Image aquisition - Basics of digital photography

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Cours ATSI
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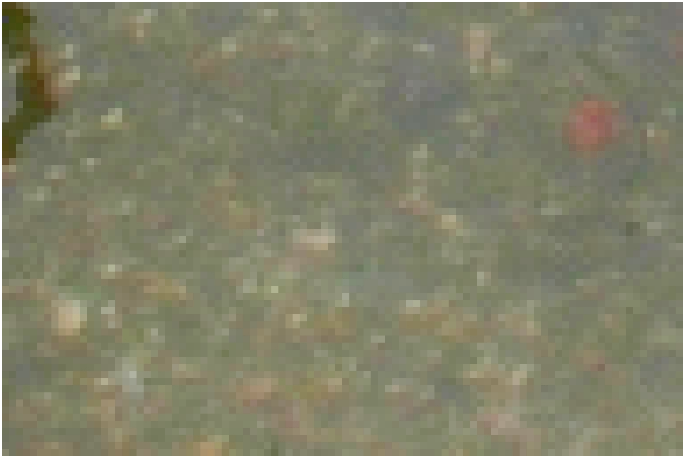
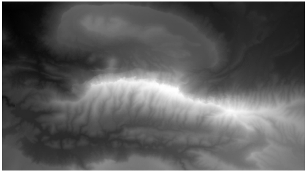
1. Examples
2. The optical system
3. Fourier transform and bidimensional sampling
4. Noise modeling



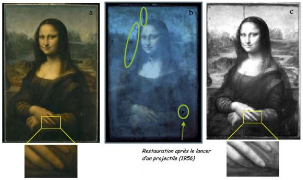

BALLESTA
SE VENDE

Studied devices :

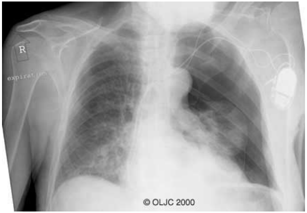


Digital elevation model

radar image

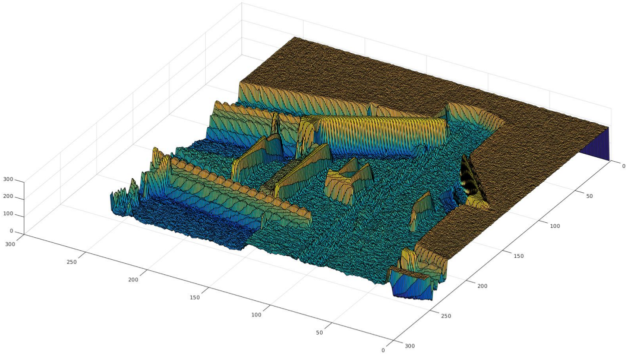


Mona Lisa : visible light, UV, IR

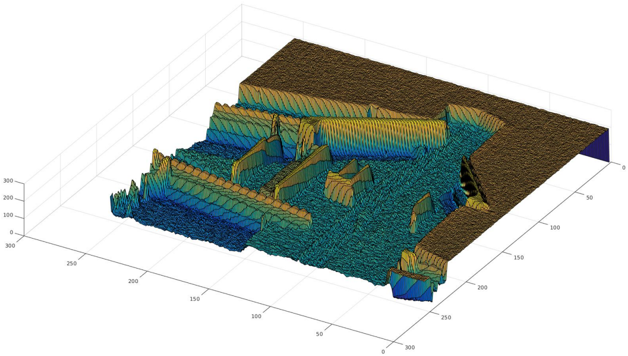


radiography (X rays)

Range image (laser)



An image, seen as a surface :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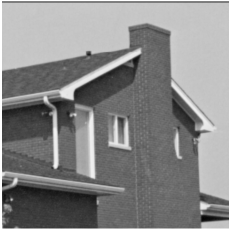

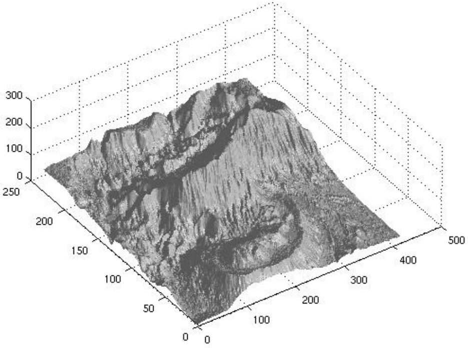
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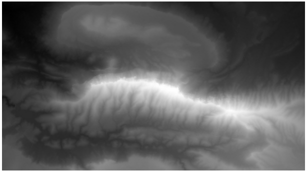



PREFECTURE

The pinhole model (Sténopé)

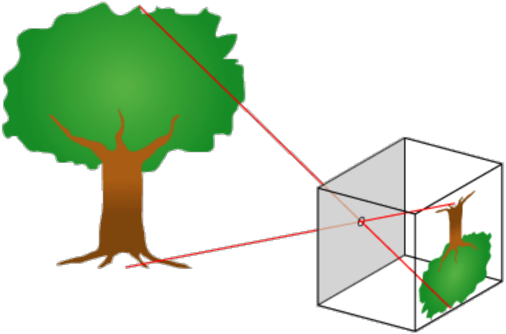
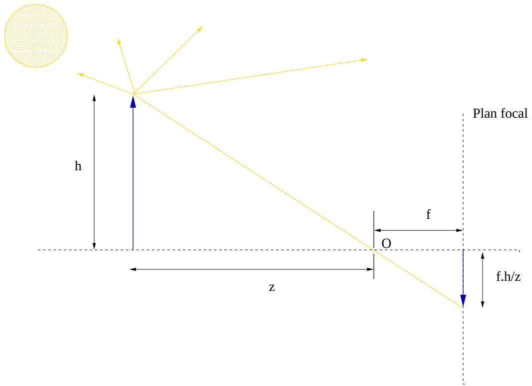


Illustration from wikipedia.org

(Aristote, Alhazen 10^{th} century, Brunelleschi begining of 15^{th} century, Da Vinci 1500, Kepler 1604 *camera obscura*)

Courtesy Manon Heffernan

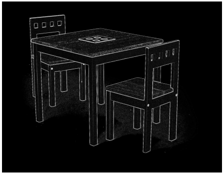
The pinhole model



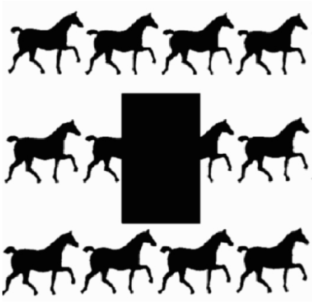
Part of the light emitted by the object is selected by the aperture O and is projected onto the focal plane. Distance f is called the focal distance.

