Gain Compression Effect on the Modulation Dynamics of an Optically Injection-Locked Semiconductor Laser using Gain Lever

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ABSTRACT

The modulation response of an optically-injected gain lever semiconductor laser is studied and calculations show that a gain-lever laser operating under medium to strong optical injection provides a unique and robust configuration for ultra large bandwidth enhancement. Modulation bandwidths above nine times the relaxation oscillation frequency of the free-running laser can be reached using injection-locking conditions that are reasonable for practical applications. The impact of the gain compression on the modulation dynamic is discussed for the first time. This work is of prime importance for the development of directly-modulated broadband optical sources for high-speed operation at 40 Gbps and beyond.

Keywords: Semiconductor laser, injection-locking, gain lever, nonlinear gain

1. INTRODUCTION

Direct modulation of semiconductor lasers at high frequencies is a major challenge in the development of low-cost fiber optic communications, in particular for future access networks.\textsuperscript{1} In these systems, the modulation bandwidth (in GHz) of a directly modulated laser (DML) is the most important figure-of-merit that determines the maximum data rate (in Gbps) achievable. In order to improve the performance and capacity of such optical networks for high-speed operation, it is necessary to first enhance the modulation efficiency and bandwidth of the optical transmitters. This must however be done in an energy-efficient way, without increasing the laser’s intensity noise or low-signal distortion, and without suffering from optical frequency deviation (frequency chirp).

The bandwidth of conventional DMLs remains strongly limited by the intrinsic relaxation oscillation frequency (ROF) of the laser gain medium, ranging from a few to tens of GHz.\textsuperscript{2} Over the past few years, tremendous efforts have been made to improve the modulation dynamics of semiconductor lasers. Indeed, most of the aforementioned effects can be attained using high-quality materials and optimization of the device structure\textsuperscript{3, 4}. Most importantly, similar and further improvements can be achieved using nonlinear photonic techniques such as the use of optical injection-locking (OIL),\textsuperscript{5} or optical gain lever (GL) schemes.\textsuperscript{6} On the first hand, OIL relies on the locking of a slave laser (SL) at the frequency of the light from a master laser (ML), which is directly injected into the SL cavity. Once the SL is perfectly locked within the stable area, OIL allows pushing the resonance peak to very high frequencies without affecting the modulation efficiency but at the expense of a pre-resonance frequency dip.

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occurring in the modulation response.\textsuperscript{7} On the other hand, GL takes advantage of the sublinear relationship between optical gain and pump current in a semiconductor laser, due to the gain saturation occurring with increasing carrier density. Consequently, the standard GL architecture relies on the use of a semiconductor laser with two separate electrical regions: a long section that is continuous-wave (CW) biased far above threshold where carriers are clamped (low differential gain) and a much shorter one modulated and pumped close to the optical transparency (large differential gain). The GL photonic chip thus provides a monolithic way to improve and shape the modulation performance of semiconductor lasers with little impact on the ROF and without any requirement on the optical properties of the laser, hence requiring little modification of its design and fabrication. Unfortunately, because the improved modulation bandwidth is achieved at the expense of linearity in the gain versus injection current curve, a major drawback of GL lasers is the increase of non-linear distortion. Very recently, the modulation response of an optically-injected gain lever (OIGL) semiconductor laser was studied for the first time using small-signal analysis of a rate equation model. Calculations showed that a GL laser operating under medium to strong optical injection constitutes a unique and robust configuration for ultra-large bandwidth enhancement. Modulation bandwidths above nine times the ROF of the free-running laser were predicted using injection-locking conditions that are reasonable for practical applications. This paper goes a step beyond by investigating the effects of the nonlinear gain through the gain compression factor on the modulation dynamic of OIGL lasers. Calculations reveal that in case of the OIGL laser, the gain compression does not impact that much the modulation performance and can even be used to properly tailor the flatness of the modulation response. This result is found to be drastically different from the sole GL laser or the OIL laser without GL. As such, this theoretical work is of prime importance for the development of directly-modulated broadband optical sources for high-speed operation in optical access networks as well as for future silicon photonics integrated circuits.

