Static/dynamic locking range dependence on grating characteristics of index-coupled dfb lasers

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ABSTRACT

The effect of weak optical injection on the threshold gain and resonant frequency of index-coupled DFB semiconductor lasers is theoretically analyzed. The locking bandwidth modification due to injection is discussed for structures having different grating characteristics. Simple formulas are derived which allow comparison of the sensitivity to optical injection for DFB lasers and Fabry-Perot lasers. A symmetrical locking bandwidth is verified and a stability analysis for weak injection is also carried out, imposing a maximum limit of injection rate.

Keywords: Optical injection, DFB lasers, locking bandwidth.

1. INTRODUCTION

Optical injection in semiconductor lasers is a popular technique allowing applications such as chirp reduction [1], laser array synchronization [2], optical generation of microwaves [3]-[5], amplitude-phase modulation conversion [6], etc. Optical injection studies [7]-[9] are usually limited to the case where the intensity distribution along the laser cavity is supposed uniform and are based on a homogenous rate-equation analysis. As a consequence, inaccurate results regarding the stable locking bandwidths are obtained when describing optical injection in distributed feedback lasers (DFB), especially those where a phase shift has been inserted to improve the side-mode suppression ratio (SMSR) and where the spatial hole-burning effect becomes important. Mode hopping in Fabry-Perot semiconductor lasers asymmetrically reduces the single mode locking range. In DFB lasers, strong mode selection due to the grating structure overcomes this problem. In fact, the stable locking bandwidth is practically symmetrical in DFB lasers. Furthermore, optical synchronization can be maintained even on the case of a non-cooperative injection where the phase relation between the injected and cavity field leads to a mode threshold gain that is higher than that of the free-running laser [9]. The analysis is based on the coupled-mode equations describing a solitary DFB laser. The static solution is found and external optical injection is supposed weak and taken into account by linearizing around the static solution. The modification of the facet reflectivity due to optical injection allows one to find the associated static locking range.

2. COUPLED-WAVE SOLUTIONS FOR WEAK INJECTION

For the sake of simplicity, only a $\lambda/4$ phase-shifted DFB laser having a real coupling coefficient κ as shown schematically on Fig. 1 is analyzed. The choice of this laser structure is justified by the fact that phase-shifted DFB lasers have been commercially available since the eighties and, additionally, the analysis for this structure can be straightforwardly extended to more general structures. Furthermore, only the first Bragg order will be taken into account.

2.1 Coupled-wave static solution in the free-running regime

The index modulation inside the laser cavity is described by dividing the laser structure into two uniform gratings, each with a spatial grating period Λ:

$$n^{(j)} = n_0 + \Delta n \cos\left(\frac{2\pi}{\Lambda} z + \Omega^{(j)}\right), \qquad (1)$$

where $j = 1$, 2 corresponds to the region of negative and positive z, respectively, and with a phase shift given by $\Omega^{(1)} = -\Omega^{(2)} = \Omega$. The amplitude modulation of the real index is specified by Δn. Such a laser is well described by the

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traveling coupled-mode wave equations [10], which are derived from the Helmholtz wave equation when the electric field $E(z,t)$ is sought in the form $E^{(j)}(z,t)=R^{(j)}(z,t)e^{-i\beta_0 z}+S^{(j)}(z,t)e^{i\beta_0 z}$ and when second derivatives are neglected:

$$\begin{aligned}
&\frac{1}{v_g}\frac{\partial R}{\partial t}+\frac{\partial R}{\partial z}=j\kappa S+(\alpha-j\delta)R+i_{spf}\\
&\frac{1}{v_g}\frac{\partial S}{\partial t}-\frac{\partial S}{\partial z}=j\kappa R+(\alpha-j\delta)S+i_{spr}\\
&\Gamma\left(\frac{dN}{dt}+\frac{N}{\tau_r}\right)+2\Gamma g_m v_g\left(\frac{F^*F+R^*R}{v_g}\right)=\frac{I}{eL}
\end{aligned} \tag{2}$$

