Pulse Characterization of Passively Mode-Locked Quantum-Dot Lasers Using a Delay Differential Equation Model Seeded With Measured Parameters

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Abstract—A delay differential equation-based model for passive mode locking in semiconductor lasers is shown to offer a powerful and versatile mathematical framework to simulate quantum-dot lasers, thereby offering an invaluable theoretical tool to study and comprehend the experimentally observed trends specific to such systems. To this end, mathematical relations are derived to transform physically measured quantities from the gain and loss spectra of the quantum-dot material into input parameters to seed the model. In the process, a novel approach toward extracting the carrier relaxation ratio for the device from the measured spectra, which enables a viable alternative to conventional pump–probe techniques, is presented. The simulation results not only support previously observed experimental results, but also offer invaluable insight into the device output dynamics and pulse characteristics that might not be readily understood using standard techniques such as autocorrelation and frequency-resolved optical gating.

Index Terms—Delay differential equations (DDEs), frequency-resolved optical gating (FROG), mode-locked semiconductor lasers, pulse asymmetry, quantum-dot lasers, semiconductor device modeling.

I. INTRODUCTION

OVER the last decade, quantum-dot mode-locked lasers (QDMLLs) have generated considerable interest as promising alternatives to quantum well (QW) sources, owing to their demonstrably superior performance characteristics [1]–[4]. Much of this interest in QDMLLs arises from the fact that certain properties unique to QD systems, such as low linewidth enhancement factors and reduced values of unsaturated gain and absorption enable a considerably wider map of operational stability, compared to their bulk and QW counterparts [3]–[6]. This is important, because most target applications require stable pulse generation over variable operating conditions. For instance, typical operating environments such as data centers require components to be able to tolerate high temperatures accompanied by rapid and considerable heat generation within the system. Consequently, devices need to be engineered to be able to sustain stable output pulses over a range of operating conditions [7]–[9]. This, in turn, necessitates the need to not only map out the parameter space of the device over a range of possible operating conditions, but also to gain insight into the functional parameters that are unique to QD systems and modify the properties of the device, compared to QW-based devices. With this objective, although a considerable amount of effort has gone into developing robust analytical and numerical models to describe passive mode locking in semiconductor lasers (and even modified specifically for QDMLLs), the information available in the literature about the properties unique to QD structures that give them superior performance characteristics relative to QW structures, and how they can be accessed and exploited so as to optimize device performance, is rather scarce. This has led to the associated theoretical and experimental work following two distinct paths, with the former focusing on developing analytical/numerical models of varying degrees of complexity to describe specific physical processes/phenomena, while the latter typically relies on a process of iterative design through an extensive testing and characterization. On the theoretical side, as more physical effects and processes are incorporated into the analysis, the model grows in complexity and evolves to encompass a large parameter space. But while this added complexity might enable a better understanding of the underlying device physics and operation, the lack of access to realistic values of many of the associated parameters at various operating conditions severely hinders the process of practical device design/engineering through numerical simulation.

Our previous work involved first deriving a condition for the onset of stable mode locking in a QDMLL [7], and subsequently, studying the temperature stability of QDMLLs based on measured temperature-dependent gain and loss spectra, where the gain and loss spectra were extracted as a function of gain section current density and absorber bias voltage, over a range of temperatures from 20 °C to 70 °C [8].
In this paper, we present a systematic approach toward using a delay differential equation (DDE)-based model for passive mode locking in a semiconductor laser [10]–[13] as a powerful and versatile tool to simulate device behavior over a range of operating conditions and to potentially serve as a quantitative map to guide device design. Recent work has shown that variants of this model can be used to study phenomena such as injection locking [14], and a multisection version of the model has been recently developed to study passive mode locking [15], [16]. Relative to the other models used to study passive mode locking in semiconductor lasers, the DDE model has a parameter space of just nine parameters that can be mostly extracted from measurable physical quantities. Hence, we derive new expressions to transform experimentally measured quantities to model parameters, such as the carrier relaxation ratio of a QDMLL. The standard technique to determine this parameter, which is defined as the ratio of the gain relaxation rate to the absorber relaxation rate, involves measuring gain and absorber recovery times using pump–probe techniques, individually for the semiconductor optical amplifier [17] corresponding to the gain section, and for the saturable absorber [18], [19] corresponding to the absorber section of the two-section laser. The approach discussed in this study provides an analytical expression for the carrier relaxation ratio as a function of static parameters that can be easily measured on a multisection QD laser test structure over temperature, current density, bias voltage, and wavelength. This allows mapping out the carrier relaxation ratio of the QDMLL over a range of operating conditions, each of which can be used as an input parameter in the DDE model in order to simulate device performance at that condition, while circumventing the need to perform pump–probe measurements.