2. RATE EQUATION MODEL

Fig. 1(a) depicts schematically the GL laser under study that is composed by two sections, sharing the same active area but electrically isolated. The first section ((a)) is very short and pumped close to the optical transparency at $I_a + I_{RF}$ as shown in Fig. 1(a) representing the evolution of the material gain as a function of the carrier density. In order to ensure the GL effect, $I_a$ is chosen such that the steady-state optical gain of section (a) ($G_{a,0}$) remains close to zero. Section (b) is CW-pumped well above the lasing threshold hence the steady-state optical gain ($G_{b,0}$) is near the threshold value ($G_{th}$). Besides, it is important to note that, parameter $b$ which represents the fractional length of section (b) is taken close to unity so as to strengthen the GL effect. The large difference between the differential gains of each section ($G_{a}' \gg G_{b}'$) leads to a significant change of the carrier density in the section (b) while keeping only a small variation of current in the modulation section (a). Fig. 1(b) depicts schematically the OIL effect. The light output from the ML is injected into the SL cavity. The two relevant parameters for OIL is the frequency detuning between the ML and SL $\Delta f = f_{ML} - f_{SL}$, where $f_{(M/S)}$, is the lasing frequency of the ML or SL and the injected power in the SL cavity $P_{ML}$.

The analysis of the OIGL semiconductor laser is traditionally described via a set of differential rate equations as in\textsuperscript{8} including one for carrier density in each section, one for the photon density and finally one for the phase. In this paper, this model is extended so as to include the gain saturation through a gain compression factor. To this end, taking into account the nonlinear gain coefficient, the rate equation model can be expressed as follows:
\begin{align}
\frac{dN_{e,-a}}{dt} &= \frac{J_a}{\epsilon D} - \frac{N_{e,-a}}{\tau_{c,a}} - G_a N_{\gamma}, \quad (1a) \\
\frac{dN_{e,-b}}{dt} &= \frac{J_b}{\epsilon D} - \frac{N_{e,-b}}{\tau_{c,b}} - G_b N_{\gamma}, \quad (1b) \\
\frac{dN_{inj}}{dt} &= \left[\Gamma \left(G_a (1-h) + G_b h\right) - \frac{1}{\tau_p}\right] N_{\gamma} + 2k_c \sqrt{N_{\gamma,inj} N_{\gamma}} \cos(\phi) + R_{sp}, \quad (1c) \\
\frac{d\phi}{dt} &= \left[\Gamma \left(G_a (1-h) + G_b h\right) - \frac{1}{\tau_p}\right] \frac{\alpha_H}{2} - \Delta\omega_{inj} - k_c \sqrt{N_{\gamma,inj} N_{\gamma}} \sin(\phi), \quad (1d)
\end{align}

where \( N_{e,-k} \) denotes the carrier density, \( J_k \) the current density, \( \tau_{c,k} \) the carrier lifetime and \( G_k \) the uncoupled material gain of section \( k \). In addition, \( N_{\gamma} \) and \( \tau_p \) are the photon density and photon lifetime of the SL cavity, respectively, while \( \alpha_H \) corresponds to the linewidth enhancement factor. Regarding the optical injection, \( N_{\gamma,inj} \) is the photon density injected with a coupling factor efficiency \( k_c \) and detuned from the free-running SL by \( \Delta\omega_{inj} = 2\pi\Delta f \), with \( \phi \) the phase offset between ML and SL. Finally, let us recall that \( h \) is the fractional length of section \( b \) while \( \Gamma \) is the optical confinement factor.

