A classical model for the noise properties of distributed optical amplifiers

Philippe Gallion*, Virginie Dallot**, Edouard Bridoux***
Ecole Nationale Supérieure des Télécommunications, CNRS, URA 820.

ABSTRACT

Adopting a new classical approach, the additive noise power generated in optical amplifiers is calculated in terms of power spectral density. The classical formalism used combines a corpuscular approach to a phase-amplitude description of the optical field. The noise contributions of the input field fluctuations, including zero-point fluctuations, and of the electron momentum fluctuations at optical frequency linked to the amplifier itself, are clearly identified. The excess of noise associated to coupling or built-in losses is determinated. The well-known result of the *Amplified Spontaneous Emission* (ASE.) is obtained for the laser amplifiers.

This description is applied to linear phase-insensitive amplifiers and to inhomogeneous, nonlinear phase-insensitive Raman amplifier, pointing out the effects of gain compression and gain distribution.

This new approach makes possible the treatment of the squeezed-state of light and the quadrature reduced noise amplifications.

Keywords: Optical Amplifier, Optical Noise, Spontaneous Emission, Noise Figure, Raman Amplifier, Gain Saturation, Gain distribution, Non linear Gain.

1. PHASE-AMPLITUDE OPTICAL NOISE REPRESENTATION

Let us consider the simultaneous reception of a deterministic optical field with complex amplitude $Aexpj\phi$ and of an additive optical band-limited stationary Gaussian noise $N(t)$ with a flat spectrum in a pass-band bandwidth equal to B_o. It is assumed that both the deterministic field and the optical noise refer to the same polarization and then, can be represented with a scalar notation.

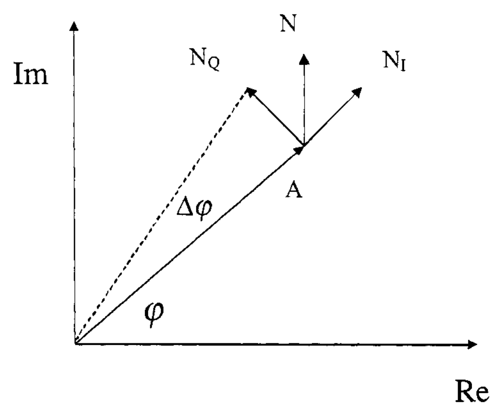


Figure 1: Phasor representation of a small random signal in addition to a deterministic field.

A two-quadrature component description of noise is mandatory to understand the noise generation in optical amplifiers. As shown on Figure 1 standard decomposition of the amplitude noise N into an in-phase $N_I(t)$ and a quadrature $N_Q(t)$ components is used **[1-6]**. Assuming an appropriate normalization in which the optical power equals the squared field, the total instantaneous power is the squared sum of the deterministic field and of the in-phase component of the noise. Under the small noise approximation, this total power can be written as:

$$P \approx (A + N_I)^2 \approx A^2 + 2.A.N_I \tag{1.1}$$

According to the well-known energy equal-repartition principle, the total noise power P_N is assumed to be equally shared between the two noise components and the resulting optical power fluctuates around its average value A^2 with the mean squared fluctuations:

$$\overline{(\Delta P)^2} \approx 4.A^2.\overline{N}_I^{\,2} = 4\overline{P}.\overline{P}_I = 2\overline{P}.\overline{P}_N \tag{1.2}$$

in which $\overline{P} = A^2$, $\overline{P}_I = \overline{P}_N/2$ and $\overline{P}_N$ are the deterministic signal power, the average power of the in-phase noise component and the total noise power, respectively. Observing that only the optical noise spectral components within the spectral range B_e on each side of the optical carrier frequency produce beating within the electrical bandwidth B_e, the optical noise bandwidth contribution is determined by $B_o = 2B_e$ and the noise power can be expressed as:

$$\overline{P_N} \approx S_N B_0 \approx 2S_N B_e \tag{1.3}$$

in which S_N is the single-sided optical noise power spectral density.

As also shown on the figure 1, while the in-phase component $N_I(t)$ induces amplitude change and therefore power fluctuations, the quadrature component $N_Q(t)$ induces phase fluctuations that can be approximated by $\Delta\varphi \approx N_Q / A$.
The mean squared phase fluctuations is thus expressed as:

$$\overline{(\Delta\varphi)^2} \approx \overline{P}_Q / \overline{P} = \overline{P}_N / 2\overline{P} \tag{1.4}$$

in which $\overline{P}_Q = \overline{P}_N/2$ is the average power of the quadrature noise component. The r.m.s. power and phase fluctuation product, independent of the signal power, is obtained:

$$\delta\varphi.\delta P = \sqrt{\overline{(\Delta\varphi)^2}}.\sqrt{\overline{(\Delta P)^2}} = 2\left(\overline{P}_I\,\overline{P}_Q\right)^{1/2} = \overline{P}_N \tag{1.5}$$

2. THE FUNDAMENTAL NOISE SOURCES

According to the quantum treatment of noise, the two fundamental noise sources are the input field fluctuations, including zero-point fluctuations, and the electron momentum fluctuations at optical frequency **[7-10]**. The well-known *Amplified Spontaneous Emission* (A.S.E.) is only a consequence of them. We propose in this section to combine a corpuscular approach with a phase-amplitude description of the optical field to derive them using classical formalism and electrical engineering notations.

