New Model of Noise Figure and RIN Transfer in Fiber Raman Amplifiers

Bruno Bristiel, Shifeng Jiang, Philippe Gallion, and Erwan Pincemin

***Abstract*—A new and complete analysis of noise figure and pump noise transfer in fiber Raman amplifiers is reported. Our approach is based on vacuum fluctuation which is the minimum level of the optical noise. As a result, the model enables us to represent both the intrinsic noise of the amplifier as the amplification of the input vacuum fluctuation, the amplification plus attenuation noise addition, and the extrinsic noise due to the noise transfer from the pump.**

***Index Terms*—Noise figure (NF), optical noise, Raman amplification, relative intensity noise (RIN), vacuum fluctuation.**

I. Introduction

DISTRIBUTED fiber Raman amplifiers (FRAs) are well known to reduce the noise generation [1] and to smooth the signal level variation along the link, reducing system vulnerability to nonlinear effects. Noise in optical amplifiers has been analyzed by means of several theoretical frameworks. However, the appropriate definition of the noise figure (NF) of optical amplifiers is still being discussed [2]–[4]. The aim of this letter is to propose an original model of noise in FRA that enables one to estimate the NF and to analyze the relative intensity noise (RIN) transfer from the pump to the signal.

The letter is planned as follows: Section II is devoted to the description of our model by using propagation equations for the signal and pump optical noise. This enables the expression of the NF in Section III and the analysis of the RIN transfer in Section IV. In Section V, the model is discussed and main conclusions are outlined.

II. Model Description

The signal P_S and pump P_P powers propagation equations [5] in a nondepleted FRA are expressed as

$$\frac{dP_S}{dz} = -\alpha_S P_S + C_R P_P P_S \tag{1}$$

$$\pm\frac{dP_P}{dz} = -\alpha_P P_P \tag{2}$$

where α_S and α_P represent the fiber attenuation coefficients for the signal and the pump, respectively. The sign $\pm$ refers to the forward or backward propagation of the pump. C_R is the fiber Raman gain efficiency. The Raman net gain G and the Raman on–off gain G_R at the FRA output $(z = L)$ are derived from (1) as $G_R(L) = \exp[C_R P_{P0} L_{\text{eff}}] = G(L)\exp(\alpha_S L)$, where P_{P0} is the launched pump power and L_{eff} the well-known effective length at the pump wavelength.

A. Intrinsic Raman Amplifier Noise

The minimum additive optical noise, which accompanies any optical field, is usually referred, in quantum electrodynamics, as the vacuum fluctuations. This noise is only observable through its cross-term product with usefull signal, referred to as shot-noise [6]. For each mode of polarization and considering that the optical bandwidth is twice the electrical bandwidth, its spectral density in the optical domain is $h\nu/2$, where $h\nu$ is the photon energy. The vacuum fluctuation, producing shot-noise in power detection, is the only intrinsic signal-independent input noise.

Fiber attenuation is a nondeterministic process, which does not act on the constantly maintained vacuum fluctuation thanks to an attenuation noise source also known as partition noise [6], [8]. Raman amplification is phase-insensitive and, thus, it is necessary to consider an intrinsic amplification noise source in order to satisfy the principle of Heisenberg [7]. Thus, for an FRA, the evolution of the spectral density N_S of the optical noise at the signal wavelength is expressed as

$$\frac{dN_S}{dz} = -\alpha_S\left[N_S - \frac{h\nu_S}{2}\right] + C_R P_P\left[N_S + \frac{h\nu_S}{2}\right]. \tag{3}$$

When the input signal is limited to the vacuum fluctuation, meaning shot-noise limitation of an hypothetical power detection, the spectral density of the output optical noise is

$$N_S = G\frac{h\nu_S}{2} + (G-1)\frac{h\nu_S}{2} + 2\alpha_S G D_{\text{inv}}\frac{h\nu_S}{2} \tag{4}$$

with

$$D_{\text{inv}} = \int_0^L \left[\frac{1}{G(z)}\right] dz.$$

The vacuum fluctuation is an input noise reference which amplification contributes to the output noise in addition to intrinsic amplification noise generation.

