Analysis of self-pulsation in distributed Bragg reflector laser based on four-wave mixing

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ABSTRACT

All-optical regeneration at 40 Gbit/s and beyond appears to be a crucial element for future transparent networks. One solution to achieve the regeneration is an all-optical clock recovery element combined with a Mach-Zehnder interferometer. Among the different approaches investigated so far to accomplish the clock recovery function, a scheme based on a single self-pulsating distributed Bragg reflector laser is of particular interest from practical and cost viewpoints. In this structure at least two longitudinal modes beat together, generating power oscillation even though the laser is DC biased. The oscillation frequency is given by the free spectral range of the structure. In order to optimize the clock recovery performance of such a laser, a model based on four-wave-mixing has been developed. It takes into account the evolution of the amplitude and the phase of the complex electric field of each longitudinal mode. From this model, a stability analysis is derived through the adiabatic approximation. The spectral density of correlated phase of these modes is calculated and compared to the uncorrelated spectral density of each mode.

Keywords: Distributed Bragg reflector, self-pulsating laser, clock recovery, four-wave mixing and phase correlation.

1. INTRODUCTION

In order to improve transmission distance, transparency, capacity and speed of optical networks, practical means for all-optical treatment of the data, such as all-optical digital logic functions and 3R (re-amplify, re-time, and re-shape) regeneration are of particular interest[1−3]. For example, the all-optical clock recovery would supersede the complicated optoelectronic schemes including a high speed photo-receiver, a high-Q filter, a power amplifier and a high speed laser or an integrated laser modulator. Among the different approaches investigated so far, a scheme based on a single self-pulsating distributed Bragg reflector (SP DBR) laser is of particular interest from practical and cost viewpoints. In such a structure, under certain bias conditions, the beating between the longitudinal modes generates a power oscillation even though the laser is DC biased. The non-linearities and the time characteristics of the semiconductor devices lead to oscillation in the radio-frequency (RF) domain. It has been demonstrated that such a SP DBR laser is able to synchronize its self-pulsation to the bit-rate of an incoming data signal, acting as an all-optical clock recovery device[4]. The optical field generated inside the laser cavity can be expressed as a monochromatic wave with a slowly time varying amplitude:

$$E_k(z,t) = A_k(t)\exp\left(-j(\omega_k t + \phi_k(t)\right)\cdot Z_k(z), \tag{1}$$

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where A_k is the amplitude of the k^{th} mode, $\omega_k = 2\pi\nu_k$ its angular frequency with ν_k the optical frequency, Z_k its longitudinal dependence, and ϕ_k its instantaneous phase fluctuation. In the case of a laser with M longitudinal modes, the beating between these modes leads to a quadratic temporal average of the total electric field with the following expression:

$$< |E_T|^2 >= \sum_{k=1}^{M} < |E_k|^2 > + \sum_{i=1}^{M}\sum_{j\neq k} 2 < E_k E_j \cos(\Omega_{kj} t + (\phi_j(t) - \phi_k(t))) >, \tag{2}$$

where the beating pulsation between two adjacent modes, Ω_{kj}, is defined as $(\omega_j - \omega_k)$. For example, for a laser with two uncorrelated modes, a RF signal at the frequency $\Omega_{kj}/2\pi$ can be observed by a photodiode, with a spectral linewidth corresponding to the sum of the two modes' spectral linewidths. However, in the case of a SP laser, the RF signal linewidth is smaller, benefiting from the phase correlation of the optical modes through the non-linear effects. Indeed, the four-wave-mixing (FWM) results as a modulation of the carrier population (CDM), leading to a nonlinear gain and refractive index modulation, affecting both the amplitude and the phase of the lasing modes. Based on a four-wave mixing formalism, we present in this paper a study of the origin of the RF oscillation in a 3 longitudinal modes SP DBR laser. The paper is organized as follows: in Part 2 the equations describing the behavior of a multimode DBR SP laser are set including the FWM nonlinearity, in Part 3 the stability analysis of this equation system is performed and the solutions are established. Power spectral density for each modes of the DBR SP laser are calculated and compared in with the spectral power density in the case of uncorrelated modes. Part 4 presents and discusses the simulation results based on our model. Finally conclusions are drawn in Part 5.