First consequence : there is no absolute perception of object sizes.

Size constancy. Ponzo illusion.

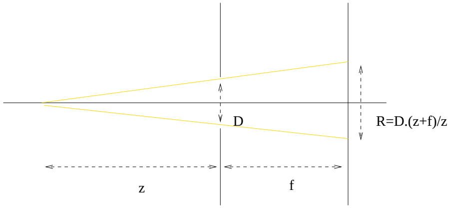


Second consequence :
Objects are mostly opaque and occlude themselves.
The occultation phenomenon → image discontinuity (edges)



Human perception is able to reconstruct hidden parts by amodal completion
(more in a forecoming lecture about inpainting)

In order to get more light : increase the aperture size



Aperture O : $\rightarrow$ light cones $\rightarrow$ blur

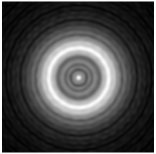
For an aperture with diameter D and S the scene at distance z, we get

$$g_z * S,$$

where g_z is the indicator function of a disk of radius $(z+f)D/z$.

If $z >> f$ each point of the scene yields approximately the same blur amount

Finer modeling : diffraction phenomenon ; only noticeable for small D values



Left : original image ; right : convolution with a low-pass filter (here a Gaussian filter)

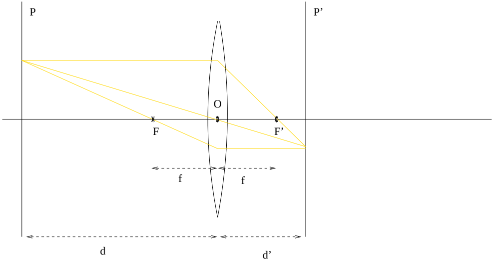
Sheila Bocchine - creative commons

A handmade pinhole

The lens

- In order to get a sharper image with large enough aperture (need of light) : use of a lense.
- We still get $g * S$, where g is the impulse response of the optical system (neglecting various aberrations and interferences).

The "thin lense" model



Rays coming from P converge on P'. We write f for the *focal distance*
$\rightarrow$ Descartes (thin lense) formula

$$\frac{1}{d} + \frac{1}{d'} = \frac{1}{f}.$$

To *focus* : adjust the position of P.
If the photographed object is not in P : out-of-focus blur $\rightarrow$ convolution.

Hypothesis :

- Light ray are close to the normal to the lense surface
- Light traveling inside the lense is neglected.

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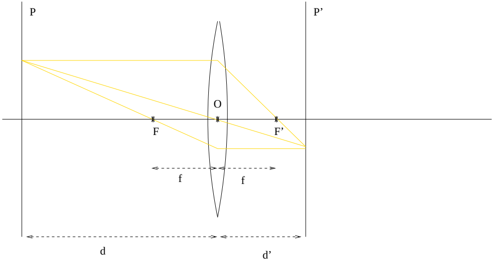

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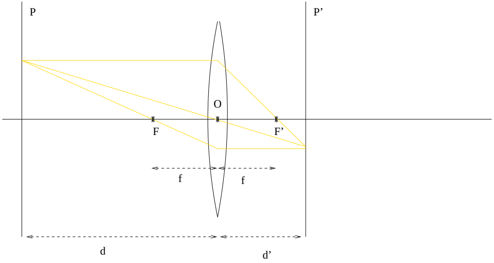

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We observe a scene

$$\tilde{s} = g_o * s,$$

where

$$g_o = g_{ouv} * g_{flou} * g_{fil}.$$

- g_{ouv} finite aperture
- g_{flou} focus inacuracy
- g_{fil} motion blur

g_o is called the Point Spread Function (PSF) (*réponse impulsionnelle* in French) of the optical system.

Limitations

An important shortcoming of the **convolution** model :
blur depends on the distance between the objects and the camera !

Depth of field

Defined as the distance between the closest and the farthest objects that are *in focus* for a given imaging device.
In focus means that the blur size is less than ϵ (e.g. 1 μm).

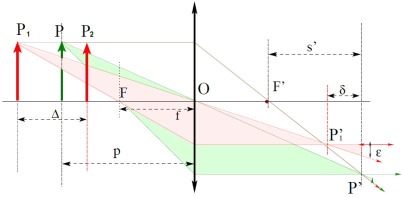

Figure by courtesy of H. Maître

Using Descartes formula, and writing D for the aperture of the system,

$$\Delta = \frac{2\epsilon D p f(p-f)}{D^2 f^2 - \epsilon^2 (p-f)^2}$$

Assuming further that $\delta << s$ and $f << p$, we get

$$\Delta \approx \frac{2\epsilon p^2}{Df}$$

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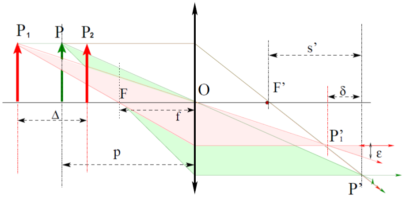

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Depth of field (DOF)

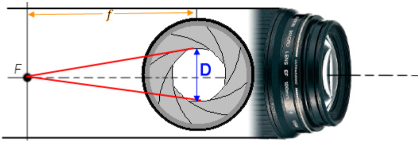
$$\Delta \approx \frac{2\epsilon p^2}{Df}$$