B. Maximum Pump Noise Transfer

The RIN transfer results from the amplification of the signal by the pump noise. Thus, it is necessary to know the evolution of the pump noise (5). If we consider now, the transfer of the pump noise in the evolution of the spectral density at the signal wavelength, (3) becomes

$$\begin{aligned}\frac{dN_P}{dz} &= \mp\,\alpha_P\left[N_P - \frac{h\nu_P}{2}\right]\\ \frac{dN_S}{dz} &= -\,\alpha_S\left[N_S - \frac{h\nu_S}{2}\right]\end{aligned} \tag{5}$$

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$$+ C_R P_P \left[N_S + \frac{h\nu_S}{2} \right] + C_R N_P P_S. \qquad (6)$$

In the optical domain, the RIN is usually expressed as the ratio of the spectral density of the optical noise over the averaged power. Taking into account both attenuation plus amplification noise sources and the pump noise transfer, the optical RIN of the signal is expressed as

$$\mathrm{RIN}_S^{\mathrm{out}} = \mathrm{RIN}_S^{\mathrm{in}} + \left[\frac{G-1}{G} + 2\alpha_S D_{\mathrm{inv}} \right] \frac{h\nu_S}{2P_{S0}} + \ln(G_R) \left[\mathrm{RIN}_P^{\mathrm{in}} + \frac{h\nu_P}{2P_{P0}} \alpha_P (L - L_{\mathrm{eff}}) \right]. \qquad (7)$$

$\mathrm{RIN}_S^{\mathrm{in}}$ and $\mathrm{RIN}_P^{\mathrm{in}}$ correspond to the input RINs for the signal and the pump, respectively.

III. Noise Figure

The original formulation of NF in the optical domain defines it as the degradation of the optical signal-to-noise ratio [2], [9]. The input noise is defined as the vacuum fluctuation. If we consider the noise as white Gaussian in the limited optical measurement bandwidth, the NF is defined as

$$\mathrm{NF} = \frac{N_S}{\frac{Gh\nu_S}{2}} = 1 + \frac{G-1}{G} + 2\alpha_S D_{\mathrm{inv}}. \qquad (8)$$

This NF formulation (8) is very simple, rigourous, and in complete agreement with the IRE standards definition [9] adopted by the IEEE. The quantum limit of a 3-dB NF for a very high gain FRA is satisfied [7]. Forward pumping schemes enable the lowest NF to be reached due to the minimization of the term D_{inv}. It has been shown that this model leads to the same NF estimation as the standard amplified spontaneous Raman scattering approach after the shot-noise correction [4], [6], [8].

The equivalent NF $\mathrm{NF}_{\mathrm{eq}}$ represents the NF that a localized preamplifier of gain G_R would experience when generating the same noise as the considered distributed FRA. The equivalent NF expression $\mathrm{NF}_{\mathrm{eq}} = \mathrm{NF}e^{-\alpha_S L}$ is derived from the NF cascading formula, also known as the Friis formulation. Simulation and experiment are in a very good agreement as shown in Fig. 1, as far as the Raman gain G_R is limited to 25 dB to avoid amplified Rayleigh backscattering. Simulation parameters appear in Table I. It is to be mentioned that the Raman amplification noise generation is temperature-dependent [1]. Thus, the NF estimation may be increased by 0.5 dB for high gain values.

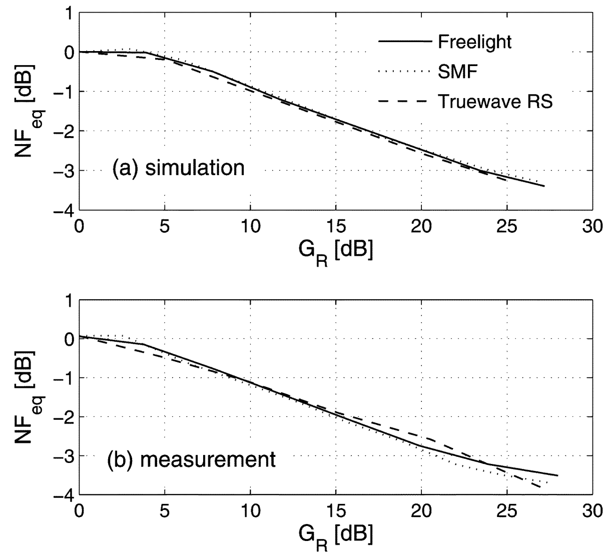

Fig. 1. Equivalent NF as a function of the on–off Raman gain for 100-km spans of different transmission fibers with a backward pumping at 1455 nm. The input signal power is −5 dBm at a wavelength of 1555 nm.