2. FOUR WAVE MIXING IN A SELF-PULSATING DBR LASER

There are five non-linear effects generating four-wave mixing (FWM) in a semiconductor components: the carrier modulation, the carrier heating, the spectral hole burning, the two-photon absorption and the Kerr effects. These last four effects can be neglected when the free spectral range between the modes involved is of the order of 40 GHz[5], so that only CDM will be taken into account. For the sake of simplicity and clarity, only three longitudinal modes ($M = 3$) propagating in the SP DBR are considered in our approach. The time evolution of the electric field is given by:

$$\begin{aligned}\frac{dE_k}{dt} &= \frac{1}{2}(1 - j\alpha_H)(G - \gamma_k)E_k \\ &+\frac{1}{2}(1 - j\alpha_H)\frac{\Gamma v_g a}{V}\left(\sum_{m=1}^{M-1}(\Delta N_m E_{k-m} + \Delta N_m^* E_{k+m})\right) \\ &-j(\omega_k - \omega_k^i)E_k + F_k(t),\end{aligned} \tag{3}$$

where α_H represents the phase-amplitude coupling factor, γ_k the cavity loss for the k^{th} mode. The instantaneous fluctuations of the electric field due to the spontaneous emission are given by Langevin source term, $F_k(t)$. $(\omega_k - \omega_k^i)$ represents the detuning of the angular frequency ω_k of the k^{th} lasing mode from its initial cavity resonance one ω_k^i. The gain, G is assumed constant over the spectral range of the three modes, and G is given by $\Gamma v_g g$ with Γ the optical confinement, v_g, the group velocity and g expressed as $a(N_0 - N_{tr})/V$, where a is the differential gain, V the volume of the active layer, N_{tr}, the carrier number at transparency, N_0 the average carrier number, satisfying, the carrier rate equation given by :

$$\frac{dN_0}{dt} = \frac{I}{e} - \frac{N_0}{\tau_e} - GP_t, \tag{4}$$

where P_t is the total photon number and is equal, in this study, to $|E_1|^2 + |E_2|^2 + |E_3|^2$. ΔN_m represents the m^{th} order of the carrier modulation. Its dependence with the frequency is given by[6] :

$$\Delta N_m = -(N_0 - N_{tr}) \cdot \frac{\sum_{i=1}^{M} E_k E_{i-m}^*}{P_t + P_s - jm\Omega\tau_e P_s} \tag{5}$$

with the photon saturation number, P_s equals to $V/(\Gamma v_g a\tau_e)$, and τ_e the carrier lifetime. Since $\Omega_{sp}\tau_e \gg 1$, the contribution to the carrier modulation of $(1+P_t/P_s)$ can be neglected. By separating real and imaginary parts of Eq. (4), six rate equations are obtained, describing the time evolution of both the amplitude A_k and the phase ϕ_k of the k^{th} mode. Based on the approximation that $A_2^2/(\Omega_{sp}\tau_e P_s) \ll 1$, the contribution to the non-linear gain corresponding only to a transfer of energy between modes is neglected, whereas the one introducing a transfer of phase are kept. We consider as well, that the amplitude of the mode 2 is larger than the one of mode 1, which is larger than the one of mode 3 in accord with some previous experimental investigations[4]. We achieve for the amplitude and for the phase the following equations :

$$\frac{dA_1}{dt} = \frac{1}{2}\Big((G-\gamma_1) + \frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\frac{A_3}{A_1}\sqrt{1+\alpha_H^2}\sin(\psi + \arctan\alpha_H)\Big)A_1 + F_{A_1} \tag{6}$$

