Classical Phase-Amplitude Description of Optical Amplifier Noise. Application to Noise Figure Derivation for Distributed Optical Amplifiers

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Abstract

A classical phase-amplitude description of the optical field is proposed to discuss the optical amplifier noise generation. This description allows pointing out the contributions to output noise of the vacuum fluctuation input noise and of the intrinsic amplification and attenuation noises mechanisms. Comparison is done with the standard Amplified Spontaneous Emission (ASE) and associated beat noise approach. The model is applied to the theoretical noise figure discussion for distributed amplifiers, as a function of built in internal loss and values of the net gain achieved, and to the derivation of the equivalent lumped amplifier noise figure.

Introduction

The control of noise accumulation is a key issue for the optical amplifier cascades used in long span optical transmission systems and in optical signal processing devices. In the former, the Raman gain distribution is well known to reduce the noise generation and to smooth the signal level variation along the link, reducing system vulnerability to nonlinearity effects.

As already pointed out, the well-known "signal-against-noise" and "noise-against-noise" beating description of optical noise correctly gives the dominant noise terms but failed to give an exact and complete description since, for instance, the heuristic addition of an output shot noise must be included afterward ¹. In such an approach, the averaged Amplified Spontaneous Emission (ASE) noise power is first calculated and its stochastic nature only appears, under implicit Gaussian assumption, in the square law process of detection by beatings of random phase but constant amplitude signals. Since the post detection noise is, in this case, no more additive, it as been also pointed out that the definitions of an Electrical Signal to Noise Ratio (ESNR) and its corresponding noise figure (NF) are un-usefully tedious and inaccurate ². For instance, the independence on incoming signal is an approximation and the standard noise figure cascading formula does not hold.

The aim of this paper is to propose a classical Rice's phase-amplitude description ³ of optical amplifier noise and its application to Optical Signal to Noise Ratio (OSNR) and noise figure derivations for distributed optical amplifiers. The classical phase-amplitude model is presented in the first section of the paper. The second one is devoted to the noise propagation equation in a traveling wave quantum amplifier. The two particular cases of a purely attenuating medium and of a perfectly transparent medium are investigated. Comparison with the standard Amplified Spontaneous Emission (ASE) and beat noise approach results is made in the third section of the paper. The fourth and the fifth sections are respectively devoted to the noise figure discussion in distributed amplifiers, as a function on built in internal loss and for various values of the net gain achieved, and to its equivalent lumped amplifier noise figure.

1. Phase-amplitude optical noise representation

A two-quadrature component description of noise is well known in the field of digital communications and is mandatory to understand the noise generation in optical amplifiers. Let us consider the optical field as the sum of a deterministic component, with complex amplitude $Aexpj\phi$, and of an additive optical band-limited stationary Gaussian noise N(t), with a flat spectrum, in a pass-band bandwidth equal to B_o ⁴⁻⁵. It is assumed that both the deterministic field and the optical noise refer to the same polarization and then, can be represented using a scalar notation. An appropriate normalization, in which the optical power equals the squared field, is also assumed

As shown on Figure 1, the classical phase-amplitude description ³ decomposition of the amplitude noise N into an inphase $N_I(t)$ and a quadrature (i.e. out of phase) $N_Q(t)$ components is used.. The minimum additive optical noise, which accompanies any optical field, is usually referred, in quantum electrodynamics, as the *zero-point field fluctuations* or the *vacuum fluctuations*.



Figure 1: Phase-amplitude representation of the optical field.

Avoiding tedious development, the corresponding spectral density is easily derived, by considering the minimum energy E_n of a quantified oscillator with frequency v:

$$E_n = (n + 1/2)hv = (hv/2) \quad \text{for} \quad n = 0 \tag{1}$$

By considering the standard integration time-bandwidth relationship of a perfect integrator $2\tau B_e = 1$, and observing that the noise passband (optical) bandwidth B_o is twice the observation baseband (electrical) bandwidth B_e , the corresponding optical noise power may be expressed as:

$$\overline{P_N} = h\nu B_e = (h\nu/2)B_0 \tag{2}$$

Writing this averaged noise power $\overline{P_N} = S_N B_0$, the corresponding single-sided optical power spectral density of noise is found to be:

$$S_N = h\nu/2 \tag{3}$$

Figure 2 presents the optical power spectral density of zero-point field fluctuations and the bandwidth considerations. Despite its dependence on the optical frequency, the optical power spectral density is considered as an Additive Gaussian White Noise (AGWN), in the narrow optical band approximation. However, thanks to its linear dependence on the optical frequency, the results of the analysis remain more general. Its is to be mentioned that, as the thermal noise in the radiofrequency range, the zero-point field fluctuation noise level do not depends of the signal level and it is to be considered as the minimum input noise in optical amplifier noise analysis, even when no other input signal is applied. It is also very convenient to use it as noise reference for noise figure definition and measurement.