2.1 Zero-point fluctuations

Let us consider now a corpuscular description of light, in which the optical signal, of frequency ν, incoming on the photodetector device, is pictured as a constant rate flow of photons of individual energy $h\nu$. As a result of the lack of correlation between the photons, the number n of photons received during an observation time τ is fluctuating according to the well-known Poisson statistic law. The mean squared fluctuations of $n = \overline{n} + \Delta n$ around its expected value, is simply equal to the expected value $\overline{n}$:

$$\overline{(\Delta n)^2} = \overline{n} \tag{2.1}$$

The corresponding shot noise or quantum noise is not a consequence of using the corpuscular description of light but a counterpart of fundamental optical field fluctuations itself. Using the proportional relationship between the number of photons and the received optical power $\overline{n} = \overline{P}\tau/h\nu = \overline{P}/(2B_e h\nu)$, the Poisson fluctuation relation can be interpreted as power instantaneous fluctuations **[4-6]**:

$$\overline{(\Delta P)^2} = 2h\nu B_e \overline{P} \tag{2.2}$$

Comparing with the power fluctuations obtained from the phase amplitude representation and given by Equation (1.2), the power fluctuations associated with the shot noise appear to be produced by the in-phase components N_I of an additive noise N with a total power:

$$\overline{P_N} = h\nu B_e \tag{2.3}$$

Observing again that optical noise spectral components within a spectral range B_e below and above the optical carrier frequency produce beating within the observation bandwidth, the optical noise bandwidth is $B_o = 2B_e$ and the corresponding single-sided optical power spectral density of noise is:

$$S_N = h\nu / 2 \tag{2.4}$$

This additive amplitude optical noise which accompanies any optical field is usually referred in quantum electrodynamics as the *zero-point field fluctuations* or the *vacuum fluctuations*. It is an elusive noise since the zero-point field fluctuations cannot be observed alone. The addition of the zero-point field fluctuations to a classical deterministic field defines a so-called *coherent state of the light* **[8-10]**. This noise is only observable through its cross term product with another signal and is not directly observable.

2.2 Noise linked to absorption, losses and beam splitting

2.2.1 Partition noise

The noise linked to localized attenuation, losses or beam splitting can be derived in a classical corpuscular approach by using the partition noise **[11]**. Let us consider for instance the transmission–reflexion problem pictured on figure 2 where an average photon number $\overline{n}$ is reaching a mirror with a reflectivity R, for a given time τ. For each photon, the probability to be transmitted is $(1\text{-}R)$ and the probability to be reflected *is* R. In these conditions the transmission–reflexion process introduces a so-called partition noise and the corresponding transmitted and reflected photon number fluctuations obey the following relations:

- for a single photon

$$\overline{\Delta n_R^2} = \overline{n_R^2} - \overline{n}_R^2 = \overline{n}_R - \overline{n}_R^2 = R(1-R) \tag{2.5}$$

- for $\overline{n}$ independent photons

$$\overline{\Delta n_{R \text{ or } T}^2} = \overline{n} R(1-R) \tag{2.6}$$

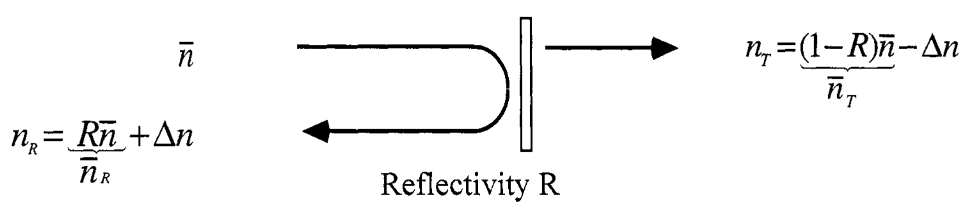


Figure 2: Partition noise in the transmission–reflexion of an optical beam

Using again the proportional relationship between the number of photons and the received optical power $\overline{n} = \overline{P}\tau / h\nu = \overline{P}/(2B_e h\nu)$, the beam partition leads to the fluctuations power instantaneous fluctuations **[5-6]**:

$$\overline{(\Delta P)^2} = 2h\nu B_e R(1-R)\overline{P} = 2h\nu B_e R\overline{P_T} \tag{2.7}$$

Where $\overline{P_T} = (1-R)\overline{P}$ is the mean transmitted power. Using the same derivation as equation (2.2) the corresponding noise power is :

$$\overline{P_R} = Rh\nu B_e \tag{2.8}$$

Using the same derivation as Equation (2.3) the corresponding single-sided noise spectral density is :

$$S_R = Rh\nu / 2 \tag{2.9}$$

2.2.2 Noise contribution of distributed absorption

For an elementary slice of width dz in a medium, with a lineic absorption coefficient α, the elementary noise contribution to the single sided power spectrum is obtained by substituting R to αdz in Equation (2.9):

$$dS_\alpha = \alpha(h\nu / 2)dz \tag{2.10}$$

2.3 Amplification noise

To avoid a quantum description of noise, we will use a heuristic derivation in which the quantum nature of the light is simply introduced by a conjugation relation between the two noise components in the form of the well-known Heisenberg uncertainty product **[12]**.

