Correlation functions derivation of the phase and power fluctuations of the optical noise of a laser oscillator with phase-amplitude coupling

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ABSTRACT

This paper shows the use of the Rice representation to obtain the correlation functions of the phase and power fluctuations of the optical noise of a laser oscillator when the Henry's factor is taken into account.

Keywords: optical noise, Rice representation , correlation functions, Henry's factor.

1. INTRODUCTION

Currently the description of the optical noise in lasers and optical amplifiers is an active field of research. Several different approaches to analyze such noise have been developed [1,2,3]. Some of these frameworks take into account the wave-like aspect of light while others deal with its particle-like nature. We have been working on developing another more engineer-oriented description based on the Rice representation [4] (also known as the in-phase quadrature representation) of the optical noise [5, 6]. The Rice representation is a well-known tool used to describe bandpass signals in radio-frequency and signal processing applications. This representation appears as a simple model for semiconductor lasers, optical amplifiers and for the analysis of their intensity and phase noises[5].
In this work we show the usefulness of the Rice representation for obtaining the different relations among the quadrature components of the optical noise through their respective auto and cross-correlation functions and their power spectral densities when the Henry's factor is taken into account [6,9].

2. DERIVATION OF THE CORRELATION FUNCTIONS OF THE PHASE AND POWER FLUCTUATIONS OF THE OPTICAL NOISE.

In this section we derive the correlation functions of the phase and power fluctuations of the optical noise of a laser oscillator signal. To derive such correlations, first we represent the oscillator signal (coherent state) by means of a deterministic phasor with a normalized amplitude $\overline{A} = \sqrt{\overline{P}}$ ($\overline{P}$ is the average power of the optical signal) and an added noise vector $N(t)$ with the Rice representation:

$$N(t) = N_I(t)\cos(2\pi\nu_0 t) - N_Q(t)\sin(2\pi\nu_0 t) \qquad (1)$$

ν_0 is the optical frequency**.**
$N_I(t)$, $N_Q(t)$ the in-phase and quadrature components of $N(t)$ are defined as [7]:

$$N_I(t) = 2N(t)\cos(2\pi\nu_0 t) * h_{LPF}(t) \qquad (2)$$

$$N_Q(t) = -2N(t)\sin(2\pi\nu_0 t) * h_{LPF}(t) \qquad (3)$$

where $*$ represents the convolution operation, and $h_{LPF}(t)$, $H_{LPF}(\nu)$ are the impulse response and transfer function, respectively, of an ideal low-pass filter with bandwidth $\Delta\nu_0$:

$$h_{LPF}(t) = 2\Delta\nu_0 \, sinc(2\Delta\nu_0 t) \qquad (4)$$

$$H_{LPF}(\nu) = rect\left(\frac{\nu}{2\Delta\nu_0}\right) \qquad (5)$$

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On a coherent state, the noise vector corresponds to the zero-point fluctuations [8], therefore its average power (p_N) is:

$$p_N = 2\pi S_N \Delta\nu \qquad (6)$$

where

$$S_N = \frac{h\nu}{2} \qquad (7)$$

is the single-sided spectral density, h is the Planck's constant, ν is the optical frequency, and $\Delta\nu$, is the optical bandwidth of the noise signal respectively.

The instantaneous phase fluctuation $\delta\varphi(t)$ of the laser oscillator signal due to the noise vector $N(t)$ is (under a small noise consideration, $\overline{A} >> N_Q(t)$):

$$\delta\varphi(t) \approx \frac{N_Q(t)}{\overline{A}} \quad (8)$$

with a mean-squared value:

$$\left\langle \delta\varphi^2(t) \right\rangle = \frac{p_q}{\overline{P}} \quad (9)$$

where $\overline{P}$ is in this case the average stimulated power dissipated in the mode of the laser oscillator, and p_I, p_Q of the in-phase and quadrature components respectively.

If we considered an equal repartition energy for the quadrature components of the optical noise we obtain:

$$\left\langle \delta\varphi^2(t) \right\rangle = \frac{P_N}{2\overline{P}} \quad (10)$$

The optical power fluctuations ($\Delta p(t)$) around its mean value $\overline{A}$ can be expressed as a function of its in-phase component as[5]:

$$\Delta p(t) = 2\overline{A} N_I(t) \quad (11)$$

The cross-correlation function of the phase and power fluctuations of the optical noise can be expressed as:

$$R_{\Delta p \Delta\varphi}(\tau) = E\left[2\overline{A} N_I(t+\tau) \left(\frac{N_Q(t) + \alpha_H N_I(t)}{\overline{A}} \right) \right] = 2\alpha_H R_{N_I N_I}(\tau) \quad (12)$$

(in this case we take into account the phase-amplitude coupling factor α_H).