Figure 2: Spectral density of vacuum fluctuation noise.

The total instantaneous power is the squared sum of the deterministic field and of the in-phase component of the noise. Under the small noise approximation and assuming an equal noise power repartition between the in-phase and the quadrature noise components, the instantaneous optical power fluctuates around its average value $\overline{P} = A^2$ with the mean squared fluctuations:

$$\overline{(\Delta P)}^2 \approx 4.A^2.\overline{N}_I^2 = 4\overline{P}.\overline{P}_I = 2\overline{P}.\overline{P}_N$$
⁽⁴⁾

in which $\overline{P}_l = \overline{P}_N/2$ is the average power of the in-phase noise component. Observing that only the optical noise spectral components, within the spectral range B_e on each side of the optical carrier frequency, produce an observable cross term within the observation (electrical) bandwidth B_e , the optical noise bandwidth contribution is determined by $B_o = 2B_e$ and the power fluctuations can be expressed as the well-known Schottky fluctuation relation:

$$\left(\Delta P\right)^2 = 2h\nu B_e \overline{P} \tag{5}$$

Equation 5 can be easily converted into the well-known Poisson shot noise fluctuation relationship for the photon number $n = P\tau/h\nu$. However, the shot noise is dependent on the signal level and may be not considered as an intrinsic noise input for the amplifier, but only as a noise reference level, as far as power measurements are only concerned. This noise reference level, required by any noise figure definition ², is usually referred as "shot limited signal". Only vacuum fluctuations, producing shot noise in power detection, are the intrinsic input noise.

As in the radiofrequency range, a noise generation is associated to any attenuation or beam partition process in the optical domain, thanks to fluctuation-dissipation theorem. For an elementary slice of width dz in a medium, with a lineic absorption coefficient α , the elementary noise contribution to the single sided power spectrum is expressed as ⁵:

$$dS = \alpha (h\nu/2)dz \tag{6}$$

For the light amplification through an elementary slice of width dz in a medium with the gain per unit of length β , the elementary noise contribution to the single sided power spectrum is expressed as ⁶⁻⁵:

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$$dS = \beta \frac{h\nu}{2} dz \tag{7}$$

2. Noise propagation equation

Let us consider a slice of width dz of an amplifier medium with both a lineic gain coefficient β and a lineic attenuation coefficient α as shown in figure 3. Using the classical phase-amplitude description of the optical field and its associated noise sources, the single-sided spectral density $S(\omega, z)$ of optical noise is found to follow the propagation equation:

$$dS_N = \underbrace{(\beta - \alpha)S_N.dz}_{\text{Noise Amplification}} + \underbrace{\left[(\beta + \alpha)(h\nu / 2).dz\right]}_{\text{Noise Generation}}$$
(8)



Figure 3: Signal and noise propagation

In the general case ⁶ the gain the coefficient β decays, thanks to pump absorption and pump depletion and the solution is expressed as incomplete gamma functions. Under assumptions of an amplification *L* shorter than the effective length corresponding to pump absorption and depletion, the general solution of equation 8 is:

$$S_{N}(z) = C \exp(\beta - \alpha) z - \frac{\beta + \alpha}{\beta - \alpha} h\nu/2$$
(9)

C is an integration constant. Assuming an input noise spectral density $S_N(0)$, the total output noise spectral density for an overall amplification *L* is found to be:

$$S_N(L) = K(G-1)(h\nu/2) + G.S_N(0)$$
(10)

G is the net gain defined as $G = \exp(\beta - \alpha)L$ and *K* the multiplicative noise excess factor as compared to the minimum added amplification noise $(G-1).(h\nu/2)$, required to fulfill minimum uncertainty product requirement ⁵. *K* in expressed as:

$$K = (\alpha + \beta) / |\beta - \alpha|. \tag{11}$$

For a purely attenuating medium with $\beta = 0$, the general equation (10) is written as:

$$S_N(L) = \underbrace{\left[S_N(0) - (h\nu/2)\right]}_{\text{Noise excess as compared}} \exp(-\alpha L) + h\nu/2 \tag{12}$$

Equation 12 shows that the excess of noise, as compared to vacuum fluctation level, vanishes out through attenation in the same way that the signal. When the vacuum fluctuation level is reached, or assumed as input noise, the noise level remains constant since the noise attenuation exactly counteracts the noise generation.