2.3.1 Conjugation relation between the two noise components

The minimum value of the r.m.s power and phase fluctuation product as given by Equation (1.5) is obtained in the particular case of zero-point field fluctuations. By using Equation (2.3) we obtain in this case:

$$\delta\varphi.\delta P = h\nu B_e \tag{2.11}$$

Introducing the photon number $n = P\tau/h\nu$ received during any observation time τ and the time-bandwidth relation $B_e = 1/2\tau$ this relation is the minimum value of the Heisenberg uncertainty product:

$$\delta\varphi.\delta n = \frac{1}{2} \tag{2.12}$$

The most popular *energy-arrival time* relation is obtained by using $\delta\varphi = 2\pi\nu\delta t$ and the energy defined as $E = P\tau$ this *phase-number* relation is converted into

$$\delta E.\delta t = h/4\pi \tag{2.13}$$

2.3.2 Noise addition necessity

An ideal noiseless phase insensitive linear amplifier only amplifies with a gain G, both the incoming signal and its fluctuations. The output number fluctuations are related to the input one by the relation $\delta n_{OUT} = G\delta n_{IN}$, while the phase fluctuations are kept unchanged $\delta\varphi_{OUT} = \delta\varphi_{IN}$. Output phase and photon number measurements fulfilling Equation (2.12) would imply that, at the same time, the input signal measurements fulfill **[6]**:

$$(\delta\varphi.\delta n)_{INPUT} = \frac{1}{2G} < \frac{1}{2} \tag{2.14}$$

Equation (2.14) is in contradiction to the Heisenberg minimum uncertainty product. The noiseless amplifier therefore cannot exist. Any amplifier must add additional output uncertainties that are introduced by an extra noise with an origin intrinsic to the amplifier itself.

2.3.3 Minimum added noise

Assuming an uncorrelated noise variance addition for each of the two quadratures, the square minimum output uncertainty product fulfilling Heisenberg relation is:

$$\delta\varphi_{OUT}^2.\delta P_{OUT}^2 = (\delta\varphi_D^2 + \delta\varphi_A^2).(\delta P_D^2 + \delta P_A^2) = G^2\overline{P}_N^2 \tag{2.15}$$

In which $\delta\varphi_A$ and δP_A are the amplifier contributions to uncertainty and $\delta\varphi_D$ and δP_D the detector ones.

Using the Equations (1.2) and (1.4.) and denoting $\overline{P}_A = \overline{P}_{IA} + \overline{P}_{QA}$ and $\overline{P}_N = \overline{P}_{ID} + \overline{P}_{QD}$ the corresponding noise powers shared between in phase and quadrature components for the amplifier noise and the detector noise respectively, we write Equation (2.15) in the form:

$$(\overline{P}_{ID} + \overline{P}_{IA})(\overline{P}_{QD} + \overline{P}_{QA}) = G^2 \overline{P}_{ID} \overline{P}_{QD} \tag{2.16}$$

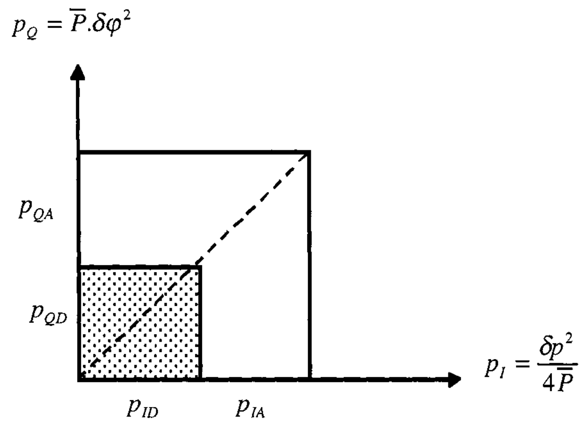


Figure 3: Two quadratures noise addition for $a=b=1/2$

Introducing the constants a and b smaller than the unit, we express the in-phase/quadrature noise power sharing in the forms **[6]**:

$$\overline{P}_{ID} = a\overline{P}_N \ ; \ \overline{P}_{QD} = (1-a)\overline{P}_N \text{ and } \overline{P}_{IA} = b\overline{P}_A \ ; \ \overline{P}_{QA} = (1-b)\overline{P}_A \tag{2.17}$$

The figure 3 shows the two quadratures noise addition for $a=b=1/2$. It is easy to show that the minimum value of the added power is obtained for $a=b$, reducing Equation (2.16) to:

$$(\overline{P}_N + \overline{P}_A)^2 = \mathrm{G}^2 \overline{P}_N^2 \tag{2.18}$$

Using at last Equation (2.3) for the minimum value of the detector noise power and Equation (2.18), the minimum extra noise power required at the output to avoid the violation of Heisenberg minimum uncertainties is obtained **[12]**:

$$\overline{P}_A = (G-1)\frac{h\nu}{2} B_O \tag{2.19}$$

This result is obtained for a phase insensitive linear amplifier with a gain G and an optical bandwidth B_O, equal to twice the observation bandwidth B_e. By using $B_O = 1/\tau$ the product $\overline{P_A}\tau = (G-1)h\nu/2$ can be interpreted as the minimum added noise energy at the output of an amplifier. For large values of gain G, it corresponds to additional noise energy of half a photon during each observation time at the input of an equivalent noiseless amplifier. This minimum value is independent of the nature of the optical amplifier used.
Using the same derivation as Equation (2.3) the corresponding single-sided noise spectral density is:

$$S_A = (G-1)\frac{h\nu}{2} \tag{2.20}$$

2.3.4 Amplification local noise contribution

For the light amplification through an elementary slice of width dz in a medium with the gain per unit of length β, the elementary noise contribution to the single sided power spectrum is obtained by substituting G to $1+\beta dz$ in Equation (2.20):

$$dS_\beta = \beta \frac{h\nu}{2} dz \tag{2.21}$$

3. EXCESS OF NOISE AND NOISE FIGURE

3.1 Minimal total noise and minimal noise figure

The extra noise generated in the amplifier has to be added to the amplification of the unavoidable input zero-point field fluctuations. The minimum overall output optical noise power spectral density is therefore:

$$S_{TOTAL} = \underbrace{(G-1)h\nu/2}_{\substack{\text{Noise generated}\\ \text{in the amplifier}}} + \underbrace{Gh\nu/2}_{\substack{\text{Amplification of input}\\ \text{zero point fluctuations}}} \tag{3.1}$$

