# Semiclassical Model of Semiconductor Laser Noise and Amplitude Noise Squeezing—Part I: Description and Application to Fabry–Perot Laser

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***Abstract*—A semiclassical model of semiconductor laser noise, based on the Green's function method, is used to derive analytical formulas for the amplitude and frequency noise spectra taking into account incoming vacuum fluctuations and noise due to internal loss. This formalism also takes into account phenomena such as gain suppression as well as spatial hole burning (SHB). The amplitude noise squeezing is studied for Fabry–Perot structures pointing out the influence of the laser structural parameters. A complete agreement with already existing quantum mechanical models is found. However, extension of the model to SHB induces limitations in the squeezing performances, which are very important for more complex structures as will be pointed out in detail in Part II.**

***Index Terms*—Laser noise, modeling, semiconductor laser.**

## I. Introduction

PHOTON statistics and squeezed states of light have attracted the interest of the scientific community for several years. First, experimental and theoretical works have shown the feasibility of squeezed light from gas lasers and optical fiber media [1], [2]. Then, Yamamoto *et al*. [3] proved theoretically and experimentally the possibility of generating amplitude squeezed light with pump-noise suppressed semiconductor lasers. Their quantum mechanical-based model for a high-reflection-coated Fabry–Perot semiconductor laser [4] is in good agreement with experimental results. Other research groups have subsequently developed quantum mechanically based models to study amplitude squeezing not only with such a simple structure but also with more complex ones like distributed feedback (DFB) or distributed Bragg reflector (DBR) lasers, for example [5], [6]. Some semiclassical descriptions of this problem have been independently developed at the same time [7], [8].

Simultaneously, a lot of research has been carried out on the structural dependencies of semiconductor laser performances and characteristics. Compared to a simple laser model, some additional physical phenomena, such as spatial hole burning (SHB) and gain suppression, modify their performance and characteristics. The Green's function method, first used for semiconductor lasers by Henry [9], is the base of models which have been developed to take into account all of these phenomena [10], [11]. Based on such works, accurate studies of the static and dynamic characteristics of inhomogeneous laser structures like single or multisection DFB and DBR lasers [12], [13] have been performed.

This paper is a generalization of such a model [14] including both the Langevin noise forces related to the carriers and photons as well as the vacuum fluctuation forces present at the laser facets and the noise originating in the loss inside the cavity. In Section II, the model is introduced and both internal and external amplitude and phase noise spectra are analytically derived.

In Section III, owing to the quasi-longitudinal homogeneity and subsequent simplifications, simple formulas are then calculated for a Fabry–Perot semiconductor laser with a linear gain assumption [13]. A comparison is made with quantum mechanical results [4], [16], [17] and semiclassical ones [7] with complete agreement.

In Section IV, the effect of SHB even in a Fabry–Perot laser is pointed out showing an increase of the laser noise and consequently a limitation of the achievable amount of squeezing compared to a simple laser model. Finally, a conclusion is drawn consistent with the perspective to the study of more complex laser structures such as DFB and DBR lasers, which are considered in the second part of this paper.

## II. Description of the Theory

Using the Green's function method, first proposed for semiconductor lasers by Henry [9], Duan *et al*. [14] solved the scalar Helmholtz wave equation for the internal electric field taking into account nonlinear gain and SHB. New noise forces for the incoming vacuum fluctuations and for the noise associated with loss inside the laser cavity are for the first time introduced in this paper.

The vacuum fluctuations are taken into account by adding to the noise forces present in the Helmholtz equation for the field two new terms

$$f_0(t)\delta(z) + f_L(t)\delta(z - L). \tag{1}$$

The first and second term represent the contribution of the vacuum fluctuations transmitted inside the cavity at the right

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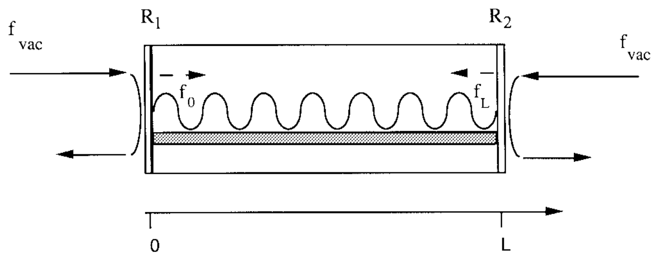



Fig. 1. Scheme of the considered laser structure with noise forces.

and left facet of the laser structure, respectively, as shown in Fig. 1. For a time scale larger than the round-trip time inside the cavity $\tau$, these forces can be approximated [17]

$$f_0(t) = \sqrt{1-R_1}\sqrt{\frac{\tau}{V}}\, f_{\mathrm{vac}}(t)$$

and

$$f_L(t) = \sqrt{1-R_2}\sqrt{\frac{\tau}{V}} f_{\mathrm{vac}}(t) \tag{2}$$

with $\langle f_{\mathrm{vac}}(t) f_{\mathrm{vac}}(t')^*\rangle = \frac{1}{2}\delta(t-t')$ [4], [17] where $V$ is the volume of the active section and $R_1$ and $R_2$ are the left and right power facet reflectivity, respectively.

The field inside the cavity is normalized so that $\langle EE^*\rangle$ represents the photon density inside the cavity and the emitted photon flux outside the cavity. The coupling of the vacuum fluctuations to the laser cavity is considered through the coefficient $\sqrt{\tau/V}$.

