

Influence of the Linewidth Enhancement Factor on the Modulation Response of a Nanostructure based Semiconductor Laser Operating under External Optical Feedback

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ABSTRACT

The knowledge of the linewidth enhancement factor (α_H -factor) is very important to understand the performance of semiconductor lasers. It affects several fundamental aspects such as the linewidth, the laser's behavior under optical feedback, the chirp under direct modulation and the occurrence of the filamentation. The dramatic variation in the α_H -factor that has been reported for quantum dot lasers makes them an interesting subject for optical feedback studies. In the particular case of QD lasers, the carrier density is not clearly clamped at threshold. The lasing wavelength can switch from the ground state to the excited state as the current injection increases meaning that a carrier accumulation occurs in the ES even though lasing in the GS is still occurring. The purpose of the paper is to show that the exploitation of the nonlinear properties arising from quantum nanostructure based semiconductor lasers operating under external optical feedback can lead, under specific conditions, to a bifurcation of the modulation properties. Starting from the generalized rate equations under optical feedback, the laser's modulation response is determined. Under the short external cavity assumption, calculations show that large variations of the α_H -factor can contribute to improve the dynamical properties such as the relaxation frequency as well as the laser's bandwidth. On the contrary, assuming the long external cavity situation, numerical results show that even small reflections in the percent range when combined to significant variations of the α_H -factor alter the laser's modulation response.

Keywords: optical feedback, nanostructures, linewidth enhancement factor, modulation response, bandwidth.

I. INTRODUCTION

Time-Delayed feedback can lead to very complex problems in a large variety of systems such as physics, economy, climatology, or electricity [1]. It is well known that the performance of a semiconductor laser can be strongly altered by any type of external optical feedback. During early stages of research, the importance of the distance between the laser facet and the external mirror reflector was pointed out in determining the nature of the semiconductor laser's response to optical feedback. Small reflections in the percent range which originate from fiber facets or any other optical elements introduced into the light path can dramatically affect the laser stability [1][2][3]. Although external optical feedback can be considered as a source of instability in some situations, it also has several beneficial effects that can improve the laser performance. For instance, controlled feedback of light can have many applications: it can be used to reduce the linewidth of the emitted light or for other applications such as encryption based on chaos, frequency tuning or velocity measurements [1][2]. This paper aims to theoretically demonstrate that external optical feedback in quantum dot (QD) semiconductor lasers can be used to improve the high-frequency properties for broadband applications (cable TV, data communications, medical, telecommunications, high power). Although injection-locking technique has already shown superior improvement in the high-speed characteristics of directly modulated lasers [4], the use of the external optical feedback can be powerful as well since it relies on a simple, compact and cheap solution which can be implemented in future integrated photonic circuits. Let us stress that the impact of the external optical feedback on the modulation properties has already been reported in the past but the presented work was built on the long external cavity assumption

and did not deal with QD nanostructure based devices [5]. Most of the work shown in this contribution will focus on the short external cavity regime for which superior improvements can be predicted. The dynamic evolution of a semiconductor laser operating in presence of external optical feedback is usually described through the following parameter [6]:

$$X_F = K_F \tau_e \sqrt{1 + \alpha_H^2} \quad (1)$$

where τ_e is the external cavity roundtrip time while $K_F = (2C_k/\tau) \sqrt{f_{ext}}$ is denoted as the feedback parameter. f_{ext} is the feedback ratio and is defined as $f_{ext} = P_1/P_0$ (with P_1 the power returned to the facet and P_0 the emitted power), C_k is the coupling coefficient from the k-facet (k=1 or 2 depending on the facet) to the external cavity, and τ is the internal roundtrip time within the laser cavity. The material α_H -factor is defined as the ratio of the partial derivatives of the real and complex parts of the complex susceptibility $\chi = \chi_r + j\chi_i$ with respect to carrier density N [7]:

$$\alpha_H = -\frac{\partial \chi_r / \partial N}{\partial \chi_i / \partial N} \equiv -\frac{4\pi}{\lambda} \frac{dn/dN}{dg/dN} \quad (2)$$

where g is the material gain. The α_H -factor depends on the ratio of the evolution of the refractive index n with the carrier density N to that of the differential gain dg/dN . The α_H -factor is used to distinguish the behavior of semiconductor lasers with respect to other types of lasers [7], and influences several fundamental aspects of semiconductor lasers, such as the linewidth [8][9], the laser's behavior under optical feedback [10], the chirp under direct modulation and the occurrence of the filamentation [11]. Above the laser's threshold, the α_H -factor increases with output power because of the reduction of the differential gain related to the gain compression [12]. Some authors have suggested that non-linear gain and/or carrier heating should have a non-negligible effect on the α_H -factor, which in turn should be considered as an optical power dependent parameter [13][14]. On one hand, in QW lasers, which are made from a nearly homogeneously broadened gain medium, the carrier density and distribution are clamped at threshold. Under the static assumption, the change of the α_H -factor is mostly due to the decrease of the differential gain from gain compression following the relation [14]:

$$\alpha_H(P) = \alpha_0 (1 + \varepsilon_p P)^{1/2} + \beta \varepsilon_p P \left(\frac{1 + \varepsilon_p P}{2 + \varepsilon_p P} \right) \quad (3)$$

where α_0 is the α_H -factor at threshold, ε_p the gain compression coefficient related to the output power P and β the parameter related to the slope of the linear gain which controls the nonlinear phase change. The situation for which $\beta=0$ corresponds to an oscillation purely located at the maximum gain peak. For a Fabry-Perot laser $\beta=0$ since the lasing mode nearly coincides with the gain peak. However, $\beta \neq 0$ for a distributed feedback (DFB) lasers which can operate at wavelengths away from the gain peak as a result of the feedback provided by the built-in grating. It can be positive or negative depending on whether the DFB laser operates on the red or the blue side of the gain peak. Typical values of β are expected to be such that $|\beta| < 1$. As an example, figure 1 shows the measured α_H -factor versus the output power for a commercial QW DFB laser. Black squares correspond to experimental data measured above the laser's threshold. The dashed line corresponds to the curve-fitting based on equation (3). Although there is a square-root dependence in (3), figure 1 shows that the device's effective α_H -factor quasi-linearly increases within the range of output powers under study (i.e. $\beta \ll 1$).