In (1a), \( R_{sp} \) represents the spontaneous emission rate that is expressed as \( R_{sp} = \Gamma \beta \left[\left(1-h\right) \frac{N_{e,-a}}{\tau_{c,a}} + h \frac{N_{e,-b}}{\tau_{c,b}}\right] \) with \( \beta \) the spontaneous emission factor. In the numerical analysis, \( R_{sp} \) is only used as a noise source to seed the simulations hence to extract the steady-state values. When the modulation dynamics are further analyzed, the spontaneous contribution is ignored as it plays little role with regards to the operating conditions under study.\(^2\)

According to,\(^2\) a logarithmic gain is then taken into account such that:

\begin{equation}
G_k(N_{e,-k}, N_{\gamma}) = \frac{G_{k,0}}{1 + \varepsilon N_{\gamma}} \ln \left(\frac{N_{e,-k} + N_a}{N_{tr} + N_a}\right), \quad (2)
\end{equation}

where \( \varepsilon \) represents the gain compression factor, \( N_{tr} \) the carrier density at transparency, \( N_a \) a fitting parameter,\(^3\) and \( G_{k,0} \) the steady-state optical gains. Once the intra-cavity photon density exceeds a certain value, the product \( \varepsilon N_{\gamma} \) (typically \( \varepsilon \approx 10^{-17} \text{ cm}^3 \)) is larger than the unity, hence the impact of the gain nonlinearities are dominant and may become problematic for high-speed applications. As already described in,\(^8\) the modulation response of the OIGL can be derived from a small-signal analysis of the rate equations by considering \( J_k, G_k, N_{e,-k}, N_{\gamma} \) and \( \phi \) as dynamical variables such that:
\[ \begin{align*}
\text{d}X &= X_1 e^{i \omega t}, \\
\text{d}G_k &= \frac{G'_{k,0}}{1 + \varepsilon N_0^y} \text{d}N_{e^{-k}} - \frac{\varepsilon G_k}{1 + \varepsilon N_0^y} \text{d}N_{\gamma},
\end{align*} \]

where \( X \) refers to one of the previous dynamical variables (except \( G_k \)). Relationship (3b) is obtained from the derivation of equation (2) with respect to \( N_\gamma \) and \( N_{e^{-k}} \). The differential gain \( G'_{k,0} \) is defined as the partial derivative of the gain with respect to carrier density: \( G'_{k,0} = \partial G_k/\partial N_{e^{-k}} = G_{k,0}/\left( N_0^y + N_a \right) \) where \( N_0^y \) and \( N_{e^{-k}} \) are defined as the steady-sates solutions of the rate equations given in set (1).

Under small-signal analysis and asymmetric-bias conditions \(( h = 1 \) and \( J_a \ll J_b \)), the novel normalized transfer function \( |R(f)|^2 \) of the OIGL laser incorporating the gain compression is expressed as follows:

\[ |R(f)|^2 = \frac{(A_0 + A_0)^2 \left( (\eta + A^2_2) f^2 \right)^2 + (A_1 f)^2}{(A_1 + A_1 f - (A_3 + A_3) f^3)^2 + [\eta (A_0 + A_0) - (A_2 + A_2 f)^2 + f^4]^2} \]

Where

\[ A_0 = \frac{\eta (\gamma_0^c)^2 g_c^c + \gamma_0^c g_c^c 3_{\phi,\alpha_H} \sigma_c^c}{16\pi^2}, \quad A_1 = \left[ \gamma_0^c (g_c^c + 1) \eta^2 + 2\eta \cos(\phi_0) (\gamma_0^c)^2 g_c^c + \sigma_c^c (\gamma_0^c g_c^c + \eta 3_{\phi,\alpha_H}) \right] \frac{1}{8\pi^3}, \]

\[ A_2 = \left[ \eta^2 + 2\eta \cos(\phi_0) (\gamma_0^c)^2 g_c^c + \sigma_c^c \right] \frac{1}{4\pi^2}, \quad A_3 = \left[ 2\eta \cos(\phi_0) + \gamma_0^c (g_c^c + 1) \right] \frac{1}{2\pi}, \]