$$\frac{dA_2}{dt} = \frac{1}{2}\Big(G-\gamma_2\Big)A_2 + F_{A_2} \tag{7}$$

$$\frac{dA_3}{dt} = \frac{1}{2}\Big((G-\gamma_3) - \frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\frac{A_1}{A_3}\sqrt{1+\alpha_H^2}\sin(\psi + \arctan\alpha_H)\Big)A_3 + F_{A_3}, \tag{8}$$

$$\frac{d\phi_1}{dt} = \frac{\alpha_H}{2}(G-\gamma_1) - \frac{1}{2}\frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\frac{A_3}{A_1}\sqrt{1+\alpha_H^2}\cos(\psi + \arctan\alpha_H) - (\omega_1 - \omega_1^i) + F_{\phi_1} \tag{9}$$

$$\frac{d\phi_2}{dt} = \frac{\alpha_H}{2}(G-\gamma_2) - (\omega_2 - \omega_2^i) + F_{\phi_2} \tag{10}$$

$$\frac{d\phi_3}{dt} = \frac{\alpha_H}{2}(G-\gamma_3) + \frac{1}{2}\frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\frac{A_1}{A_3}\sqrt{1+\alpha_H^2}\cos(\psi + \arctan\alpha_H) - (\omega_3 - \omega_3^i) + F_{\phi_3}, \tag{11}$$

where F_{A_k} and F_{ϕ_k} represent the instantaneous fluctuation of the amplitude and the phase respectively; and ψ is equal to $2\phi_2 - \phi_1 - \phi_3$. The equations (6)-(11) are similar to those describing an injection-locked laser. The time-rate of change of the mode is made of a classical contribution involving the phase amplitude coupling factor and the gain, but also a term of injection with a phase difference given by $\psi + \arctan(\alpha_H)$. If we introduce the terms of injection rate, ρ_1 and ρ_3 such that:

$$\rho_1 = \frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\sqrt{1+\alpha_H^2}\frac{A_3}{A_1} \tag{12}$$

$$\rho_3 = \frac{G}{\Omega_{sp}\tau_e}\frac{A_2^2}{P_s}\sqrt{1+\alpha_H^2}\frac{A_1}{A_3}, \tag{13}$$

we can express Eqs.(6)-(11) as:

$$\frac{dA_1}{dt} = \frac{1}{2}\Big((G-\gamma_1) + \rho_1\cos(\psi + \arctan\alpha_H - \frac{\pi}{2})\Big)A_1 + F_{A_1} \tag{14}$$

$$\frac{dA_2}{dt} = \frac{1}{2}\Big(G-\gamma_2\Big)A_2 + F_{A_2} \tag{15}$$

$$\frac{dA_3}{dt} = \frac{1}{2}\Big((G-\gamma_3) + \rho_3\cos(\psi + \arctan\alpha_H - \frac{\pi}{2})\Big)A_3 + F_{A_1} \tag{16}$$

$$\frac{d\phi_{21}}{dt} = \frac{1}{2}\Big(\alpha_H(\Delta G_2 - \Delta G_1) - \rho_1\sin(\psi + \arctan\alpha_H - \frac{\pi}{2})\Big) - \Delta\omega_{21}^i + F_{\phi_{21}} \tag{17}$$

$$\frac{d\phi_{32}}{dt} = \frac{1}{2}\Big(\alpha_H(\Delta G_3 - \Delta G_2) - \rho_3\sin(\psi + \arctan\alpha_H - \frac{\pi}{2})\Big) - \Delta\omega_{32}^i + F_{\phi_{32}}, \tag{18}$$

where $\Delta\omega_{jk}^i = (\omega_k^i - \omega_j^i)$, representing the cavity resonance frequency detuning between modes k and j, $\Delta G_k = G - \gamma_k$, $\phi_{21} = \phi_2 - \phi_1$ and $\phi_{32} = \phi_3 - \phi_2$, representing the relative phases.

