Control of Dynamical Instability in Semiconductor Quantum Nanostructures Diode Lasers: Role of Phase-Amplitude Coupling

Pramod Kumar[1*] and Frédéric Grillot[2,3]

[1]Femtosecond Laser Laboratory, Indian Institute of Science Education and Research (IISER) Mohali, Manauli-140306, Punjab, India.
[2]Université Européenne de Bretagne, INSA, CNRS-Laboratoire FOTON, 20 avenue des buttes de Coesmes, 35708 Rennes Cedex 7, France
[3]Télécom ParisTech Ecole Nationale Supérieure des Télécommunications Laboratoire CNRS 5141,46 rue Barrault, 75634 Paris Cedex 13, France
[*]Corresponding author: pk6965@gmail.com

We present extensive numerical comparative investigations of the complex dynamics of Edge Emitting semiconductor diode lasers and quantum dot Nanostructures lasers subject to optical injection in dependence on the line-width enhancement factor α which accounts for the amplitude-phase coupling of the optical field. The variation of α leads to conspicuous changes of the dynamics of the systems, which are characterized and investigated as a function of optical injection strength η and α for the fixed coupled-cavity delay time τ. In particular, we provide a qualitative understanding of the physical mechanisms underlying the observed dynamical behavior and its dependence on α. Our analysis is based on the observation that the cross-correlation and Bifurcation measure unveils the signature of enhancement of amplitude-death islands in which the coupled lasers mutually stay in ultimate stable phase-locked states. The amplitude death and existence of multiple amplitude death islands could be implemented for diode lasers stabilization applications.
.

PACS number(s):42.65.Sf, 42.55.Px

INTRODUCTION

The semiconductor diode lasers are known to be very sensitive to the external optical perturbations such as, for example, optical self-feedback, optoelectronic feedback, optical injection. When the diode laser subjected to optical injection by another diode laser then the radiation emitted from mutually delay-coupled diode lasers is well behaving, well understandable, well classifiable in terms of complex nonlinear dynamics. On the one hand these dynamical instabilities are undesired features and disturb the many applications where one needs the constant stable power but one the other hand they may allow for new methods for secure communications using chaos synchronization. So the systematic study and control of these nonlinear dynamics provides fundamental insight into the underlying physics of the system. On the basis of which one can redesign the device or improve the processing, or simply exploit the dynamical performance of a system to one's advantages.

Amplitude death [3,4] is one of the collective fascinating phenomena, where coupled lasers drive each other to a fixed point and stop the oscillatory dynamics. The coupling induced stabilization is known to occur via either the change the stability of unstable fixed points which are already exists in the absence of coupling or the stationary state can be entirely newly created by the coupling. That is why, the routes by which the two coupled lasers reach their stable states in the presence of finite-delayed interaction remain poorly understood because of very diverse time scales involved in the dynamics. . In this sprit, we are going to address the question whether these dynamical behaviors of a coupled diode lasers can be controlled via Amplitude death and what dynamics one encountered during the route to reach the stable state.

DIODE LASERS SYSTEM AND THE MODEL

For numerical simulations, the time evolutions of the complex electric field $E_j(t)$ and the carrier density $N_j(t)$ (with the threshold value subtracted out) averaged spatially over the laser medium for each laser are modeled by [1]
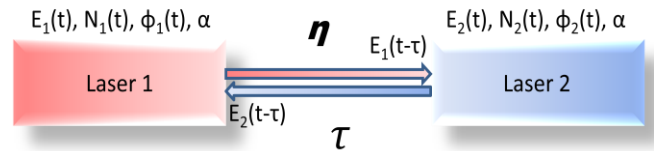

FIG.1. (Color online) Schematic diagram of a Delay-coupled diode laser system.

$$\frac{dE_j(t)}{dt} = (1+i\alpha)N_j(t)E_j(t) + \eta E_k(t-\tau)e^{-i\omega_k\tau}, \ j \neq k,$$

$$T\frac{dN_j(t)}{dt} = J_j - N_j(t) - (1+2N_j(t))\left|E_j(t)\right|^2, \ j,k = 1,2.$$

where η is the coupling parameter, Js are the injected current densities (with the threshold value subtracted out), T is the ratio of the carrier lifetime to the photon lifetime, the delay time $\tau = 2L/c$ is the round-trip time taken by the light to cover the distance L between the lasers, and α is the

linewidth enhancement factor of the diode lasers. In order to scan and understand the different dynamical regimes, we use correlation measure [1].We observe the route to amplitude death of low frequency complex dynamics in the output power of a diode laser (slave) when subjected to optical injection from another diode laser (master).

