Bit-Error-Rate Evaluation of the Distributed Raman Amplified Transmission Systems in the Presence of Double Rayleigh Backscattering Noise

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***Abstract*—A new exact bit-error-rate evaluation method for distributed Raman amplification transmission systems in the presence of both amplified spontaneous emission and double Rayleigh backscattering (DRB) is developed based on a study of statistical characterization of DRB.**

***Index Terms*—Bit-error rate (BER), double Rayleigh backscattering (DRB), Raman amplification.**

I. Introduction

Distributed Raman amplification (DRA) offers significant improvement of optical signal-to-noise ratio when compared to lumped erbium amplification [1]. It is well-known that the double Rayleigh backscattering (DRB) is a major noise source in DRA, and extensive studies have been carried out on the impact of DRB [2]–[7], [12]. Actually, in the presence of DRB, the bit-error rate (BER) is widely evaluated using the Q-factor method, which is based on the Gaussian approximation (GA) of the photocurrent probability density functions (pdfs) [11], [12]. Although there exists a semi-analytical (SA) method for the exact BER evaluation in the presence of amplified spontaneous emission (ASE) [8], [9], to our knowledge, there is still not an exact BER evaluation method taking into account both DRB and ASE. We think that it is mainly due to the shortage of a complete statistical characterization of DRB. This letter will therefore begin with the statistical characterization of DRB, in Section II. Then, the formulas for the BER evaluation will be presented in Section III. Finally, an example will be given in Section IV.

II. Statistical Properties of the DRB Noise

In order to investigate the statistical properties of the DRB noise, we propose to model the temporal-spatial propagation of the forward and backward fields by the following two equations, where the pump is depolarized or unpolarized:

$$\left(\partial_z + v_s^{-1}\partial_t\right) A_+ = g_s(z)A_+ + \rho(z)A_- \tag{1a}$$

$$\left(-\partial_z + v_s^{-1}\partial_t\right) A_- = g_s(z)A_- - \rho^*(z)A_+ \tag{1b}$$

with $g_s = [C_R P_p(z) - \alpha_s]/2$. In (1), $A_\pm(z,t)$ are the forward and backward traveling field envelopes; v_s and α_s are the signal group velocity and attenuation coefficient; C_R and P_p are the Raman coefficient and pump power; and ρ is the differential Rayleigh backscattering coefficient modeled as a delta correlated zero-mean circular complex Gaussian (ccG) stochastic process [4], [5], with[1] $\langle\rho^*(x)\rho(z)\rangle = \gamma_R\delta(x-z)$, where γ_R is the Rayleigh backscattering coefficient. Note that the coefficient of the second term of the right-hand side (RHS) of (1b) is written as $-\rho^*$ to verify the coupled mode theory [10], requiring that the total power must be conserved, $d/dz(|A_+|^2-|A_-|^2)=0$, when $g_s=0$. The DRB field is the first-order perturbation solution of (1a) and (1b), neglecting the discrete reflections. At the output ($z=L$), we have[2] $\tilde{A}_{\rm DRB}(f) = \tilde{H}_{\rm DRB}(f)\tilde{A}_L(f)$, where $A_{\rm DRB}$ is DRB noise field, A_L is the output signal without Rayleigh scattering, and $H_{\rm DRB}$ can be written as[3]

$$\tilde{H}_{\rm DRB}(f) = \frac{\gamma_R L}{2} - \int_0^L\int_0^z \frac{G(z)\rho(x)\rho^*(z)}{G(x)} e^{j4\pi\frac{z-x}{v_s}f}\,dx\,dz \tag{2}$$

where G is the amplifier gain [3]. Since $H_{\rm DRB}$ is causal, i.e., $H_{\rm DRB}(t<0)=0$, it will be called as DRB filter hereafter.

