Systematic Investigation of the Alpha Parameter Influence on the Critical Feedback Level in QD Lasers

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ABSTRACT

The dramatic variation in the linewidth enhancement factor ($\alpha_{\text{tf}}$-factor) that has been reported for quantum dot lasers makes them an interesting subject for optical feedback studies. A low $\alpha_{\text{tf}}$-factor combined with a high damping factor is especially interesting because it should increase the tolerance to optical feedback in these devices and may offer potential advantages for direct modulation. In the particular case of QD lasers, the carrier density is not clearly clamped at threshold. The lasing wavelength can switch from the ground state (GS) to the excited state (ES) as the current injection increases meaning that a carrier accumulation occurs in the ES even though lasing in the GS is still occurring. The filling of the ES inevitably enhances the $\alpha_{\text{tf}}$-factor of the GS above threshold as experimentally and numerically shown. Consequently, this strong variation of the GS $\alpha_{\text{tf}}$-factor in comparison to QW devices, should theoretically produce a significant variation in the onset of coherence collapse due to feedback. This coherence collapse regime, in which the laser is subject to instabilities, is incompatible with data transmission because of the induced high bit-error rate. One method to investigate the tolerance to optical feedback is to compare experiment with the theoretical work introduced by Petermann. It will be presented that under specific conditions, i.e., in the case of a strong enhancement in the $\alpha_{\text{tf}}$-factor, the feedback sensitivity of the laser can vary by as much as 10dB within the same device.

Keywords: optical feedback, coherence collapse, quantum well, quantum dash, linewidth enhancement factor.

1. INTRODUCTION

The extension of optical networks to local residential subscribers requires the development of extremely low-cost laser transmitter sources [1]. While wafer fabrication allows for large-scale production, which drastically reduces the cost per laser, packaging remains a cost bottleneck, as it is not supported by parallel processing. Cost reduction must therefore be based on packaging simplification, such as flip-chip bonding and direct coupling of the laser into the fiber [2]. The characteristic problem of semiconductor lasers is their sensitivity to optical feedback. Fiber optic communication systems can be limited by unwanted optical feedback arising at fiber facets and junctions [3][4]. Although Faraday isolators are used extensively to reduce back reflections by as much as 60dB, the elimination of the isolator remains a challenge and is desirable because it leads to simplified packaging and will greatly reduce costs [5]. The performance of a semiconductor laser is strongly altered by any type of external optical feedback. During early stages of research, the importance of the distance between the laser facet and the external mirror reflector was pointed out in determining the nature of the semiconductor laser’s response to optical feedback [6]. Small reflections in the percent range which originate from fiber facets or any other optical elements introduced into the light path can dramatically affect the laser stability. Although external optical feedback can be considered as a source of instability in some situations, it also has several beneficial effects that can improve the laser performance. At the extremes of very weak and very strong optical feedback, linewidth narrowing and noise suppression can occur. This advantage, along with the large gain bandwidth of the semiconductor laser, can produce a highly tunable, narrow linewidth source that attracts many applications in spectroscopy, metrology and telecommunications [7].

Five distinct regimes based on spectral observation were reported for 1.55-µm distributed feedback (DFB) semiconductor lasers [8]. At the lowest feedback level, regime I, the laser operates on a single external cavity mode that emerges from the solitary laser mode. Depending on the phase of the feedback, the laser linewidth can be narrowed or
broadened. Within regime II, the mode appears to split into two modes arising from rapid mode hopping. Noise-induced hopping between two external cavity modes is the underlying reason for this behavior. The transition to regime II has been shown to correspond to multiple solutions to the steady state equation that determines the frequency of the laser. This condition is satisfied when the parameter $X_F$ is equal to unity in the following expression:

$$X_F = K_F \tau_e \sqrt{1 + \alpha_H^2}$$

where $\tau_e$ is the external cavity roundtrip time while $K_F = \left(2C/\tau_e\right) \sqrt{R_p}$ is denoted as the feedback parameter. $R_p$, is the feedback ratio and is defined as $R_p = P_1/P_0$ (with $P_1$ the power returned to the facet and $P_0$ the emitted power), $C$ is the coupling coefficient from the laser facet to the external cavity, and $\tau$ is the internal roundtrip time within the laser cavity. The $\alpha_H$-factor is defined by:

$$\alpha_H = \frac{4\pi}{\lambda} \frac{dn}{dN} \frac{dg}{dN}$$

where $g$ is the material gain. The $\alpha_H$-factor depends on the ratio of the evolution of the refractive index $n$ with the carrier density $N$ to that of the differential gain $dg/dn$. The $\alpha_H$-factor is used to distinguish the behavior of semiconductor lasers with respect to other types of lasers [9], and influences several fundamental aspects of semiconductor lasers, such as the linewidth [10] and laser behavior under optical feedback [11].