In using the DDE model in its original form, we find that the important operational characteristics of the device under study are captured without the need for additional equations or parameters, thereby preserving its relatively simple formalism. For example, our analysis is shown to simulate experimental trends accurately, without the need to include additional equations or parameters, such as pulse narrowing with increasing absorber bias. In addition, it will be shown that the simulations accurately predict commonly observed experimental phenomena, such as pulse narrowing with increasing absorber bias. In (1) and (2) aforesaid, the subscript \( r \) corresponds to the gain (absorber) section, \( E(t,z) \) describes the optical field in the two sections. The variables \( N_{r,q}(t,z) \) are carrier densities in the gain/absorber sections, while the variables \( N_{r,q}^{\tau} \) are the carrier densities at transparency in the corresponding section. The parameter \( v \) is the light group velocity, assumed constant and equal in both sections. The parameters \( \alpha_g, \alpha_q, \Gamma_g, \Gamma_q, \) and \( \gamma_g, \gamma_q \) are, respectively, the linewidth enhancement factors, differential gains, transverse modal fill factors, and carrier density relaxation rates in the gain and absorber sections. The parameters \( J_g \) and \( J_q \) denote the injection current densities in the gain and absorber sections, respectively.

If a transformation is now performed to a frame of reference moving with the propagating pulse, the 2-D (spatiotemporal) system of PDEs is simplified to a 1-D (temporal) system of DDEs [11], [12]

\[
\frac{dA(\tau)}{d\tau} = \gamma \sqrt{\kappa} \exp \left[ \frac{1}{2} \left( \frac{1}{2} - \frac{i \alpha_q}{2} \right) G(\tau - T) \right]
\]

\[
- \left( \frac{1}{2} - \frac{i \alpha_q}{2} \right) Q(\tau - T) \right)] A(\tau - T) - \gamma A(\tau) \tag{3}
\]

\[
\frac{dG(\tau)}{d\tau} = g_0 - \Gamma G(\tau) - e^{-Q(\tau)} \left( e^{G(\tau) - 1} \right) |A(\tau)|^2 \tag{4}
\]

\[
\frac{dQ(\tau)}{d\tau} = q_0 - Q(\tau) - s \left( 1 - e^{-Q(\tau)} \right) |A(\tau)|^2. \tag{5}
\]

In (3)–(5) previously, \( \gamma \) incorporates the effect of spectral filtering and ensures that a finite number of modes participate in the mode-locking process, \( \alpha_q \) and \( \alpha_q \) are the linewidth enhancement factors in the gain and absorber media, respectively, and \( \kappa \) accounts for linear cavity losses. The delay parameter incorporates history, which appears in the first equation of the model through the evaluation of optical field \( A \), saturable gain \( G \), and saturable loss \( Q \) at \( \tau - T \), where \( T \) is the normalized
round trip time. The carrier relaxation ratio \( \Gamma = (\tau_{\text{abs}}/\tau_{\text{gain}}) \) is the ratio between the absorber and gain relaxation times, and \( s \) is the saturation parameter, defined as the ratio of the differential absorption to the differential gain. Finally, \( g_0 \) and \( q_0 \) are the unsaturated gain and absorption, respectively.

A recent experiment performed by our group [8] made use of the segmented contact method [22], [23] to obtain modal gain and total loss spectra as a function of current density, over absorber bias voltage and temperature for the QD material comprising the device. These curves were used to extract the modal gain \( (g_{\text{mod}}) \) and unsaturated absorption \( (a_0) \) as a function of current density at each bias condition.

Now, from (2) in [8], we have the following threshold condition for lasing:

\[
(g_{\text{mod}} (J) - \alpha_i) L_g - (a_0 + \alpha_i) L_a = \alpha_m (L_a + L_g)
\] (6)

where, \( \alpha_m \) and \( \alpha_i \) represent, respectively, mirror losses and internal losses, and \( L_g \) and \( L_a \) represent, respectively, the gain and absorber section lengths, with

\[
\alpha_m = \left(\frac{1}{L}\right) \ln \left(\frac{1}{R_1 R_2}\right)
\] (7)

where \( (L_a + L_g) = L \) and \( R_1, R_2 \) denote mirror reflectivities. Substituting (7) into (6), we get

\[
(g_{\text{mod}} (J) - \alpha_i) L_g = (a_0 + \alpha_i) L_a - \left(\frac{1}{2}\right) \ln (R_1 R_2).
\] (8)