For large values of the gain G the corresponding and equivalent total input noise at the input of an ideal noiseless amplifier is thus twice the minimum value associated to zero point fluctuations given by Equation (2.4):

$$S_{TOTAL} = h\nu \tag{3.2}$$

For this reason, the *Noise Figure F* of an optical amplifier, expressing the noise added to the amplified input noise by the mean of a multiplying factor, has a minimum value equal to 2, in the high gain limit **[13-14]**.
This noise corresponds to an overall noise energy of one photon during each observation time at the input of an equivalent noiseless amplifier.

3.2 Amplifier noise excess

The present optical amplifiers add a larger amount of noise and operate above the fundamental limit expressed by Equation (3.2). The main reasons are that the net gain G is usually the result of the subtraction of the local total gain and loss coefficients while their noise contributions add. This can be expressed by multiplying the added noise contribution by a factor K, greater than 1, leading to the noise power density:

$$S_{TOTAL} = K(G-1)\frac{h\nu}{2} + G\frac{h\nu}{2} \tag{3.3}$$

An alternative approach is to assume a noise free input signal and to make reference to the unavoidable output shot noise resulting from the output zero point fluctuations. Equation (3.3) is then rewritten in the form:

$$S_{TOTAL} = F(G-1)\frac{h\nu}{2} + \frac{h\nu}{2} \quad \text{with} \quad F = K + 1 \tag{3.4}$$

The first term in Equation (3.4) is the total output noise supplementing the minimum output zero point fluctuations. It is not the added noise supplementing the amplified zero point fluctuations that is partly included. F is the optical noise figure of the amplifier. It has to be pointed out that the minimum value of F obtained for an ideal amplifier is 2, while the minimum value of K is 1. It is a result of not considering the elusive zero-point fluctuations as an input noise producing a part of the output noise but as a property of the amplifier since they are present at the input even when no signal is detectable. This factor 2 limit is not directly related to polarization, bandwidth or double cross-term considerations, as sometimes believed, but results from Heisenberg conjugation between the two noise quadratures.

3.3 Amplified Spontaneous Emission

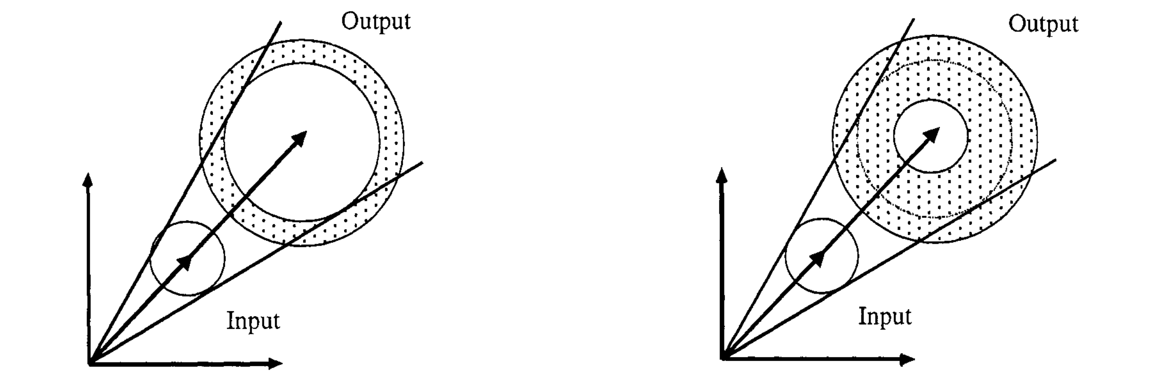
The output optical noise of laser amplifier is usually described in terms of *Amplified Spontaneous Emission* (ASE). The average amplified spontaneous emission power for a single polarization in a single sided optical bandwidth B_o is **[15-18]**:

$$\overline{P}_{ASE} = n_{SP}(G-1)h\nu B_o \tag{3.5}$$

in which n_{SP} is the population inversion factor. This mean power value is not the noise itself, as it has been sometimes considered to be the case, but the root mean square of the power fluctuations of a Gaussian process with the single sided optical spectral density:

$$S_{ASE} = 2n_{SP}(G-1)\frac{h\nu}{2} \tag{3.6}$$

The noise figure is in this case $F = 2n_{SP}$ with a lower limit of 2 for the fully inverted situation. In this case, the 2 factor is explained by considering the input zero-point fluctuations as one of the sources of the spontaneous emission in the amplifier, while the other is produced by momentum fluctuations of the electrons at optical frequencies associated to the gain process itself.
Since a part of it is produced by input noise amplification, the amplified spontaneous emission is not the noise added to the amplified input fluctuations but the noise added to the zero-point output fluctuations. A 2 factor must eventually multiply this value to take into account the two orthogonal polarization states. This additive optical signal on the receiver generates its own shot noise contribution, a noise beating with the useful signal and also a noise resulting from its own power fluctuations, interpreted as noise against noise beating. The Figure 4 shows the comparison of the classical additive noise and ASE noise descriptions.