where $\beta_0=\pi/\Lambda$ is the Bragg propagation constant, I is the injection current, α the net gain, δ the phase detuning, g_m the material optical-field gain per unit distance, which is taken nonlinear, τ_r is the spontaneous recombination time constant including the Schockley-Read-Hall, the bimolecular and Auger recombination mechanisms, and the terms i_{sp} represent the spontaneous excitation, evaluated from the bipolar radiative recombination, taken as locally proportional to N^2 and include the spontaneous confinement factor which is similar to the gain confinement factor Γ. The latter gives an effective gain below the material gain and reduces the effective phase shift associated with the complex gain. The normalized detuning of the oscillating mode from the Bragg wave vector β_0 is written as $\delta=n\omega/c-\pi/\Lambda$. $R(z)e^{i\omega t}$ and $S(z)e^{i\omega t}$ are the counter-propagating waves expressed as:

$$\begin{aligned}
R^{(j)}(z)&=r_1^{(j)}e^{\gamma z}+r_2^{(j)}e^{-\gamma z}\\
S^{(j)}(z)&=s_1^{(j)}e^{\gamma z}+s_2^{(j)}e^{-\gamma z}
\end{aligned}. \tag{3}$$

The static solution is found by the typical method of substituting the proposed solution (3) in the coupled-wave equations (2) and then applying boundary conditions at $z=\pm L/2,\ 0$. This procedure yields the dispersion relation, $\gamma^2=(\alpha-j\delta)^2+\kappa^2$, and the following eigenvalue equation:

$$e^{j\Omega}\left[1+pe^{\gamma L}\frac{\rho_r-p}{1-\rho_r p}\right]\left[p+e^{-\gamma L}\frac{1-\rho_l p}{\rho_l-p}\right]=e^{-j\Omega}\left[1+pe^{-\gamma L}\frac{1-\rho_l p}{\rho_l-p}\right]\left[p+e^{\gamma L}\frac{\rho_r-p}{1-\rho_r p}\right] \tag{4}$$

where γ is the complex propagation constant, α the threshold gain, parameter p is defined by $p=(-\gamma+\alpha-i\delta)/i\kappa$, and ρ_r and ρ_l are related to the amplitude reflectivities, $\hat{\rho}_r$ and $\hat{\rho}_l$, of the right and left facets, respectively, by:

$$\rho_r=\hat{\rho}_r e^{-j(2\beta_0 L-\Omega)} \text{ and } \rho_l=\hat{\rho}_l e^{-j(2\beta_0 L-\Omega)} \tag{5}$$

The multiple solutions of (4) represent the different cavity modes associated with the laser structure but only the mode with the lowest threshold gain will oscillate. If the phase shift is exactly $\Omega=\pi/2$, the modes with lowest threshold gain for any real coupling strength κ lie at the Bragg frequency ($\delta=0$), meaning that the imaginary part of the propagation constant is null. Fig. 2a illustrates this threshold gain as a function of coupling strength. At first sight, it may seem that the coupling strength-length product should be specified as high as possible in order to obtain the lowest threshold current. Nevertheless, the gain margin between the lowest gain mode and its neighbor is maximal at around $\kappa L=2.1$.

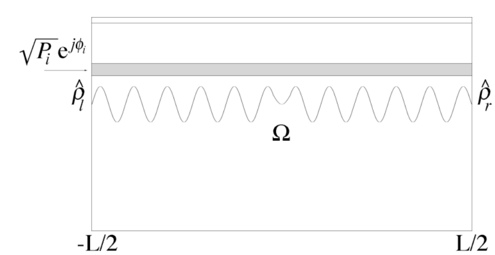

Fig. 1. Phase-shifted index-coupled DFB laser heterostructure.