On the other hand, in QD nanostructure lasers, the lasing wavelength can switch from the ground state (GS) to the excited state (ES) as the injected current increases. This accumulation of carrier in the ES arises even though lasing in the GS is still occurring. As a result, the filling of the ES inevitably enhances the effective α_H -factor of the GS transition introducing a non-linear dependence with the injected current. It turns out that this interplay between the filling of lower energy transitions and higher ones is also important to the above-threshold α_H -factor. In case of a nanostructure-based laser, the α_H -factor power-dependence can be expressed as follows [12]:

$$\alpha_H(P) = \alpha_0(1 + \varepsilon_P P) + \frac{\alpha_1}{1 - \frac{g_{th}}{g_{max} - g_{th}} \varepsilon_P P} \quad (4)$$

with g_{max} , the maximum gain and g_{th} the gain at threshold. The first term in (4) denotes the gain compression effect at the GS (similar to QW lasers) while the second is the contribution from the carrier filling in the ES that is related to the gain saturation in the GS. Thus, when the injection current increases, the α_H -factor can balloon up to giant values as the lower energy states of the QDs are saturated. After ballooning, the α_H -factor may even plummet to negative values at reasonable powers.

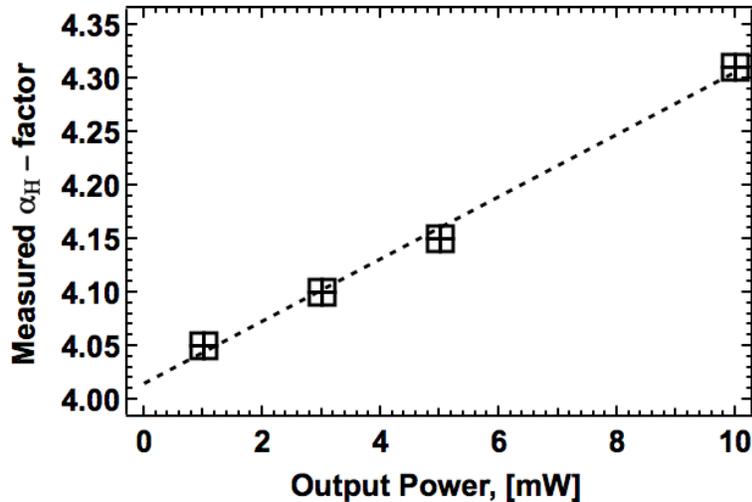


Fig.1. Measurement of the above-threshold α_H -factor of a typical commercial QW DFB laser (black squares). The dashed line corresponds to the curve-fitting based on equation (3).

The purpose of the paper is to show that the exploitation of the nonlinear properties arising from QD nanostructure based semiconductor lasers operating under external optical feedback can lead under specific conditions to a bifurcation of the modulation properties. Starting from the generalized rate equations in presence of external optical feedback, it is shown that the small-signal analysis allows to extract the modulation response and to successfully model the key operating parameters of the system. The novelty of the derivation is mostly based on the incorporation of the non-linear gain through the free-running damping rate and relaxation oscillation frequency, along with the impact of non-linear gain compression for the QD device under investigation, thereby accounting for the unique properties introduced by QD physics. On one hand, the short external cavity regime shows that large variations of the α_H -factor when combined with a proper feedback level can significantly enhance the relaxation frequency and the modulation bandwidth. On the other hand, assuming a long external cavity, numerical results show that even small reflections in the percent range when combined to significant variations of the α_H -factor alter the laser's modulation response. Calculations point out that a detrimental ripple arises around the relaxation peak creating an overshoot in the modulation response. This ripple always increases both with feedback level and with the α_H -factor. These preliminary results point out the major role of the α_H -factor in the feedback sensitivity and can be of first importance in optical telecommunications especially for broadband applications. Also, from a technological point of view, it is important to emphasize that the control of the microwave properties through an external feedback loop relies on a simple, ultra-compact and cheap solution.

II. GENERALIZED RATE EQUATIONS UNDER OPTICAL FEEDBACK

In what follows, it is assumed that the cavity medium is isotropic and the laser perfectly index guided. In addition to these conditions, the longitudinal axis only is explicitly taken into account. Both transverse and lateral variations are accounted for by the effective dielectric constant ε . Figure 2 shows a basic scheme of the QD laser operating in presence

of external optical feedback with L the length of the laser cavity (with internal roundtrip time $\tau=2nL/c$) and L_e the length of the external cavity (with roundtrip time $\tau_e=2n_eL_e/c$). Parameters n (n_e respectively) and c account for the optical index in the laser cavity (in the external cavity respectively) as well as for the velocity of light in vacuum.

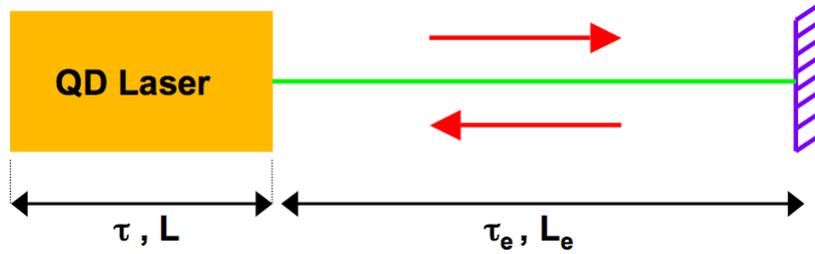


Fig. 2. Basic scheme of the nanostructure QD laser operating in presence of external optical feedback

The derivation starts with the wave equation for the electromagnetic field. From Maxwell's equations under the previous assumptions, the complex Fourier component $E_\omega(z)$ of the electric field in the laser cavity is governed by the one-dimensional scalar wave equation [15][16][17]:

$$\nabla_z^2 E_\omega(z) + k_0^2 \varepsilon E_\omega(z) = F_\omega(z) \quad (5)$$