In the context of a system of two delay-coupled diode lasers and Quantum dot nanostructures lasers, we emphasize the effect of α on the control of complex dynamical instabilities near the phase flip transition regimes as a function of coupled-cavity time delay τ and the optical injection strength η. Shifting of different phase-correlated dynamics, such as phase-flip Bifurcation and Strange Bifurcation [1, 2], was observed in our previous work when η is varied for a particular τ. In this present work, we show that these phase flip transitions does not occur abruptly at a particular value of η and α. Instead we find a coupling strength region around the phase flip transition where the co-existence of multi-attractors occurs as shown in figure 2 and figure 3. We show that the phase flip Bifurcation occurs from in-phase amplitude death to anti-phase amplitude death and the Strange Bifurcation occur from anti-phase to in-phase transition regimes [1]. The existence of multiple attractors near the regime of strange bifurcation has raised the issue of whether the noise plays a crucial role or not. We also study the effect of the phase-amplitude coupling factor [10] on the dynamics in the amplitude death regime and extends the study for Nanostructures quantum dot lasers. One of the Key features of semi-conducteur lasers is their 3-dB modulation bandwidth, which is limited due to the presence of strong relaxation oscillation damping rates. This work also theoretically focuses on the impact of strong injection in a quantum nanostructure semiconductor laser with large variation of the phase-amplitude coupling factor through the non-linear dynamics and the modulation response at zero-detuning.The combination of the strong injection, optimized phase-amplitude coupling factor and the zero-detuning case shows the possibility to reach the stable state, which is fully suitable for laser control applications.

INFLUENCE OF THE α IN AN INJECTION-LOCKED EDGE-EMITTING DIODE LASER

We start by analyzing the modulation properties of the coupled laser system from the rate equations [1]. The behavior of the correlation function versus coupling strength η, α are shown in figure 2 and figure 3, using the numerical solutions of the of equations . In Figure 4, the symbol AD represents the signature of ultimate death state while BS and MS represent the Bistability and multistability between in-phase amplitude-death island and anti-phase amplitude-death island respectively. Recall that

the phase-space plot of this ultimate death state is represented as AD in figure 5, and the MD dynamics is shown in figure 6.
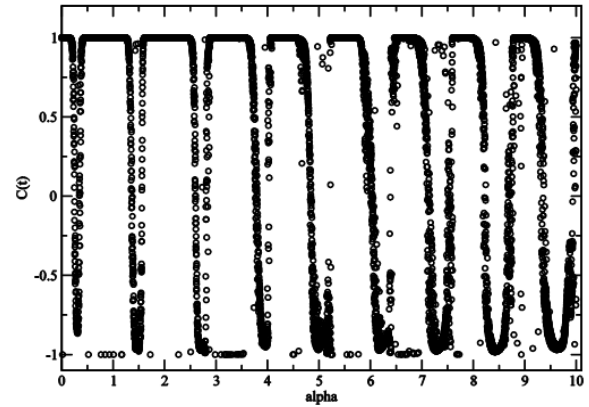