2.2 Gain and frequency variations due to external optical injection

The method presented by Favre [11] for describing external optical feedback in DFB lasers is adapted here for describing optical injection. In his analysis, the external mirror reflectivity is taken into account for calculating the equivalent facet reflectivity. In our case, the equivalent reflectivity is approximated to include optical injection. Assuming that injection occurs at the left facet,

$$\hat{\rho}_{leq}(z) = \hat{\rho}_l + \left(1 - \hat{\rho}_l^2\right)\sqrt{\frac{P_i(z)}{P(z)}}e^{-j\theta(z)} \tag{6}$$

where P_i/P is the local power density ratio between the injected and cavity fields distributed throughout the cavity and $\theta = \phi_i - \phi$ is the phase difference between these fields. The change in the reflectivity due to injection resulting from (5) and (6) is:

$$\Delta\rho_l(z) = \left(1 - \hat{\rho}_l^2\right)\sqrt{\frac{P_i(z)}{P(z)}}e^{-j\theta(z)}e^{-j(2\beta_0 L - \Omega)} \tag{7}$$

On the one hand, assuming a weak injection rate ($\sqrt{P_i/P} << 1$), the eigenvalue equation (4) can be linearized about γ_0, solution of (4) without injection. This leads to a relation between the variation of the complex propagation constant $\Delta\gamma = \gamma - \gamma_0$ and the reflectivity variation $\Delta\rho_l$. On the other hand, differentiation of the dispersion relation allows us to express the gain $\Delta\alpha = \alpha - \alpha_0$ and phase change $\Delta\delta = \delta - \delta_0$ as:

$$\Delta\alpha L - j\Delta\delta L = \frac{\gamma_0}{\alpha_0 - j\delta_0}\Delta\gamma L \tag{8}$$

The only variable in the right-hand side of equation (8) that depends on the injection conditions is $\Delta\gamma$. So we can assume a proportionality relation [11] and write (8) as:

$$\Delta\alpha L - j\Delta\delta L = \frac{C_l}{2}\sqrt{\frac{P_i}{P}}e^{-j\theta} \tag{9}$$

where C_l is a complex coefficient that depends only on solitary DFB laser modal characteristics, e.g. the grating position with respect to the facet, the reflectivity, the threshold gain or the phase deviation, and can be calculated using the facet reflectivity change (7) and the logarithmic differentiation of the eigenvalue equation (4). The coefficient specifically gives information on how sensitive the laser structure is to an external optical injection as will be later discussed. For the $\lambda/4$ phase-shifted laser, the coefficient is, after some manipulation, determined as:

$$C = \frac{\gamma_0}{\alpha_0}\frac{(1-p_0^2)\cosh^2(\gamma_0 L/2)}{[(-e^{-\gamma_0 L} - (2p_0^2 - 1)e^{\gamma_0 L} - 2][4p_0^2/(1-p_0^2)/i\kappa] + p_0(1+p_0^2)e^{\gamma_0 L} - (1-p_0^2)e^{-\gamma_0 L}/p_0} \tag{10}$$

In order to determine the static-locking bandwidth, the gain and frequency deviations from the free-running regime must be analyzed. To do so, we proceed by obtaining the phase deviation $\Delta\delta$ from the Bragg detuning definition. Let ΔN and $\Delta\omega$ denote the carrier density and resonant frequency changes, respectively. The phase deviation is:

$$\Delta\delta = \frac{n_g}{c}\Delta\omega + \frac{\omega}{c}\frac{\partial n}{\partial N}\Delta N \tag{11}$$

where n_g is the group refractive index. Relating the carrier density variations ΔN to the gain variation per unit of time with $\Delta N \partial G/\partial N = 2\Delta\alpha c/n_g$ the frequency variation is obtained from (11) as

$$\Delta\omega = \frac{c}{n_g L}(\Delta\delta L - \alpha_H \Delta\alpha L) \tag{12}$$

The linewidth enhancement factor α_H expresses the coupling between the real and imaginary parts of the permittivity in a semiconductor laser and is defined as:

$$\alpha_H = \frac{2\omega}{n_g} \frac{\partial n / \partial N}{\partial G / \partial N} \tag{13}$$