In (5), $\nabla_z^2 = \partial^2/\partial z^2$ is the Laplacian operator for the longitudinal coordinate z , $\omega/2\pi$ the lasing frequency, $k_0=\omega/c$ the wavenumber with ε the complex dielectric constant and $F_\omega(z)$ the frequential Langevin force term accounting for the distributed spontaneous emission. It has been shown that the propagation wave equation can be solved by using Green's functions theory, and that the general solution of equation (5) can be written through the integral relation [15][16][17]

$$E_\omega(z) = \int_{(L)} G_\omega(z, z') F_\omega(z') dz' \quad (6)$$

In (6), the integration is done over the total laser cavity length L and includes the Green's function $G_\omega(z, z')$ whose expression is given by [15]:

$$G_\omega(z, z') = \frac{Z_+(z_>)Z_-(z_<)}{W(\omega, N(z), \rho_k)} \quad (7)$$

where $z_<$ and $z_>$ correspond to the lesser or greater values of z and z' , $Z_+(z)$ and $Z_-(z)$ are two independent solutions of the homogeneous part of (5) with respect to the boundary conditions respectively on the left and on the right facet. Finally, it is important to point out that in (7), $W(\omega, N(z), \rho_k) = 0$ is the Wronskian term of the previous solutions depending on the lasing frequency $\omega/2\pi$, the carrier density $N(z)$ and the amplitude reflectivity ρ_k of the k -facet (with $k = 1, 2$ depending on the facet). The dependence of the Wronskian on the facet reflectivity will be used to take into account external optical feedback coming from a distant reflecting point of amplitude reflectivity $\sqrt{f_{ext}} \ll 1$. By injecting (7) into (6), the general solution giving the electric field in the laser cavity becomes:

$$E_\omega(z) = \int_{(L)} \frac{Z_+(z_>)Z_-(z_<)}{W(\omega, N(z), \rho_k)} F_\omega(z') dz' \quad (8)$$

It is well-known that the oscillation condition corresponds to a zero in the Wronskian term which serves to determine the laser longitudinal mode. Such a condition can be written as follows:

$$W(\omega_0, N_0(z), \rho_k) = 0 \quad (9)$$

As the Wronskian is complex, both the lasing frequency $\omega_0/2\pi$ and the carrier density distribution at threshold N_0 are

completely determined from (9). Assuming that the semiconductor laser operates only in one longitudinal mode, the field distribution can be simplified and is denoted by $Z_+(z)=Z_-(z)=Z_0(z)$. When the laser is exposed to external feedback, the lasing frequency and the carrier density distribution deviates from their original values. As a result, the new Wronskian describing the lasing conditions under feedback can be developed as [18] [19]:

$$W(\omega, N(z), \rho_{k,eq}) = W(\omega_0, N_0(z), \rho_k) + \frac{\partial W}{\partial \omega} \Delta \omega + \frac{1}{L} \int_{(L)} \frac{\partial W}{\partial N} \Delta N dz + \frac{\partial W}{\partial \rho_k} \Delta \rho_k \quad (10)$$

with $\Delta \omega = \omega - \omega_0$, $\Delta N = N - N_0$ and $\Delta \rho_k = \rho_{k,eq} - \rho_k$ is the variation of the k-facet reflectivity induced by external optical feedback [20], [21]:

$$\Delta \rho_k = (1 - \rho_k^2) \sqrt{f_{ext}} e^{-j\omega \tau_e} \quad (11)$$

In (11), $\omega/2\pi$ is the lasing frequency in presence of optical feedback. As it has been mentioned at the beginning of this section, spatial hole burning (SHB) effects are also taken into account in (10) through the integral term over the cavity length. Let us stress that the inclusion of the SHB effects are of first importance when DFB lasers are considered as shown in reference [18]. By manipulating (8) and (10), the rate equation for the electric field can be expressed after the inverse Fourier transform such as [17][18]:

$$\frac{d\xi_0(t)}{dt} = \left[-\frac{j}{L} \int_{(L)} \frac{\partial W / \partial N}{\partial W / \partial \omega} \Delta N dz \right] \xi_0(t) - j \frac{\partial W / \partial \rho_k}{\partial W / \partial \omega} \sqrt{f_{ext}} (1 - \rho_k^2) \xi_0(t - \tau_e) + F(t) \quad (12)$$

where $\xi_0(t)$ represents the slowly varying envelope of the electrical field in the laser cavity:

$$\xi_0(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \xi_\omega e^{j(\omega - \omega_0)t} d\omega \quad (13)$$

with $\xi_\omega = E_\omega / Z_0(z)$, and $F(t)$ the Langevin force in the time domain. It is important to note that (12) can be applied to any type of semiconductor lasers. The third term of (12) extends the known Green's function approach [16], [17] to the case of external optical feedback and constitutes a generalization of the Lang and Kobayashi rate equations. To convert the field complex amplitude rate equation into photon density S and phase ϕ rate equations, let us write the complex electrical field as:

$$\xi_0(t) = \sqrt{S(t)} e^{j\varphi(t)} \quad (14)$$

where $S(t)$ is the photon number inside the cavity and $\varphi(t) = \phi(t) + (\omega - \omega_0)t$ the phase of the electrical field. By injecting (14) into (12) and after having separated the real and imaginary parts, the dynamic evolution of the electric field of a semiconductor laser operating under external optical feedback is given by [18][19]:

$$\frac{dS}{dt} = \frac{2}{L} \int_{(L)} W_{Ni} \Delta N dz S + 2 \operatorname{Im} \left[W_p \sqrt{f_{ext}} (1 - \rho_k^2) \frac{\xi_0(t - \tau_e)}{\xi_0(t)} \right] P + F_S(t) \quad (15)$$

$$\frac{d\phi}{dt} = \omega_0 - \omega - \frac{1}{L} \int_{(L)} W_{Nr} \Delta N dz - \operatorname{Re} \left[W_p \sqrt{f_{ext}} (1 - \rho_k^2) \frac{\xi_0(t - \tau_e)}{\xi_0(t)} \right] + F_\phi(t) \quad (16)$$

with,

$$W_{Nr} = \operatorname{Re} \left(\frac{\partial W / \partial N}{\partial W / \partial \omega} \right), W_{Ni} = \operatorname{Im} \left(\frac{\partial W / \partial N}{\partial W / \partial \omega} \right) \text{ and } W_p = \frac{\partial W / \partial \rho_k}{\partial W / \partial \omega}$$