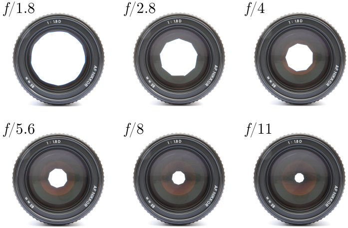
DOF therefore depends on

- aperture size
- focal f (the zoom factor)
- object distance from the camera

Ouverture

D diameter of the *diaphragm* ; f focal distance ;
the aperture value (f-number) is defined as $N = f/D$ (generally a geometric progression)





images wikipedia.org

When N increases (at constant f) :

- Depth of field increases (previous formula)
- Diffraction becomes more visible
- Vigneting (see next slides) decreases

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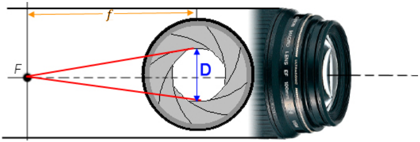

images wikipedia.org

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Depth of field

f-number $N = 1,4$

Depth of field

f-number $N = 13$

Artistic blur

f/2.8 - by bahramr - creative commons

The focal length has a strong impact on the image perception

Diverses focales en format 24 × 36 depuis un même point de vue : influence sur la taille de l'image

28 mm :
courte focale,
image de petite taille.

50 mm :
focale normale.

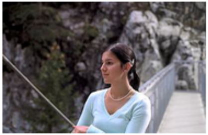
70 mm

210 mm :
longue focale,
image de grande taille.

Changement de focale et de point de vue : influence sur la perspective

24 mm :
point de vue rapproché,
perspective exagérée.

50 mm :
perspective « normale ».



100 mm

200 mm :
point de vue éloigné,
perspective écrasée.

(source : Wikipedia)

Beyond the convolution

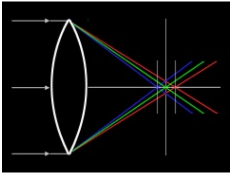
Among phenomena not accounted for by the convolutional model (linear, translation invariant) :

- Geometric distortions

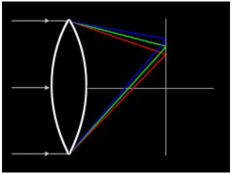
Pincushion (zoom)
French “coussinet”

Barril (wide angle)
French “barillet”

- Chromatic aberrations



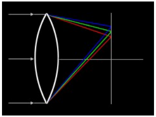
Longitudinal



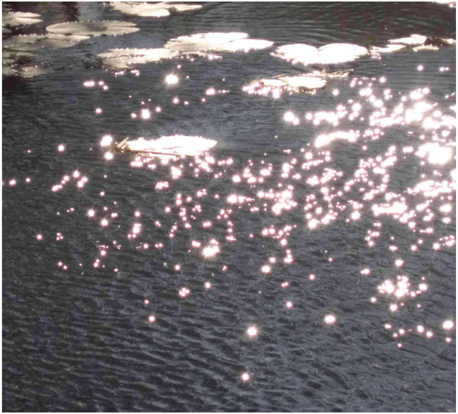
Lateral

- Vignetting

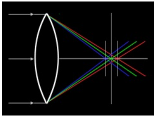
Transversal aberration



Zoom factor depends on the wavelength



Lateral aberration



Position of the focal plan depends on the wavelength

Before and after correction of the vignetting
dxo.com

Another acquisition parameter : exposure time

- Can compensate for a lack of light or a small aperture
- Should be short when objects are moving
- Otherwise : motion blur

Sampling

- After light is focused by the optical system $\rightarrow$ light information is acquired on the focal plane
- Digital sensor : array of photoreceptors, converting photons into electrons (semiconductor components, silicon)
- *Sampling* of the light signal

$$\left(f : \mathbb{R}^2 \mapsto \mathbb{R}\right) \rightarrow (\{f(k)\}_{k \in \Omega}) .$$

- Integration (counting) of photons by the sensor $\rightarrow$ convolution with g_{capt}, the indicator function of the sensor
  Can be modeled as a modification of the PSF

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Integration by the sensor

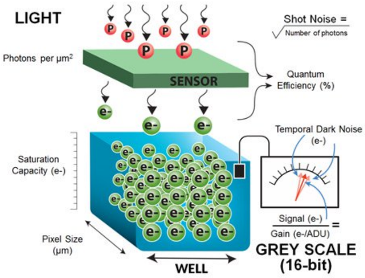
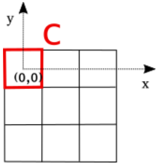


image credit : www.flir.com

At each photoreceptor cell : couting of photons
$\neq$ pointwise values

Integration by the sensor



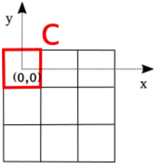
$I_{i,j}$: captured image value at position (i,j)

$$
\begin{aligned}
I_{i,j} &= \int_{C+(i,j)} I(x,y)dxdy \\
&= \int_{\mathbb{R}^2} I(x,y)\mathbb{1}_C(x-i,y-j)dxdy \\
&= \int_{\mathbb{R}^2} I(x,y)\mathbb{1}_C(i-x,j-y)dxdy \\
&= (I * \mathbb{1}_C)(i,j)
\end{aligned}
$$

$\rightarrow$ sampling of $I * \mathbb{1}_C$

the PSF g_0 is modified as $g_0 * \mathbb{1}_C$

Integration by the sensor



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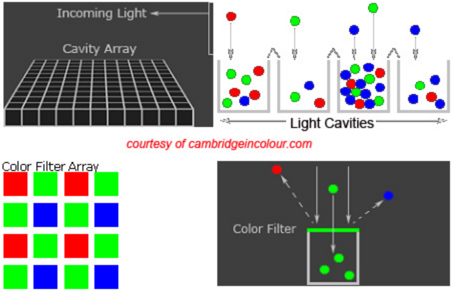
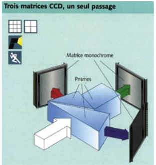
Color images : Bayer frame



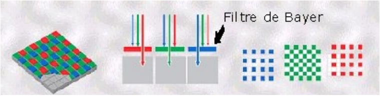
Typical size of sensors :
1 (smartphone) to 10 (single lens reflex, full format) micrometers.