TABLE I
MEASURED FIBER PARAMETERS

Fiber type	Freelight^{TM}	SMF	Truewave-RS^{TM}
α_S(dB/km)	0.2	0.2	0.2
α_P(dB/km)	0.260	0.263	0.256
C_R(W^{-1}.km^{-1})	0.54	0.42	0.69
Chromatic dispersion @1555nm (ps/nm/km)	10	15	5

IV. Pump Noise Transfer

The maximum RIN transfer treated before does not represent a real case of propagation. The pump and the signal do not propagate necessarily in the same direction and with the same group velocities. Thus, fiber chromatic dispersion must be included in the pump noise transfer analysis as in [10] and [11]. In this section, we only analyze the RIN transfer and, thus, the amplification of the input vacuum fluctuation, the amplification and attenuation noise sources are separated from the model to simplify the analysis.

The pump RIN causes nonnegligible temporal power fluctuations. Thus, we can represent the spectral densities of the RIN as $N_S^{\mathrm{rin}}(z,t) = N_S^{\mathrm{rin}}(z)e^{i\omega t}$ and $N_P^{\mathrm{rin}}(z,t) = N_P^{\mathrm{rin}}(z)e^{i\omega t}$ for the signal and the pump, respectively. ω is the RIN modualtion pulsation when the optical carrier is in baseband. The signal is forward propagating and the pump is either forward (upper signs) or backward propagating (lower signs). The propagation equations of N_S^{rin} and N_P^{rin} in time and space can be described as

$$\left(\frac{\partial}{\partial z} + \frac{1}{V_S}\frac{\partial}{\partial t} \right) N_S^{\mathrm{rin}}(z,t)e^{-i\omega t} = \left(-\alpha_S + C_R P_P \right) N_S^{\mathrm{rin}}(z) + C_R N_P^{\mathrm{rin}}(z) P_S \qquad (9)$$

$$\left(\frac{\partial}{\partial z} \pm \frac{1}{V_P}\frac{\partial}{\partial t} \right) N_P^{\mathrm{rin}}(z,t)e^{-i\omega t} = \mp \alpha_P \left(N_P^{\mathrm{rin}}(z) - \frac{h\nu_P}{2} \right) \qquad (10)$$

where V_S and V_P are the group velocities of the signal and the pump, respectively. These propagations, (9) and (10), are simplified by choosing a frame of reference moving with the signal $(z = V_S t)$. Thus, the set of partial differential equations only depends on the space and is rewritten as

$$\frac{d}{dz} N_S^{\mathrm{rin}} = \left(-\alpha_S + C_R P_P \right) N_S^{\mathrm{rin}} + C_R N_P^{\mathrm{rin}} P_S \qquad (11)$$

$$\frac{d}{dz} N_P^{\mathrm{rin}} + i\omega\beta^{\pm} N_P^{\mathrm{rin}} = \mp \alpha_P \left(N_P^{\mathrm{rin}} - \frac{h\nu_P}{2} \right) \qquad (12)$$

where $\beta^{\pm} = 1/V_S \pm 1/V_P$. Note that $\beta^- \approx 2/V_S$ for a backward pumping and $\beta^+ = D(\lambda_S - \lambda_P)$ for a forward propagating pump, where D is the chromatic dispersion at the signal

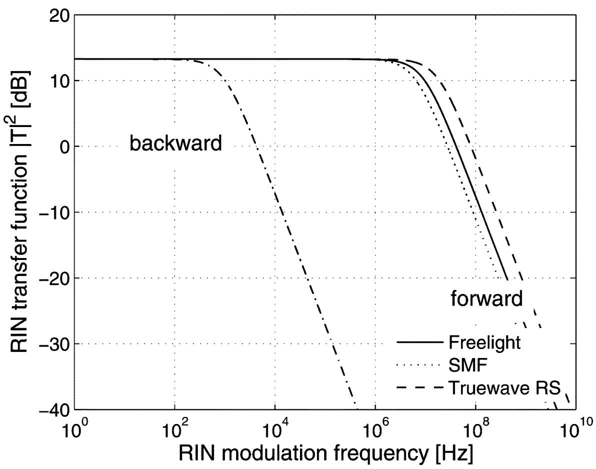

Fig. 2. RIN transfer functions T, in the electrical domain, as a function of the RIN modulation frequency. Both forward and backward pumping schemes are considered for a transmission span of 100 km and $G_R = 20$ dB.