According to (12), the system described by (15) and (16) constitutes again a generalization of the well-known Lang and Kobayashi rate equations [21] and can be used to study the static and dynamical behavior of every type of laser structures as well as the sensitivity to external optical feedback. The dynamic evolution of the carrier density is governed by the usual rate equation:

$$\frac{dN}{dt} = \frac{I(t)}{e} - \frac{N(t)}{\tau_0} - GS(t) \quad (17)$$

where $N(t)$, τ_0 , and $I(t)$ respectively are the carrier density within the active zone, the carrier density lifetime and the pump current. The optical gain G in the active region is linked to the carrier density through the relation:

$$G(N) = \Gamma \frac{\partial g}{\partial N} \left(\frac{N - N_t}{1 + \varepsilon_S S} \right) = \Gamma \frac{\partial g}{\partial N} \left(\frac{N - N_t}{1 + \varepsilon_P P} \right) \quad (18)$$

where $\partial g / \partial N$ is the differential gain, N_t the carrier density at the transparency and P the output power. The confinement factor Γ takes into account the fraction of the optical power in the active region. In order to derive the modulation response the generalized rate equations have been linearized via a small-signal analysis. When considering a laser diode with a sinusoidal modulation $\delta I(t)$ of the injection current around the mean current $\langle I \rangle$:

$$I(t) = \langle I \rangle + \delta I(t) = \langle I \rangle + \text{Re}(\Delta I e^{j\omega_m t}) \quad (19)$$

with the modulation frequency ω_m , then the output power $P = h\nu\alpha_m v_g (V/\Gamma)S$ (where $h\nu$ is the energy per photon, V the cavity volume, $\alpha_m v_g$ is the energy loss through the mirrors with α_m is the mirror loss and v_g the group velocity), the phase and the electron density N will also vary around their mean values.

$$P(t) = \langle P \rangle + \delta P(t) = \langle P \rangle + \text{Re}(\Delta P e^{j\omega_m t}) \quad (20)$$

$$\phi(t) = \delta\omega t + \delta\phi(t) = \delta\omega t + \text{Re}(\Delta\phi e^{j\omega_m t}) \quad (21)$$

$$N(t) = N_{th} + \delta N(t) = N_{th} + \text{Re}(\Delta N e^{j\omega_m t}) \quad (22)$$

In (22), N_{th} denotes the carrier density at threshold and $\delta\omega t$ is the external reflection phase difference obtained by assuming the mode with the minimum linewidth [23]. Injecting relationships (19), (20), (21) and (22) into the set of equations (15), (16) and (17), the transfer function for intensity modulation can be expressed as follows [22]:

$$H_K(j\omega) \equiv \frac{e}{\tau_p} \frac{\Delta P(j\omega)}{\Delta I(j\omega)} = (1 - H_f(j\omega)) \left[\frac{H_{fr}(j\omega)}{1 - H_{fr}(j\omega)H_f(j\omega)} \right] H_p(j\omega) \quad (22)$$

In (22), $H_{fr}(j\omega)$ is the normalized transfer function of the laser diode without optical feedback (free-running case), e is the elementary charge of the electron and τ_p is the photon lifetime within the laser cavity. The solitary laser diode is characterized by the relaxation resonance frequency ω_r and the damping frequency ω_d , which is given by,

$$H_{fr}(j\omega) = \frac{1}{1 + j \frac{\omega_m}{\omega_d} + \left(j \frac{\omega_m}{\omega_r} \right)^2} \quad (23)$$

Transfer function $H_p(j\omega)$ accounts for the parasitic RC and carrier transport effects where ω_c is considered as a free-running characteristic constant that can be extracted along with other free-running parameters. Parasitic transfer function can be expressed such as,

$$H_p(j\omega) = \frac{1}{1 + j \frac{\omega_m}{\omega_c}} \quad (24)$$

In (22) the parameter $H_f(j\omega)$ represents the influence of the external optical feedback and is written as follows,

$$H_f(j\omega) = \frac{2j}{\tau} |C_k| \sqrt{f_{ext} [1 + \alpha_H^2]} \left(\frac{1 - e^{-j\omega_m \tau}}{\omega_m} \right) \quad (25)$$

with $\alpha_H = \alpha_{GS} + \alpha_{ES}$ the above threshold α_H -factor of the GS transition which takes into account both the gain compression at the GS and the carrier filling in the ES. Coupling strength coefficient C_k of the k-facet is given by,

$$C_k = \frac{j\tau}{2} (1 - \rho_k^2) |W_p| \quad (26)$$

Equation (26) constitutes a general expression of the coupling strength coefficient, which takes into account the coupling between the k-facet to an external cavity. The coupling strength coefficient serves to quantify the sensitivity to external optical feedback of both the threshold gain and frequency variations of a semiconductor laser as well as to determine its coherence collapse threshold [19][20][21][24]. Equation (26) shows the possibility to calculate the coupling strength coefficient of any type of laser structures. In the following, the modulation response of a QD nanostructure based laser will be extracted for various situations. The objective is to show that the exploitation of the enhanced nonlinearities arising from QD nanostructures when properly associated to external optical feedback can lead to different scenarios of their dynamical properties. More particularly the short external cavity regime shows that the combination of large variations of the α_H -factor and optical feedback can be leveraged to our benefit to enhance the relaxation frequency and the modulation bandwidth.

III. NUMERICAL RESULTS AND DISCUSSION

A. Laser description and free-running case

The semiconductor laser under study is a QD Fabry-Perot grown on an n^+ -InP substrate. A detailed description of the laser structure is provided elsewhere [25]. The laser has a 4- μm wide ridge waveguide and 500- μm cleaved cavity length. The nominal emission wavelength of this Fabry-Perot device is around 1560 nm and the threshold current was measured to be 45 mA at room temperature. Figure 3 shows the calculated GS α_H -factor (evaluated at the GS-wavelength) based on relationship (4) in which gain compression effects at the GS as well as the carrier filling from the ES have been taken into account. When increasing the pump current, the device α_H -factor is enhanced from 1 to about 14, which results in a strong phase-amplitude coupling within the laser cavity. These calculated values are in good agreement with measurements reported in reference [10]. It is important to note that such variations cannot be observed in a QW laser because the gain is nearly-homogeneously broadened [12]. Consequently, it will be shown in the following that such enhanced nonlinearities arising from the QD laser can be leveraged to our benefits to enhance the modulation properties when properly combined with controlled optical feedback.