Alternatives for color capture

- Tri-CCD



- Foveon sensor



$\rightarrow$ color lecture (next week)

Reminder on Fourier analysis

- Let $f \in L^1(\mathbb{R}^2)$,

$$\mathcal{F}(f)(\omega) = \hat{f}(\omega) = \int_{\mathbb{R}^2} f(\mathbf{x}) \exp(-i\mathbf{x}.\omega) d\mathbf{x}.$$

- Alternative definition (signal processing) with $\exp(-2i\pi\mathbf{x}.\omega)$.
- $\hat{f}$ is continuous and $f \to 0$ at infinity.
- From now we assume that f is smooth enough and decrease quickly at infinity :

$$\forall p, q \in \mathbb{N}^2, |\mathbf{x}|^p \partial_q f \leq C.$$

$$\mathcal{F}^{-1}(f) = \frac{1}{(2\pi)^2} \int_{\mathbb{R}^2} f(\mathbf{x}) \exp(i\mathbf{x}.\omega).$$

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- $\mathcal{F}(f * g) = \mathcal{F}(f).\mathcal{F}(g)$ and $\mathcal{F}(f.g) = (2\pi)^{-2}\mathcal{F}(f) * \mathcal{F}(g)$.
- $\mathcal{F}(f(\mathbf{x} - \mathbf{a})) = \exp(-i\omega.\mathbf{a})\mathcal{F}(f)$ and $\mathcal{F}(\exp(i\mathbf{a}.\mathbf{x}).f)(\omega) = \mathcal{F}(f)(\omega - \mathbf{a})$
- In the following, we'll apply these formulae to the Dirac's impulse (δ) without further justifications

Mathematical framework : theory of generalized functions (*distributions* in French) (e.g. "Analyse de Fourier et applications", C. Gasquet et P. Witomski, Dunod, 2000)

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Modeling of the sampling operation

- The sampling of a function f at point x is modeled as

$$f.\delta_x$$

  where δ_x is the Dirac's impulse at x
- $\mathcal{F}(\delta_x)(\omega) = \exp(-ix\omega)$
- Under some conditions, relationships between convolutions and multiplication remain true for generalized functions
- We can show that the *Dirac comb*, $\sum_{j\in\mathbb{Z}} \delta_{aj}$ is in $\mathcal{S}'$ and that

$$\mathcal{F}(\sum_{j\in\mathbb{Z}} \delta_{aj})(\omega) = \sum_{j\in\mathbb{Z}} \exp(iaj\omega) = \frac{2\pi}{a} \sum_{j\in\mathbb{Z}} \delta_{\frac{2\pi j}{a}}.$$

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- $\mathcal{F}(\delta_x)(\omega) = \exp(-ix\omega)$
- Under some conditions, relationships between convolutions and multiplication remain true for generalized functions
- We can show that the *Dirac comb*, $\sum_{j\in\mathbb{Z}} \delta_{aj}$ is in $\mathcal{S}'$ and that

$$\mathcal{F}(\sum_{j\in\mathbb{Z}} \delta_{aj})(\omega) = \sum_{j\in\mathbb{Z}} \exp(iaj\omega) = \frac{2\pi}{a} \sum_{j\in\mathbb{Z}} \delta_{\frac{2\pi j}{a}}.$$

Modeling of the sampling operation

- The sampling of a function f at point x is modeled as

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Acquisition model II

Back to the modeling of image acquisition :

$$u = (g_0 * s).\Pi_\Gamma$$

where

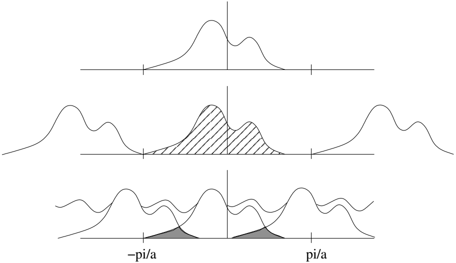
- s scene (function of L^1),
- g_0 : PSF of the system and sensor integration ($g_{ouv} * g_{flou} * g_{fil} * g_{capt}$),
- $\Pi_\Gamma = \sum_{\gamma \in \Gamma} \delta_\gamma$, impulse fonction along positions on Γ

Aliasing in dimension 1

We consider $\Gamma = a\mathbb{Z}$, donc $\Pi_\Gamma = \sum_{j\in\mathbb{Z}} \delta_{aj}$,
Then $u = f.\Pi_\Gamma$ and

$$\mathcal{F}(u) = \frac{1}{a}\mathcal{F}(f) * \sum_{j\in\mathbb{Z}} \delta_{\frac{2\pi j}{a}},$$

$$\mathcal{F}(u)(\omega) = \frac{1}{a}\sum_{j\in\mathbb{Z}} \mathcal{F}(f)\left(\omega - \frac{2\pi j}{a}\right).$$



Aliasing in dimension 1

Consequence :

- if $\text{Supp}(\mathcal{F}(f)) \subset [-\pi/a, \pi/a]$, then

$$\mathcal{F}(f.\Pi_\Gamma).\mathbb{1}_{[-\pi/a,\pi/a]} = \mathcal{F}(f),$$

  we therefore can recover f because

$$f = \mathcal{F}^{-1}\left(\mathcal{F}(f.\Pi_\Gamma).\mathbb{1}_{[-\pi/a,\pi/a]}\right) = f.\Pi_\Gamma * \mathcal{F}^{-1}(\mathbb{1}_{[-\pi/a,\pi/a]})$$

  but $\mathcal{F}^{-1}(\mathbb{1}_{[-\pi/a,\pi/a]}) = \frac{\sin(\pi x/a)}{\pi x/a}$, so that

$$f(x) = \sum_{j\in\mathbb{Z}} f(ja)\frac{\sin((x-ja))\pi/a}{(x-ja))\pi/a}.$$

- otherwise, the spectrum of $f.\Pi_\Gamma$ is degraded in an irreversible way

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Shannon theorem : dimension 1