The solution of the scalar Helmholtz equation of the field leads to the following equation for the slowly varying amplitude of the internal field $\beta_0$ and using the same method and notations from [14] by expanding the Wronskian around the linear operation point

$$\begin{aligned}\frac{d\beta_0}{dt} =& j(\overline{\omega}-\omega_0)\beta_0(t) - j\int_0^L W_N\,\Delta N(z) \\ &\cdot \beta_0(t)\,dz + F_{\beta'}(t)\end{aligned} \tag{3}$$

with

$$F_{\beta'}(t) = k_0 n\,\frac{Z_0(0)}{\partial W/\partial\omega}\, f_0(t) + k_0 n\,\frac{Z_0(L)}{\partial W/\partial\omega}\, f_L(t)$$

where

$$W_N = \frac{\partial W}{\partial N}\bigg/\frac{\partial W}{\partial\omega}$$

with $\omega_0$ the laser frequency at the linear gain operation point, $\overline{\omega}$ the actual lasing frequency modified by gain suppression, and $F_{\beta'}(t)$ the Langevin force associated with the complex amplitude of the field in the time domain.

The derivatives of the Wronskian with respect to the frequency and the carrier density are given by [14]

$$\frac{\partial W}{\partial\omega} = \frac{2k_0 n n_g}{c}\int_0^L Z_0^2(z)\,dz$$

and

$$\frac{\partial W}{\partial N} = k_0 n g_d(-\alpha_H + j) Z_0^2(z) \tag{4}$$

where $g_d$ is the differential gain and $\alpha_H$ the linear material linewidth enhancement factor.

To take into account the noise due to internal loss, an additive Langevin noise forces has to be introduced [4], [13]

$$\langle f_{\mathrm{loss}}(t) f_{\mathrm{loss}}(t')^*\rangle = \frac{1}{2\tau_{P\mathrm{loss}} V}\,\delta(t-t') \tag{5}$$

with

$$\frac{1}{\tau_P} = \frac{1}{\tau_{P\mathrm{loss}}} + \frac{1}{\tau_{P\mathrm{mirror}}} = v_g\alpha_{\mathrm{int}} + v_g\alpha_{\mathrm{mirror}}$$

where $\alpha_{\mathrm{int}}$ is the internal loss, $\alpha_{\mathrm{mirror}}$ the distributed mirror loss, and $\tau_p$ the photon lifetime inside the cavity.

The resolution of (4) leads to two differential equations for the photon density $P$ and field phase $\Phi$ defined via $\beta_0(t) = \sqrt{P(t)}\exp(j\phi(t))$:

$$\begin{aligned}\frac{dP}{dt} =& 2\int_0^L \mathrm{Im}(W_N)\Delta N(z)\,dz\,P(t) \\ &+ \mathrm{Re}(G_{\mathrm{NL}})(N, S)P(t) + F_{P'}(t) \\ \frac{d\phi}{dt} =& \omega_0 - \overline{\omega} - \int_0^L \mathrm{Re}(W_N)\Delta N(z)\,dz \\ &+ \tfrac{1}{2}\mathrm{Im}(G_{\mathrm{NL}})(N, S) + F_{\phi'}(t)\end{aligned} \tag{6}$$

where Re stands for the real part of the complex force and Im stands for the imaginary one.

$G_{\mathrm{NL}}$ represents an effective nonlinear gain defined as [12]

$$G_{\mathrm{NL}}(N, S) = v_g\,\frac{\displaystyle\int_0^L Z_0^2(z) g_{\mathrm{NL}}(N, S)\,dz}{\displaystyle\int_0^L Z_0^2(z)\,dz} \quad \text{with } g = g_{\mathrm{L}} + g_{\mathrm{NL}}. \tag{7}$$

The modified Langevin noise forces are equal to

$$\begin{aligned}F_{P'}(t) =& F_P(t) + 2\sqrt{P}\,\mathrm{Re}\left[k_0 n\sqrt{\frac{\tau}{V}}\,\frac{Z_0(0)}{\partial W/\partial\omega}\, f_0(t)\right] \\ &+ 2\sqrt{P}\,\mathrm{Re}\left[k_0 n\sqrt{\frac{\tau}{V}}\,\frac{Z_0(L)}{\partial W/\partial\omega}\, f_L(t)\right] \\ &+ 2\sqrt{P}\,\mathrm{Re}\,[f_{\mathrm{loss}}(t)]\end{aligned} \tag{8a}$$

$$\begin{aligned}F_{\phi'}(t) =& F_\phi(t) + \frac{1}{\sqrt{P}}\,\mathrm{Im}\left[k_0 n\sqrt{\frac{\tau}{V}}\,\frac{Z_0(0)}{\partial W/\partial\omega}\, f_0(t)\right] \\ &+ \frac{1}{\sqrt{P}}\,\mathrm{Im}\left[k_0 n\sqrt{\frac{\tau}{V}}\,\frac{Z_0(L)}{\partial W/\partial\omega}\, f_L(t)\right] \\ &+ \frac{1}{\sqrt{P}}\,\mathrm{Im}\,[f_{\mathrm{loss}}(t)].\end{aligned} \tag{8b}$$

The Langevin forces $F_P$ and $F_\Phi$ are calculated using the phasor description, as introduced by Henry [9], [14].