FIG. 2. Plot of the cross-correlation C(t) versus α for a fixed time delay $\tau = 14$ (in units of cavity photon lifetime).
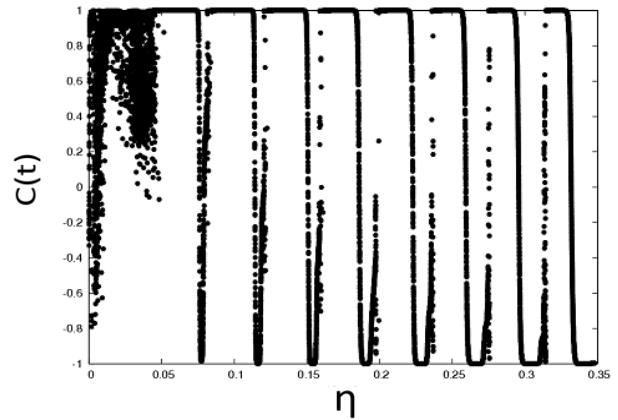

FIG. 3. Plot of the cross-correlation C(t) versus coupling strength, η for a fixed time delay $\tau = 14$ (in units of cavity photon lifetime).
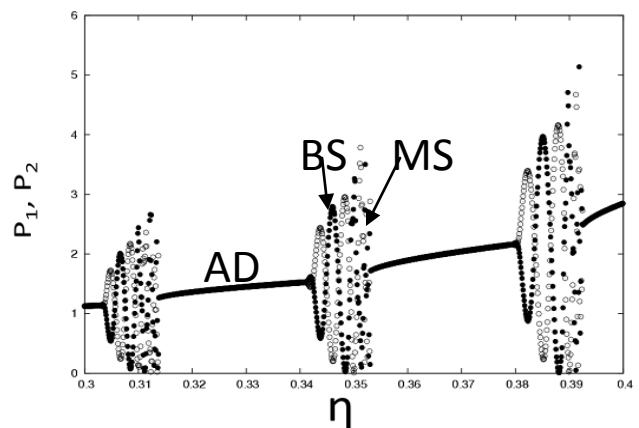

FIG. 4. Bifurcation Plots of laser output powers P1 (open circles) and P2 (filled circles) versus coupling strength η for a fixed time delay $\tau = 14$ (in units of cavity photon lifetime).
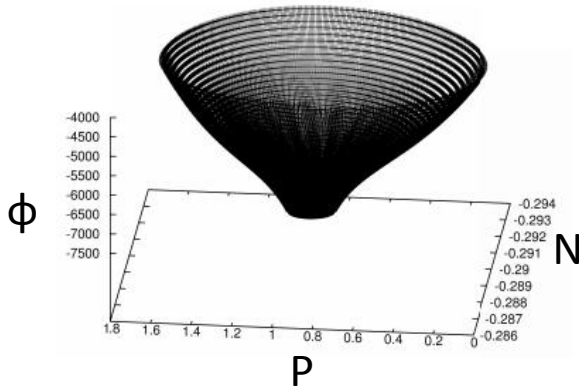
FIG. 5. Plot of Amplitude death attractor of the AD state shown in figure 4 in the parameter space of P(t), N(t), φ(t) for a fixed time delay τ = 14.
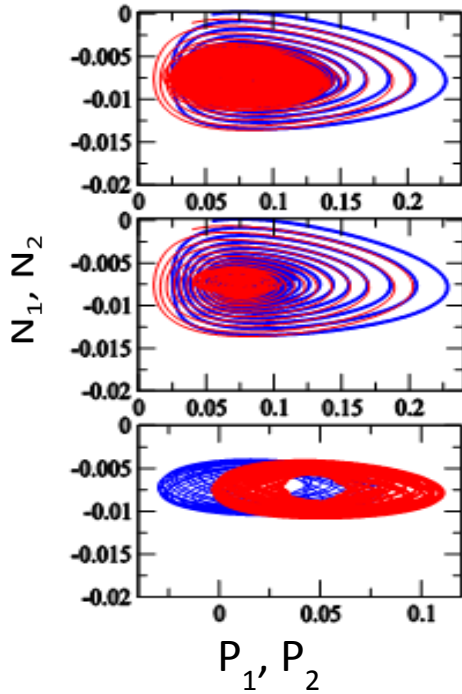

FIG. 6. (Color online) phase space plots of the multi-attractors dynamics (MS state in figure 4.) between two stable locking states as the initial condition are varied for fixed η, τ and initial transients of 10^5 data points are discarded.