3. ANALYSIS

3.1. Steady state condition

In order to determine the validity limit of this system of differential equations, the time-rate of change and the amplitude and phase noise sources terms are set to zero. A quadratic equation is then obtained:

$$\frac{1+\alpha_H^2}{4}X^2 - \alpha_H \Delta\omega^i X + (\Delta\omega^{i\,2} - \frac{1}{4}(\rho_3 - \rho_1)^2) = 0, \tag{19}$$

where X is equal to $(2\Delta G_2 - \Delta G_1 - \Delta G_3)$ and the spectral detuning between the modes, $\Delta\omega^i$ is given by $\Delta\omega_{21}^i - \Delta\omega_{32}^i$. This equation has a real root if only its discriminant is positive. This implies that:

$$|\Delta\omega^i| \quad \leq \quad \frac{\sqrt{1+\alpha_H^2}}{2}|\rho_3 - \rho_1| = \Delta\omega_l. \tag{20}$$

Eq. (20) relates the maximal value of $\Delta\omega^i$ to the intermodal half-locking bandwidth[7] $\Delta\omega_l$. While $|\Delta\omega^i|$ is smaller than $\Delta\omega_l$, the system converges towards steady state solutions in the injected regime. At contrary, if $|\Delta\omega^i|$ is larger than $\Delta\omega_l$, the system does not produce a stable solution since the modulation lateral bandwidths are not sufficiently close to the modes to realize an injection-locking. It is worth noticing that $\Delta\omega_l$ is function of the injection rate, related to the amplitude ratio of mode 1 and 3. As the discrepancy between A_1 and A_3 increases, $\Delta\omega_l$ increases. A self-pulsating behavior is achieved when the main mode is surrounded by asymmetric modes, as previously demonstrated.[4]

3.2. Small-signal analysis

Small perturbation analysis is performed within the steady state condition defined in Eq. (20). In order to simplify this calculation, we assume that the intensity of the electric field of each mode instantaneously follows any changes in both gain and refractive index of the cavity. This implies that the fluctuation in the phase and amplitude are uniquely originated by the spontaneous emission coupled to the modes. This will affect the amplitude and the phase of each mode. Consequently, the adiabatic approximation allows to consider the fluctuations at frequencies around the self-pulsation frequency. The small signal analysis of the Eqs. (17) and (18) leads to the following equations for the relative phases :

$$\frac{d\delta\phi_{21}}{dt} = \frac{\alpha_H}{2}\left(\delta(\Delta G_2) - \delta(\Delta G_1)\right) - \frac{1}{2}\rho_1 \cos(\overline{\theta}) \cdot (\delta\phi_{21} - \delta\phi_{32}) + F_{\phi_{21}} \tag{21}$$

$$\frac{d\delta\phi_{32}}{dt} = \frac{\alpha_H}{2}\left(\delta(\Delta G_3) - \delta(\Delta G_2)\right) - \frac{1}{2}\rho_3 \cos(\overline{\theta}) \cdot (\delta\phi_{21} - \delta\phi_{32}) + F_{\phi_{32}}, \tag{22}$$

where the static phase, $\overline{\theta}$ is equal to $(\psi^0 + \arctan\alpha_H - \frac{\pi}{2})$. If we applied the adiabatic approximation to the rate equation of the photon number of the k^{th} mode, it is possible to substitute $\delta(\Delta G_k)$ by the ratio between the photon noise source, F_{P_k} and the photon number, P_k such as $P_k = A_k^2$. The time rate of change of the fluctuation of the relative phases can then be written as:

$$\frac{d\delta\phi_{21}}{dt} = \frac{\alpha_H}{2}\left(\frac{F_{P_1}}{P_1} - \frac{F_{P_2}}{P_2}\right) - \frac{1}{2}\rho_1 \cos(\overline{\theta}) \cdot (\delta\phi_{21} - \delta\phi_{32}) + F_{\phi_{21}} \tag{23}$$

$$\frac{d\delta\phi_{32}}{dt} = \frac{\alpha_H}{2}\left(\frac{F_{P_2}}{P_2} - \frac{F_{P_3}}{P_3}\right) - \frac{1}{2}\rho_3 \cos(\overline{\theta}) \cdot (\delta\phi_{21} - \delta\phi_{32}) + F_{\phi_{32}}. \tag{24}$$