In the DDE model, \( G \) and \( Q \) describe the saturable gain and loss introduced by the gain and absorber sections, respectively, and \( \kappa < 1 \) describes the total roundtrip nonresonant linear intensity losses. Thus, the threshold condition for lasing is given by

\[
\kappa e^{(G-Q)} = 1.
\] (9)

From (8) and (9), we find

\[
G = (g_{\text{mod}} (J) - \alpha_i) L_g
\] (10)

\[
Q = (a_0 + \alpha_i) L_a
\] (11)

\[
\kappa = \sqrt{R_1 R_2}.
\] (12)

Next, we derive expressions relating the saturable gain and loss \((G \text{ and } Q, \text{respectively})\) to the unsaturated gain and absorption parameters \((g_0 \text{ and } q_0, \text{respectively})\) from the definitions given in [11] (see the Appendix for complete derivations).

The following relations are obtained:

\[
g_0 = \Gamma G
\] (13)

where \( \Gamma = (\tau_{\text{abs}}/\tau_{\text{gain}}) \).

Similarly

\[
q_0 = (Q/s)
\] (14)

where

\[
s = \frac{g_0 \Gamma q}{g_0 \Gamma q} = \frac{[g_{\text{mod}} (J)/\partial J]_{g_{\text{mod}} (J)=0}}{[g_{\text{mod}} (J)/\partial J]}. \quad (15)
\]

It has previously been found that the differential gain at optical transparency, seen in the numerator, is a good estimate for \( g_0 \). We also make the reasonable assumption that the optical confinement factors in the gain and absorber sections are equal, so that \( \Gamma_g = \Gamma_a \).

Further, in order to enforce the condition for stable, fundamental mode-locked operation (as opposed to \( Q \) switched, harmonic mode locked or continuous wave operation), we perform a linear stability analysis similar to that shown in [11] (Fig. 5), and arrive at the relation

\[
q_0 = mg_0
\] (16)

where \( m \) represents the slope of a line in the \( q_0 - q_0 \) plane, chosen to lie entirely within the region corresponding to the fundamental mode-locking regime. This yields an expression for the carrier relaxation ratio \( \Gamma \) which depends solely on measurable static device parameters defined previously in (10) and (11)

\[
\Gamma = \left(\frac{Q}{m G s}\right) = \frac{(a_0 + \alpha_i) L_a}{m (g_{\text{mod}} (J) - \alpha_i) L_g s}.
\] (17)

For the simulations in this paper, \( m \) was chosen to be 2. The carrier relaxation ratio of the device is, thus, extracted as a function of its measured modal gain and absorption parameters and its physical dimensions.

III. RESULTS AND DISCUSSION

A. Description of Simulated Device

The device studied in this study is an 8-stack, dots-in-a-well (DWELL) laser structure grown by elemental source molecular beam epitaxy (MBE), with a 7-mm gain section and a 1-mm absorber section generating a 4.96-GHz pulse repetition rate under fundamental mode locking. A schematic of the device with important details of the epitaxial layer structure is shown in Fig. 1. The parameters appearing on the right-hand side of (17) can be readily measured as a function of gain-section current density, wavelength, absorber reverse bias and temperature using the segmented contact method [22], [23]. Previously, we used these measurable, namely, the gain and loss characteristics, to study the impact of temperature on the stability of QDMLLS [8].

B. Extraction of the Carrier Relaxation Ratio

The model parameter values used in determining \( \Gamma \) are summarized in Table I (3-V absorber bias and device temperatures of 20 °C, 30 °C, 40 °C, 50 °C, and 60 °C) and represent values
at the onset of mode locking, i.e., the point at which a pulse was observed on an autocorrelator (AC) [8].

This data is plotted in Fig. 2 (solid squares), along with the values of absorption recovery times reported in [19] (solid circles) as a function of temperature. The similarity of the two plots suggests that the trend exhibited by (17) for $\tau_a$ over temperature follows the trend exhibited by the absorption recovery time $\tau_b$.

As a comparison, the pump–probe characterization of the absorber recovery times reported in [19] measured a decrease in the absorption recovery time with increasing temperature for a constant reverse bias of 2 V (the device studied in that work comprised a 190-nm active region, which corresponds to an electric field of 105 kV/cm). For a similar temperature range, with a 3-V reverse bias (288-nm active region, yielding an electric field of 104 kV/cm) and a similar QD epitaxial structure, we extract a comparable decrease in $\tau_a$, as seen from Fig. 2. While it is known that both gain and absorption recovery times decrease with temperature [19], [24], the similarity in the trends discussed earlier, together with the decreasing value of $\Gamma$ in Table I is strongly suggestive that $\tau_a$ is the dominant factor in determining the behavior of $\Gamma$. Thus, it may be inferred that a decreasing value of $\Gamma$ with increasing temperature suggests that the absorber recovery time decreases by a significantly larger factor than the corresponding reduction in the gain recovery time.