\[ A_1' = \frac{2\pi}{3_{\phi,\alpha_H} \gamma_b^c} + \frac{2\pi}{\gamma_b^c}, \quad A_2' = \frac{4\pi^2}{\gamma_b^c 3_{\phi,\alpha_H}}, \]

\[ \sigma_c^c = \frac{1}{1 + \varepsilon N_0^y} \left( \frac{1}{\tau_p} - 2\eta \cos(\phi_0) \right) \left( \gamma_b^c - \frac{1}{\tau_{c,b}} \right), \quad 3_{\phi,\alpha_H} = \cos(\phi_0) - \alpha_H \sin(\phi_0), \]

\[ \gamma_k^c = \frac{\gamma_k}{1 + \varepsilon N_0^y}, \quad \gamma_b^c = \frac{\varepsilon N_0^y}{\tau_{c,b} (1 + \varepsilon N_0^y)}, \quad g_c^c = \frac{\gamma_b^c}{\gamma_k^c}, \]

and

\[ A_0' = \frac{\chi_c^c (\gamma_0^c)^2 g_c^c 3_{\phi,\alpha_H}}{8\pi^3}, \quad A_1' = \frac{\chi_c^c}{8\pi^3} \left[ \gamma_0^c (g_c^c + 1) \eta 3_{\phi,\alpha_H} + (\gamma_0^c)^2 g_c^c \right], \]

\[ A_2' = \frac{\chi_c^c}{2\pi} (\gamma_0^c 3_{\phi,\alpha_H} + \gamma_0^c (g_c^c + 1)) \quad A_3' = \frac{\chi_c^c}{2\pi}, \]

\[ \chi_c^c = \frac{\varepsilon N_0^y}{1 + \varepsilon N_0^y} \left( \frac{1}{\tau_p} - 2\eta \cos(\phi_0) \right). \]

Equation (4) is nearly similar to the one found in our previous analysis without gain compression. Indeed, we find here the same coefficients \( A_0, A_1, A_2, A_3, A_1' \) and \( A_3' \) where the \( \sigma \)-coefficient, the damping rate factor \( \gamma_k \), and the damping rate ratio \( g \) depend on \( \varepsilon \) (equation (5)). Additionally, four new \( \varepsilon \)-dependent coefficients are introduced \( A_0', A_1', A_2', A_3' \). In what follows, simulations investigate the effects of the gain compression on the modulation dynamics for the free-running case without GL as well as for the GL and OIGL laser configurations.

### 3. Simulation Results

Tab. 1 gives all the material and laser parameters used in the calculations. All simulations are performed assuming three different values of the optical injection, both with GL effect \((g = 10)\) and without \((g = 1)\). The
Table 1: Material and Laser Parameters

<table>
<thead>
<tr>
<th>Simulation parameters</th>
<th>Symbols</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Cavity length</td>
<td>$L$</td>
<td>$500 \times 10^{-6}$ m</td>
</tr>
<tr>
<td>Mirror reflectivity</td>
<td>$R_1 = R_2$</td>
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</tr>
<tr>
<td>Bias current (section a)</td>
<td>$I_a$</td>
<td>18 mA</td>
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<tr>
<td>Bias current (section b)</td>
<td>$I_b$</td>
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</tr>
<tr>
<td>Steady-state optical gain (section b)</td>
<td>$G_{0,b}$</td>
<td>$1.8 \times 10^{13}$ s$^{-1}$</td>
</tr>
<tr>
<td>Internal modal losses</td>
<td>$\alpha_i$</td>
<td>$14 \times 10^{2}$ m$^{-1}$</td>
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<tr>
<td>Mirror losses</td>
<td>$\alpha_m$</td>
<td>$22.8 \times 10^{2}$ m$^{-1}$</td>
</tr>
<tr>
<td>Optical index</td>
<td>$n_g$</td>
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<tr>
<td>Carrier lifetime</td>
<td>$\tau_c$</td>
<td>$0.1 \times 10^{-9}$ s</td>
</tr>
<tr>
<td>Photon lifetime</td>
<td>$\tau_p$</td>
<td>$3.17 \times 10^{12}$ s</td>
</tr>
<tr>
<td>Linewidth Enhancement Factor</td>
<td>$\alpha_H$</td>
<td>2</td>
</tr>
<tr>
<td>Coupling S-M factor</td>
<td>$k_c$</td>
<td>$1 \times 10^{11}$ s$^{-1}$</td>
</tr>
<tr>
<td>Damping rate factor (section b)</td>
<td>$\gamma_b$</td>
<td>20 GHz</td>
</tr>
</tbody>
</table>

