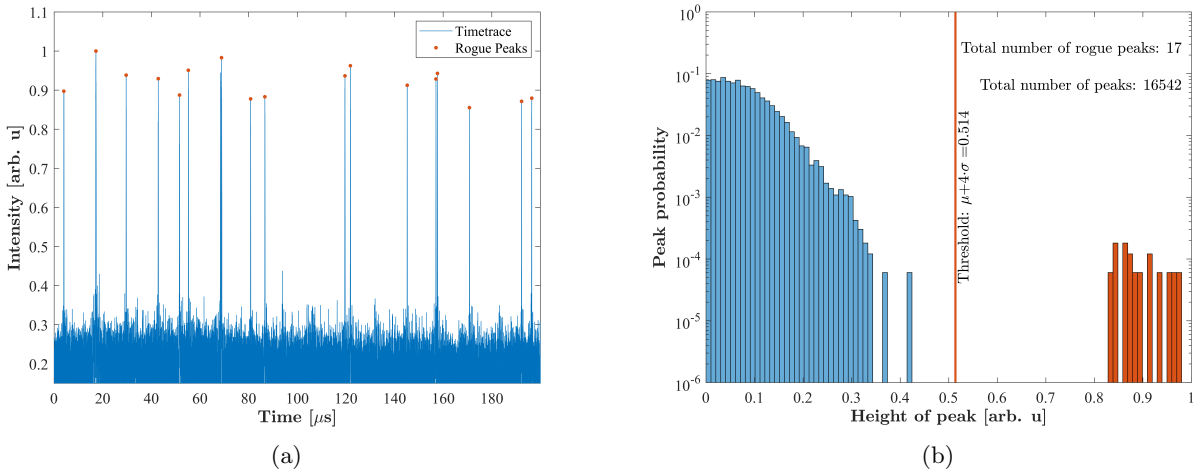


Figure 2: (a) Experimental time trace with 17 dragon-king events meeting the rogue wave criterion; (b) Histogram of the retrieved maxima intensity and the related threshold for the criterion $\mu + 4 \times \sigma$.

lasers,^{27,28} pump-modulated fiber lasers,²⁹ Kerr lens mode-locked Ti:Sapphire lasers,²³ fiber ring lasers³⁰ and optically injected semiconductor lasers.¹⁰ For the latter, the laser under study was a VCSEL biased below the limit of single transverse mode emission and its output wavelength was around 980 nm. Another similar laser was used for the injection process. Close to the zero detuning condition, the slave laser can generate giant intensity pulses. A careful analysis of the probability density function confirmed a long-tailed distribution of these extreme pulses, which can therefore be accounted as rogue waves. Simulations based on a simple noise-free rate equation model gave results in qualitative agreement with the experiments, allowing for the interpretation of the sporadic high amplitude pulses as the result of a deterministic nonlinear process.

The experimental setup to observe rogue waves in a mid-infrared QCL is described in Ref. 25. The time trace in Fig. 1 a) shows sudden bursts surging high above the mean value of the QCL's intensity. The rogue spikes are illustrated with a red dot and this means that the maximum of the spike is larger than 0.443. Further details about the distribution of the bursts can be found in Fig. 1 b), which describes the logarithmic probability for a spike of a given magnitude. The observation time was chosen as a compromise between the number of displayed extreme events and a sufficient resolution to ensure that the retrieved events were not artifacts. It can be seen that most of the events are gathered in the linear part of the distribution (between 0 and 0.3). However, there is a clear long-tail distribution in the statistics which is incompatible with a regular distribution of maxima and this means that events with a large amplitude appear more frequently than expected. Within the statistics, the rogue events are underlined with a red color, meaning that they are above the $\mu + 4 \times \sigma$ threshold. In total, there are 200 rogue peaks among more than 40000 peaks in the time trace lasting for 2.1 ms.