After linearization around the operation point and a Fourier transform analysis, the spectra of the photon number and phase fluctuations are found to be

$$\begin{aligned}\Delta P(\Omega) =& \frac{F_{P'}}{j\Omega - G_{\mathrm{NLPr}}\overline{P} + \displaystyle\int_0^L U_2(z)A(z)|Z_0(z)|^2\,dz} \\ &+ \int_0^L H_p(z, \Omega)[\Delta J(z) + F_N]\,dz\end{aligned} \tag{9a}$$

and

$$\Delta\omega(\Omega) = \frac{\int_0^L U_1(z)A(z)|Z_0(z)|^2\,dz + \frac{1}{2}G_{\mathrm{NL}Pi}}{j\Omega - G_{\mathrm{NLPr}}\overline{P} + \int_0^L U_2(z)A(z)|Z_0(z)|^2\,dz} F_{P'} + F_{\phi'} + 2\pi \int_0^L H_f(z,\,\Omega)[\Delta J(z) + F_N]\,dz. \quad (9b)$$

The following abbreviations are used thereafter:

$$C_1(\Omega) = \frac{1}{j\Omega - G_{\mathrm{NLPr}}\,\overline{P} + \int_0^L U_2(z)A(z)|Z_0(z)|^2\,dz} \quad (10a)$$

$$C_2(\Omega) = \frac{\int_0^L U_1(z)A(z)|Z_0(z)|^2\,dz + \frac{1}{2}G_{\mathrm{NL}\,Pi}}{j\Omega - G_{\mathrm{NLPr}}\overline{P} + \int_0^L U_2(z)A(z)|Z_0(z)|^2\,dz} \quad (10b)$$

$$Z_1(0) = k_0 n \frac{Z_0(0)}{\frac{\partial W}{\partial\omega}} \qquad Z_1(L) = k_0 n \frac{Z_0(L)}{\frac{\partial W}{\partial\omega}}. \quad (10c)$$

A straightforward calculation leads to the internal amplitude and frequency noise double-sided spectra which are given by

$$\begin{aligned} S_{A\mathrm{int}}(\Omega) &= \frac{\langle \Delta P(\Omega)\,\Delta P(\Omega)^*\rangle}{4\overline{P}} \\ &= L\int_0^L \frac{D_{NN}(z,\,\Omega)}{4\overline{P}} \cdot |H_P(z,\,\Omega)|^2\,dz + \frac{D_{PP}}{4\overline{P}}\,|C_1(\Omega)|^2 \\ &\quad + 2\mathrm{Re}\left[\int_0^L \frac{D_{PN}(z,\,\Omega)}{4\overline{P}}\,C_1(\Omega)\,H_p(z,\Omega)^*\,dz\right] \end{aligned} \quad (11a)$$

$$\begin{aligned} S_{\omega\mathrm{int}}(\Omega) &= \langle \Delta\omega(\Omega)\,\Delta\omega(\Omega)^*\rangle \\ &= 2\,\mathrm{Re}\left[\int_0^L D_{PN}(z,\,\Omega)C_2(\Omega)H_f(z,\,\Omega)^*\,dz\right] \\ &\quad + L\int_0^L D_{NN}(z,\,\Omega)|H_f(z,\,\Omega)|^2\,dz \\ &\quad + D_{PP}|C_2(\Omega)|^2 + D_{\phi\phi} \end{aligned} \quad (11b)$$

where * denotes the complex conjugate.

The modified correlation factors of the new Langevin force $F_{P'}$ and $F_{\phi'}$ are given by

$$2D_{PP} = 2\frac{\overline{P}}{V}\left\{R_{\mathrm{sp}} + \left[\frac{1}{\tau_{P\mathrm{loss}}} + \tau(1-R_1)|Z_1(0)|^2 + \tau(1-R_2)|Z_1(L)|^2\right]\right\} \quad (12a)$$

$$2D_{\phi\phi} = \frac{1}{2\overline{P}V}\left\{R_{\mathrm{sp}} + \left[\frac{1}{\tau_{P\mathrm{loss}}} + \tau(1-R_1)|Z_1(0)|^2 + \tau(1-R_2)|Z_1(L)|^2\right]\right\} \quad (12b)$$

where $R_{\mathrm{sp}}$ is the rate of spontaneous emission coupled into the lasing mode

$$\begin{aligned} R_{\mathrm{sp}} &= \frac{4\omega_0^2}{c^3}\,\frac{\int_0^L Z^* n g n_{\mathrm{sp}} Z\,dz \int_0^L Z^* n n_g Z\,dz}{\left|\frac{\partial W}{\partial\omega}\right|^2} \\ &= K n_{\mathrm{sp}} v_g g. \end{aligned} \quad (12c)$$

Please note that the spontaneous emission rate is enhanced by a factor $K$, as defined in [9] and [18], in contrast to an homogeneous laser. This factor $K$ takes into account the longitudinal inhomogeneities of the cavity, the laser output power coupling and the filtering of spontaneous emission of spontaneous emission by the cavity [18]. This factor is also called the longitudinal Petermann factor which is different than the one introduced by Petermann [19], which takes into account lateral effects, even if they are defined with the same formula. According to this formalism, the square modulus of the two coefficients $Z_1(0)$ and $Z_1(L)$ can be considered as some sort of local Petermann factors which are applied in this case to the incoming vacuum fluctuations.