Finally, considering (12), the resonant frequency and gain variations are obtained by separating (9) into its real and imaginary parts:

$$\Delta\omega = \rho(1+\alpha_H^2)^{1/2} \sin\left(\theta - \arg C_l - \tan^{-1}\alpha_H\right) \tag{14a}$$

$$\Delta G = 2\rho\cos\left(\theta - \arg C_l\right) \tag{14b}$$

where $\rho = c/(2n_g L)|C_l|\sqrt{P/P_i}$ is the normalized injection rate. The locking bandwidth is determined from (14a) as $|\Delta\omega| \le \rho(1+\alpha_H^2)^{1/2}$. The gain deviation due to optical injection can reduce or increase the threshold gain depending on the phase difference between the injected and cavity field. Equation (14b) is maximal for non-cooperative injection, when the threshold gain is increased, and (14b) is minimal when the injected field cooperates with the cavity mode reducing the threshold gain. Fig. 2a depicts the threshold gain increase as the injection rate increases when the gain deviation is maximal. In homogeneous Fabry-Perot lasers, the phase difference between the injected and cavity field could be such that gain margin between the main and side mode is inversed, thus resulting in an asymmetrical locking bandwidth. This is contrary to what is usually observed in DFB lasers because they have a strong side-mode suppression ratio and wide stop-band. Consequently, the increase in threshold gain due to injection does not affect the gain margin between the cavity and side mode. The resulting symmetrical locking bandwidth versus injection rate diagram (Fig. 2c) shows a strong dependence on coefficient C_l via the index-coupling strength. In Fabry-Perot lasers, the complex coefficient C_l, is simply a constant that depends on the amplitude reflectivity, whereas in DFB lasers, the coefficient strongly depends on its grating structure and consequently so does the normalized injection rate ρ.

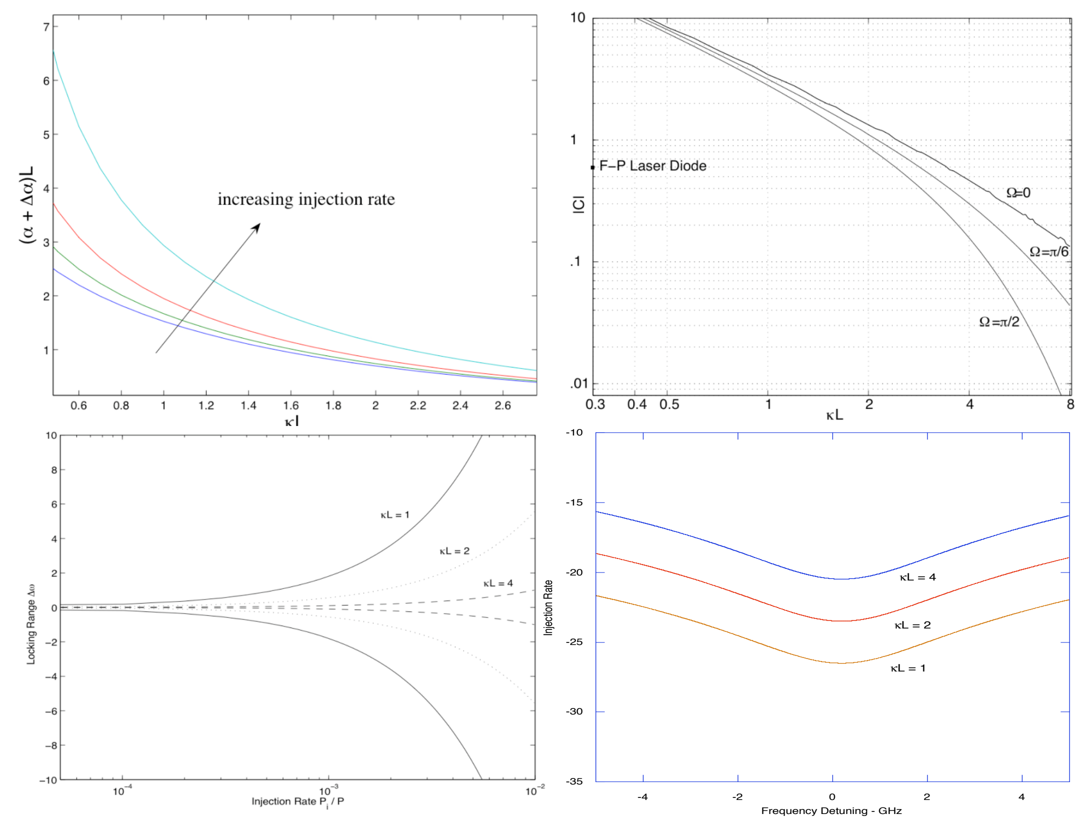

Fig. 2. (a) Threshold gain in the free running regime (lower curve) and its modification due to injection. (b) Coefficient C dependence on the coupling strength for various phase shifts. (c) Static locking range and (d) static/dynamic limit.