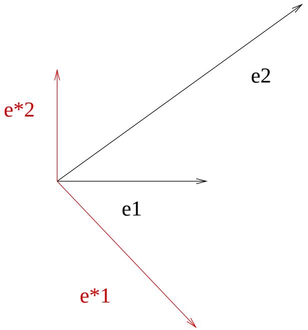
Let $f \in L^2(\mathbb{R})$ such that $\mathcal{F}(f)(\omega) = 0$ for $\omega \notin [-\pi/a, \pi/a]$, then

$$f(x) = \sum_{j \in \mathbb{Z}} f(ja) \frac{\sin((x - ja))\pi/a}{(x - ja)\pi/a}.$$

- Convergence in $L^2(\mathbb{R}^2)$
- $\mathcal{F}(f)$ has bounded suport $\rightarrow f \in C^\infty$: formula is also valid for pointwise equality
- Pointwise equality $f \in L^1(\mathbb{R})$ (uniform convergence).

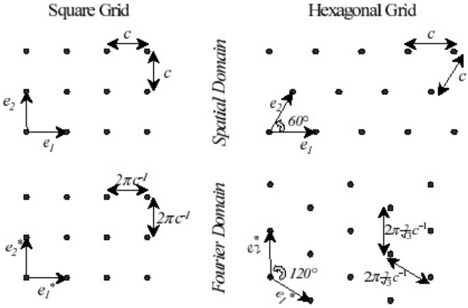
Dimension 2 : the dual grid

Let $\Gamma = \mathbb{Z}.\mathbf{e}_1 + \mathbb{Z}.\mathbf{e}_2$, with $\mathbf{e}_1, \mathbf{e}_2 \in \mathbb{R}^2$ not aligned.
The dual grid is $\Gamma^* = \mathbb{Z}.\mathbf{e}_1^* + \mathbb{Z}.\mathbf{e}_2^*$,
where $\mathbf{e}_i.\mathbf{e}_j^* = 2\pi\delta_{i,j}$.



Grid examples

- Square grid $\leftrightarrow$ square grid
- Hexagonal grid $\leftrightarrow$ hexagonal grid



Aliasing in dimension 2

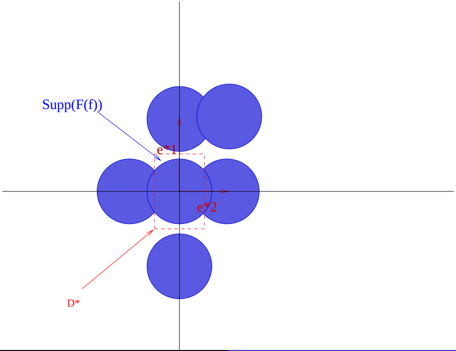
Let $\Gamma = \mathbf{e}_1\mathbb{Z} + \mathbf{e}_2\mathbb{Z}$ and $\Gamma^* = \mathbf{e}_1^*\mathbb{Z} + \mathbf{e}_2^*\mathbb{Z}$, $\Pi_\Gamma = \sum_{j,k\in\mathbb{Z}} \delta_{j\mathbf{e}_1, k\mathbf{e}_2}$.
We can shox that $\mathcal{F}(\Pi_\Gamma) = |\mathbf{e}_1^* \wedge \mathbf{e}_2^*|\Pi_{\Gamma^*}$.
The sampled image is $u = f.\Pi_\Gamma$ et

$$\mathcal{F}(u) = |\mathbf{e}_1^* \wedge \mathbf{e}_2^*|\mathcal{F}(f) * \Pi_{\Gamma^*}$$

so that

$$\mathcal{F}(u)(\omega) = |\mathbf{e}_1^* \wedge \mathbf{e}_2^*| \sum_{\gamma\in\Gamma^*} \mathcal{F}(f)(\omega - \gamma)$$

Aliasing occurs if $\gamma +$ Supp$(\mathcal{F}(f))$ are not disjoint, for $\gamma \in \Gamma^*$.



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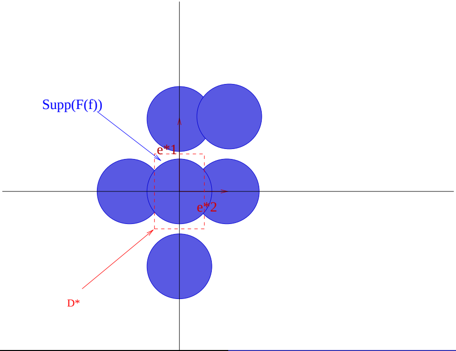

Shannon theorem in dimension 2

Let $K \subset \mathbb{R}^2$ and $f \in L^2(\mathbb{R}^2)$ such that

1. $\forall \gamma, \gamma' \in \Gamma^*, \gamma + K \bigcap \gamma' + K = \emptyset$
2. Supp$(\mathcal{F}(f)) \subset K$,
   then

$$f(\mathbf{x}) = \sum_{\gamma \in \Gamma} f(\gamma) s(\mathbf{x} - \gamma),$$

where

$$s = \frac{1}{|K|} \mathcal{F}^{-1}(\mathbb{1}_K).$$

Remarks

- High frequency of the scene s (e.g. edges) are mitigated by the PSF g_o.
- If this is not enough for sampling (i.e. Supp($\mathcal{F}(f)$) $\notin K$) we can add some low-pass filter : $h * f$ where Supp($F(h)$) $\in K$.
  This is usually done by adding micro-lenses on top of each sensor
- if the sampling frequency increases, then sensor size must be decreased
  $\rightarrow$ increase of the size of the support of $\mathcal{F}(h_{capt})$ ...

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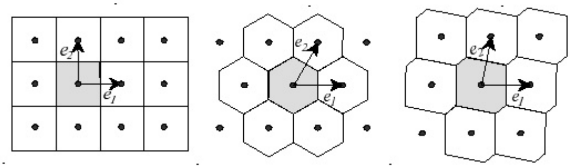
Sampling an image ; an optimization problem

When designing an acquisition system : several parameters, including Γ^* and K.