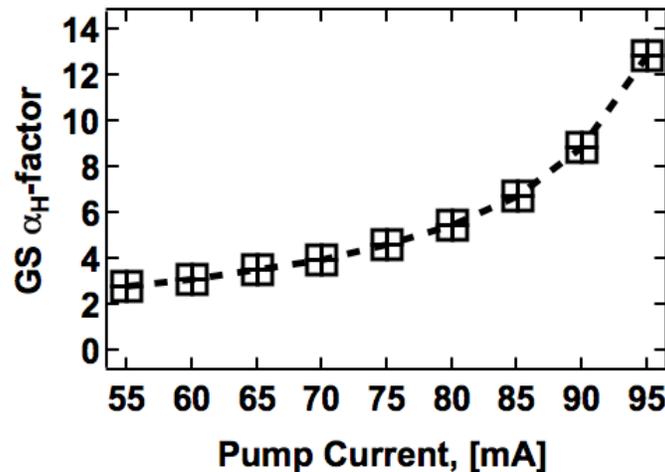


Fig. 3. Calculated above-threshold GS α_H -factor of the QD laser under study

Figure 4 shows the calculated transfer function of the quantum nanostructure based laser in the free-running case (e.g. $f_{\text{ext}}=0 \Leftrightarrow H_f(j\omega)=0$) both in amplitude (a) and in phase (b). The modulation frequency ω_m is varied from 0 GHz to 10 GHz. The transfer function is curve-fitted based on the know-experimental parameters already published in reference [10]. The parasitic term is $\omega_c=67$ GHz as reported in reference [4]. As shown in figure 4(a) the maximum of the transfer function increases from 2.2 GHz to 4.2 GHz while the maximum modulation bandwidth is about 6.6 GHz at the highest pump current. Figure 4(b) shows that the phase response varies from 0° to -180° but the frequency difference required for the commutation gets larger when increasing the pump level. This effect can be attributed to the damping frequency, (imaginary term in relationship (23)) which is enhanced at higher electrical injection.

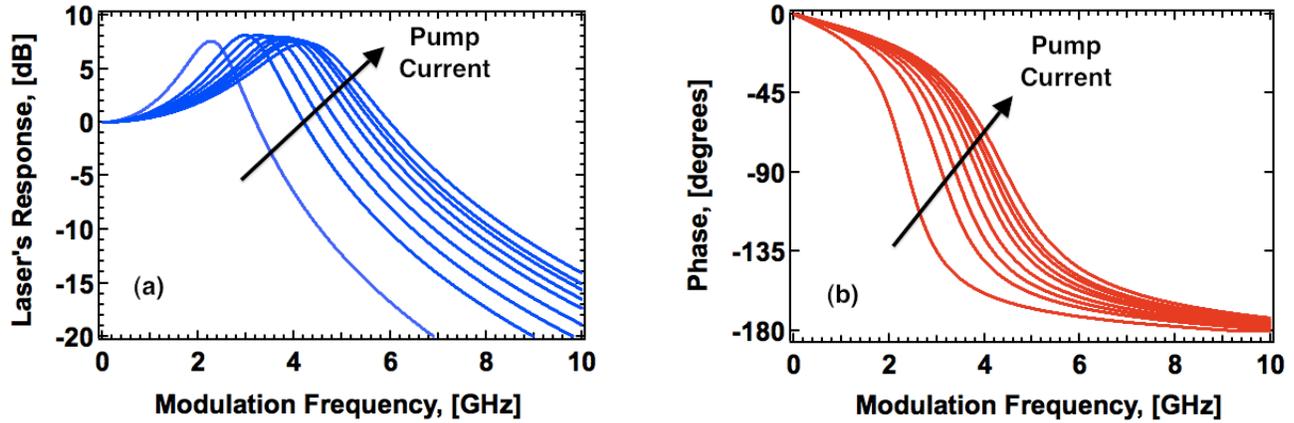


Fig. 4. Calculated transfer function of the QD laser in the free-running case (a) amplitude, (b) phase

B. Short external cavity configuration

The short external cavity situation is fulfilled when $\omega_r\tau_c \ll 1$. In such a configuration, the frequency related to the external cavity (ω_c) is larger than the laser's relaxation frequency (ω_r). For instance, assuming external cavity lengths of about 1 mm, 2 mm and 5 mm, the frequencies related to the external cavity (assuming $n_e=1$) are as large as 150 GHz, 75 GHz and 30 GHz. These values being much larger than the relaxation frequencies of the free-running laser (e.g. $\omega_c \gg \omega_r$) whatever the pump level, the use of the optical feedback can be leveraged to our benefits in order to improve the modulation properties of the QD laser. Figures 5 and 6 show the calculated modulation responses of the laser operating under optical feedback for various external cavity lengths: (a) $L_c=1$ mm, (b) $L_c=2$ mm, (c) $L_c=5$ mm. In the simulations the α_H -factor is in the range from 1 to 14. Figure 5 is calculated for a constant optical feedback of $f_{\text{ext}}=10^{-3}$ while figure 6 corresponds to the maximum feedback strength under study such as $f_{\text{ext}}=10^{-2}$. The phase evolution under such a configuration will be discussed in another paper. Results show that the frequency peak related to the maximum of the transfer function as well as the modulation bandwidth can be tuned to higher values in all configurations. Thus, for $L_c=2$ mm and $f_{\text{ext}}=10^{-3}$, the amplitude of the modulation response is enhanced by several decibels when increasing the α_H -factor. When $L_c=2$ mm, $f_{\text{ext}}=10^{-2}$ and $\alpha_H=14$, the relaxation peak is close to 9 GHz (for 4.2 GHz in the free-running case). As regards the modulation bandwidth, it gets as large as 13 GHz to be compared to the 6.6 GHz in the free-running case. When $L_c=5$ mm, $f_{\text{ext}}=10^{-2}$ and $\alpha_H=14$, the transfer function gets flat which is a suitable configuration for broadband applications (cable TV, data communications, medical, telecommunications, high power). These calculations demonstrate, that a judicious combination between optical feedback and α_H -factor can lead to an efficient regeneration of the laser modulation properties. These effects are well enhanced in QD nanostructure lasers for which large variations of the above-threshold α_H -factor are reported as compared to their QW counterparts.