These equations show that the evolution of the phase detuning between the modes 2 and 1 is linked to the one between the modes 3 and 2, and also to the ratio of the amplitude of modes 3 and 1, through ρ_1 and ρ_3.

3.3. Relative phase noise power spectra

The above system of differential equations can be easily solved in the frequency domain using the Fourier transformation. The relative phases fluctuations are expressed as a function of the electrical analysis frequency, Ω' such as $\Omega' = \Omega - \Omega_{sp}$. In the frequency domain, Eqs. (23) and (24) are expressed as follows:

$$\widetilde{\delta\phi_{21}}(\Omega') = \frac{j\Omega\left(\widetilde{\delta\phi_2}^0 - \widetilde{\delta\phi_1}^0\right) + \frac{\rho_1}{2}\cos(\overline{\theta})\widetilde{\delta\phi_{32}}}{j\Omega' + \frac{\rho_1}{2}\cos(\overline{\theta})} \tag{25}$$

$$\widetilde{\delta\phi_{32}}(\Omega') = \frac{j\Omega\left(\widetilde{\delta\phi_3}^0 - \widetilde{\delta\phi_2}^0\right) + \frac{\rho_3}{2}\cos(\overline{\theta})\widetilde{\delta\phi_{21}}}{j\Omega' + \frac{\rho_3}{2}\cos(\overline{\theta})}, \tag{26}$$

where $\widetilde{\delta\phi_k}^0$ is the Fourier transform of the phase fluctuation of k^{th} mode in absence of four-wave mixing and is equal to $1/(j\Omega')(\widetilde{F}_{\phi_k} - \frac{\alpha_H}{2}\frac{\widetilde{F}_{P_k}}{P_k}8$. Thus, it is now possible to express the power spectral density of the relative phases noise as a function of the spectral densities of the three uncorrelated modes. The spectral densities for ϕ_{21} and ϕ_{32} are expressed as follows:

$$S_{\phi_{21}}(\Omega') = \left(S^0_{\phi_2} + S^0_{\phi_1}\right)\frac{1}{1+\left(\frac{\rho_1\cos(\overline{\theta})}{2\Omega'}\right)^2} + S_{\phi_{32}}\frac{1}{1+\left(\frac{2\Omega'}{\rho_1\cos(\overline{\theta})}\right)^2} \tag{27}$$

$$S_{\phi_{32}}(\Omega') = \left(S^0_{\phi_3} + S^0_{\phi_2}\right)\frac{1}{1+\left(\frac{\rho_3\cos(\overline{\theta})}{2\Omega'}\right)^2} + S_{\phi_{21}}\frac{1}{1+\left(\frac{2\Omega'}{\rho_3\cos(\overline{\theta})}\right)^2}. \tag{28}$$

These expressions, obtained in the case of an adiabatic approach, demonstrate qualitatively the existence of a phase correlation between the different modes involved in our model. Indeed, the non-correlation in phase between modes leads theoretically to $S_{\phi_{ij}} = S^0_{\phi_i} + S^0_{\phi_j}$, which is not the case here. Besides, Eqs. (27) and (28) give some information about the behavior of the spectral densities of the relative phases noise. At high analysis frequency, they behave as the sum of the spectral densities of the modes in absence of four-wave mixing. At lower analysis frequency, the contribution of the uncorrelated spectral densities vanishes and $S_{\phi_{21}}(\Omega')$ converges towards $S_{\phi_{32}}(\Omega')$.
From Eqs. (27) and (28), it is also possible to express the sum $S_{\phi_{21}} + S_{\phi_{32}}$ as a function of$S^0_{\phi_1}$, $S^0_{\phi_2}$ and $S^0_{\phi_3}$.