- K contains some proportion of Supp$(\mathcal{F}(u))$
- A possibility is to first design Γ^*, then choose K as the Voronoi cell associated to Γ^*, i.e.

$$K = \{\beta \in \mathbb{R}^2 : \forall \gamma \in \Gamma^* ||\beta|| \leq ||\beta + \gamma||\}.$$

Reasonable if the spectrum is isotropic and decreasing



(a) square grid (b) hexagonal grid (c) intermediate grid

Sampling quality

The sampling density is defined as

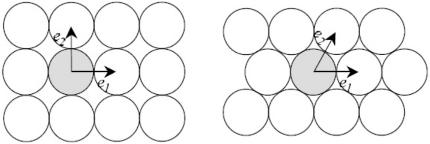
$$D(\Gamma) = (|\mathbf{e}_1 \wedge \mathbf{e}_2|)^{-1} = 2\pi|\mathbf{e}_1^* \wedge \mathbf{e}_2^*|.$$

For a given acquisition system and a given image (with known $\mathcal{F}(u)$) the quality of the sampling process (assuming Shannon conditions) can be measured as

$$\frac{|\text{Supp}(\mathcal{F}(u))|}{2\pi D(\Gamma)}$$

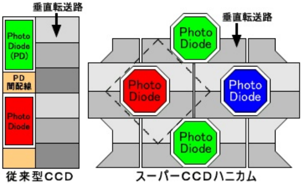
Example 1 : hexagonal grid and isotropic spectrum

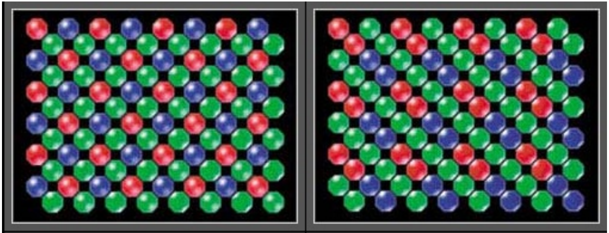
Assuming that Supp$(\mathcal{F}(f))$ is a disk, we can show that the hexagonal grid is the most efficient (about 15% more than the square grid)



Example 2 : *super CCD* (Fujifilm)

Stagged rows → larger sensors for a given horizontal and vertical resolution.

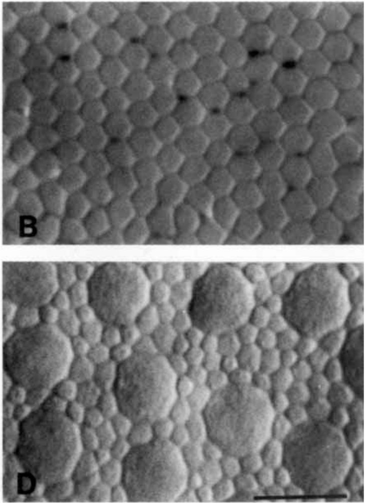




Conventional Super CCD color filter array layout

Super CCD EXR color filter array layout

Spatial layout of human photoreceptors



B : cones on the fovea D : peripheric cones and rods
(more info in the forecoming lecture on color)

Fig. de Curcio et al. 1990

Further reading :

- *Echantillonnage, interpolation et détection*, PhD thesis, Andrés Almansa, ENS Cachan, 2002
- *Sampling and reconstruction of wave-number limited functions in N-dimensional Euclidean space*, D. Petersen et D. Middleton, Information and control, 5, pp.279-323, 1962

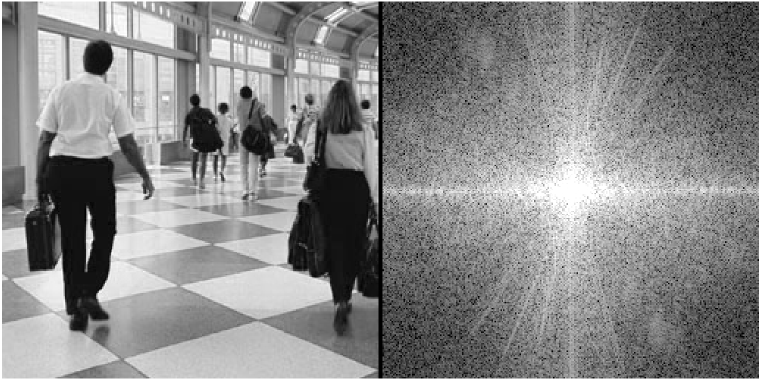
Discrete Fourier transform

In practice, we deal with discrete images : $\{u_{k,l}\}_{k=1,\dots,M;l=1,\dots,N}$.
We consider the Discrete Fourier Transform (DFT) of $u_{k,l}$ is defined as :

$$\hat{u}_{m,n} = \frac{1}{N^2} \sum_{k=0}^{N-1} \sum_{l=0}^{N-1} u_{k,l} \omega_N^{-mk} \omega_N^{-nl},$$

where $\omega_N = \exp\left(\frac{2i\pi}{N}\right)$.