The other correlation factors of the Langevin noise forces for the photons and the carriers are given by [12]

$$\begin{aligned} 2D_{NN}(z,\,\Omega) &= \frac{2\left\{v_g g(z) n_{\mathrm{sp}}\overline{S}(z) + \frac{R[\overline{N}(z)]}{\tau_e}\right\}}{V} \\ 2D_{NP}(z,\,\Omega) &= -\frac{2v_g g(z) n_{\mathrm{sp}}\overline{S}(z)}{V}. \end{aligned} \quad (13)$$

These formulas were obtained by considering explicitely a $z$ dependence in the usually used formulas [9]. Simulations show, for a homogeneous one-section laser, a very small dependence of these coefficients on $z$ and consequently these last coefficients can be considered constant all along the cavity and their value is set equal to their average value. However, in the case of multisection lasers or inhomogeneous pumping, this dependence must be considered.

We shall now concentrate on the external field $E_{\mathrm{ext}}$ at the right facet of the laser, as depicted in Fig. 1:

$$E_{\mathrm{ext}}(t) = \sqrt{1-R_2}\,\sqrt{\frac{V}{\tau}}\,\beta_0(t) - \sqrt{R_2}\,f_{\mathrm{vac}}(t) \quad (14a)$$

with

$$\beta_0 = A_{\mathrm{int}}\,\exp{(j\phi)} \qquad E_{\mathrm{ext}} = A_{\mathrm{ext}}\exp{(j\psi)} \quad (14b)$$

where $E_{\mathrm{ext}}$ is associated with the emitted photon flux and $\beta_0$ is associated with the photon density inside the cavity as in [14].

Using these notations and the boundary conditions at the laser facet, the amplitude and phase variations of the external field are equal to

$$\delta A_{\mathrm{ext}} = \sqrt{\frac{(1-R_2)V}{\tau}}\,\delta A_{\mathrm{int}}(t) - \sqrt{R_2}\,\mathrm{Re}\,[f_{\mathrm{vac}}(t)] \quad (15a)$$

$$\delta\psi = \delta\varphi - \frac{\sqrt{R_2}}{2\overline{A}_{\mathrm{ext}}}\,\mathrm{Im}\,[f_{\mathrm{vac}}(t)]. \quad (15b)$$

The internal and external phases $\phi$ and $\psi$ are assumed to have a zero mean.

TABLE I
LASER PARAMETERS USED IN THE SIMULATIONS

| Parameters | Symbols | Values |
|---|---|---|
| Cavity length | $L$ | 300 $\mu$m |
| Thickness of the active layer | $d$ | 0.15 $\mu$m |
| Width of the active layer | $w$ | 2.0 $\mu$m |
| Effective refractive index | $n$ | 3.50 |
| Group index | $n_g$ | 3.56 |
| Linear linewidth enhancement factor | $\alpha_H$ | 4.00 |
| Bragg wavelength | $\lambda_{B0}$ | 1.50 $\mu$m |
| Fabry–Perot laser wavelength | $\lambda_{\rm FP}$ | 1.55 $\mu$m |
| Spontaneous carrier lifetime | $\tau_e$ | 2 ns |
| Differential gain | $g_d$ | $1.5 \cdot 10^{-16}$ cm$^2$ |
| Transparency carrier density | $N_0$ | $1.0 \cdot 10^{18}$ cm$^{-3}$ |
| Inversion population factor | $n_{\rm sp}$ | 2 |

The external amplitude and phase noise spectra are obtained as

$$S_{A\rm ext}(\Omega) = \frac{(1-R_2)V}{\tau}\, S_{A\rm int}(\Omega) + \frac{R_2}{4} - \frac{\sqrt{R_2}(1-R_2)}{2} \cdot \mathrm{Re}\,[Z_1(L)]\,\mathrm{Re}\,[C_2(\Omega)] \tag{16a}$$

and

$$S_{\omega\rm ext}(\Omega) = S_{\omega\rm int}(\Omega) + \frac{\Omega^2 R_2}{4\,\overline{A}_{\rm ext}^2} + \frac{\sqrt{R_2}\,\sqrt{1-R_2}}{2} \cdot \frac{\Omega\,\mathrm{Im}\,[Z_1(L)]}{\overline{A}_{\rm ext}}\,\{1 - \mathrm{Im}\,[C_2(\Omega)]\}. \tag{16b}$$

The minus sign in (16a) shows that noise compensation for the amplitude noise between internal fluctuations and reflected vacuum fluctuations is possible and consequently explaining why amplitude noise squeezing occurs.

Before applying this model to complex structures (see [20]), we shall now demonstrate its validity by thoroughly studying the more simple case of a Fabry–Perot laser structure assuming a linear gain approximation for the sake of comparison and simplicity.

## III. ANALYTICAL STUDY OF THE FABRY–PEROT LASER STRUCTURE

For such a structure with symmetric facet reflectivities, the modulus square of the internal field is approximated as a constant. Consequently, many laser parameters (shown in Table I) exhibit no longitudinal dependencies and many simplifications occur in the modulation responses and noise spectra.