Table II shows the variation of the extracted relaxation ratio at a fixed temperature of 20 °C, for absorber bias voltages of 0, −3, and −5 V, wherein, a trend similar to that seen with increasing temperature is observed for $\Gamma$.

Again, the carrier relaxation ratio shows a decrease with increasing reverse bias. This trend agrees with the results obtained in [18] using pump–probe techniques, and supports the experimentally observed phenomenon that stronger bias voltages produce narrower pulses, given that pulse trimming is achieved primarily with faster absorption recovery rates, which shortens the net gain window during which pulses are generated.

In view of the previous discussion, it can be seen that the trends exhibited by the carrier relaxation ratio $\Gamma$ extracted from gain–loss data acquired by the segmented contact method using the expression derived previously in (17) follow the trends that can be expected by individually measuring $\tau_a$ and $\tau_c$ using pump–probe techniques, for a range of operating temperatures and bias voltages. Thus, it is reasonable to expect that this method of extracting $\Gamma$ from gain and loss data provides a viable alternative to pump–probe measurements.

### TABLE I
**MEASURED/EXTRACTED TEMPERATURE-DEPENDENT PARAMETER VALUES (DIMENSIONLESS) AT THE ONSET OF MODE LOCKING FOR A −3-V ABSORBER BIAS**

<table>
<thead>
<tr>
<th>$T(°C)$</th>
<th>$G$</th>
<th>$Q$</th>
<th>$G-Q$</th>
<th>$Q/G$</th>
<th>$s$</th>
<th>$\Gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>4.22</td>
<td>3.20</td>
<td>1.02</td>
<td>0.76</td>
<td>4.65</td>
<td>0.08</td>
</tr>
<tr>
<td>30</td>
<td>4.18</td>
<td>3.33</td>
<td>0.85</td>
<td>0.79</td>
<td>5.89</td>
<td>0.07</td>
</tr>
<tr>
<td>40</td>
<td>3.95</td>
<td>3.43</td>
<td>0.52</td>
<td>0.87</td>
<td>8.47</td>
<td>0.05</td>
</tr>
<tr>
<td>50</td>
<td>3.33</td>
<td>3.03</td>
<td>0.30</td>
<td>0.91</td>
<td>10.68</td>
<td>0.04</td>
</tr>
<tr>
<td>60</td>
<td>2.89</td>
<td>2.43</td>
<td>0.46</td>
<td>0.84</td>
<td>12.20</td>
<td>0.03</td>
</tr>
</tbody>
</table>

C. Transformation of Measured Values to Model Parameters

The expressions derived in Section II, particularly for the carrier relaxation ratio $\Gamma$, provide a convenient technique to accurately determine dimensionless parameters for input into the DDE model. Such a situation may be envisioned for a case where device performance needs to be simulated over a range of operating conditions.

A close examination of the system of DDEs given in (3)–(5) reveals the parameter space involved. Thus, in order to model a QDMLL realistically, it is imperative to constrain as many of the parameters as possible to values obtained from measurements on an actual QD device. The simulation results, obtained by seeding the model with these parameters (transformed to dimensionless form, using the relations derived in Section II), can then be used to interpret and characterize the mode-locking performance of the same device.

Since the onset of mode locking occurs at or beyond threshold, threshold values provide a convenient set of initial conditions for (3)–(5). The first step, then, is to use values measured at threshold in (10) and (11) to calculate $G(0)$ and $Q(0)$, and then use the values of $G(0)$ and $Q(0)$ in (13) and (14) to calculate the corresponding unsaturated parameters, $g_0$ and $q_0$, at threshold. Further, parameters $\kappa$ and $s$ can be directly found from (12) and (15), respectively, while the carrier relaxation ratio $\Gamma$ is extracted from (17). Next, the absorber relaxation time is used to infer the delay parameter $T$ and the spectral filtering coefficient $\gamma$ for each operating condition as follows. The delay parameter $T$ is calculated for each case by simply scaling the cavity round trip time to the absorber relaxation time for that case. In all cases, the absorber relaxation time was estimated from the experimental results in [18] and [19] for a saturable absorber with an epitaxial layer structure similar to the device simulated in this work, following from the discussion in the previous section. For instance, at an operating temperature of 20 °C, the bias voltage applied on the absorber section was expressed as an electric field strength across the active region. This value was then converted back to a bias voltage for the saturable absorber structure studied.
in [18], and the corresponding value of absorber relaxation time was found from [18] (Fig. 2). For higher temperatures, the exponential expression for thermionic emission given in [18] (eqn. (2)) was used to estimate the same. Also, the spectral filtering coefficient \( \gamma \) was estimated from the measured optical spectrum of the device as the product of the number of cavity modes taking part in the mode-locking process and the cavity intermode frequency spacing \( (T^{−1}) \). Finally, the linewidth enhancement parameters in the gain and absorber sections were chosen such that the steady-state pulsewidth obtained from simulation was consistent with the typical pulsewidth measured on an AC for the actual device.