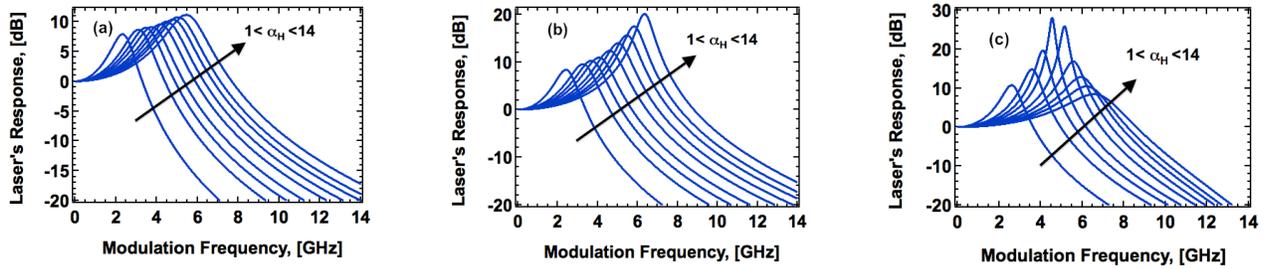


Fig. 5. Calculated transfer function of the QD laser in presence of controlled external optical feedback $f_{\text{ext}}=10^{-3}$. The α_H -factor is varied from 1 to 14 and simulations are done for different external cavity lengths (a) $L_e=1$ mm, (b) $L_e=2$ mm and (c) $L_e=5$ mm

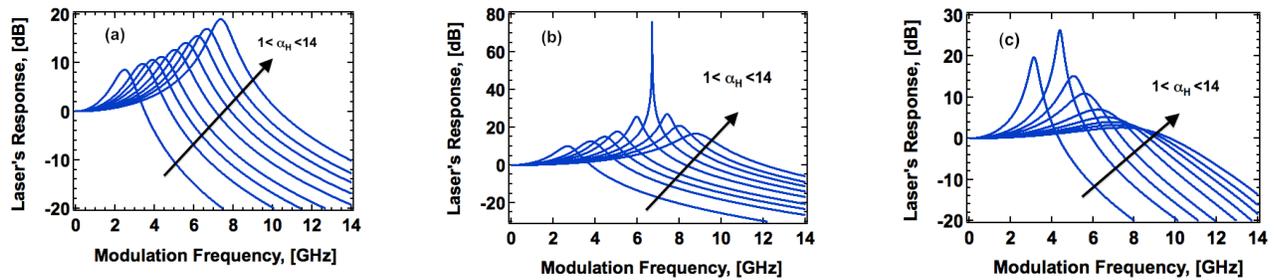


Fig. 6. Calculated transfer function of the QD laser in presence of controlled external optical feedback $f_{\text{ext}}=10^{-2}$. The α_H -factor is varied from 1 to 14 and simulations are done for different external cavity lengths (a) $L_e=1$ mm, (b) $L_e=2$ mm and (c) $L_e=5$ mm

Figures 7 and 8 show the calculated transfer functions for various feedback levels ranging from 10^{-6} to 10^{-2} . On the contrary to the previous results, the pump current is now constant meaning that the α_H -factor does not change. Figure 7 is obtained for an external cavity length of $L_e=2$ mm while figure 8 is for $L_e=5$ mm. Case (a) corresponds to the lowest α_H -factor ($\alpha_H=1$) while case (b) is for the largest α_H value ($\alpha_H=14$). When varying feedback, the frequency peak and the modulation bandwidth can be also nicely controlled. At low α_H values, the modulation responses are not deeply affected; only slight changes are predicted when increasing the feedback. Let us stress that, between figures 7(a) and 8(a), the amplitude of the response is also enhanced. However, when $L_e=5$ mm, the frequency of the external cavity getting smaller, more effects on the laser's response are expected as shown in the figure 8(b). In figure 7(b) the frequency peak is increased from about 4.2 GHz to almost 9 GHz leading to a modulation bandwidth as high as 13 GHz. Figure 8(b) shows similar results but with enhanced differences. For instance, the amplitude of the modulation response is increased by a factor of two for $f_{\text{ext}}=10^{-4}$ while a relatively flat response can be reached from $f_{\text{ext}}=10^{-3}$. It is however important to pay attention to the amplitude of the laser's response. If the amplitude is too large, which can happen under certain feedback conditions, this overshoot in the intensity response can be detrimental for some applications. Figure 9 summarizes part of the previous results on the variations of the frequency peak related to the maximum of the transfer function by taking into account both variations of the feedback strength and of the α_H -factor. Simulations show that, under the short external cavity situation, a proper combination between the optical feedback strength and large α_H values can be leveraged to our benefits so as to increase the dynamic properties of QD nanostructure lasers.

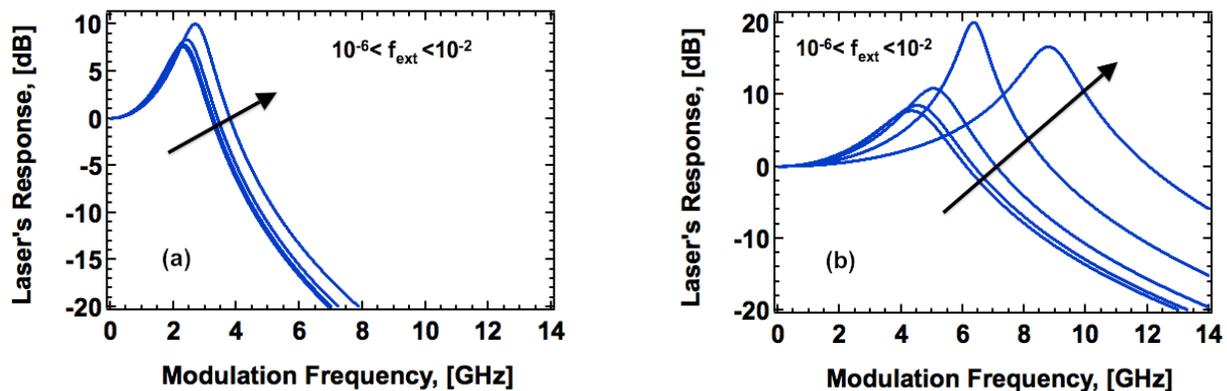


Fig. 7. Calculated transfer function of the QD laser in presence of controlled external optical feedback and for $L_c=2$ mm. The α_H -factor is set to 1 (a) and to 14 (b) and simulations are done for different feedback levels $10^{-6}<f_{ext}<10^{-2}$