Under this assumption, the overall amplitude and phase modulation transfer functions are

$$H_P(\Omega) = \frac{\overline{P} v_g g_d}{L\left(\Omega_0^2 - \Omega^2 + \dfrac{j\,\Omega}{\tau_r}\right)}$$

$$H_f(\Omega) = \frac{j v_g g_d \alpha_H \Omega}{L\left(\Omega_0^2 - \Omega^2 + \dfrac{j\Omega}{\tau_r}\right)}$$

with

$$\Omega_0^2 = \overline{P} v_g^2 g_d g \quad \text{and} \quad \frac{1}{\tau_r} = \frac{1}{\tau_e} + v_g g_d \overline{P} \tag{17}$$

where $\overline{P}$ represents the steady-state photon density in the cavity, $L$ the length of the cavity, and $\tau_e$ the spontaneous electron lifetime.

All the Langevin force diffusion coefficients are also $z$-independent. The value of the carrier noise Langevin force $F_N$ depends on the pump noise of the current source used for the laser. With a Poissonian pump noise, the diffusion coefficient $D_{NN}$ is expressed as [14]

$$2D_{NN} = \frac{2n_{\rm sp} v_g g \overline{P} + 2\,\dfrac{\overline{N}}{\tau_e}}{V} \tag{18}$$

where $\overline{N}$ represents the steady state carrier density in the cavity.

For complete pump noise suppression it results in [5]

$$2D_{NN} = \frac{2\left(n_{\rm sp} - \dfrac{1}{2}\right) v_g g \overline{P} + \dfrac{\overline{N}}{\tau_e}}{V}. \tag{19}$$

For the calculation of the diffusion coefficients $D_{PP}$ and $D_{\phi\phi}$, the Wronskian and its derivative with respect to the frequency have to be determined.

The internal field in a Fabry–Perot structure is given by [9]

$$Z_0(z) = \sqrt{R_1}\,\exp(-j\gamma z) + \exp(j\gamma z) \tag{20}$$

with

$$\gamma = k_0 n + j\,\frac{g - \alpha_L}{2} \quad \text{and} \quad \sqrt{R_1 R_2}\,\exp(-2j\gamma L) = 1.$$

Using the following formula for the Wronskian of a such a structure:

$$W = 2j\gamma\sqrt{R_1}\,[\sqrt{R_1 R_2}\,\exp(-2j\gamma L) - 1] \tag{21}$$

the two parameters $Z_1(0)$ and $Z_1(L)$ are then given by

$$Z_1(0) = \frac{1+\sqrt{R_1}}{2\sqrt{R_1}}\,\frac{k_0\,n}{\gamma\tau}$$

and

$$Z_1(L) = \frac{1+\sqrt{R_2}}{2\sqrt{R_2}}\,\frac{k_0 n}{\gamma\tau}. \tag{22}$$

In the following expressions, the imaginary part of $\gamma$ is neglected compared to its real part since the gain is very close to the loss inside the cavity. This results in a real $Z_1(0)$ and $Z_1(L)$ which leads to simplifications of the noise forces. According to their definitions, these two factors are operating as local longitudinal Petermann factor for the incoming vacuum fluctuations. The longitudinal Petermann factor [9], [18] enhances spontaneous emission inside the cavity and has to be considered in the Langevin diffusion coefficient. For a Fabry–Perot laser, it is given by [9]

$$R_{\rm sp} = K n_{\rm sp} v_g g \quad \text{with} \quad K = \left[\frac{(R_1+R_2)(1-R_1R_2)}{2R_1R_2\ln(R_1R_2)}\right]^2. \tag{23}$$

Finally, the Langevin diffusion coefficients are equal to

$$2D_{PP} = \frac{2\overline{P}}{V}\left\{R_{\rm sp} + \left[\frac{1}{\tau_{P\rm loss}} + \frac{1-R_1}{\tau}\left(\frac{1+\sqrt{R_1}}{2\sqrt{R_1}}\right)^2 + \left(\frac{1+\sqrt{R_2}}{2\sqrt{R_2}}\right)^2\frac{1-R_2}{\tau}\right]\right\} \quad (24a)$$

$$2D_{\phi\phi} = \frac{2D_{PP}}{4\overline{P}^2}. \quad (24b)$$

In the case of high reflection coefficients at the two facets of the lasers, these two diffusion coefficients are given by

$$2D_{PP} = \frac{2\overline{P}}{V}\left(R_{\rm sp} + \frac{1}{\tau_P}\right)$$
$$2D_{\phi\phi} = \frac{1}{2\overline{P}V}\left(R_{\rm sp} + \frac{1}{\tau_P}\right). \quad (25)$$

Only the additive term (8) linked to transmitted vacuum fluctuations leads to the correct result. With the new value of $D_{PP}$, consistent noise and amplitude squeezing are obtained, which is not possible with the values normally used [9], [14].

The $z$-independent coefficients $C_1$ and $C_2$ are for this case

$$C_1(\Omega) = \frac{j\Omega + \frac{1}{\tau_r}}{(\Omega_0^2 - \Omega^2) + \frac{j\Omega}{\tau_r}}$$
$$C_2(\Omega) = -\frac{v_g^2 g_d g \alpha_H}{(\Omega_0^2 - \Omega^2) + \frac{j\Omega}{\tau_r}}. \quad (26)$$

### A. Laser Internal Noise

For the sake of comparison, we now consider the case of a Fabry–Perot semiconductor laser with high facet reflectivities in order to compare the results to already published ones [4], [7], [13], [16], [17].