$$S_{NNout} = \underbrace{(2n_{SP}-1)(G-1)h\nu/2}_{\text{Noise generated in the amplifier}} + \underbrace{Gh\nu/2}_{\text{Amplification of input zero point fluctuations}}$$

Classical additive noise description

$$S_{NNout} = \underbrace{2n_{SP}(G-1)h\nu/2}_{\text{Noise excess as compared to minimun uncertainty}} + \underbrace{h\nu/2}_{\text{unavoidable zero point fluctuations at the output}}$$

ASE noise description

Figure 4: Phasor comparison of the classical additive noise and ASE noise descriptions.
The doted areas correspond to the noise added by the amplifier

4. INFLUENCE OF SIGNAL INDUCED GAIN NONLINEARITIES

In 1989 Yamamoto and Mukai pointed out theoretically the effects of gain sensitivity on noise generation in optical amplifiers **[17]**. The *Optical Phase-Sensitive linear Amplifier* (OPSA.) was introduced as a "theoretical noiseless amplifier with a noise figure of 0 dB". Several experimental works have then illustrated the effects of gain compression on noise generation in saturated *Erbium Doped Fiber Amplifier* (EDFA) and *Semiconductor Optical Amplifier* (SOA) **[19-24]**.
These works have been a first and promising step toward the improvement of noise performances in the amplification process. However few works then have tried to understand and describe theoretically the amplification processes involved in such sensitive optical amplifiers and to evaluate their role in noise generation mechanisms. By applying the classical formalism previously presented, the sections 4 and 5 are devoted to the understanding and modeling of the influence of gain nonlinearities and gain distribution, in optical amplifier noise generation.
In this section, the effects of signal induced gain nonlinearities on noise generation are evaluated in a saturated optical amplifier, such as a SOA or an EDFA. The results are compared to those obtained in the linear amplification regime.

For input powers beyond the saturation power P_S the gain of SOA is signal power-dependent. This results from various processes such as carrier heating, spectral and spatial hole burning. The gain is decreasing with an increasing input signal power. In this situation, a noise reduction has been reported **[25-34]**. The signal power dependence of the gain is expressed in our formalism by the gain multiplicative factor ($1-\varepsilon P(z)$). The gain compression coefficient ε is assumed to be homogeneous and frequency-independent over the signal linewidth. The net gain coefficient is expressed as $g=\left[(\beta-\alpha)-\varepsilon\beta P(z)\right]$. By using the various noise contributions given by Equations (2.10) and (2.21), the differential equation for the total noise power spectral density $S_N(\omega,z)$ is:

$$S_N(\omega,z+dz)=\underbrace{\left[1+(\beta-\alpha).dz\right]S_N(\omega,z)+\left[(\alpha+\beta).dz\right].(h\nu/2)}_{\text{Linear gain contribution}}$$

$$\underbrace{-\left[\varepsilon\beta P(z)dz\right]S_N(\omega,z)-\left[\varepsilon\beta P(z)dz\right].(h\nu/2)}_{\text{Nonlinear gain contribution}} \quad (4.1)$$

By integrating Equation (4.1) along the total amplification length L, the total noise power spectral density, if we assume an incident power spectral density $S_N(\omega,0)$, is:

$$S_N(\omega,L)=A.G_L S_N(\omega,0)+\left[K_{SPL}A(G_L-1)-\varepsilon\beta P(0)AG_L\left\{K_{SPL}\frac{(G_L-1)}{G_L g_L}+L(1-K_{SPL})\right\}\right].\frac{h\nu}{2} \quad (4.2)$$

while the total output power is given by :

$$P(L)=\frac{g_L G_L P(0)}{g_L+\varepsilon\beta(G_L-1)P(0)}=AG_L P(0) \quad (4.3)$$

In these equations, the factor $A=g_L/(g_L+\varepsilon\beta(G_L-1)P(0))$, $g_L=\beta-\alpha$, $K_{SPL}=(\beta+\alpha)/(\beta-\alpha)$, $P(0)$ and $G_L=exp(gL)$ are respectively the global net gain reduction factor, the linear net gain coefficient, the linear excess noise factor, the input signal power and the global gain in the amplification linear regime. In Equation (4.2) the contribution of the incident noise $S_N(\omega,0)$ is reduced by the factor A as well as the intrinsic noise generated inside the amplifier. The last term in the bracket expresses an extra noise reduction as compared to the amplification linear regime.
The noise produced in a gain compressed optical amplifier has been numerically estimated along the amplification length for various K_{SPL} values (i.e. for various values of the ratio α/β) and for different values of $\varepsilon P(0)$. For a given K_{SPL} value, the gain compression leads to a noise reduction up to a critical amplifier length L_C that depends on the K_{SPL} value but not on the gain suppression factor. Beyond L_C the noise quantity overcomes the noise value obtained in the linear regime and no reduction of noise can be obtained. Below L_C the noise generated in a gain compressed optical amplifier is weaker as the ratio α/β is small and as $\varepsilon P(0)$ is large. The nonlinear gain suppression acts clearly as a reductor of intensity noise improving the signal to noise ratio in certain conditions. This effect can be understood as a self-rejection of local optical power fluctuations through gain compression.