In the frequency domain, the DRB filter is a zero-mean stochastic process. We can find its autocorrelation function (AKF) and prove that the DRB filter is stationary in the frequency domain with a correlation width of order of the inverse of the signal round-trip time in fiber $T_R = v_s/2L$. In the time domain, the DRB filter is a real, zero-mean and delta correlated stochastic process and with a time span equal to T_R. The DRB noise power spectral density (PSD) at the end of transmission can be written as

$$S_{\rm DRB}(f) = |\tilde{H}_{\rm DRB}(f)|^2 S_L(f) \approx K_{\rm DRB}S_L(f) \tag{3a}$$

with the so-called Rayleigh crosstalk coefficient [3]

$$K_{\rm DRB} = \langle|\tilde{H}_{\rm DRB}(f)|^2\rangle = \gamma_R^2 \int_0^L\int_0^z \frac{G^2(z)}{G^2(x)}dx\,dz \tag{3b}$$

[1]In this letter, the symbol $\langle\cdot\rangle$ denotes ensemble average over the fiber samples and $E[\cdot]$ will denote the expectation in time.

[2]In this letter, '$\sim$' denotes the Fourier transformation.

[3]The first term of (2) marks $\langle H_{\rm DRB}\rangle$ equal to zero, and so the deterministic part of DRB field, proportional to the directly transmitted signal, is removed. Equation (2) resembles [5, eq. (6)] or [12, eq. (15.27)]. But $\rho(x)\rho^*(z)$ is written as $\rho(x)\rho(z)$ there. This is not in agreement with the coupled mode theory.

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where S_L is the transmitted signal PSD. It is worth noting that the second step of (3a) holds because we are only interested in the integrals of $S_{\rm DRB}$. When the signal carrier linewidth B_C is far larger than the DRB filter correlation width, i.e., $B_C \gg T_R^{-1}$, the integral of $K_{\rm DRB}S_L(f)$ is practically a good estimation of that of $S_{\rm DRB}(f)$. Since the signal carrier linewidth is practically of order of megahertz or more whereas $1/T_R \sim$ kHz, this condition is generally verified.

By neglecting the amplitude noise, we can write the directly transmitted signal field of a random bit stream as

$$A_L(t)=\sum_n c_n X_n(t)=\sum_n c_n X_L(t-nT_B)\exp(i\phi_L) \quad (4)$$

where c_n is a discrete random variable, X_L is the output signal pulse shape, T_B is the bit time, and ϕ_L represents the signal phase noise. The DRB noise can then be written as

$$A_{\rm DRB}(t) = \sum_n c_n[H_{\rm DRB} \otimes X_n](t) \quad (5)$$

where $\otimes$ stands for the convolution. Since the delta correlated DRB filter has a time span generally far larger than the bit time, i.e., $T_R \gg T_B$, (5) implies that the DRB noise is a sum of a large number of statistically independent random variables. According to the central limit theorem [13], this means that the DRB noise can be considered as a ccG process. Moreover, (5) implies also that the DRB noise is mainly constituted of contributions of the bits shifted by a large number of bit times. Therefore, DRB noise is statistically quasi-independent of the signal.

The DRB and ASE fields can all be treated as zero-mean ccG variables, but the later is delta-correlated whereas the former is not. Moreover, in the above, we have, in fact, assumed that the DRB noise and signal states of polarization (SOP) are preserved. In [6] and [12], it has been shown that the DRB noise has the same SOP as the output signal field, and, unlike the unpolarized ASE, its degree of polarization is $1/9$. So we have the coherency matrix of the total noise field

$$E\left[\mathbf{A}_N(t+\tau)\mathbf{A}_N^{\dagger}(t)\right]=S_A\delta(\tau)\mathbf{I}_2+R_D(\tau)\begin{pmatrix}5/9 & 0\\ 0 & 4/9\end{pmatrix} \quad (6)$$

where $\mathbf{I}_2$ is the 2×2 identity matrix, $\mathbf{A}_N$ and $\mathbf{A}_N^{\dagger}$ are, respectively, the Jones (column) vector of the total noise field and its conjugate transpose; R_D is the AKF of DRB noise, or the inverse Fourier transform of $S_{\rm DRB}$; and S_A is the ASE noise PSD on one polarization, which suffers also from the Rayleigh backscattering and can be calculated using the average power formulas [2], [12]. From (6), we see that the two noise field components are not correlated. This implies that, being ccG variables, they are statistically independent [13].