Now, having reduced the number of free parameters to two (namely, the linewidth enhancement factors in the gain and absorber sections), we set the two to be equal in order to simplify the analysis. Previous work [11], [25] has shown that this case corresponds to the most stable operating point, according to the DDE model. While this assumption could result in some departure from the actual values that the linewidth enhancement factors in the gain and absorber sections might take, the primary objective here is to study the broad trends, and how they might affect device performance. Further, it is reasonable to expect that any such departure may be marginal close to threshold, considering that the device under study in this study has a significantly longer gain section, so that the device behavior can be expected to be dominated by the gain section component. This value is then fit to the deconvolved AC pulsewidth measurement.

The results shown in the following were obtained as follows. The parameter values measured/extracted as discussed previously were used as initial conditions for the simulations. The system of equations defining the model (3), (4), and (5) was then integrated over two thousand round trip times, in order to enable the system to reach steady-state dynamics.

Table III lists the parameter values used to seed the simulations with parameters measured/extracted at saturable absorber bias conditions of 0, −3, and −5 V, at a temperature of 20 °C. As mentioned at the beginning of this section, the parameter values were obtained from the experimental data presented in [8], and made dimensionless for input into the model. The parameter values presented in Table III correspond to threshold conditions, i.e., current density values of 357 A/cm² at 0 V, 462 A/cm² at −3 V, and 537 A/cm² at −5 V (see ref: [8, Tables I and II]). As shown previously, the model is successful in capturing well-known trends in QDMLLS such as the effect of an increased absorber bias in trimming the pulses [26]. This paper discusses the mode-locking behavior and associated trends at an operating temperature of 20 °C in more detail including the effect of the linewidth enhancement factor, by seeding the model with parameters extracted at three different absorber bias voltages, as detailed in Table III previously. As will be seen in Sections III-D and III-E in the following, the results predicted by simulation showed very good agreement with experimental observations.

One of the important predictive capabilities of the DDE model is the ability to gain insight into the range of values of the linewidth enhancement factor close to threshold for stable, fundamental mode-locking to be likely. It is seen that the linewidth enhancement factors in the gain and absorber sections have a profound influence on pulse structure, so that past a certain value of the linewidth enhancement factor, instabilities emerge in the pulse structure. Typical examples of instabilities of this type include pulse splitting and satellite pulses.

Consequently, sudden changes in pulse structure were used to determine mathematical ranges for the linewidth enhancement factor for stable, fundamental mode locking to be likely. Table IV summarizes the mathematically likely ranges for \( \alpha_g \) and \( \alpha_q \) discussed previously, and the values used in each case to obtain a pulsewidth closest to what was obtained experimentally from AC measurements.

For instance, for the 5-V reverse bias case, the mathematically permissible range of \( \alpha_g \) and \( \alpha_q \) extends to 1.1, past which the structure of individual pulses exhibits instabilities such as splitting, as shown in Fig. 3 for \( \alpha_{g,q} = 1.2 \) (black pulse train), in sharp contrast to the stable pulses obtained with \( \alpha_{g,q} = 0.5 \) (green pulse train). As presented in Table IV, the general trend with increasing reverse bias is to reduce the range of acceptable linewidth enhancement factors for stable locking, although the device itself operates at a larger value with increasing reverse bias because of the consequent increase in threshold. Although a larger linewidth enhancement factor results in pulse broadening due to self-phase modulation, the improved pulse trimming at higher reverse bias in the absorber can help counteract this effect. The physics of these processes are all accounted for in the model.