The double-sided amplitude noise of the internal field is described by the following equation:

$$S_{A_{\rm int}}(\Omega) = \frac{A_{{\rm int}\,P}\Omega^2 + B_{{\rm int}\,P}}{(\Omega_0^2 - \Omega^2)^2 + \frac{\Omega^2}{\tau_r^2}} \quad (27)$$

with

$$A_{{\rm int}\,P} = 2D_{PP}$$
$$B_{{\rm int}\,P} = \frac{2D_{PP}}{\tau_r^2} + 2D_{NN}(\overline{P}v_g g_d)^2 + 4\overline{P}D_{NP}\frac{v_g g_d}{\tau_r}.$$

At high pumping level $R = I/I_{th} - 1$, the internal amplitude noise spectrum at low frequency is equal to

$$S_{A\rm int}(0) = \frac{\tau_P}{4\overline{P}}(2D_{PP} + 2D_{NN} + 4D_{NP}). \quad (28)$$

For a high-reflection-coated Fabry–Perot laser and an arbitrary value of the laser internal losses, (29) results in

$$S_{A\rm int}(0) = \begin{cases} \frac{\tau_P}{2} & \text{Poissonian pump} \\ \frac{\tau_P}{4} & \text{Quiet pump} \end{cases} \quad (29)$$

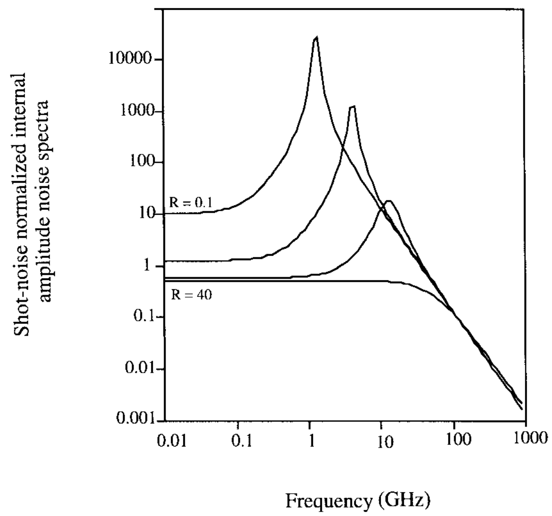



Fig. 2. Quietly pumped high-reflection-coated Fabry–Perot laser internal amplitude noise spectra normalized by the shot noise level for different values of the pumping level $R$ defined as $I/I_{th} - 1$ equal to 0.1, 1, 10, and 40.

which gives for the photon number $PV$ when the laser is normally pumped $\langle\Delta(PV)^2\rangle = (\overline{P}V)$.

When the laser is pumped with a Poissonian pump, the internal field fluctuations are at the shot noise level and the variance of photon number statistics is equal to the average photon number inside the cavity [4], [7].

Previously used values for $D_{PP}$ would result in an internal amplitude noise equal to zero for a Poissonian pump and even in a negative amplitude noise if the laser is quietly pumped. This shows that the vacuum fluctuation forces are necessary to give an appropriate description of semiconductor laser noise spectra.

Fig. 2 shows internal amplitude noise spectra for a pump-noise-suppressed Fabry–Perot laser structure with high-reflection-coated facets and no internal loss. These results are in complete agreement with the formerly published ones [4], [7], [16], [17], with a maximum achievable reduction of 50% of the internal amplitude noise at low frequency. This level of maximum squeezing is independent of the laser internal loss. However, for a given current or emitted power, a laser with a nonnegligible internal loss would have more internal amplitude noise as compared to the shot-noise level.

Using the same approach, the internal frequency noise is written as

$$\frac{A_{{\rm int}\,\phi}\Omega^2 + B_{{\rm int}\,\phi}}{(\Omega_0^2 - \Omega^2)^2 + \frac{\Omega^2}{\tau_r^2}} + C_{{\rm int}\,\phi} \quad (30)$$

with

$$A_{{\rm int}\,\phi} = -\frac{PD_{NN}v_g g_d \alpha_H}{2}$$
$$B_{{\rm int}\,\phi} = -\frac{Pv_g^2 g_d^2 \tau_{in}\alpha_H[|Z_1(0)|^2 + |Z_1(L)|^2]}{2}$$
$$C_{{\rm int}\,\phi} = D_{\phi\phi}.$$

Fig. 3 shows internal frequency noise for the same Fabry–Perot structure and for different pump levels. A good agreement with already published results [4] is also

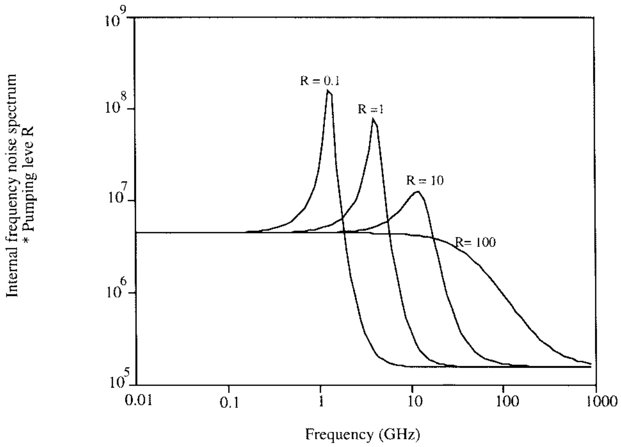



Fig. 3. Quietly pumped high-reflection-coated Fabry–Perot laser internal frequency noise spectra multiplied by $R$ for $R = 0.1$, 1, 10, and 100.