Using equations (2) and (3) in (12) we have:

$$R_{\Delta p \Delta\varphi}(\tau) = -16\alpha_H \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E[N(t-m+\tau)N(t-n)] cos(2\pi f \nu_0 (t-m+\tau)) \cdot sin(2\pi\nu_0(t-n)) h_{LPF}(m) h_{LPF}(n) dm dn \cdot \qquad (13)$$

if we considered $N(t)$ as a wide sense stationary process, then we will have:

$$R_{\Delta p \Delta\varphi}(\tau) = -8\alpha_H \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} R_{NN}(\tau - m + n) \cdot [sin(2\pi\nu_0(\tau - m + n)) + sin(2\pi\nu_0(2t + \tau - m - n))] \cdot \cdot h_{LPF}(m) h_{LPF}(n) dm dn \quad (14)$$

where $R_{NN}(k)$ is the autocorrelation function of the optical noise process during a period of time k.

In order to solve the equation (14) we make several change of variables and knowing that the impulse response $h_{LPF}(t)$ is an even function of t we have, after several manipulations:

$$R_{\Delta P \Delta\varphi}(\tau) = \alpha_H \, sinc(2\Delta\nu\tau) * R_{NN}(\tau) cos(2\pi\nu_0\tau) \quad (15)$$

Its corresponding spectral density is:

$$S_{\Delta P \Delta\varphi}(\nu) = \frac{h\nu\alpha_H}{4} rect\left(\frac{\nu}{2\Delta\nu} \right) \quad (16)$$

then

$$R_{\Delta P \Delta\varphi}(\tau) = \frac{h\nu\alpha_H}{4} sinc(2\Delta\nu\tau) \quad (17)$$

The autocorrelation function of the phase fluctuation due to the quadrature component of optical noise can be expressed as:

$$R_{\Delta\varphi\Delta\varphi}(\tau) = E\left[\frac{\left(N_Q(t+\tau)+\alpha_H N_I(t+\tau)\right)}{\bar{A}} \cdot \frac{\left(N_Q(t)+\alpha_H N_I(t)\right)}{\bar{A}}\right] \quad (18)$$

and after several manipulations

$$R_{\Delta\varphi\Delta\varphi}(\tau) = \left[1+\alpha_H{}^2\right]\frac{h\nu}{8\bar{P}} sinc(2\Delta\nu\tau) \quad (19)$$

Its corresponding spectral density is:

$$S_{\Delta\varphi\Delta\varphi}(\nu) = \left[1+\alpha_H{}^2\right]\frac{h\nu}{8\bar{P}} rect\left(\frac{\nu}{2\Delta\nu}\right) \quad (20)$$

The autocorrelation function of the power noise component due to the in-phase component of the optical noise vector can be expressed as:

$$R_{\Delta P\Delta P}(\tau) = 4\bar{P}R_{N_I N_I}(\tau) \quad (21)$$

using equation (2) in equation (22) we have:

$$R_{\Delta P\Delta P}(\tau) = 2\bar{P}h_{LPF}(\tau) * R_{NN}(\tau)\left[cos\left(2\pi\nu_0(\tau)\right)\right] \quad (23)$$

Its corresponding spectral density is:

$$S_{\Delta P\Delta P}(\nu) = \frac{\bar{P}h\nu}{2} rect\left(\frac{\nu}{2\Delta\nu}\right) \quad (24)$$

then

$$R_{\Delta P\Delta P}(\tau) = \frac{\bar{P}h\nu}{2} sinc(2\Delta\nu\tau) \quad (25)$$

III. CONCLUSION

The Rice representation of the optical field has been used to obtain the auto and crosscorrelation functions and the corresponding spectral densities of its quadrature noise components when the Henry's phase-amplitude coupling factor is taken into account. The Rice representation appears as being a simple model for semiconductor lasers and for the analysis of their respective intensity and phase noises.

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