### D. DDE Simulations as an Indicator of Lower Unsaturated Gain and Absorption in QD Devices

Since the unsaturated gain and loss parameters are expected to be lower for QD structures in comparison to QW structures,
TABLE IV
Likely $\alpha$-PARAMETER RANGES FOR STABLE, FUNDAMENTAL ML OVER BIAS VOLTAGE AT 20 °C

<table>
<thead>
<tr>
<th>V</th>
<th>Range of $\alpha_p$, $\alpha_s$</th>
<th>$\alpha_p = \alpha_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0 &lt; \alpha_p \leq 2.6$</td>
<td>0.1</td>
</tr>
<tr>
<td>-3</td>
<td>$0 &lt; \alpha_p \leq 2.1$</td>
<td>0.2</td>
</tr>
<tr>
<td>-5</td>
<td>$0 &lt; \alpha_p \leq 1.2$</td>
<td>0.5</td>
</tr>
</tbody>
</table>

the effect of varying this pair of parameters was important to this study.

For an operating temperature of 20 °C and a 0 V reverse bias, the values of $g_0$ and $q_0$ can be extracted from the parameter values listed in the first column of Table III using (13) and (14). Next, keeping all other parameters unchanged from Table III, $g_0$ and $q_0$ are each scaled up by a factor of 10. As seen clearly from Fig. 4(a), this leads to significant instabilities in the steady-state output. Moreover, under the influence of an applied reverse bias on the absorber section or an elevated temperature [8], the simulation results predict stable, fundamental mode locking for an even smaller range of unsaturated gain and absorption.

For instance, for the cases $T = 20$ °C, −3-V bias [see Fig. 4(b)] and $T = 60$ °C, 0-V bias [see Fig. 4(c)], keeping all the parameter values unchanged, $g_0$ and $q_0$ were each scaled up by a factor of 5, whereby, the output is seen to have switched to higher order mode locking.

This suggests that even a fivefold increase in the values of $g_0$ and $q_0$ (which represents values typical of QW devices) is impractical for stable, fundamental mode locking for the QD device under study. This is strongly suggestive that in order for QD systems to achieve stable pulsation at the fundamental repetition rate at elevated temperatures or under an applied reverse bias, the values of unsaturated gain and absorption must be on the order of the values extracted in Tables I–III. Higher values tend to push the system out of the stable, fundamental mode-locking regime. In other words, the simulations show that lower values of unsaturated gain and absorption in the QD device under study compared to a QW version are necessary to achieve stable, fundamental mode locking.

Finally, Table V gives a comparison between typical pulsewidths obtained experimentally from AC measurements (assuming a sech$^2$-intensity profile [13]), and the corresponding values obtained by simulation using the DDE model. Comparing the last two columns shows very good agreement between the two.

However, to address subtleties in the DDE model regarding pulse shape, and not just width, a more complete characterization of the pulse is required. In the next section, FROG is used to measure the evolution of the pulse shape with increasing absorber bias and compared to the DDE model.

E. Correlation With FROG Measurements

The schematic diagram of the collinear second-harmonic generation (SHG) FROG system is shown in Fig. 5 [28]. The optical output of the laser is collected with an optical head, which integrates a lens, an isolator, and a short 1-m single-mode polarization-maintaining (PM) fiber pigtail, and then is coupled into the FROG system through a PM fiber. The
pulse train is sent into a free-space Michelson interferometer to produce pulse pairs with variable delay that are coupled into an aperiodically poled lithium niobate (A-PPLN) waveguide with a 6-cm long poling region to produce a SHG signal. The variable delay was produced by a voice coil-actuated servo having a delay resolution of 3.5 fs. The servo can be controlled either digitally via computer or by an analog waveform. The two delay pulses were sent collinearly into an A-PPLN waveguide.

The A-PPLN chip improves SHG conversion efficiency of the mode-locked diode lasers to a level that allows real-time data acquisition. By comparing the original QDMLL wavelength spectrum with the value computed from the FROG measurement, the bandwidth of the A-PPLN is verified to be sufficiently wide. At each relative time delay between the two pulse replicas, the SHG spectrum is recorded using an Ocean Optics QE65000 spectrometer. Because of the collinear geometry, interference fringes are produced. By applying an ac modulation to the servo position, the delay is modulated, and the fringes are averaged out if a sufficiently long integration time is used. The dc background from the low-pass filtered FROG trace is removed using background subtraction [29], [30]. The data acquisition, preprocessing, and processing were done using a commercial FROG software that was developed by Mesa Photonics.

FROG measurements on the device under study in this study revealed a curious trend. As seen from Fig. 6(a), a bias condition of a 95-mA gain current and a −3 V bias on the absorber section yielded a temporal pulse shape that is typical for such devices, with a steep rising edge, and a slower falling edge.