III. BER Formulations

The fact that the two polarizations of DRB noise are ccG random variables makes technically possible an SA analysis of the photocurrent pdf. For the sake of simplicity, we assume that the photoreceiver is ideal and all other noises are neglected. Therefore, the photocurrent consists only in the contributions of the two independent polarizations, i.e., $i=i_x+i_y$. Setting the receiver responsivity to one, we have

$$i_k(t) = \iint_{\Re^2} A_k(t-\tau_1)K(\tau_1,\tau_2)A_k^*(t-\tau_2)d\tau_1 d\tau_2 \quad (7)$$

with $k=x,y$, and the integration kernel

$$K(\tau_1,\tau_2) = \int_{\Re} H_o(\tau_1-\tau_3)H_e(\tau_3)H_o(\tau_2-\tau_3)d\tau_3. \quad (8)$$

In (7), A_k is the optical field component before the optical filter; and in (8), H_o and H_e are the optical and electrical filters before and after the photodiode, respectively.

Since K is self-adjoint, we can obtain a complete set of normalized orthogonal functions $\{\varphi_n\}$, by the equation [14]

$$\int_{\Re} K(\tau_1,\tau_2)\varphi_n(\tau_2)d\tau_2 = \lambda_n\varphi_n(\tau_1) \quad (9)$$

with real eigenvalues λ_n. On this set, we can expand the field as

$$\begin{aligned} A_k(t-\tau) &= \sum_n A_{k,n}(t)\varphi_n(\tau)\\ &= \sum_n [S_{k,n}(t)+N_{k,n}(t)]\varphi_n(\tau) \end{aligned} \quad (10)$$

where $S_{k,n}$ and $N_{k,n}$ are the signal and total noise projections. The vector (column) $\mathbf{A}_k$ is then a complex Gaussian random vector with mean $\mathbf{S}_k$ and covariance matrix $\mathbf{C}_k$ given by

$$C_{k,nm} = \iint_{\Re^2} \varphi_n^*(\tau_1)R_k(\tau_1-\tau_2)\varphi_m(\tau_2)d\tau_1 d\tau_2 \quad (11)$$

where R_k is the noise AKF on one polarization given by (6).

Since the two noises are all ccG, we can find the moment generation function (MGF) [13] for the photocurrent, given under the matrix form as

$$\Phi(t,z) = E[e^{(i(t)z)}] = \frac{\exp\left[\mathbf{S}_x^{\dagger}(t)(\mathbf{I}-z\mathbf{D}\mathbf{C}_x)^{-1}\mathbf{D}\mathbf{S}_x(t)z\right]}{\det[\mathbf{I}-z\mathbf{D}\mathbf{C}_x]\det[\mathbf{I}-z\mathbf{D}\mathbf{C}_y]} \quad (12)$$

where $\det[\cdot]$ stands for the determinant, $\mathbf{I}$ is the identity matrix, and $\mathbf{D}$ is a diagonal matrix with $D_{nn}=\lambda_n$. Notice that, when there is no DRB, we have $\mathbf{C}_x=\mathbf{C}_y=S_A\mathbf{I}$, and this is in agreement with the result of previous work where there is only ASE [8], [9]. From (12), the mean and variance of the photocurrent can be calculated directly [7], [12], [13]. The photocurrent pdf is the inverse Fourier transform of $\Phi(j2\pi s)$ [13]. This can be achieved either numerically or by using the residue theorem and the steepest descent method for the space signal pdf $f_0(x)$ and the mark signal pdf $f_1(x)$ [8], [9]. We denote the optimum decision level, at which $f_0(i_0)=f_1(i_0)$, as i_0. Then, if the space and mark signals are equally probable, the BER can be calculated by

$$\mathbf{BER} = \frac{1}{2}\int_{-\infty}^{i_o} f_1(x)dx + \frac{1}{2}\int_{i_o}^{+\infty} f_0(x)\,dx. \quad (13)$$