3. DRAGON-KING EVENTS

The notion of dragon-king events was first introduced in 2009³¹ even if corresponding events have long been spotted.³² Dragon-king have been observed in many different contexts, ranging from avalanches³³ to bursting neurons.³⁴ They are also characterized by giant events, but in this case, these events follow a power law except the occasional very large events which have a higher probability of occurrence. One main characteristic of rogue waves distributions is that they are scale free, which means that events of arbitrarily large sizes are caused by the same dynamical mechanisms governing the occurrence of small and giant events. In contrast, giant pulses related to dragon-king events cannot be explained by a scale free distribution. This means that dragon-king events possess a distinct formation mechanism which may ease the prediction of these giant pulses.³⁵

As it is a sub-category of rogue waves, no study of the dragon-king events in optical systems have been reported so far. This does not mean that some of the aforementioned efforts on optical rogue waves do not partially contain dragon-king among conventional rogue waves. In our study of rogue waves in QCLs, we have observed

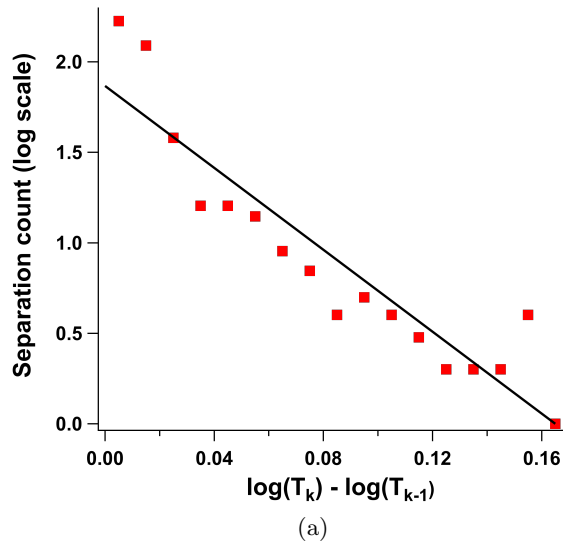


Figure 3: Statistical distribution of the time intervals between rogue events in log scale and the associated linear regression.

a quite rare configuration where giant pulses do not follow the conventional PDF (like the one in Fig. 1). In this configuration, giant pulses all have more or less the same amplitude and consequently, there is a gap in the PDF. As this case rarely occurred, we were not able to compute enough data to draw a PDF showing a local minimum and then a local maximum for a very large value of the amplitude. Figure 2 shows the time trace with the potential dragon-king events and the related PDF. The waveform looks very similar to the one in Fig. 1, except that no pulses with intermediate size occur. Only 17 giant events were spotted amid the 200 μs time trace, composed of more than 16000 peaks.

4. LONG-SCALE PRECURSORS OF ROGUE WAVES

In the following, we are going to focus on methods to predict giant pulses in QCLs. One of the characteristics of rogue waves is that the time interval between the events we is not easily foreseen. Other studies have shown that the separation time (S) between rogue events follows a Poissonian distribution³⁶ when the time elapsed between these events (numbered $k - 1$ and k) is written in log scale: $S = \log(T_k) - \log(T_{k-1})$. This leads to a linear distribution for of the inter rogue-wave time intervals.³⁷ A deviation of the statistics is sometimes found for high feedback ratios,³⁸ and in this case, the curve is characterized by two slopes instead of one. In our case, Fig. 3 shows the evolution of the separation count in log scale with S when taking into account 419 rogue waves intervals from four different time traces. The evolution is linear as encountered in other semiconductor lasers. Even if we are in a configuration with a very high feedback strength, it seems like a linear regression with one slope is in good agreement with our data, contrary to the result given in Ref. 38 for a high feedback strength. In some semiconductor lasers exhibiting rogue waves, these extreme events do not really occur as isolated pulses because they keep the signature of the time-delayed feedback.^{38,39} On the one hand, this means that they can mostly be predictable and this can be useful in order to prevent harmful rogue waves in optical systems. On the other hand, the bursting process relies on the external cavity period which is in the nanosecond range, so this makes the prediction difficult to achieve at such fast scale. In our configuration with QCLs, we also have an external cavity with a period of less than 10 ns but the observed dynamics occur at a longer time scale. The typical time scale of the extreme burst is in the microsecond range and there is also a clear long-term signature when looking at the global picture. Figure 4 a) shows the superposition of the rogue events of Fig. 1 when they are all centered at $t = 250 \mu\text{s}$. Rogue events are more likely to occur in some areas, especially every 12.8 μs or every 6.4 μs as illustrated with the purple solid line for the average of all the superpositions. We were not able to link these large time scales to any of the parameters of our experimental apparatus and further study is required to understand why extreme events are more likely to occur in a given time interval. A closer look at