In regime III the laser re-stabilizes in a single external cavity mode (the lowest linewidth mode) with constant power. As the feedback level is further increased, the impact of optical feedback becomes independent of the external cavity length and the laser undergoes a transition to a chaotic state characterized by coherence collapse and denoted as regime IV [12]. Coherence collapse is the common name given to describe the irregular dynamics occurring when the laser is operated above threshold, and has been greatly studied over the last twenty years. This regime has been described as co-existing chaotic attractors [13] and as an important source of noise [14][15]. The main consequence of the coherence collapse regime on the semiconductor laser is a drastic collapse of the laser’s coherence time leading to an enhancement of the laser linewidth (up to several GHz). For lasers used as an optical transmitter, coherence collapse has been experimentally [5] and theoretically [3] demonstrated to cause a strong degradation in the bit error rate (BER). As the feedback level is further increased, the laser enters regime V, which is characterized by single-mode, constant intensity and narrow linewidth operation. This regime can only be reached when laser diodes with antireflection-coated facets are used.

This paper reports the dynamic properties of a multi-mode Fabry-Perot (FP) Quantum Dash (QDash) laser emitting on the GS around 1560nm. In particular, it is shown that the GS $\alpha_H$-factor drastically increases from $\sim1$ to $\sim14$ as the bias current goes beyond the threshold value. Consequently, the feedback sensitivity of the laser, which is evaluated through the onset of its coherence collapse, is found to vary by $\sim20$-dB over the range of current investigated. This paper shows that the above threshold variation of the $\alpha_H$-factor can significantly alter the device’s feedback sensitivity and has to be considered as a significant input for the realization of isolator-free transmissions under direct modulation.

2. PREDICTING THE ONSET OF THE COHERENCE COLLAPSE

One method to investigate the onset of the coherence collapse is to use a model derived from the microwave modulation characteristics of laser diodes [16]. Starting from the rate equations for a single-mode laser diode under the influence of external optical feedback evaluated through small-signal analysis, it was shown that the modulation transfer function can be expressed as:

$$H_K(j\omega_m) = (1 - K(j\omega_m)) \frac{H(j\omega_m)}{1 - K(j\omega_m)H(j\omega_m)}$$

where $\omega_m$ is the modulation frequency. In (3), $H(j\omega_m)$ is the normalized transfer function of the laser diode in the absence of optical feedback given by:
\[ H(j\omega_m) = \frac{1}{\left(\frac{j\omega_m}{\omega_r}\right) + \frac{j\omega_m}{\omega_d} + 1} \]  

where \( \omega_r \) and \( \omega_d \) are the relaxation and damping resonance frequencies, respectively. Expression (3) is derived for the minimum linewidth mode, and has most stable external cavity mode as demonstrated in [17]. The modification of the solitary laser modulation response due to weak feedback when \( \alpha_f > 1 \) is found to be [18]:

\[ K(j\omega_m) = \frac{K_F}{\omega_m} \sqrt{1 + \alpha_H^2 \left(1 - e^{-j\omega_m\tau}\right)} \]  

Examination of (3) reveals that the system will become unstable due to a small perturbation if an unstable pole occurs. The existence of such a pole does not necessarily mean that coherence collapse occurs, as the critical feedback regime is described by a complicated dynamic process. However, the minimum feedback level at which an unstable pole occurs does correspond to the onset of the coherence collapse. As a result, considering the case of a long external cavity (e.g. \( \omega_r\tau_r \gg 1 \)) as well as the condition \( \omega_r < \omega_p \), the unstable pole occurs in (3) when \( K(j\omega_m)H(j\omega_m) = 1 \). This condition leads to an analytical expression for the onset of coherence collapse expressed by:

\[ R_{pc} = \left(\frac{\gamma \tau_i}{4C}\right)^2 f(\alpha_H) \]  

with \( \gamma = \omega_r^2/\omega_d \) the damping factor in rad/s and \( f(\alpha_H) = \sqrt{1 + \alpha_H^2} \) a function linked to the \( \alpha_H \)-factor. Expression (6) depends only on the solitary laser response and the \( \alpha_H \)-factor. There is no explicit dependence on the external cavity length since a long-cavity asymptotic assumption was used in the derivation. For the case of a short external cavity (\( \omega_r\tau_r \ll 1 \)), the onset of the coherence collapse is found to be dependent on the external cavity length as demonstrated in [19]. In validating (6) against numerical simulations of an external cavity laser diode, it was shown that the expression deviated from the numerical results for low \( \alpha_H \)-factor values. Consequently, the expression for the critical feedback level was found to be empirically driven by the same expression but with \( f(\alpha_H) \) given by \( g(\alpha_H) \) and expressed as:

\[ g(\alpha_H) = \left(1 + \alpha_H^2\right)/\alpha_H^4 \]. This relation predicts that coherence collapse does not occur if \( \alpha_H \to 0 \), as explained through Henry and Kazarinov’s ellipse whose eccentricity decreases with the \( \alpha_H \)-factor [20]. For the \( \alpha_H=0 \) case, all the fixed points describing the stability of the system (modes and antimodes) are located on a circle around the solitary laser mode. Under this condition, the minimum gain mode and the minimum linewidth mode are the same and do not compete with each other and coherence collapse could theoretically not occur [17].

3. RESULTS AND DISCUSSION

1.1 Description of the feedback loop

The experimental apparatus to measure the coherence collapse threshold is depicted in fig. 1. The setup is based on a 50/50 4-port optical fiber coupler. Emitted light was injected into port 1 using a single-mode lensed fiber in order to avoid excess uncontrolled feedback. The optical feedback was created with a high-reflectivity dielectric-coated fibre (R>95%) located at port 2. The feedback level was controlled via a variable attenuator and its value was determined by measuring the optical power at port 4 (back reflection monitoring). The effect of the optical feedback was analyzed at port 3 via a high-resolution (10pm) optical spectrum analyzer (OSA). A polarization controller was used to make the feedback beam’s polarization identical to that of the emitted wave in order to maximize the feedback effects.
The roundtrip time between the laser and the external reflector is about ~10-30ns. As a consequence, the condition $f_r \tau_c >> 1$ is fulfilled (through the long external cavity) and conditions removing the dependence on the feedback phase and the external cavity length are met. In order to improve the accuracy of the measurements at low output powers, an erbium-doped-fibre-amplifier (EDFA) was used with a narrow band filter to eliminate the excess noise. The EDFA is positioned between the laser facet and the polarization controller (not shown in fig. 1). The amount of injected feedback into the laser is defined by the ratio of $R = P_1/P_0$, where $P_1$ is the power returned to the facet and $P_0$ is the power emitted from the facet. The amount of light that effectively returns into the laser depends on the optical coupling loss of the device to the fiber, which was estimated to be about 4dB. The device is epoxy-mounted on a heat sink and the temperature is controlled at 20°C. As the feedback is increased a few decibels beyond regime III, relaxation oscillation sidebands appear on the emission line. These sidebands then grow and broaden with the feedback rate leading to the coherence collapse. Thus, the determination of the onset of the coherence collapse was performed by observing the condition where the laser linewidth begins to significantly broaden as shown in [12].

1.2 Results and discussion

The device under study was grown on an n+ InP substrate and its structure is described as a 500µm long cleaved-cavity ridge waveguide (RWG) with a 4-µm wide stripe. Details of the structure have already been published elsewhere [21]. Fig. 2 shows the light-current characteristic measured at room temperature. The threshold current leading to a GS-emission at 1.55µm is ~45mA and the external differential efficiency is about 0.2W/A. Beyond a pump current of ~100-mA, ES lasing emission occurs. As observed in fig. 1, the onset of ES lasing leads to a kink in the light-current characteristics as well as a modification of the slope efficiency [22].

Next, the frequency response of the laser is measured since the damping rate is needed in the calculation of the critical feedback level as shown in (6). In fig. 3, the resonance frequency squared, $f_r^2$, is plotted as function of the output power for the device under study. The experimental dependence of the relaxation oscillation frequency deviates from the
expected linear dependence and is modelled via the following relation [23]:

\[ f_r^2 = \frac{AP}{1 + \frac{P}{P_{sat}}} = \frac{AP}{1 + \varepsilon_P P} \]  

(7)

where \( \varepsilon_P P = P/P_{sat} \) with \( \varepsilon_P \) representing the gain compression coefficient related to the output power \( P \). This value means that at this level of output power, non-linear effects are significant. In the case for the device under test, the curve-fit leads to a \( P_{sat} \approx 17 \text{mW} \) and a gain compression coefficient of approximately \( \varepsilon_P = 1/P_{sat} \approx 0.06 \text{mW}^{-1} \). The maximum of the resonance frequency can be directly deduced from the curve-fitting as \( \Omega = (AP_{sat})^{1/2} \) and was expected to be \( \approx 7.6 \text{GHz} \).