For an exact and local attenuation and gain compensation $\beta = \alpha$, the general equation (10) is written as:

$$S_N(L) = 2\alpha (h\nu/2)L + S_N(0)$$
⁽¹³⁾

Under this constant signal assumption, the spectral density of the ouput noise inceases linearly as a function of the lineic coefficient of the builtin loss.

3. Comparison with the corrected Amplified Spontaneous Emission (ASE) description

Comparison with the Amplified Spontaneous description (ASE) description is displayed in the figure 4. In the particular case of a homogeneous laser amplifier with a vacuum fluctuation input noise, the classical noise description is equivalent to the corrected standard Amplified Spontaneous Emission (ASE) and we have $F = K + 1 = 2n_{sp}$, where n_{sp} is referred as the population inversion factor. As already mentioned the ASE description must also be corrected by addition of the vacuum fluctuation spectral density at the output. This correction is equivalent to the output shot noise addition usually performed, when power detection is considered. However the classical noise description allows a more general treatment including for instance non laser amplifier, input noise, pump noise transfer, phase noise or a different noise power repartition between the two field noise components.



Figure 4: Comparison of classical noise and amplified spontaneous emission (ASE) description.

4. Noise figure discussion

Using the definition of signal to noise ratio and noise figure F in the optical domain ² and making reference to a coherent state input, i.e. a vacuum fluctuations, input noise, $S_N(0) = h\nu/2$, the output total noise is expressed as:

$$S_N(L) = FG(h\nu/2) \tag{14}$$

Using the standard definition, the noise figure is expressed as:

$$F = K \frac{(G-1)}{G} + 1 \approx K + 1 \text{ for } G >> 1$$
(15)

Introducing the built in internal loss attenuation $A = \exp(-\alpha L)$, the noise figure can be also expressed as:

$$F = \frac{\ln G - 2\ln A}{\ln G} \cdot \frac{G - 1}{G} + 1 \tag{16}$$



Figure 5: Noise figure as a function of fiber loss for various values of the net gain

The noise figure as a function of fiber loss, for various values of the achieved net gain is shown on Figure 5. The noise figure is obviously found to be less than the "3db limit" for the low values of the built in loss and of the achieved net gain.

For a purely attenuating medium with $\beta = 0$, implying G=A, and a signal propagation at a constant level, the general equation 16 is written as:

$$F = 2\alpha L + 1 \tag{17}$$

For instance, for $a = 0.046 \text{ km}^{-1}$ (i.e.0.2dB/km) and a transmission length $L = 100 \text{ km} (A_{dB} = 20 \text{ dB})$, the corresponding noise figure expressed in dB is $F_{dB} = 10.2 \text{ dB}$

For an exact and local attenuation and gain compensation $\beta = \alpha$ implying G=I the general equation (10) is written as:

$$F = 1/A \tag{18}$$

5. Equivalent lumped amplifier noise figure

Performances of a distributed amplifier as usually expressed in terms of the noise figure F_{LUMP} , for an hypothetic lumped amplifier, localized after the corresponding attenuating section, and producing the same amount of noise. Observing that the noise figure F_{FIBER} of a pure attenuation fiber is related to its attenuation coefficient by $F_{FIBER} = 1/A$ and using the standard cascading noise figure formula, this noise figure is expressed as:

$$F_{LUMP} = AF. \tag{19}$$



Figure 6: Equivalent lumped post amplification

This value is strongly dependant on attenuation of the fiber and may be obviously less than the 3dB (F = 2) high gain limit of an ideal amplifier for which $K = n_{SP} = 1$. This equivalent noise figure may be of course also negative, when expressed in dB.

Conclusion

The proposed classical phase-amplitude description of the optical field, including the vacuum fluctuations as a minimum input noise, allows pointing out the contributions of input noise and intrinsic amplifier noises mechanisms, including amplification and built in attenuation noise, to output noise of optical amplifiers. Comparison is done with the standard Amplified Spontaneous Emission (ASE) and the associated beat noise approach. The model has been applied to the noise figure discussion in distributed amplifiers, as a function of built in attenuation and for various values of the net gain achieved. The particular cases of a purely attenuating medium and of a perfectly transparent medium have been discussed and the equivalent lumped amplifier noise figure as been derived. This classical noise description allows a more general treatment including for instance non laser amplification, input noise, pump noise transfer, phase noise and line width, and a different noise power repartition between the two field noise components, for noise reduced amplifier analysis and design.

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