An image and its DFT

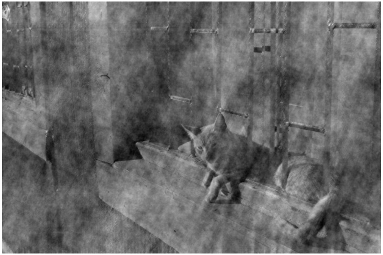


An image and the logarithm of the modulus of its DFT

Digression

Image A

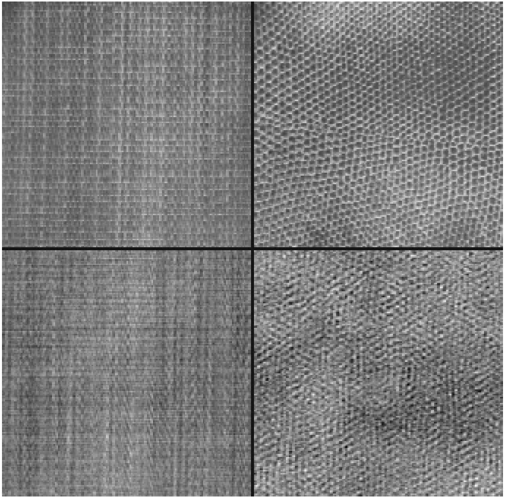
Image B



Phase of A and modulus of B

Phase of B and modulus of A

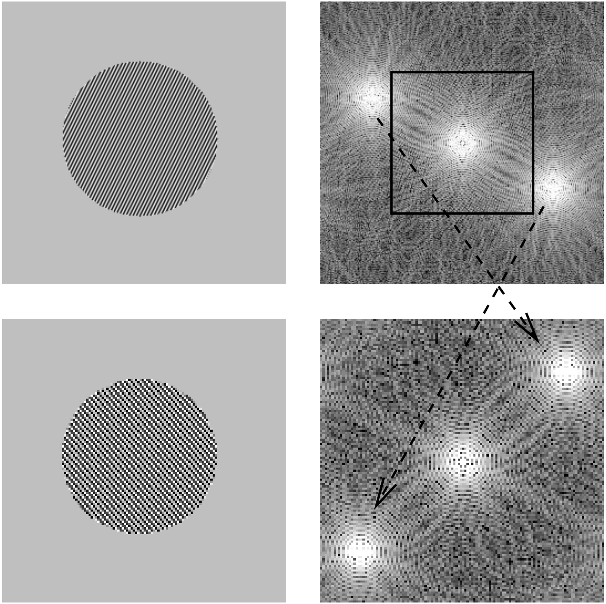
Digression (2)

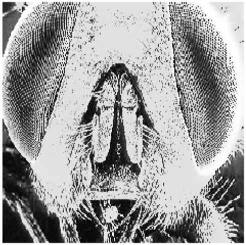


Images on the bottom row are obtained by replacing the phases of the DFT by random (uniform) phases.

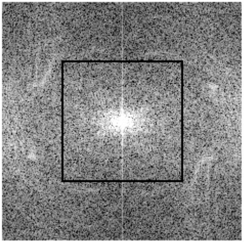
More information in the forecoming lecture about texture

A synthetic example of aliasing



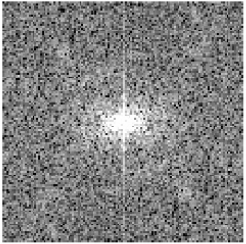


(a) Image originale



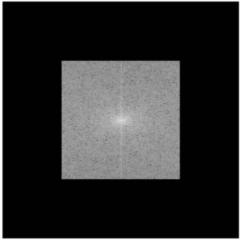
(b) Sa TFD, non nulle en dehors du carré visible en surimpression

(c) Image sous-échantillonnée d'un facteur 2



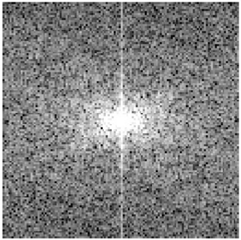
(d) La TFD correspondante, sur laquelle il y a repliement

(a) Image obtenue par TFD inverse de b



(b) Image obtenue en mettant à zéro les hautes fréquences de 11.1.3-a

(c) Sous-échantillonnage : le repliement a disparu



(d) TFD de c

A real life example

Image from a camera with no micro-lenses in front of sensors

Zoom

Image from a camera with no micro-lenses in front of sensors

Zoom

Image from a camera with no micro-lenses in front of sensors

Further aliasing

Left : original image,
Right : sub-sampling by a factor of 2

Apparition du ringing

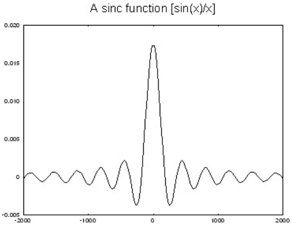
Left : original image,
Right : sub-sampling by a factor of 2 after multiplication by the indicator function of a square

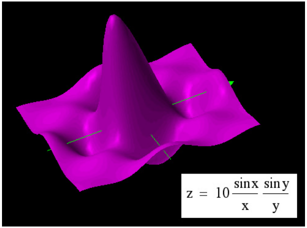
The ringing phenomenon

To be in Shannon conditions we can apply a square filter in the Fourier domain : a multiplication in the frequency domain by $\mathbb{1}_K$. The resulting image is then

$$\mathcal{F}^{-1}(\mathcal{F}(f)\mathbb{1}_K) = f * \mathcal{F}^{-1}(\mathbb{1}_K).$$

K being the indicator function of a square of side a, $F^{-1}(\mathbb{1}_K)$ is the product of 2 sinc functions :





Ringing

Acquisition model II

Back to the modeling of image acquisition :

$$u = (g_0 * s).\Pi_\Gamma$$

where

- s scene (function of L^1),
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Acquisition model III

$$u = h\left[Q\left((g_o * s).\Pi_\Gamma + b\right)\right]$$

where

- s scene (function of L^1),
- g_0 : PSF of the system and sensor integration ($g_{ouv} * g_{flou} * g_{fil} * g_{capt}$),
- $\Pi_\Gamma = \sum_{\gamma\in\Gamma} \delta_\gamma$,
- b additive noise for $(i,j) \in \Gamma$, $\{$b(i,j)$\}$ is a collection of i.i.d. random variables
- h an increasing function : a "contrast change"
- Q a *quantization* operator

For a detailed analysis of contrast change and quantization :
$\rightarrow$ next lecture

Classical noise models

Let v be the "clean" discrete image and u its noisy version

- Additive noise :
  $u(i,j) = v(i,j) + b(i,j)$
  where the $b(i,j)$ are i.i.d. random variable, usually Gaussian
  This model is usually wrong ! (see next)
- Impulse noise :
  $u(i,j) = v(i,j).A(i,j) + (1 - A(i,j)).B(i,j)$,
  where $A(i,j)$ are i.i.d. Bernoulli and $B(i,j)$ arbitrary i.i.d. variables ("salt and pepper" if B takes values in $\{0, M\}$)
  Used e.g. to model transmission uncertainty
- Multiplicative noise (also called speckle) :
  $u(i,j) = v(i,j).b(i,j)$
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  typically observed with radar images