INFLUENCE OF THE α IN AN INJECTION-LOCKED QUANTUM DOT DIODE LASER

Injection-locking of semiconductor lasers is one of the most attractive research topics since this method induces superior improvement in the high-speed characteristics of directly modulated lasers such as increasing the modulation bandwidth, suppressing nonlinear distortion, relative intensity noise, mode hopping and reducing chirp [5][6]. Previous work has focused on realizing high modulation bandwidths and associated design strategies, analyzed the modulation properties of the coupled system in the spectral domain, and numerically investigated the modulation response of the injection-locked system. In order to understand the limiting factors in an injection-locked system it is important to investigate the governing theory that can be obtained by properly modeling the impact of characteristic parameters such as the so-called linewidth enhancement factor (the α-parameter)[7]. A dimensionless volume averaged normalized approach to theoretically evaluate the nonlinear dynamics as a function of the injected field ratio and/or the detuning frequency for varied slave laser bias cases can be described as follows [8]:

$$\frac{dY}{d\tau} = ZY - \varepsilon Y\left(Y^2 - P\right) + \eta\cos\theta$$

$$\frac{d\theta}{d\tau} = \alpha Z - \alpha\varepsilon\left(Y^2 - P\right) - \frac{\eta}{Y}\sin\theta - \Delta\Omega$$

$$T\frac{dZ}{d\tau} = P - Z - Y^2\left(1 + 2Z - 2\varepsilon Y^2 + 2\varepsilon P\right) + \eta\cos\theta$$

with Y, describing the normalized field magnitude and Z the normalized carrier density. The T-parameter is the ratio of the cavity decay rate to the spontaneous carrier relaxation rate. Parameter P is proportional to the pumping current above threshold while coefficient ε accounts for the nonlinear carrier contribution to relaxation rate. The detuning and phase offset between the master and the slave are denoted ΔΩ and θ, respectively. The normalized injection strength is $\eta = \eta_0\sqrt{P}/\gamma_c$ with γ_C the cavity decay rate and η_0 the maximum injection strength. In solving the coupled normalized differential equations, the normalized field magnitude Y is not at steady state, and is thus represented as a dependent term in the normalized field magnitude and phase rate equations. In what follows, this model is used for "the stability analysis of a quantum dot laser.

The quantum dot laser under study is a ridge waveguide with 500-µm cleaved cavity length. The ground state (GS) emission wavelength is at 1560 nm [6].Because of the carrier filling in the lasing and non-lasing higher energy levels, gain compression effects are much larger as compared to bulk or quantum well materials[9]. Consequently, it has been shown that the α-factor in quantum dot lasers cannot be considered as a constant parameter since it strongly varies from a laser to another and also with optical output power. Figure 9 shows the bifurcation diagrams calculated at a constant pump current but for different values of the α-parameter ranging from 1