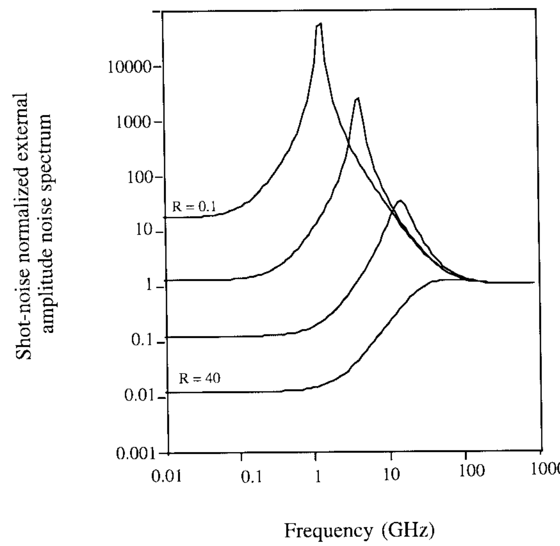



Fig. 4. Quietly pumped high-reflection-coated Fabry–Perot laser external amplitude noise spectra normalized by the shot noise level for $R = 0.1$, 1, 10, and 40.

found. The variance of the phase fluctuations is greater than its corresponding shot-noise level, which means that this state of light produced is not a minimum uncertainty state.

### B. Laser External Noise

Using (16a) and (24), the amplitude noise of the external field is given by

$$S_{A_{\text{ext}}}(\Omega) = \frac{R_2}{4} - \frac{(1-R_2)(1+\sqrt{R_2})}{4\tau\tau_r} \frac{\Omega_0^2}{(\Omega_0^2-\Omega^2)^2 + \dfrac{\Omega^2}{\tau_r^2}} + \frac{(1-R_2)V}{\tau} \frac{A_{\text{int}\,P}\,\Omega^2 + B_{\text{int}\,P}}{(\Omega_0^2-\Omega^2)^2 + \dfrac{\Omega^2}{\tau_r^2}}. \tag{31}$$

With the same method, the external frequency noise is obtained

$$S_{\omega\text{ext}}(\Omega) = \frac{A_{\text{int}\,\phi}\Omega^2 + B_{\text{int}\,\phi}}{(\Omega_0^2-\Omega^2)^2 + \dfrac{\Omega^2}{\tau_r^2}} + C_{\text{int}\,\phi} + \frac{\Omega^2}{2\overline{A}_{\text{ext}}^2}. \tag{32}$$

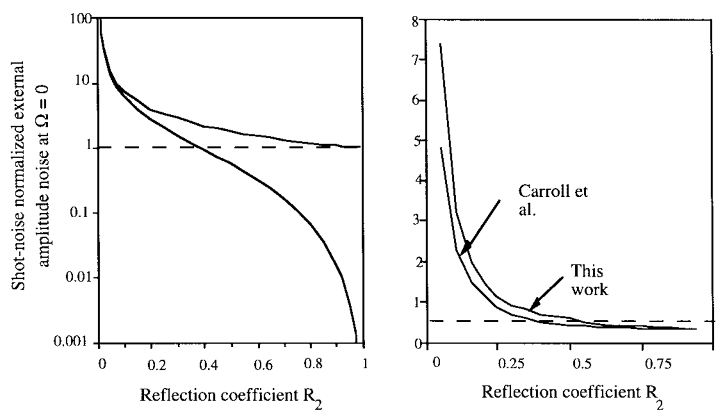
Using these equations and the definition of all the parameters, the external field amplitude noise value at low frequency and high pump rate is equal to

$$S_{A\text{ext}}(0) = \left(\frac{1-R_2}{\tau}\right) S_{A\text{int}}(0) + \frac{R_2}{4} - \frac{(1+R_2)}{4}\,\frac{1-R_2}{\tau}\,\tau_P. \tag{33}$$

For a Fabry–Perot laser with high reflection coefficients, the external amplitude noise spectra for high pump rates are then given by

$$S_{A\,\text{ext}}(\Omega) = \begin{cases} \dfrac{1}{4} & \text{for a poissonian pump} \\ \dfrac{1}{4}\left(1 - \dfrac{v_g\alpha_{\text{ext}}}{v_g\alpha_{\text{int}} + v_g\alpha_{\text{int}}}\right) & \text{for a quiet pump.} \end{cases} \tag{34}$$

This formula is exactly the same as already obtained with quantum mechanically based models [4], [16], [17]. In the case of a Poissonian pump, the external amplitude noise at low frequencies tends to the shot-noise level and with a quiet pumping, the sub-shot-noise level can be reached. For a laser without internal loss, complete compression is possible, as seen in (34) or in Fig. 4.




(a) (b)

Fig. 5. External noise at $\Omega = 0$ normalized by the shot noise for a Fabry–Perot laser without internal loss emitting a high power as (a) a function of the power reflection coefficient $R_2$ with $R_1$ close to 1 and for (b) a symmetric laser with $R_2 = R_1$ with a quiet pumping source.