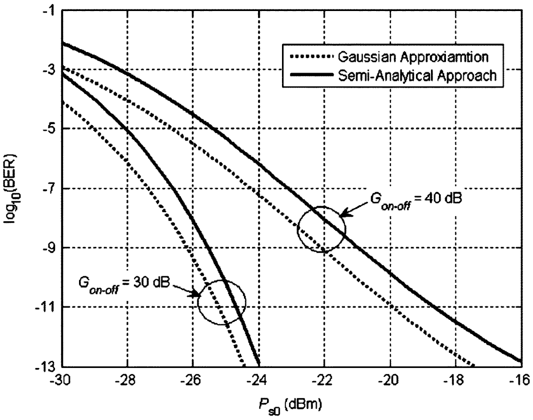

Fig. 1. BER as a function of the input signal power. SSMF features: $C_R = 0.42$ l/km/W, $\alpha_p = 0.25$ dB/km, $\alpha_s = 0.20$ dB/km, and $\gamma_R = 6.0\,10^{-8}$ m^{-1}.

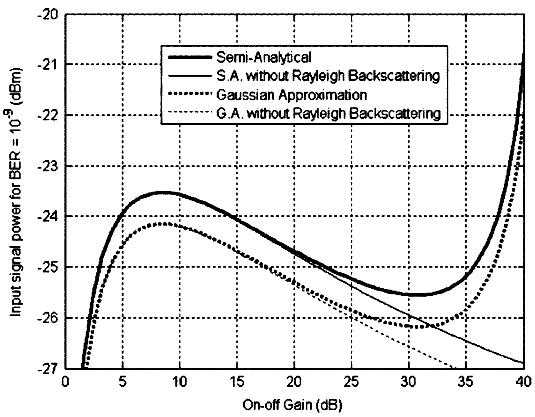

Fig. 2. Sensitivity as a function of DRA ON–OFF gain.

IV. Example and Discussion

In this section, we will use the SA formulas of Section III to evaluate the BER and sensitivity of a counterpumped DRA-based, 100-km standard single-mode fiber (SSMF) single span, direct-detection system using 40-Gb/s return-to-zero (RZ) format at 1555 nm. The pump is assumed to be nondepleted. The AKF of the RZ signal can be found in the literature [12], [15]. The optical filter is Lorentzian, with a 3-dB bandwidth of 0.4 nm ($f_{3\text{ dB}} = 50$ GHz). The electrical filter is chosen to be a second-order Butterworth filter [15] with $f_{3\text{ dB}} = 30$ GHz. The signal power pulse shape is Gaussian with a full-width at half-maximum temporal width of 6.25 ps (1/4 bit duration). The decision time is at the maximum point of the output pulse of the electronic circuit. The pdfs for mark and space signals are calculated numerically using (12).

Fig. 1 shows the BER as a function of the input signal power P_{s0}, the signal power at $z = 0$, where the DRA ON–OFF gains are chosen to be 30 and 40 dB. Fig. 2 shows the sensitivity (input signal power for BER $= 10^{-9}$) as a function of the ON–OFF gain, where the fiber parameters are the same as in Fig. 1. The values obtained without Rayleigh backscattering are also plotted in Fig. 2 for comparison. We see that the DRB seriously deteriorates the sensitivity at high gains and is negligible at low gains, i.e., $G_{\text{on-off}} < 20$ dB, which corresponds to the majority of practical cases. In Figs. 1 and 2, the BER and sensitivities were also estimated using GA [11], [12]. We see that GA can lead to a sensitivity over-estimation of the order of 1 dB.

V. Conclusion

We found that, under the generally satisfied condition $B_s \gg B_C \gg T_R^{-1}$, the DRB noise field components are ccG random variables statistically independent to the signal and their PSD can be considered as identical to that of the signal. The MGF of the photocurrent is then obtained based on these statistical properties. An example of BER and sensitivity evaluations of a DRA-based 40-Gb/s system using direct-detection is given with the formulas presented in this letter. It is also shown that the usually adopted GA of the pdf leads to a sensitivity over estimation of order of 1 dB. Finally, the ideal receiver assumption is not restrictive to the method. We just need to add the ASE noise of the optical preamplifier to (6) and the Gaussian receiver noise current term [9] to (12).

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