$$S_{\phi_{21}} + S_{\phi_{32}} = 2S^0_{\phi_2} + S^0_{\phi_1} + S^0_{\phi_3} + \frac{b^2 - a^2}{1 + a^2 + b^2}\left(S^0_{\phi_1} - S^0_{\phi_3}\right) \tag{29}$$

with $a = \frac{\rho_1\cos(\overline{\theta})}{2\Omega'}$ and $b = \frac{\rho_3\cos(\overline{\theta})}{2\Omega'}$. Therefore it can be proven that the sum of the spectral densities of phases detunings, $S_{\phi_{32}}$ and $S_{\phi_{21}}$, is smaller than the sum of the uncorrelated phases, $2S^0_{\phi_2} + S^0_{\phi_1} + S^0_{\phi_3}$. Indeed, this condition is always satisfied because $S^0_{\phi_1}$ is smaller, by definition, than $S^0_{\phi_3}$ when A_1 is greater than A_3, and reciprocally. This relation shows that the phase correlation induced by FWM entails a reduction of the spectral linewidth of the self-pulsating signal.

4. NUMERICAL RESULTS

After linearization and Fourier transformation of the Eqs. (4), (6)-(11), the following equation is achieved:

$$\begin{pmatrix}\widetilde{F}_{A_1}\\ \widetilde{F}_{A_2}\\ \widetilde{F}_{A_3}\\ \widetilde{F}_{\phi_1}\\ \widetilde{F}_{\phi_2}\\ \widetilde{F}_{\phi_3}\\ \widetilde{F}_N\end{pmatrix} = (j\Omega' I - M)\cdot\begin{pmatrix}\delta\widetilde{A}_1\\ \delta\widetilde{A}_2\\ \delta\widetilde{A}_3\\ \delta\widetilde{\phi}_1\\ \delta\widetilde{\phi}_2\\ \delta\widetilde{\phi}_3\\ \delta\widetilde{N}_0\end{pmatrix}, \tag{30}$$

where M is 7×7 matrix and I the identity matrix. For any values of Ω' satisfying that $\det(j\Omega' I - M) \neq 0$, we can express the phase fluctuation of the three modes as a function of the Langevin terms. With Q the inverse matrix of $(j\Omega' I - M)$, the phase fluctuations are given directly by:

$$\delta\widetilde{\phi}_1 = q_{4,1}\widetilde{F}_{A_1} + q_{4,2}\widetilde{F}_{A_2} + q_{4,3}\widetilde{F}_{A_3} + q_{4,4}\widetilde{F}_{\phi_1} + q_{4,5}\widetilde{F}_{\phi_2} + q_{4,6}\widetilde{F}_{\phi_3} + q_{4,7}\widetilde{F}_N \tag{31}$$

$$\delta\widetilde{\phi}_2 = q_{5,1}\widetilde{F}_{A_1} + q_{5,2}\widetilde{F}_{A_2} + q_{5,3}\widetilde{F}_{A_3} + q_{5,4}\widetilde{F}_{\phi_1} + q_{5,5}\widetilde{F}_{\phi_2} + q_{5,6}\widetilde{F}_{\phi_3} + q_{5,7}\widetilde{F}_N \tag{32}$$

$$\delta\widetilde{\phi}_3 = q_{6,1}\widetilde{F}_{A_1} + q_{6,2}\widetilde{F}_{A_2} + q_{6,3}\widetilde{F}_{A_3} + q_{6,4}\widetilde{F}_{\phi_1} + q_{6,5}\widetilde{F}_{\phi_2} + q_{6,6}\widetilde{F}_{\phi_3} + q_{6,7}\widetilde{F}_N. \tag{33}$$