However, keeping the gain current fixed, when the absorber bias was increased to −5 V, the asymmetry of the pulse was observed to flip as seen in Fig. 6(b). Specifically, the trailing edge of the pulse is seen to show a faster decay than the rise time of the leading edge. An identical trend was seen at higher absorber bias, as seen in Fig. 6(c) and (d), where the pulse asymmetry reversal in going from an absorber bias of −3 to −5 V was observed for a gain current of 100 mA.

Simulation results of the DDE model seeded with physically measured parameters provide an invaluable tool to help understand this trend. Fig. 7 shows simulation results for the −5-V absorber bias condition near threshold at 20 °C, with \( \alpha_g = \alpha_q = 1.1 \) (solid green plots), and \( \alpha_g = \alpha_q = 1.6 \) (dashed red plots). The switching of pulse asymmetry can be clearly seen in Fig. 7(a), from the green, solid pulse, to the red, dashed pulse.

The reversal in asymmetry may be interpreted as follows. At higher bias voltage, the gain section requires a higher threshold current, so that the higher concentration of injected carriers induces an increase in the linewidth enhancement factor. Combined with the length of the gain section, this results in significant pulse broadening due to dispersion and self-phase modulation as the pulse propagates through the gain section. Simultaneously, the applied bias on the absorber section has the effect of expediting the sweep-out rate of the photogenerated carriers, leading to a faster recovery time. A careful comparison of the plots in Fig. 7(b) and (c) reveals the subtle distinguishing features between the two cases.

In the first case (\( \alpha_g = \alpha_q = 1.1 \), solid green curve), the pulse is “less broadened” in the gain section, owing to less self-phase modulation due to the relatively lower linewidth enhancement factor. Thus, the pulses exiting the gain section are expected to be narrower in the former case. The steeper leading (rising) edge associated with a narrower pulse causes the absorber to saturate more rapidly, as seen from the green curves in Fig. 7(b).

By contrast, in the second case (\( \alpha_g = \alpha_q = 1.6 \), dashed red curve), a “more broadened” pulse exiting the gain section with a less steep leading edge profile is not able to saturate the absorber quite as fast, so that the absorber saturation is noticeably more gradual than in the former case, as seen from the dashed red curves in Fig. 7(b).

The trailing (falling) edge profile is governed by the absorber dynamics. On inspection of the saturable loss profiles shown in Fig. 7(b), one can observe that the absorber recovery is slightly more rapid for the dashed red plot (\( \alpha_g = \alpha_q = 1.6 \)). Thus, due to the fact that the trailing edge experiences more loss at its tail, it is “better trimmed,” so that the pulse has a steeper trailing (falling) edge. The combined effect of the gain and absorber sections is that in the former case (\( \alpha_g = \alpha_q = 1.1 \)), the leading
edge of the pulse is steeper, while the trailing edge shows a relatively gradual decay, whereas in the latter case (\(\alpha_g = \alpha_g = 1.6\)), the leading edge is less steep, the trailing edge is better trimmed, and the asymmetry is reversed.

Pulse asymmetry is a characteristic feature of model-locked two-section quantum-dot lasers [31]. The variation of pulse asymmetry has been studied before in dye lasers [32], and simulated for a microchip laser system [33], and holds promise for applications such as chirp compensation and pulse compression. It is in such instances that simulation results of the DDE model seeded with measured parameters can provide an invaluable aid to complement the diagnostic techniques aforementioned.

IV. Conclusions

A (DDE) model for passive mode locking in semiconductor lasers, when seeded with QD laser-measured parameters, has been shown to accurately simulate experimental trends and observations. New expressions have been derived to transform quantities experimentally measured from gain and loss spectra to parameters suitable for input into the model. In particular, a novel approach for estimating the carrier relaxation ratio of a device from static gain and loss measurements has been discussed as a convenient alternative to conventional pump–probe techniques. The results of the simulations have been found to support experimental observations. The effect of increasing unsaturated gain and absorption, while retaining the values extracted from QD gain/loss measurements for the other parameters, was to predict departures from stable, fundamental mode locking in the output. Thus, it was shown that QDMLLs must operate with generally lower values of unsaturated gain and loss than QW devices. The simulations also predict a much lower range of values of linewidth enhancement factor for stable, fundamental mode locking to be probable, when a reverse bias is applied on the absorber section. In addition, the simulations yield information that is often not accessible from experimental techniques such as autocorrelation. Finally, the model provides invaluable insight into understanding abrupt transitions in the dynamics of the device output, such as the sudden switching of pulse asymmetry. This phenomenon was confirmed for the first time with direct FROG measurements of the laser.