5. INFLUENCE OF THE GAIN DISTRIBUTION INDUCED BY PUMP ABSORPTION

The work presented in this section, points out the effect of gain distribution on signal noise generation for the amplification Raman process by using the classical formalism. The involved processes are different since the output signal power and the output signal noise power at frequency ν_S depend on the noise component and power properties of the pump at frequency ν_P **[35]**. The noise and gain performances of Raman amplification in forward and backward configuration are compared.
Since these two configurations are significantly different it is difficult to describe them by using the same modeling. In forward configuration the signal experiments an instantaneous value of the pump along the propagation inside the amplifier, inducing pump and signal noise correlation. In the backward configuration the signal experiments a time average of the pump fluctuations during propagation along the amplifier length making a noise power or spectral

density description simpler. For these reasons the backward configuration is well known to be less sensitive to the pump noise. On the other hand the pump depletion by the signal is easier to take into account in forward configuration, thanks to the space and time jointed propagation of signal and the of pump. The pump depletion, acting as an additional pump loss mechanism is difficult to take into account in backward configuration without a space and time description.

To clearly identify the effects of gain distribution and keep the modeling simple a time average space analysis is performed. The pump depletion induced by the signal is neglected in backward configuration since it is more related to work developed in section 4. Furthermore, shot noise limited pump conditions are assumed to minimize pump noise treatment discrepancy between the two configurations.

Signal and signal noise powers, in an optical bandwidth B_0, at the output of a Raman amplifier are numerically calculated. The considered Raman amplifiers are assumed to have homogeneous gain and loss coefficients all along the effective amplification length. Loss coefficients are frequency dependent and are identified as α_S and α_P at signal and pump frequency. The loss contributions induced by Rayleigh backscattering, with loss coefficient α_{RS}, α_{RP} at signal and pump frequency respectively, are taken into account in the process of noise generation and signal propagation [**37**]. The pump is supposed attenuated by the different loss processes but the depletion resulting from the signal amplification is neglected in backward configuration. The incident signal and the input pump are assumed shot noise limited. The fiber is assumed to be single mode with a constant effective interaction area.

For an amplifying slice with the Raman amplification coefficient. g_R, the signal power P_S and the associated noise power N_S are given by:

$$P_S(z+dz) = P_S(z) - (\alpha_{RS} + \alpha_S)P_S(z)dz + g_R P_P(z) P_S(z)dz \tag{4.4}$$

$$N_S(z+dz) = N_S(z) + \left[-\alpha_{RS} - \alpha_S + g_R P_P(z)\right] N_S(z)dz + g_R N_P(z) P_S(z)dz$$
$$+\left[\alpha_S + g_R P_P(z) + g_R N_P(z)\right]\left(h\nu_S / 2\right) B_0 dz. \tag{4.5}$$

Similarly the signal power P_P and the associated intensity power noise N_P are written as:

$$P_P(z+dz) = P_P(z) - (\alpha_{RP} + \alpha_P)P_P(z)dz - (\nu_P / \nu_S) g_R P_S(z) P_P(z)dz \tag{4.6}$$

$$N_P(z+dz) = N_P(z) - \left[\alpha_{RP} + \alpha_P + (\nu_P / \nu_S) g_R P_S(z)\right] N_P(z)dz$$
$$-(\nu_P / \nu_S) g_R N_S(z) P_P(z)dz + \alpha_P (h\nu_P / 2) B_0 dz. \tag{4.7}$$

The last terms in equations (4.5) and (4.7) are the contributions of zero-point fluctuations. They are commonly included in the A.S.E contribution and are independent of the incident field. Using these equations the output signal power and the associated output intensity power noise are evaluated. The total gain and optical Signal-to-Noise Ratio (SNR) are then calculated for different amplification lengths in backward (Figure 5) and forward configuration (Figure 6). The noise figure F defined according to the IEEE standard [**31**] can be directly deduced from SNR since the initial incident signal and associated noise are constant at the input.

From the two figures, we can see that, whichever the configuration is, the amplifier is saturating when signal is propagating along the amplification length because the pump is attenuated. Moreover in both configurations, the pump acts as a noise generator and degrades the SNR.

Comparing the two configurations, it appears that the forward configuration is less efficient in terms of gain and of noise performances than the backward configuration. In forward configuration, the gain is localized at the input of the amplifier since the pump propagates conjointly with the signal. The latter therefore rapidly depletes it. The amplification length is so shorter and less efficient (smaller gain factor) than in backward configuration. The SNR in forward configuration rapidly degrades with the amplification length since, far from the effective propagation length of the pump, the signal is no more amplified. Then the signal is altered by loss processes and is very sensitive to pump fluctuations.