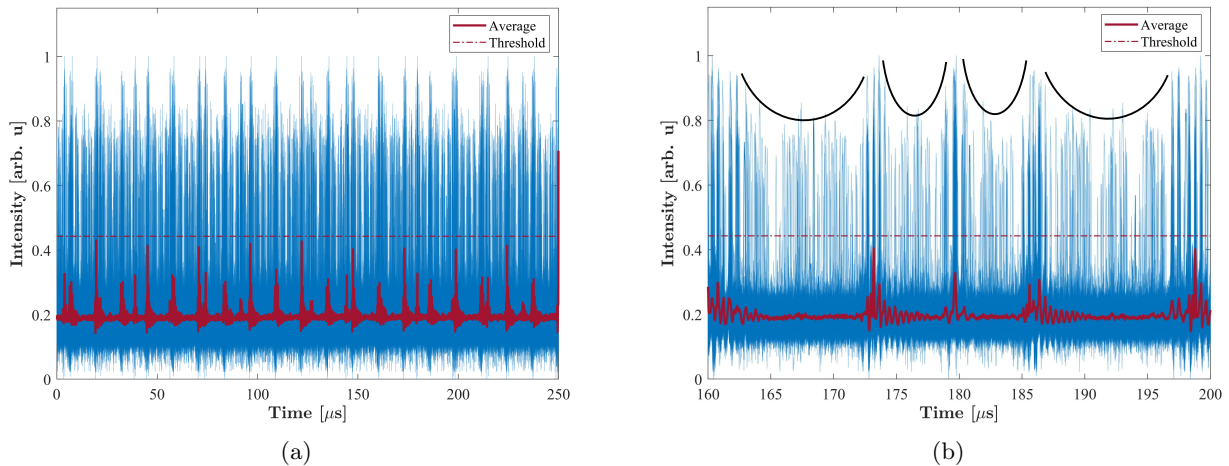


Figure 4: (a) Time series centered on the maximum of local bursts (the reference value is $t = 250 \mu\text{s}$) and superposition of 200 rogue events. The corresponding averaged output power is displayed with a purple solid line and the threshold corresponding to $\mu + 4 \times \sigma$ is displayed with a dash-dot line; (b) Same diagram with a close-up on several areas of the superposition exhibiting regions where the largest pulses are more likely to occur even though rogue waves can occur anywhere. The black solid curves enclose forbidden regions for intermediate-size pulses.

the distribution of rogue peaks in Fig. 4 b) shows that extreme bursts, which are depicted in blue and are above the dash-dot purple line, can almost occur any time. However, the largest rogue peaks only occur around the aforementioned time intervals and this is emphasized by the black solid curves where almost no events surging above 0.8 occur. This result is relevant for applications where optical rogue waves can be harmful, because this means that the largest events (and consequently the most hazardous) can be well predicted once a first one has been detected. Moreover, the time scale in the order of the microsecond (1000 times larger than that found in laser diodes) eases the control process. It is interesting to notice that similar long-scale dynamics have already been studied in other semiconductor lasers under external optical feedback or optical injection. More precisely, highly damped semiconductor lasers, like quantum dot lasers and QCLs, have been recently found to generate square waves with a period in the microsecond range.^{7,40} One of the possible explanations for such behavior could be related to the Van der Pol-Fitzhugh-Nagumo model,⁴¹ consisting in a general scenario of relaxation oscillations which are characterized by the competition of two timescales: the timescale of carriers in the nanosecond range and a thermal timescale in the microsecond range. In the case of dragon-king events, we were not able to compute a relevant diagram for S and for the superposition of the spikes because the number of retrieved rogue waves is too sparse to clearly see any relationship.

5. CONCLUSIONS

In this paper, giant spikes in a QCL under external optical feedback are investigated owing to the PDF and the statistical distribution of the time intervals. Two kinds of extreme pulses are discussed, namely dragon-king events and rogue waves, and the latter is more precisely analyzed because it is the most common case, in an experimental point of view. In order to predict and have a high reliable control over these rogue spikes, we unveil the birth of precursors taking always place before these events occur. The main advantage of rogue waves in QCLs is that their characteristic time is longer than the one typically observed in other semiconductor lasers. The reason for such a difference could be explained by the strong thermal effects in QCLs, thus introducing a more complex coupling than in usual time-delay dynamics. This hypothesis however needs further modelling and experiments to be confirmed. Overall, we believe that these results are useful for improving the efficiency of mid-infrared remote sensing or countermeasure systems as well as for future research on neurophotonics clusters aiming at reproducing synaptic transmissions in an all-optical system.

Acknowledgments

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