The coefficient is graphed on Fig. 2b. The equivalent for a Fabry-Perot laser is shown for comparison. It can be noted the laser structure becomes less sensitive to external injection as the coupling strength is increased. Furthermore, a $\lambda/4$ shift produces the structure that is the least sensitive to external injection given that the corresponding *C*-curve drops the fastest for a coupling strength $\kappa L > 2$.

Table 1. Parameters used on all calculations

	symbol	value
free space wavelength	λ	1.55 μm
Group refractive index	n_g	3.7
nominal length of laser	L	400 μm
average value kappa L product;	κL	2
confinement factor for electrons	Γ	0.35
confinement factor for spontaneous emission	Γ_{spont}	0.35
guide loss parameter	aL	1.6
linear nonradiative	A	0
spontaneous recombination coeff	B	10^{-10} cm^2
Auger recombination coeff	C	3×10^{-29} cm^5
electron density at tranparency	N_0	1.5×10^{18} m^{-3}
differential gain length product at transparency	$g_m^{'}$	20
Henry's linewidth factor	α_H	3
gain saturation factor	ε	0.05
Complex Coeficient	C_l	$0.874e^{-j\pi/2}$

2.3 Static/dynamic limit of the stable-locking bandwidth

A static solution to the coupled-wave equations (14) describing an injected DFB laser requires the necessary condition that the frequency detuning between the master and slave laser remain bounded, i.e. $|\Delta\omega| \le \rho\sqrt{1+\alpha_H^2}$, which follows from (13). In F-P lasers [7]-[8], the competition between amplified spontaneous emission and forced oscillations at the master laser frequency is accompanied with a net gain whose sign depends on the phase detuning. For a positive net gain resulting from injection, the locking bandwidth is reduced because the amplified spontaneous emission of a cavity free mode is divergent while the injected mode is constantly attenuated by a coherent injection in phase opposition. Consequently, mode hopping is observed. In contrast, DFB lasers, having a high side-mode suppression ratio, preserve dynamic stability all throughout the static locking range [9]. Nevertheless, increasing the injection rate beyond a certain limit will cause laser operation in an unstable regime resulting in chaotic behavior. Classical systems involving differential equations impose a stability analysis based on the Routh-Hurwitz criterion as in [8] to establish the limit of the linear approximation we made valid for weak injection. This leads in establishing the static/dynamic limit of the locking bandwidth defined in terms of the modified damping time associated to relaxation oscillations:

$$\frac{1}{\tau_r} = \frac{1}{\tau_{r0}} + 2\rho(1+\alpha_H^2)^{1/2}\cos\left(\theta + \arg C + \tan^{-1}\alpha_H\right) \tag{15}$$

where

$$\frac{1}{\tau_{r0}} = A + 2BN_0 + 3CN_0^2 + \frac{G_m^{'}P_0}{1+\varepsilon P_0}\left(1 + \frac{\varepsilon}{G_m^{'}\tau_p^{'}}\right) \tag{16}$$

is the damping constant of the free-running laser. Coefficients A, B, and C are, respectively, the linear nonradiative, the bipolar radiative, and the Auger recombination coefficients. G'_m is the differential power gain, ε the gain-saturation parameter, and τ'_p the effective photon lifetime including the distribution losses. The last term in (16) represents the nonlinear gain saturation, which damps the relaxation oscillation and tends to stabilize the single-mode laser operation [7]. An infinite damping time in (15) defines the hyperbola depicted in Fig. 2d. Its position depends on the coupling strength κL, the phase-amplitude coupling factor α_H, and the phase shift Ω via the complex constant C_l. For a desired point of operation within the locking bandwidth, instantaneous power fluctuations of the master laser will not cause unlocking of the slave laser for an increased coupling strength; thus, providing a more stable measurement workbench. It should be kept in mind, however, that a high coupling strength reduces the locking bandwidth (Fig. 2c). On the contrary, if the injection locking application forsaken requires fast switching between stable-synchronized operation and optically induced modulation, lasers with low coupling factors are better for this task.