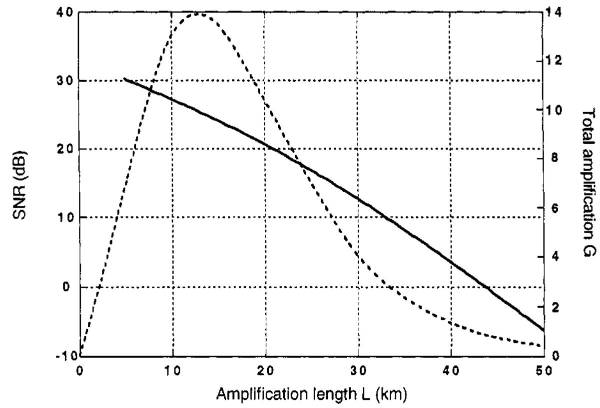


Figure 5: Gain and noise performances in Raman amplifiers in backward configuration for different fiber lengths. (ν_P=206THz, ν_S=192THz, g_R=8.10^{-14} m/W, A=70.10^{-12} μm^2, α_P=α_S=0.4dB/km, α_{RP}=α_{RS}=0.7dB.μm^4/km, B_0=5.10^{6}GHz, $P_P(0)$=0.6mW, $P_S(0)$=0.06mW). Gain (dashed line), SNR (plain line)

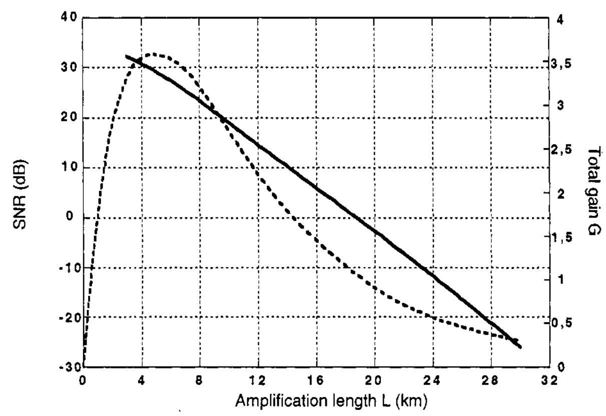


Figure 6: Gain and noise performances in Raman amplifiers in forward configuration for different fibber lengths. (ν_P=206THz, ν_S=192THz, g_R=8.10^{-14} m/W, A=70.10^{-12} μm^2, α_P=α_S=0.4dB/km, α_{RP}=α_{RS}=0.7dB.μm^4/km, B_0=5.10^{6}GHz, $P_P(0)$=0.6mW, $P_S(0)$=0.06mW). Gain (dashed line), SNR (plain line)

As previously mentioned the noise performances are better in backward configuration since the signal is less sensitive to pump fluctuations. The differences observed between the two configurations show the consequences of the distribution of the gain on noise generation. In backward configuration, pump and signal have an opposite direction of propagation implying a better distribution of the gain, than in forward configuration, and then a better SNR.
This comparison between the two configurations shows how the distribution of the gain is also a parameter which determines the noise quantity produced in optical amplifiers.
From these theoretical works on the evaluation of noise amount at the output of different optical amplifiers, it has been shown the influence of gain distribution and gain nonlinearities on noise generation. It has been in this way put to the fore the importance of controlling such parameters to achieve successfully an amplification process with reduced noise effects and so the generation of squeezed-states at the output of amplifiers.