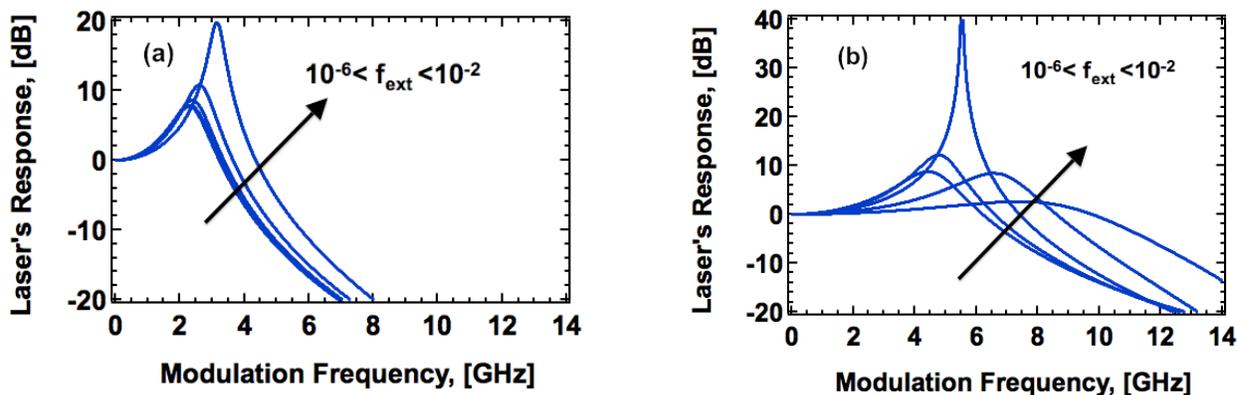


Fig. 8. Calculated transfer function of the QD laser in presence of controlled external optical feedback and for $L_c=5$ mm. The α_H -factor is set to 1 (a) and to 14 (b) and simulations are done for different feedback levels $10^{-6}<f_{ext}<10^{-2}$

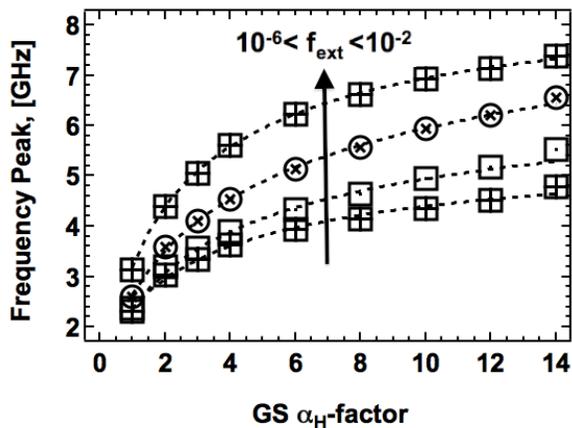


Fig. 9. Calculated relaxation peak of the QD laser in presence of controlled external optical feedback as a function of the GS α_H -factor and for various feedback levels $10^{-6}<f_{ext}<10^{-2}$

C. Long external cavity configuration

The long cavity situation occurs when $\omega_r \tau_c \gg 1$. Figure 10 shows both the amplitude and the phase of the transfer function for an external cavity length of $L_e=1$ m and for a feedback strength of $f_{\text{ext}}=10^{-5}$. Case (a) still corresponds to the situation with the lowest α_H ($\alpha_H=1$) while case (b) is the one for the largest α_H ($\alpha_H=14$). In both situations the black solid line represents the modulation response and the phase evolution in the free-running case. These results show that a parasitic reflection coming from a long external cavity behaves differently as compared to the short external cavity configuration. Figures 10(a) and 11(a) point out that even with an extremely weak optical feedback, the transfer function is already disturbed since slight oscillations start arising. Such a situation gets worse when increasing the value of α_H -factor as shown in figures 10(b) and 11(b). Inset of the figure 10(b) corresponds to a zoom recorded close to the maximum of the laser's response: the oscillations are clearly related to the external cavity modes with a periodicity of about 100 MHz. As the frequency of the external cavity is now much smaller than the relaxation frequency of the laser ($\omega_e \ll \omega_r$), the shape of the modulation response cannot be controlled under the long external cavity situation. No improvements nor in the relaxation peak or in the modulation bandwidth can be obtained under such a configuration as originally reported in reference [22]. However, we show here that this situation could be emphasized with the use of quantum nanostructure based laser because of their large variations in the α_H -factor.

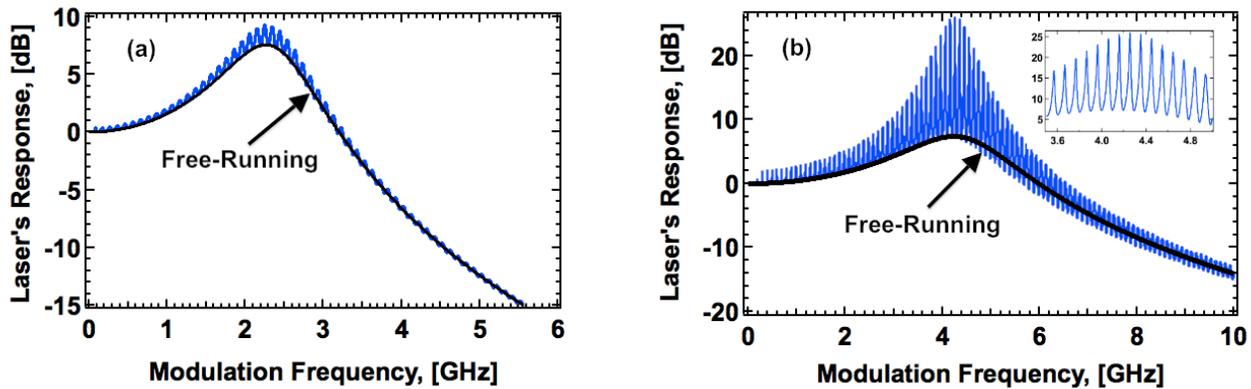


Fig. 10. Calculated transfer function of the QD laser in presence of controlled external optical feedback with long external cavity $L_e=1$ m and $f_{\text{ext}}=10^{-5}$; (a) $\alpha_H=1$ and (b) $\alpha_H=14$