![Graph of square of resonance frequency vs. output power](image)

**Fig. 3.** The square of the resonance frequency versus the output power

In fig. 4, the evolution of the damping rate against the relaxation frequency squared leads to a K-factor of 0.45ns as well as an effective carrier lifetime of 0.16ns. The maximum intrinsic modulation bandwidth \( f_{max} = 2\pi \sqrt{2/K} \) is 19.7GHz.

![Graph of damping factor vs. square of relaxation frequency](image)

**Fig. 4.** The damping factor versus the square of the relaxation frequency

The above-threshold GS \( \alpha_{int} \)-factor was then measured using the injection locking technique, which is based on the asymmetry of the stable locking region over a range of detuning on both positive and negative side of the locked mode [24]. The ratio of positive to negative detuning should theoretically remain the same for any value of side mode suppression ratio (SMSR), which was kept at 35dB for this measurement. The measured GS \( \alpha_{int} \)-factor as a function of bias current is depicted in fig. 5. It was observed that the GS \( \alpha_{int} \)-factor increased from \( \approx 1.0 \) to \( \approx 14 \) as the bias current was increased from the threshold value to 105mA. This enhancement is mostly attributed to the plasma effect as well as to the carrier filling of the non-lasing states [25] which results in a differential gain reduction above threshold.
Consequently, this strong degradation of the GS $\alpha_{tr}$-factor with the bias current should produce a significant variation in the laser’s feedback sensitivity.

Fig. 5. The above threshold GS $\alpha_{tr}$-factor versus the bias current.

In fig. 6, the measured onset of the coherence collapse is shown (black squares) as a function of the bias current at room temperature. Note that dashed and dotted lines in fig. 6 are added for visual help only. The feedback sensitivity of the laser is found to vary by ~20dB over the range of examined current levels as the $\alpha_{tr}$-factor increases at higher bias currents. In order to compare the experimental data, the onset of coherence collapse is calculated (white squares) by substituting the measured relaxation frequency, damping factor and $\alpha_{tr}$-factor values directly in equation (6). Assuming a laser with cleaved facets, the coupling coefficient from the facet to the external cavity, $C = (1 - R)/2\sqrt{R}$ is calculated to be 0.6 and the internal round trip time in the laser cavity is about ~10ps.

Fig. 6. Coherence collapse thresholds as a function of the bias current.

As shown in fig. 6, expression (6) leads to a large discrepancy between measurements and calculations, whose minimum value is ~11dB at 65mA. It is worth noting that for $\alpha_{tr}$-factors approaching unity (below 60mA), the critical feedback level saturates. This saturation is generated by the function $f(\alpha_{tr})$, which converges to 1 as $\alpha_{tr}$ gets smaller. Experimentally, the trend does not saturate at this level of bias current since the resistance to optical feedback keeps
increasing, demonstrating that the critical feedback level can be up-shifted for lower \( \alpha_{tr} \)-factors. In order to account for the \( \alpha_{tr} \)-factor approaching unity, the empirical function \( g(\alpha_{tr}) \) has been included in (6), and the results are also depicted in fig. 6 (black circles). The calculated critical feedback level is now up-shifted for the lower values of the \( \alpha_{tr} \)-factor. At low bias currents, the measured values are found to be in a better agreement with calculations. Although (6) does not match the quantitative values in fig. 6, it qualitatively reproduces the up-shifting observed for small \( \alpha_{tr} \)-factors. This effect can be explained through the variation of the \( \alpha_{tr} \)-factor, which changes \( g(\alpha_{tr}) \) by a factor of \( \sim 500 \). Thus, at low bias currents, the feedback sensitivity is mostly driven by the \( g(\alpha_{tr}) \) function and not by the variations in the damping factor. Despite the fact that (6) was derived empirically under the assumption of weak optical feedback, it is found to exhibit a better accuracy for ultra-low \( \alpha_{tr} \)-factors. The discrepancy between the experimental data and theoretical prediction is decreased from 14dB to 7dB at 55mA. When extrapolating the dotted line in fig. 6 to 45mA, the calculated values will be very close to the experimental data. In the following section, it will be shown that a better accuracy between calculations and measurements can be obtained by using another analytical relation.