REFERENCES

1. S.O. Rice, *Statistical properties of a sine-wave plus tandem noise*, Bell Syst. Tech. J., **27**, 109, 1948.

2. J.R. Pierce and E.C. Posner, *Introduction to Communications Sciences and Systems*, Plenum Press, New York, 1980.
3. A. Yariv, *Introduction to optical electronics*, Holt Rinehart and Winston, New-York, 1976.
4. B.O. Nilsson, " Noise mechanisms in laser diodes", *IEEE Trans. on electron devices*, **41**, 11, pp. 2139-2150, 1994.
5. P. Gallion (1999). "A classical corpuscular approach to optical noise". OSA, Trends in Optics and Photonics (TOPS), Vol XXX : Optical Amplifier and their applications, Edited by Susumu Kinoshita, Jeffrey C. Livas & M.
6. P. Gallion, "Basics of Digital Optical Communications" to be published, 2001.
7. H. A. Haus, J. A. Mullen, " Quantum noise in linear amplifiers". *Phys. Rev.*, **128**, 5, pp. 2407-2413, 1962.
8. R. Loudon, *The Quantum Theory of Light*, second edition, Oxford University New York, 1983.
9. H.A. Haus, "From classical to quantum noise", *J. Opt. Soc.Am. B*, **12**, 11, pp. 2019-2036, 1995.
10. J.P. Gordon (1999). "Noise in optical systems" OSA, Trends in Optics and Photonics (TOPS), Vol XXX : Optical Amplifier and their applications, Edited by Susumu Kinoshita, Jeffrey C. Livas & Gerlas van der Hoven, Optical Society of America, pp. 12-35, Washington DC.
11. Ross, *Laser Receiver: Devices, Techniques, Systems*, John Wiley & Sons New York, 1966.
12. H. Heffner, "The fundamental noise limit of linear amplifiers". *Proc. of the IRE*, pp. 1604-1608, 1962.
13. H. Haus, "The noise figure of optical amplifiers", *IEEE Photonics Techno. Lett.*, **10**, 11, pp. 1602-1604, 1998.
14. D.M. Baney, P. Gallion and R.S. Tucker, "Theory and Measurement Techniques for the Noise Figure of Optical Amplifiers", *Optical Fiber Technology*, 6, pp. 122-154, 2000.
15. Y, Yamamoto, "Characteristics of AlGaAs Fabry-Perot cavity type laser amplifiers", *IEEE J.Quantum Electron,* QE 16, pp 1047-1052, 1980.
16. J. C. Simon, "Semiconductor laser amplifiers for single-mode optical fiber communications", *J. Opt. Comm.*, **41,** 2, pp. 51-62, 1983.
17. Y. Yamamoto, T. Mukai, " Fundamentals of optical amplifiers". *Opt. and Qu. Electron.*, **21**, S1-S15, 1989.
18. E. Desurvire, *Erbium Doped Fiber Amplifiers, Principles and Applications*, John Wiley & sons, New York, 1994.
19. T. Yamatoya, F. Koyama, K. Iga, " Noise suppression and intensity modulation using gain-saturated SOA", *Optical Amplifiers and Their Applications*, OSA Technical Digest (Optical Society of America, Washington DC, 2000*)* pp. OMD12.1- OMD12.3.
20. M. Tsurusawa, M. Usami and Y. Matsushima, " Demonstration of optical noise reduction using nonlinear absorption in a SLA", *IEEE J. of selected topics in Qu. Electron.,* **5**, 3, pp. 861-865, 1999.
21. S.A.E. Lewis, S.V. Chernikov, J.R. Taylor, 'Gain saturation in silica-fibre Raman amplifier', *Electron. Lett.*, **35**, 11, pp. 923-924, 1999.
22. M. Zhao, G. Morthier, R. Baets, J. Dekoster, 'Investigation of the intensity noise reduction using a saturated SOA in spectrum sliced WDM systems', *CLEO 2001*, CThB3.
23. C.C. Wang, G.J. Cowle, 'Optical gain control of EDFA with a saturable absorber', *IEEE Photon. Technolog. Lett.,* **12**, 5, pp. 483-485, 2000.
24. S. Takeda, S. Watanabe, H. Kuwatsuka, 'All-optical noise suppression by limiter amplification in DFB-LD', *APCC/OECC 1999*, pp. 432-435 vol. 1.
25. J.A.Levenson, I. Abram, Th. Rivera, Ph. Grangier, " Reduction of quantum noise in optical parametric amplification", *J. Opt. Soc.Am. B*, **10**, 11, pp. 2233-2238, 1993.
26. H.P. Yuen, " Reduction of quantum fluctuation and suppression of the Gordon-Haus effect with phase-sensitive linear amplifiers", *Opt. Lett.*, **17**, 1, pp. 73-75, 1992.
27. A. Takada, W. Imajuku, " Amplitude noise suppression using a high gain phase-sensitive amplifier as a limiting amplifier", *Electron. Lett.*, **32**, 7, pp. 677-679, 1996.
28. A. Takada, W. Imajuku, " In-line optical phase sensitive amplifier with pump light source controlled by OPLL", *J. of Lightwave technol.*, **17**, 4, pp. 637-646, 1999.
29. H. Masuda, S. Kawai, K. Aida, "Ultra-wide band hybrid amplifier comprising distributed Raman amplifier and EDFA', *Electron. Lett.*, **34**, 13, pp. 1342-1344, 1998.
30. C.R.S. Fludger, V. Handerek, R.J. Mears, 'Fundamental noise limits in broadband Raman amplifiers', *OFC 2001*, pp. MA5.1-MA5.3.
31. A.F. Evans, J. Grochocinski, A. Rahman, C. Reynolds, M. Vasilyev, 'Distributed amplification : How Raman gain impacts other fiber nonlinearities', *OFC 2001*, pp. MA7.1-MA7.3.
32. D. Levandovsky, M Vasilyev, P. Kumar, 'Amplitude squeezing of light by means of a phase-sensitive fiber parametric amplifier', *Opt. Lett.*, **24**, 14, pp. 984-986, 1999.

33. Y. Su, L. Wang, A. Agarwal, P. Kumar, 'All-optical limiter using gain flattened fibre parametric amplifier', *Electron. Lett.*, **36**, 13, pp. 1103-1105, 2000.
34. A. Hirano, T. Kataoka, S. Kuwahara, M. Asobe, Y. Yamabayashi, 'All-optical limiter circuit based on FWM in optical fibres', *Electron. Lett.*, **34**, 14, pp. 1410-1411, 1998.
35. G. P. Agrawal, *Nonlinear Fiber Optics* , chap. 8, AT&T – Academic press Inc, 1989.
36. E. Dianov, M.V. Grekov and al., "Highly efficient 1.3µm Raman fibre amplifier", *Electron. Lett.* **34**, pp. 669-670, 1998.
37. P. B. Hansen, L. Eskildsen & al., "Rayleigh scattering limitations in distributed Raman preamplifiers", *IEEE Photon. Techn. Lett.* **10,** 1, pp. 159-161, 1998.

*gallion@com.enst.fr; phone:(+33)(0)1 45 81 77 02; fax: (+33)(0)1 45 89 00 20; Ecole Nationale Supérieure des Télecommunications, CNRS, URA 820, 46 rue Barrault, 75634 Paris, FRANCE.
**dallot@com.enst.fr; phone:(+33)(0)1 45 81 78 98; fax: (+33)(0)1 45 89 00 20; Ecole Nationale Supérieure des Télecommunications, CNRS, URA 820, 46 rue Barrault, 75634 Paris, FRANCE.
***bridoux@com.enst.fr phone:(+33)(0)1 45 81 74 94; fax: (+33)(0)1 45 89 00 20; Ecole Nationale Supérieure des Télecommunications, CNRS, URA 820, 46 rue Barrault, 75634 Paris, FRANCE.