Appendix

Following from (6)–(12) in Section II, here we derive expressions relating the saturable gain and loss \((G\) and \(Q\), respectively) to the unsaturated gain and absorption parameters \((g_0\) and \(g_0\), respectively). From the definitions given in [11], we have \((\zeta\) and \(\tau\) represent, respectively, spatial and temporal coordinates in the frame of reference of the pulse)

\[
G(\tau) = \int_{\zeta_1}^{\zeta_4} N_g(\tau, \zeta) d\zeta
\]

\[
= \int_{\zeta_1}^{\zeta_4} \frac{\varphi_g \Gamma_g}{\gamma_g} [N_g(\tau, \zeta) - N_g^{tr}] d\zeta
\]

\[
g_0 = \int_{\zeta_1}^{\zeta_4} j_0 d\zeta = \int_{\zeta_1}^{\zeta_4} \frac{\varphi_g \Gamma_g}{\gamma_g} (J_g - N_g N_g^{tr}) d\zeta
\]

substituting for \(J_g = \gamma_g N_g\), we obtain

\[
= \int_{\zeta_1}^{\zeta_4} \frac{\varphi_g \Gamma_g}{\gamma_g} [\gamma_g N_g(\tau, \zeta) - \gamma_g N_g^{tr}] d\zeta
\]

\[
g_0 = \Gamma \int_{\zeta_1}^{\zeta_4} \frac{\varphi_g \Gamma_g}{\gamma_g} \zeta [N_g(\tau, \zeta) - N_g^{tr}] d\zeta
\]

where \(\Gamma = (\tau_q / \tau_g)\).

Comparing (1) and (2), we get

\[
g_0 = \Gamma G.
\]
Similarly
\[ Q (\tau) = -\int_{\zeta_0}^{\zeta} n_q (\tau, \zeta) d\zeta \]
\[ = -\int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} \left[ N_q (\tau, \zeta) - N^\text{tr}_q \right] d\zeta \]
\[ Q (\tau) = -\int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} \left[ \frac{q_N (\tau, \zeta) - q_{N^\text{tr}}} \right] d\zeta \]
substituting for \( q_N N_q = J_q \), we obtain
\[ -\int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} \left[ J_q (\tau, \zeta) - \frac{\gamma q_{N^\text{tr}}} \right] d\zeta \]
\[ Q (\tau) = \int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} N^\text{tr} q d\zeta, \text{ since } J_q (\tau, \zeta) = 0 \]
\[ Q (\tau) = \int_{\zeta_0}^{\zeta} \left( \frac{v}{\gamma} \right) g_{1/2} g_{\alpha} g_{\text{mod}} N^\text{tr} q d\zeta \]
\[ = s \int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} N^\text{tr} q d\zeta \]
where
\[ s \equiv \frac{g_{G \alpha} m G s}{g_{1/2} g_{\alpha}} = \left[ \frac{\partial g_{\text{mod}} (J)/\partial J}{g_{\text{mod}} (J)/\partial J} \right]_{g_{\text{mod}} (J) = 0} \]
Also
\[ q_0 = \int_{\zeta_0}^{\zeta} J_q d\zeta = \int_{\zeta_0}^{\zeta} \frac{v g_{1/2} g_{\alpha}}{\gamma} N^\text{tr} q d\zeta \]
Comparing (4) and (6), we get
\[ q_0 = \left( Q / s \right) . \]
Finally, we constrain the regime of operation to stable, fundamental mode locking using a dynamical stability analysis similar to that presented in [11] (Fig. 5). A suitable constraint to achieve stable, fundamental mode locking in terms of \( g_{\alpha} \) and \( q_0 \), is given by the locus of points lying on the line \( q_0 = m q_0 \), where \( m \) represents the slope of a line in the \( q_0 - q_0 \) plane, chosen to lie entirely within the region corresponding to the fundamental mode-locking regime.

Thus, we have, \( \frac{Q}{g_{\alpha}} = m \Rightarrow \left( \frac{Q}{g_{\alpha}} \right) = m \). Therefore, using (10) and (11) from Section II, we obtain
\[ \Gamma = \left( \frac{Q}{m G s} \right) = \left( \frac{a_0 + \alpha_i}{m g_{\text{mod}} (J) - \alpha_i} \right) L_s . \]

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REFERENCES
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RAGHUNATHAN et al.: PULSE CHARACTERIZATION OF PASSIVELY MODE-LOCKED QUANTUM-DOT LASERS

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