wavelength. The analytical solution for the spectral density of the transferred RIN to the signal is expressed as

$$N_S^{\text{rin}}(L) = GP_{S0}C_R N_P^{\text{rin}}(0)F(\omega) + GP_{S0}C_R\frac{h\nu_P}{2}(L - F(\omega))\left(1 - \frac{i\omega\beta^{\pm}F(\omega)}{1 - e^{-\alpha_P L}}\right) \quad (13)$$

where P_{S0} is the input signal power and $F(\omega)$ defined as

$$F^{\pm}(\omega) = \frac{1 - e^{-\alpha_P L}}{\alpha_P + i\omega\beta^{\pm}}. \quad (14)$$

Using (14), the RIN transferred from the pump to the signal in the optical domain, is expressed as

$$\text{RIN}_S^{\text{out}} = T^{\pm}(\omega)\left[\text{RIN}_P^{\text{in}} + \frac{h\nu_P}{2P_{P0}}\alpha_P(L - F(\omega))\right]. \quad (15)$$

Assuming a long-haul distributed FRA, the electrical RIN transfer functions are defined as

$$\left|T^{\pm}(\omega)\right|^2 \approx \frac{\ln^2(G_R)\left|F^{\pm}(\omega)\right|^2}{L_{\text{eff}}^2}. \quad (16)$$

For low RIN modulation frequencies, the RIN transfer is a maximum, as expressed in (7) and shown in Fig. 2. The resulting electrical RIN transfer functions are second-order low-pass filters with cutoff frequencies $\alpha_P/2\pi\beta^{\pm}$ of the order of kilohertz and megahertz for backward and forward pumping, respectively. Pump vacuum fluctuation is a technological limit for the laser intensity noise and its very low RIN value $h\nu_P/2P_{P0}$ compared to the total RIN of existing pump lasers, makes its transfer negligible.

The quality factor degradation ΔQ can be used to analyze the degradation in performance of a transmission system. The penalty due to RIN transfer is expressed as [10]

$$\Delta Q = \sqrt{1 + Q^2\int_{f_1}^{f_2}\text{RIN}_S^2 df} \quad (17)$$

TABLE II
VALUES OF THE MAXIMUM PUMP RIN LEVELS $|\text{RIN}_P^{\text{in}}|^2$

Fiber type	Freelight^{TM}	SMF	Truewave-RS^{TM}
backward	-78.3 dB/Hz	-78.3 dB/Hz	-78.2 dB/Hz
forward	-118 dB/Hz	-116.4 dB/Hz	-121 dB/Hz

where the receiver bandwidth is between f_1 and f_2 and Q is the baseline quality factor. Considering a flat RIN level RIN_P^{in}, integration accross the signal bandwidth gives

$$\int_{f_1}^{f_2}\text{RIN}_S^2 df = \left(\text{RIN}_P^{\text{in}}\ln(G_R)\right)^2 \cdot\frac{\alpha_P}{2\pi\beta^{\pm}}\left[\arctan\left(\frac{2\pi\beta^{\pm}f_2}{\alpha_P}\right) - \arctan\left(\frac{2\pi\beta^{\pm}f_1}{\alpha_P}\right)\right]. \quad (18)$$

Transfer of pump vacuum fluctuation has been neglected in (18). This result is similar to the quality factor degradation expressed in [10]. The electrical RIN level requierement (see Table II) on the Raman pump module for a penalty of 0.1 dB Q in 20 GHz of electrical bandwidth is around -110 and -80 dB/Hz for forward and backward pumping, respectively.

V. CONCLUSION

A new model of noise has been presented. It enables the analysis of the NF in complete agreement with the IEEE standard, and the study of the noise transfer from the Raman pump. In the saturation regime, the model can be numerically developed by taking into account the pump depletion. This model can be also used for other optical amplifiers such as parametric amplifiers, also sensitive to the pump noise transfer.

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