Gain compression factor $\varepsilon$ is varied between $10^{-17}$ cm$^3$ and $5 \cdot 10^{-16}$ cm$^3$. Like the gain compression induces a variation of the ROF, the latter is systematically recalculated for each value of $\varepsilon$ and then used to normalize the corresponding frequency detuning.

![Figure 2: Modulation transfer functions of the free-running GL laser calculated from 4 and for $\varepsilon = \{0, 10^{-17}, 5 \cdot 10^{-17}, 10^{-16}, 5 \cdot 10^{-16}\}$ with (a) $g = 1$ and (b) $g = 10$. The red dotted curve represents the reference ($g = 1, \varepsilon = 0$)](image)

The modulation response is first studied for different values of $\varepsilon$ without injection, i.e with $K = 0$ and $\Delta f/f_R = 0$ (Fig. 2). To this end, Fig 2(a) shows the effect of the gain compression on the free-running response and without GL. First, simulations show that the ROF is affected by the compression hence decreasing from 13 GHz for $\varepsilon = 0$ cm$^3$ down to 0.8 GHz for $\varepsilon = 10^{-16}$ cm$^3$. When the compression is too large i.e $\varepsilon = 5 \cdot 10^{-16}$ cm$^3$, the ROF can even completely disappear. Besides, as already reported in, the decrease of the ROF is associated to a large diminution of the 3-dB modulation bandwidth. For instance, between $\varepsilon = 0$ cm$^3$ and $\varepsilon = 10^{-16}$ cm$^3$.
the modulation bandwidth is cut by about 25%, from 20 GHz to 15 GHz. Finally, it is important to stress that the modulation efficiency is also drastically reduced, from about 12 dB ($\varepsilon = 0$ cm$^3$) to almost 0 dB ($\varepsilon = 10^{-16}$ cm$^3$). This data set confirms the validity of the model for a free-running laser operating without GL effect as it reproduces well the effect of the gain nonlinearities on the modulation dynamics.

Fig. 2(b) now depicts the modulation responses without optical injection but with strong GL ($g = 10$). In this configuration, although the increase of the gain compression still induces a modification of the ROF, its impact on the modulation bandwidth is reduced. For instance, for $g = 10$, the 3-dB bandwidth decreases by only 4% from 47 GHz ($\varepsilon = 0$ cm$^3$) to 45 GHz for ($\varepsilon = 10^{-16}$ cm$^3$) instead of 25% for $g = 1$. It is also important to note that the modulation efficiency obtained for $g = 10$ and $\varepsilon = 10^{-16}$ cm$^3$ (brown curve) is similar to the one found with $g = 1$ and $\varepsilon = 0$ cm$^3$ (red dotted curve). As a consequence, the simulations show that by damping the relaxation oscillations and reducing the modulation efficiency at this resonance frequency, the gain compression in fact balances the major drawback of the GL and this without greatly affecting the modulation bandwidth.