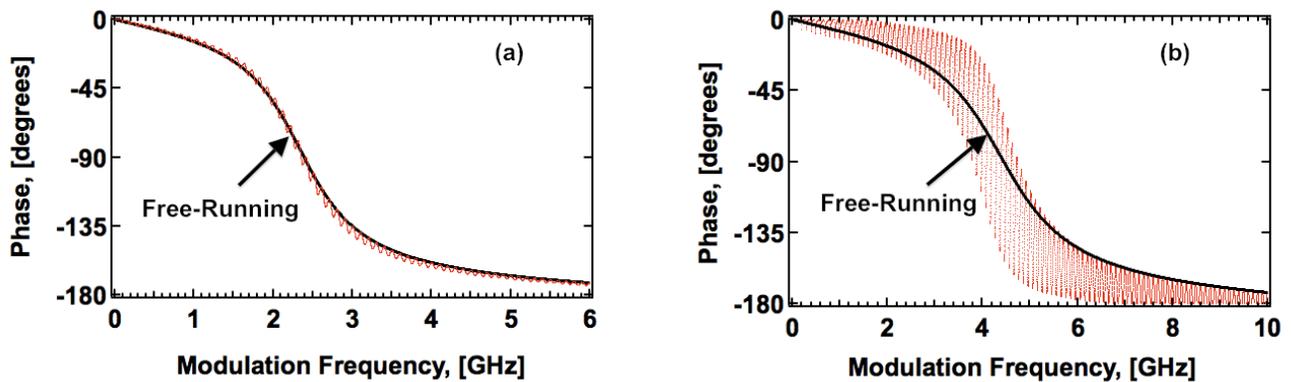


Fig. 11. Calculated phase of the transfer function of the QD laser in presence of controlled external optical feedback under long external cavity $L_e=1$ m and for $f_{\text{ext}}=10^{-5}$; (a) $\alpha_H=1$ and (b) $\alpha_H=14$

Figure 12 shows more results on the ripple dependence with both f_{ext} and α_H . When f_{ext} is constant, the ripple increases with α_H ; for instance for $f_{\text{ext}}=10^{-5}$ the ripple is enhanced from 5 dB to 16 dB (when α_H varies from 1 to 14). On the other hand, when α_H is constant, the increase of the optical feedback strength enhances the ripple as well; for instance for $\alpha_H=1$, the ripple is enhanced from 0 dB to 5 dB (when f_{ext} varies from zero to 10^{-5}). As a conclusion, these simulations point out that the use of a long external cavity leads to the occurrence of a parasitic noise located at the frequency peak. Although this behaviour has already been pointed out in reference [22], calculations presented in this paper shows that the enhanced nonlinearities arising from the QD nanostructure make the laser's response much more sensitive to optical feedback even at a very small level. The amplitude of the noise increasing both with the feedback strength and with the value of the α_H -factor, this phenomenon is detrimental for the detection.

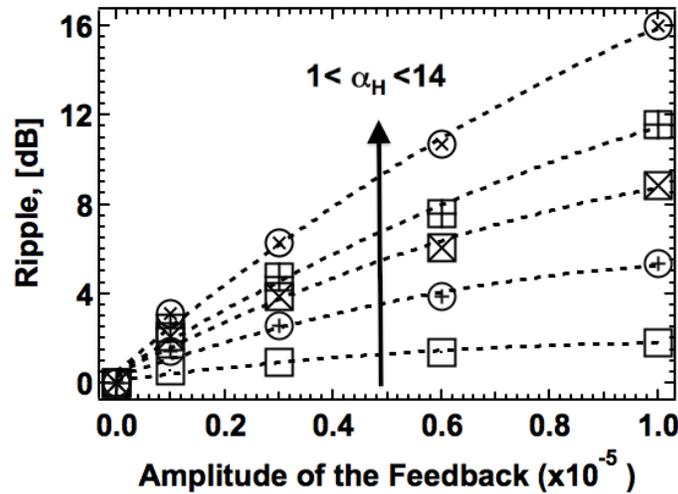


Fig. 12. Calculated ripple amplitude as a function of the external optical feedback strength under long external cavity $L_e=1$ m and for various α_H -factor from 1 to 14

IV. CONCLUSIONS

This paper shows that the exploitation of the enhanced nonlinearities in QD nanostructures lasers operating under external optical feedback can lead to different scenarios of their dynamical properties. On one hand, the short external cavity regime exhibits that large variations of the α_H -factor when combined with a proper optical feedback level can drastically enhance the relaxation frequency and the modulation bandwidth. This technique has a great advantage because it does not require a fine control of the feedback level and from a technological point of view, the method is simple, compact and cheap to fabricate. The model implicitly incorporates the nonlinear gain through the relaxation oscillation frequency and damping rate of the free-running laser. The presented model can be used to confidently extract microwave characteristics and operating parameters of the system in presence of external optical feedback. On the other hand, the long external cavity regime leads to the occurrence of a ripple arising around the relaxation peak and creating an overshoot in the laser's modulation response. This ripple always increases both with feedback level and the α_H -factor. These preliminary results point out the major role of the α_H -factor and are of first importance for broadband applications (cable TV, data communications, medical, telecommunications, high power). Calculations show, that a proper combination between optical feedback and α_H -factor can lead to the regeneration of the modulation properties. These effects are well enhanced in QD nanostructure lasers for which large variations of the above-threshold α_H -factor are reported as compared to their QW counterparts.

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