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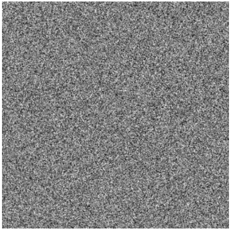
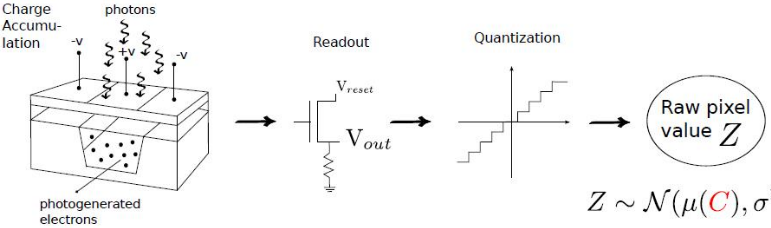
Additive noise


+

=


In practice, the i.i.d. hypothesis is wrong !

Real life examples

Source : www.dpreview.com



$$Z \sim \mathcal{N}(\mu(C), \sigma^2(C))$$

C irradiance
τ exposure time
g camera gain
a photo-response non-uniformity factor
μ_r, σ_R^2 readout noise mean and variance

$$\mu(C) = ga\tau C + \mu_R$$
$$\sigma^2(C) = g^2 a\tau C + \sigma_R^2$$

Illustration Cecilia Aguerrebere

Origins of noise

- *shot noise* Number of photons emitted by a source : Poisson law with parameter $C\tau$
  with C the *radiance* (average number of photons emitted by time unit) and τ the exposure time 0
- Dark current
  Residual emission of electrons (thermical origin) : Poisson law with parameter d_τ, depending on τ and the temperature
- Readout noise
  Error during the reading of voltage (fluctuations of the reference voltage)
- Spatial non-uniformity
  of sensors response (PRNU)
  of thermical noise (DCNU)

Complete model

$$I_{noise} = g(Poiss(C\tau) + Poiss(d_\tau)) + N_{out} + Q$$

with

- $Poiss(\lambda)$: Poisson distribution with mean λ
- C radiance (photons / time)
- g : gain
- τ : exposure time
- d_τ : average of the *dark current*
- N_{out} : readout noise Guassian with mean μ_R and variance σ_R^2).
- Q : quantization noise (uniform)

Spatial variations could be added (DCNU and PRNU)

First simplification

Complete model

$$I_{noise} = g(Poiss(C\tau) + Poiss(d_\tau)) + N_{out} + Q$$

Further realistic hypothesis :

- Quantization noise Q is small vs readout noise
- Dark current can be neglected for short exposure times ($< 1s$)

Then

$$I_{noise} = gPoiss(C\tau) + N_{out}$$

Gaussian approximation

$$I_{noise} = gPoiss(C\tau) + N_{out}$$

- $P(\lambda) \approx N(\lambda, \lambda)$ for large values of λ (in practice a few dozen)
- In usual photographic conditions this approximation is valid (false for low photon count, e.g. astrophotography)

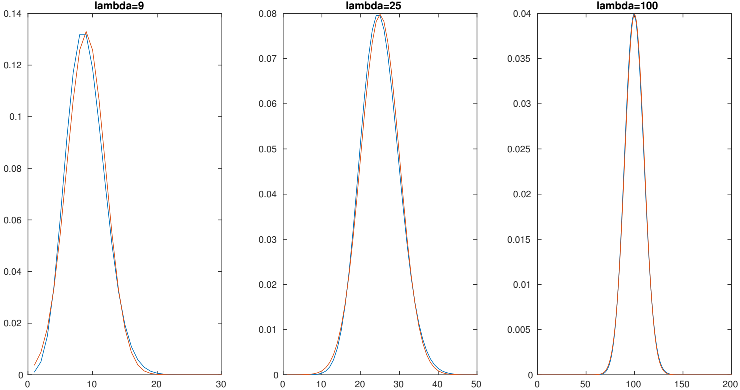
Gaussian approximation :

$$N(gC\tau + \mu_R, g^2 C\tau + \sigma_R^2)$$

with μ_R and σ_R the mean and variance of readout noise

Application : HDR image creation (forecoming lecture)

Gaussian approximation



Blue : Poisson ; red : Gaussian

Conclusion : noise variance **depends on the signal**.
In order to account for this, two possible solutions :

- direct use of the noise model (e.g. to compute statistical estimators) utilisation directe du modèle de bruit (e.g. pour dériver des estimateurs statistiques) : cf HDR lecture to come
- modification of the signal to obtain an i.i.d. Gaussian noise *variance stabilisation* (e.g. Anscomb transform $x \mapsto 2\sqrt{x+3/8}$)

Further reading :

- *Study of the digital camera acquisition process and statistical modeling of the sensor raw data*, C. Aguerrebere et al., preprint HAL, 2013
- *Optimal inversion of the generalized Anscomb transformation for Poisson-Gausian noise*, A. Mäkitalo et A. Foi, IEEE Image Processing, 22, 1, pp.91-103, 2013

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To be discussed in forecoming lectures

Image acquisition pipeline

- today

$$s \xrightarrow{optics+sensorintegration} g_0 * s \xrightarrow{sampling,noise} (g_0 * s).\Pi_{gamma} + b$$

- course 2, radiometry :
  quantization $\rightarrow$ RAW image

$$Q\left[(g_0 * s).\Pi_{gamma} + b\right]$$

- courses 5 and 6 : denoising
- course 2 color :
  demosaicking $RAW \rightarrow u$
  with u a 3-channels image
- courses 2 : color transforms

$$u \xrightarrow{whitebalance} T_w u \xrightarrow{changeofcolorspace} T_s T_w u \xrightarrow{tone\ \ mapping} h(T_s T_w u),$$

with T_w a diagonal matrix,
and h a non-linear function (course 2)