![Modulation transfer functions](image_url)

**Figure 3:** Modulation transfer functions of the optically injection-locked gain-lever (OIGL) laser calculated from Eq. 4 for $\varepsilon = \{0, 10^{-17}, 5 \cdot 10^{-17}, 10^{-16}, 5 \cdot 10^{-16}\}$ with $\{K = 3.5, \Delta f/f_R = 0.97\}$ with (a) $g = 1$ and (b) $g = 10$. Red dotted curve represents the response function for $\{g = 1, \varepsilon = 0, K = 0, \Delta f/f_R^0 = 0\}$ in both (a) and (b) cases.

Fig. 3 (a) and Fig. 4 (a) shows modulation responses under OIL only with $\{K = 3.5, \Delta f/f_R = 0.97\}$ and $\{K = 8, \Delta f/f_R = 3.3\}$, respectively. Blue curves confirm the validity of the semi-analytical model since the resonance frequency of the injection-locked oscillator is shifted towards higher frequencies as the detuning and injection strength increase, but at the expense of a pre-resonance frequency dip occurring in the strong optical injection cases. Although a larger gain compression coefficient slightly enhances the resonance frequency, both the modulation bandwidth and the amplitude efficiency are considerably reduced, as for in the free-running case. Indeed, for $\varepsilon = 10^{-17}$ cm$^3$ and under strong injection, the reduction of the modulation bandwidth does not exceed 4% (Fig 4(a)). However, the situation is much different under weak injection where the 3-dB bandwidth is cut by approximately 36% for the same level of gain compression (Fig 3(a)). This effect is even worse for $\varepsilon > 10^{-17}$ cm$^3$ for which the bandwidth is reduced by up to 70% for both weak and strong optical injection.
Figure 4: Modulation transfer functions of the optically injection-locked gain-lever (OIGL) laser calculated from Eq. 4 for \( \varepsilon = \{0, 10^{-17}, 5 \cdot 10^{-17}, 10^{-16}, 5 \cdot 10^{-16}\} \) with \( K = 8, \Delta f/f_R = 3.3 \) and (a) \( g = 1 \) and (b) \( g = 10 \). Red dotted curve represents the response function for \( \{g = 1, \varepsilon = 0, K = 0, \Delta f/f_R = 0\} \) in both (a) and (b) cases.

As demonstrated in, the GL constitutes a promising solution to compensate the drawbacks induced by both the optical injection and the gain compression. For instance, Fig. 3(b) and 4(b) show the modulation responses of the OIGL laser. As aforementioned stated, the GL effect offsets the 3-dB bandwidth and improves the flatness of the modulation transfer function. Thus, no matter what the optical injection conditions are i.e. weak (Fig. 3(b)) or strong (Fig. 4(b)), the decrease of the modulation bandwidth is less than 4\% by comparison with the case without gain compression. The best situation is actually found for \( \varepsilon = 10^{-16} \) cm\(^3\) (brown curve), where the modulation transfer function is nearly flat, without any significant resonance peak. For this particular value of the gain compression, the modulation efficiency appears always lower than what is found without GL, OIL and gain compression (red dotted curve). Consequently, improved dynamic characteristics can be further expected by using semiconductor materials such as quantum dots, which typically exhibit compression factors ranging between \( \varepsilon = 10^{-16} \) cm\(^3\) and \( \varepsilon = 10^{-15} \) cm\(^3\).

4. CONCLUSIONS

This paper investigates for the first time the effects of gain nonlinearities in an OIGL laser. By using a small-signal analysis of a rate equations model, a novel formulation of the modulation response is derived. Calculations show that the gain compression is not necessarily a limiting factor affecting the modulation dynamics of the OIGL laser. By considering a practical injection strength, a high GL effect and a compression value of about \( \varepsilon = 10^{-16} \) cm\(^3\), modulation bandwidths close to 85 GHz are anticipated associated to tolerable modulation efficiencies (inf 10 dB). Further work will also investigate the potential of using OIGL with quantum dot semiconductor materials.

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