to 15(assuming $\Delta\Omega=0$). The objective of the calculations is to show the effects of a large α-factor on the laser's stability. Numerical results point out that taking into account such variations reveals strong modifications in the bifurcation diagram. On one hand, at low α (case (a)), the laser is always stable while forα=3 (case (b)), period one oscillation starts occurring. On the other hand, for α>3 (cases (c), (d), (e) and (f)),the bifurcation diagram exhibits chaos (at low injection ratio) followed by a cascade of periodic regimes converging to a stability area (at very high injection ratio). From a general point of view, simulations reveal that the smallest the α-parameter, the better. As far as the engineering point of view is concerned, these results are very important for the optimization of the microwave properties. Although the optical injection is used to purify the relaxation frequency as well as the modulation bandwidth, these numerical results point out that a large α-factor is detrimental for the laser's stability and can induce severe degradations in the microwave properties. As a conclusion, in order to maintain a wide stability area with optical injection associated to good microwave properties, a low α-factor is mandatory in quantum dot laser.
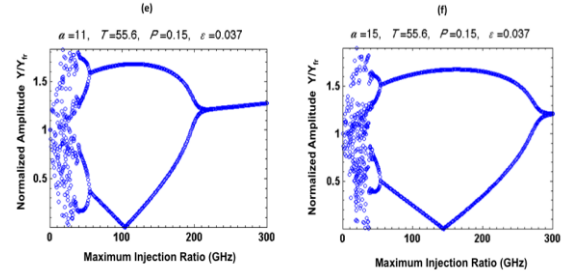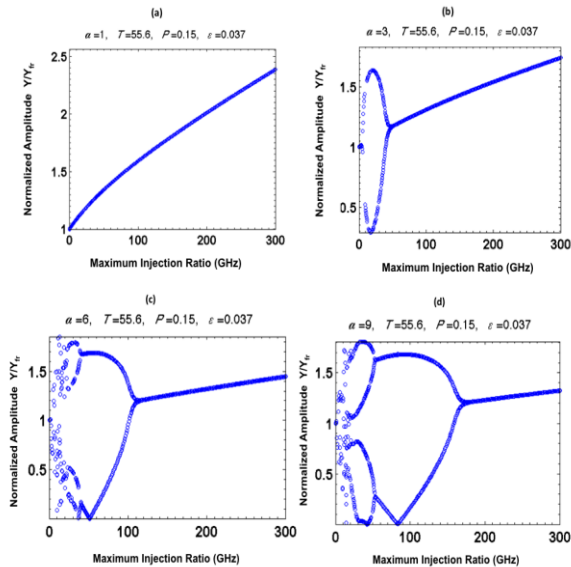

FIG. 7. Calculated bifurcations diagrams of a quantum dot laser with a variable linewidth enhancement factor

CONCLUSIONS

This new concept stabilizes the laser emission by directly controlling the nonlinearity and, consequently, the stability properties of the system. In general, semiconductor lasers with a sufficiently low alpha would be most interesting for practical applications due to the possibility of chirpless operation, and the insensitivity to delayed optical feedback or injection. The different dynamics and the strange bifurcation among them is investigated as a function of coupled-cavity time delay τ and the optical injection strength η. The correlation measure gives the signature of variation in amplitude death islands of complex dynamics of the delay-coupled lasers. The shrinkage of 'in-phase death state' and enlargement of 'out-of-phase death state' are observed and analyzed when η, α are varied. We provide detailed information about the effect of the variation of α on the dynamics of the system over wide ranges of relevant parameters. In particular, we give numerical evidence that the stability of the system increases with decreasing α. This last point is particularly predominant in a quantum dot laser for which a larger α-factor is usually observed.

[1] P. Kumar, A. Prasad and R. Ghosh, J. Phys. B: At. Mol. Opt. Phys.**42,** 145401 (2009).

[2] P. Kumar, A. Prasad and R. Ghosh, J. Phys. B: At. Mol. Opt. Phys.**41,** 135402 (2008).

[3] V. Resmi, G. Ambika, R. E. Amritkar, Phys. Rev. E 84, 046212 (2011).

[4] N. Punetha, R. Karnatak, A. Prasad, J. Kurths, and R. Ramaswamy, Phys. Rev. E **84**, 046212 (2011)

[5] T. B. Simpson, J. M. Liu, A. Gavrielides, IEEE Photon. Technol. Lett. **7**, pp. 709-711, (1995).

[6] N. A. Naderi, M. Pochet, F. Grillot, V. Kovanis, N. B. Terry and L. F. Lester, IEEE J. Quantum Electron. **15**, pp. 563-571, (2009).

[7] C. H. Henry, IEEE J. Quantum Electron.**18**, pp. 259–264, (1982).

[8]T. Erneux, V. Kovanis, A. Gavrielides and P. M. Alsing, Phys. Rev. A. **53**, 4372-4380, (1996).

[9] F. Grillot, B. Dagens, J-G Provost, H. Su, and L. F. Lester, IEEE J. Quantum Electron.**44,**pp. 946-951, (2008).

[10] P. Kumar, F. Grillot, "Signature of Multistability near the Phase-flip transition regimes in a mutually-delay-coupled diode laser system", 2012(to be submitted).