Using the Eqs.(31)-(33) and the properties of the Langevin forces, the power spectral density of the phase noise of the k^{th} mode can be written as follows:

$$\begin{aligned} S_{\phi_k}(\Omega') = {} & |q_{k,1}|^2 < \widetilde{F}_{A_1} \cdot \widetilde{F}^*_{A_1} > + |q_{k,2}|^2 < \widetilde{F}_{A_2} \cdot \widetilde{F}^*_{A_2} > + |q_{k,3}|^2 < \widetilde{F}_{A_3} \cdot \widetilde{F}^*_{A_3} > \\ & + |q_{k,4}|^2 < \widetilde{F}_{\phi_1} \cdot \widetilde{F}^*_{\phi_{21}} > + |q_{k,5}|^2 < \widetilde{F}_{\phi_2} \cdot \widetilde{F}^*_{\phi_2} > + |q_{k,6}|^2 < \widetilde{F}_{\phi_3} \cdot \widetilde{F}^*_{\phi_3} > + |q_{k,7}|^2 < \widetilde{F}_N \cdot \widetilde{F}^*_N > \end{aligned} \tag{34}$$

where the spectral densities of the Langevin forces are:

$$< \widetilde{F}_{A_k} \cdot \widetilde{F}^*_{A_k} > = \frac{R_{sp}}{2}, \tag{35}$$

$$< \widetilde{F}_N \cdot \widetilde{F}^*_N > = \frac{I}{e} \tag{36}$$

$$< \widetilde{F}_{\phi_k} \cdot \widetilde{F}^*_{\phi_k} > = \frac{R_{sp}}{2{A^0_k}^2}, \tag{37}$$

where R_{sp} is the spontaneous emission rate. Fig.1 and Fig.2 show an example of calculation results obtained with three different modes such as $A_2 > A_1 > A_3$, for three different values of the cavity resonance frequency detuning $\Delta\omega^i = (2\omega^i_2 - \omega^i_1 - \omega^i_3)$: 0, $\frac{\Delta\omega_l}{2}$, $\Delta\omega_l$. The other parameters used for these calculations are listed in Table 1.

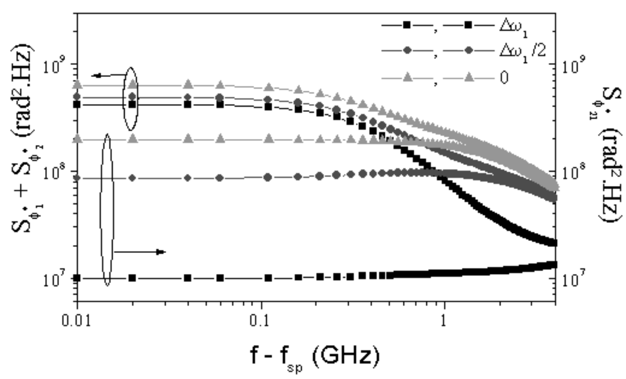

Figure 1. FM noise spectra of the relative phase ϕ_{21} and of the sum of the individual FM noise spectra of modes 1 and 2, in terms of analysis frequency.

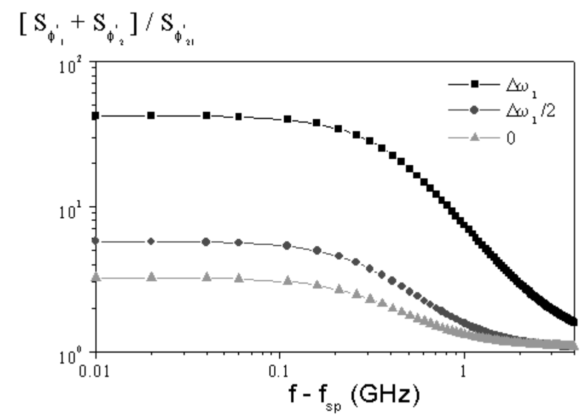

Figure 2. Ratio between FM noise spectral densities of the sum of the individual FM noise spectra of modes 1 and 2, and the ones of the relative phase ϕ_{21}, in terms of analysis frequency.