IV. IMPACT OF THE ABOVE THRESHOLD \( \alpha_{tr} \)-FACTOR

In QDash lasers, the lasing wavelength can switch from the GS to the ES as the injected current increases, indicating that a carrier accumulation occurs in the ES even though lasing in the GS is still occurring. The filling of the ES enhances the \( \alpha_{tr} \)-factor of the GS introducing an additional dependence with the injected current. Thus, considering the gain variation at the GS and the ES, the \( \alpha_{tr} \)-factor can be written as [25]:

\[
\alpha_{tr}(P) = \alpha_{GS}(P, P_{sat}) + \alpha_{ES}(P, P_{sat})
\]  

(8)

where \( P \) is the output power and \( P_{sat} \) is the saturation power at which the effects of the gain compression occur. The first term in (8) denotes the gain compression effect at the GS while the second term accounts for the contribution from the carrier filling in the ES. Based on the Lang and Kobayashi rate equations, the variation of the angular frequency induced by external optical feedback leads to another expression of the onset of the coherence collapse, which can be written as follows [26]:

\[
R_{Pc}(P) = \left( \frac{\tau_i}{\sqrt{2C}} \right)^2 \frac{\omega_P^2(P)}{1 + \alpha_{H}^2(P)}
\]  

(9)

Equation (9) shows that the onset of the coherence collapse occurs when the maximum feedback induced frequency shift exceeds the relaxation frequency. It differs from (6) in which the damping rate is one of the dominant terms. Then, substituting (8) into (9), the variation in the onset of the coherence collapse regime can be expressed as the following relation:

\[
R_{Pc}(dB) = R_{Pc0} + 10 \log \left( \frac{1}{1 + \left[ \alpha_{ES}(\alpha_{ES} + 2\alpha_{GS})/(1 + \alpha_{GS}^2) \right]^{1/2}} \right)
\]  

(10a)

with,

\[
R_{Pc0}(P) = \left( \frac{\tau_i}{\sqrt{2C}} \right)^2 \frac{\omega_P^2(P)}{1 + \alpha_{GS}^2(P)}
\]  

(10b)

the contribution of the GS only. The second term in (10a) occurs when the contribution of the ES described by the second term in (8) is taken into account. Fig. 7 shows the calculated coherence collapse threshold as a function of the bias current. On one hand, when only the contribution of the GS is considered, the critical feedback level follows the conventional trend increasing with bias current as with the laser’s relaxation frequency [27]. On the other hand, when
considering only the contribution of the ES in the coherence collapse threshold, an opposite trend is observed. This contribution can be seen as a perturbation that results in a shift in the overall coherence collapse threshold. Thus, when both the GS and ES contributions are considered in the overall coherence collapse threshold, the calculated coherence collapse threshold is in a qualitative agreement with experimental values. It is important to emphasize that although (6) has been most commonly used in the feedback field, (9) leads to a better agreement with experimental data as shown by fig. 7 compared to fig. 6.

This QDash FP laser exhibits an improved resistance to optical feedback when decreasing the bias current. This effect is induced from the \( \alpha_{fr} \) factor variations which are much more significant over the whole range of current \( (1 < \alpha_{H} < 14) \). This experimental result is in good agreement with [28] in which it has been shown that when the \( \alpha_{fr} \) factor tends to zero, the critical feedback level is up-shifted but should remain finite. Experimental results indicate that this saturation occurs at a feedback level larger than 18dB. In order to explain the increase of the \( \alpha_{fr} \) factor, the contributions of the GS and the ES terms in (8) need to be considered simultaneously. Consequently, the experimental trend depicted in fig. 6 does not agree with the relaxation frequency variations even at low bias current levels for which the coherence collapse is up-shifted. These different behaviors are specific to QDash structures in which non-linear effects associated with the ES are more emphasized. This phenomenon can make QDash lasers more sensitive to optical feedback, which results in larger variations in the onset of the coherence collapse as shown by equation (10a) compared to that of the QW devices.

**IV. CONCLUSIONS**

The onset of the coherence collapse regime has been investigated experimentally and theoretically in a 1.55-\( \mu \)m QDash FP semiconductor laser. The model based on the laser transfer function has been found to underestimate the onset of the critical feedback level whereas the other one based on external cavity mode stability analysis exhibits a good agreement with the experimental data. Although the Petermann’s model has been widely used in the field of optical feedback, it does not yield a strong agreement with QDash based semiconductor devices except for the cases where an ultra low \( \alpha_{fr} \) factor is considered. The paper also highlighted how variations of the \( \alpha_{fr} \)-factor above threshold can affect the optical feedback sensitivity for QDash lasers. Thus, it has been demonstrated that the route to chaos can be improved or degraded with regards to the \( \alpha_{fr} \)-factor variations above threshold. When compared with QW structures, QDash devices may not systematically exhibit significant improvements in sensitivity to optical feedback due to their larger and non-linear variations in the \( \alpha_{fr} \)-factors.
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