3. DISCUSSION AND CONCLUSIONS

In section 2.2, the static locking bandwidth was determined and it was found that it strongly depends on the grating structure. This fact is of great consequence for a stable locking range: lasers having a weak coupling ($\kappa L \leq 1$) exhibit an enhanced locking bandwidth and will work well with applications like amplifiers and optical-phase modulators where given frequency detunings caused by either direct modulation of the laser diode current or modulation of the injected field, must not cause unlocking of the slave laser. For strong coupling ($\kappa L \geq 4$), the bandwidth is largely reduced making this lasers attractive for applications requiring narrow-band filters like dense multi-channel systems. For as to intermediate values of the coupling factor ($1 < \kappa L < 4$), a wide range of applications may be adequate including chirp reduction, phase correlation, laser array synchronization, etc.; where a symmetrical locking bandwidth is important and modulation bandwidth will ot exceed the locking bandwidth.

The effect of weak external optical injection on threshold gain and resonant frequency has been investigated. The structures considered in this study correspond to index-coupled phase-shifted DFB lasers. Nevertheless, the method applied here can be readily extended to more complex laser structures, including DBR and vertical-cavity surface emitting lasers. A matrix development method is underway and should permit the easy calculation of the C_l coefficient in order to account perhaps for laser structures having multiple sections.

REFERENCES

1. H.L.T. Lee, R.J. Ram, O. Kjebon, and R. Schatz, "Bandwidth enhancement and chirp reduction in DBR lasers by strong optical injection ," *CLEO*, pp. 99-100, 2000.
2. M. Tamburrini, M. Parent, L. Goldberg, and D. Stillwell, "Optical feed for a phased array microwave antenna," *IEE Electron. Lett.*, vol. 23, no. 13, pp. 680-681, 1987.
3. L. Goldberg, H. F. Taylor, J. F. Weller and D.M. Bloom, "Microwave Signal Generation with Injection-Locked Laser Diodes," *Elec. Lett.*, vol 19, pp. 491-493, june 1983.
4. R.-P.Braun, G. Grosskopf, D. Rohde, F.Schmidt, and G.Walf, "Fiber-optic millimeter-wave generation at 64 GHz and spectral efficient data transmission for mobile communications," in *Proc. OFC*, 1998, pp. 17-18.
5. P. Bouyer, T.L. Gustavson, K.G. Haritos and M.A. Kasevich, "Microwave signal generation with optical injection locking," *Optics Lett.*, vol 21, pp. 1502-1504, sept. 1996.
6. O. Lidoyne, P. Gallion et D. Erasme, "Modulation Properties of an Injection-Locked Semiconductor Laser," *IEEE J. Quantum Elec.*, vol 27, pp. 344-351, 1991.
7. O. Lidoyne, P. Gallion, C. Chabran and G. Debarge, "Locking range, phase noise and power spectrum of an injection-locked semiconductor laser," *IEE Proceedings*, vol. 137, no. 3, 1990.
8. I. Petitbon, P. Gallion, G. Debarge and C. Chabran, "Locking Bandwidth and Relaxation Oscillations of an Injection-Locked Semiconductor Laser," *IEEE J. Quantum Elec.*, vol 24, pp. 148-154, 1988.
9. R. Hui, A. D'Ottavi, A. Mecozzi and P. Spano, "Injection Locking in Distributed Feedback Semiconductor Lasers," *IEEE J. Quantum Elec.* , vol. 27, pp. 1688-1695, 1991.
10. H. Kogelnik and C.V. Shank, "Coupled-Wave Theory of Distributed Feedback Laser," *J. Appl. Phys.*, vol 43, pp. 2327-1335, 1972.
11. F. Favre, "Theoretical Analysis of External Optical Feedback on DFB Semiconductor Lasers," *IEEE J. Quantum Elec.*, vol 23, pp. 81-88, jan. 1987.