Fig.1 compares the FM noise spectra obtained for the phase detuning ϕ_{21} with the sum of the individual FM noise spectra of modes 1 and 2, in terms of analysis frequency around f_{sp}. Fig.2 represents the ratio, in a logarithmic scale, between the curves plotted in Fig.1.

Firstly, Fig.1 shows that the FM noise spectra of the phase detuning ϕ_{21} are different from the sum of the individual FM noise spectra of modes 1 and 2. They also tend to the latters for high values of analysis frequency, as it was expressed by Eq. (27). This demonstrates quantitatively the existence of phase correlation, as previously explained in section 3.3. Secondly, Fig.2 shows that the FM noise spectra of the phase detuning ϕ_{21} are always smaller than the sum of the individual FM noise spectra of modes 1 and 2. This indicates that the FM noise spectra of the phase detuning ϕ_{21} have been drastically reduced through the FWM non-linear process in the vicinity of the self-pulsation frequency. So, it shows that FWM, by inducing a phase correlation between the modes, is responsible for the reduction of the spectral linewidth of the self-pulsating signal generated.

Table 1. Parameter used for Fig.1 and Fig.2.

Parameters	Symbols	Values	Unit
Width and thickness	$w.d$	$0, 1.10^{-12}$	m^2
Length	L	1.10^{-3}	m
Volume	V	1.10^{-16}	m^3
Optical confinement	Γ	0.3	
Linewidth enhancement factor	α_H	5	
Speed of light	c	3.10^8	$m.s^{-1}$
Refractive index	n	3.4	
Group index	n_g	4	
Carrier lifetime	τ_e	2×10^{-9}	s
Photon lifetime	τ_p	2×10^{-12}	s
Spontaneous emission rate	R_{sp}	10^{12}	s^{-1}
Carrier number at transparency	n_{tr}	1×10^{24}	m^{-3}
Average carrier number	n_0	1.7×10^{24}	m^{-3}
Differential gain	a	3.2×10^{-20}	m^2
Mode 1 optical losses	γ_1	1.86×10^{11}	s^{-1}
Mode 2 optical losses	γ_2	1.70×10^{11}	s^{-1}
Mode 3 optical losses	γ_2	1.80×10^{11}	s^{-1}
Saturation power	P_{sat}	7×10^4	
Self-pulsation frequency	Ω_{sp}	40.10^9	Hz
Amplitude of mode 1	A_1	$A_2/2$	
Amplitude of mode 2	A_2	$3/2\sqrt{P_{sat}}$	
Amplitude of mode 3	A_3	$2A_2/5$	

5. DISCUSSIONS AND CONCLUSIONS

Self-pulsation in DBR laser has already been demonstrated. It is an oscillation of the output power due to the carrier modulation resulting from the four-wave-mixing of adjacent longitudinal modes selected by the DBR mirror. Nevertheless, the phase correlation between modes inside the cavity, which is necessary to the generation of self-pulsation, has not been clearly demonstrated yet. In this paper, a theoretical work based on the rate equations of three modes has been developed to study the time evolution of their phase and their amplitude. From a steady-state analysis, it was possible to determine some criteria that the three modes have to fulfil in order to achieve the self-pulsation. From a small-signal analysis, we were able to extract different phase noise spectral densities and to study the stability of our differential equation system. We also managed to prove, qualitatively and quantitatively, that the phases of the different modes are partially correlated through the four-wave mixing in this type of self-pulsating laser. Our analysis can well explain the experimental results obtained on these lasers, and will allow to design high performance SP DBR for clock recovery applications.

ACKNOWLEDGMENTS

Dr. Pascal Landais is supported by Enterprise Ireland under the Research Innovation Fund.

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