Let us now consider more in detail the influence of the laser facet reflectivities for a laser without internal loss. As shown in Fig. 5(a), where $R_1$ is taken close to 1, the closer $R_2$ is to 1, the more squeezing is possible. Fig. 5(b) presents our results for the case of a symmetric Fabry–Perot laser as compared to the ones published by Carroll *et al.* [6]. As can be expected, only 50% of squeezing is reachable even if (33) and the one for [6] differ in their form. A very good agreement is found but a discrepancy appears for low reflection coefficients.

Fig. 6 presents external frequency noise spectra of a high-reflection-coated Fabry–Perot laser. Internal and external frequency noise spectra are close at low frequency but differ

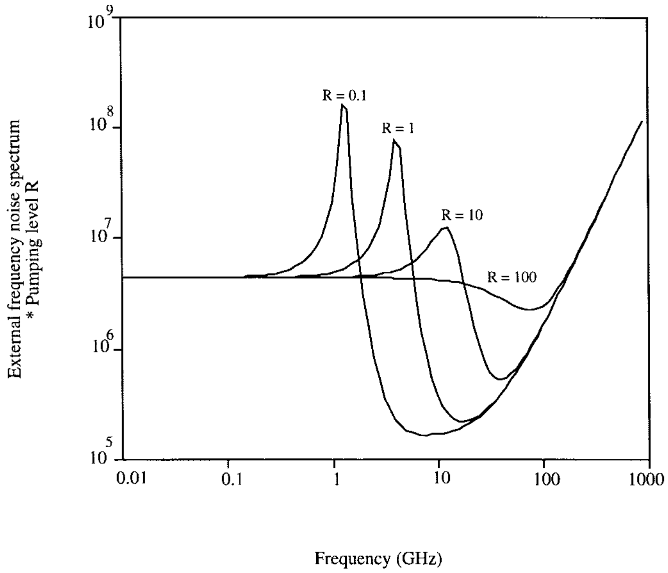



Fig. 6. Quietly pumped high-reflection-coated Fabry–Perot laser external frequency noise multiplied by $R$ for $R = 0.1$, 1, 10, and 100.

strongly at high frequency, where the major contribution comes from the incoming vacuum fluctuations. It has also to be noticed that the linewidth of the internal and external field is the same, i.e., the frequency noise at zero frequency. These results agree with external phase spectra already calculated for a quiet pumped Fabry–Perot laser [4].

## IV. Fabry–Perot Laser Noise and SHB

SHB or the inhomogeneous distribution of the field inside the cavity plays a key role for complex laser structures such as DFB and DBR and influences their static and dynamic performances [21], [22].

It is usually considered that SHB does not play a key role for Fabry–Perot laser structures. But in fact, this is only true if the facet reflectivities are equal and not too small. For an asymmetric or low-reflection-coated laser, this is no longer true.

In our model, SHB influences in our model three noise parameters: 1) the spontaneous emission via the longitudinal Petermann factor [9], [18]; 2) the incoming vacuum fluctuations by the local Petermann factor as introduced previously; and 3) by modifying the correlation between the transmitted internal field fluctuations and the reflected vacuum fluctuations at the laser facet.

According to our simulations, as well as for DFB lasers [23], SHB limits the squeezing performances of such a simple structure which a Fabry–Perot laser is. Fig. 7 shows for a Fabry–Perot laser with 30% power reflectivities on each facet the shot-noise normalized external amplitude noise at low frequency as a function of the pump level $R$ with and without SHB. A limitation on the squeezing performances at high pump level can be directly seen. These results also point out that the maximum achievable squeezing high above threshold is no more given only by the efficiency of the laser [24] but is modified by effects such as SHB. As already shown

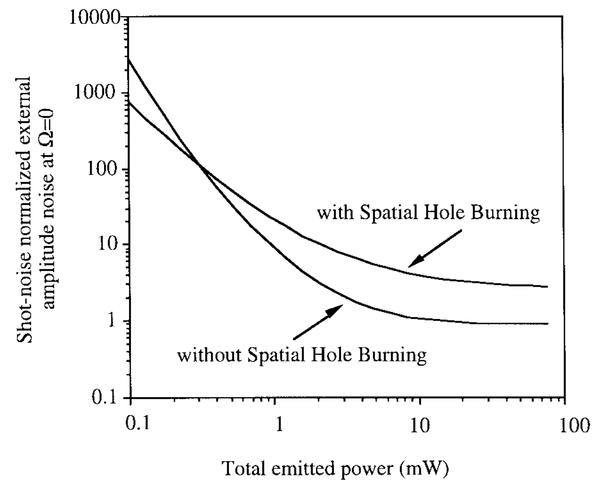



Fig. 7. Shot-noise normalized external amplitude noise at $\Omega = 0$ as a function of the pump level $R$ with and without SHB for a symmetric Fabry–Perot laser with 30% power reflectivity.

[24], gain suppression plays a similar role and introduces further limitations in the amplitude squeezing performances of a Fabry–Perot laser.

The results of Fig. 5(b) should be modified by adding an additive noise term which increases as the reflection coefficient $R_2$ gets close to zero. In this case where the laser has no internal loss, the conclusions from Fig. 5(b) are the same with and without SHB. It is no more the case when internal loss is considered. Previous calculations [4] as well as ours show in a simple description that when SHB is not considered, a low facet reflection coefficient gives the best squeezing performances. But, if SHB is considered, a low reflection coefficient will introduce a very inhomogeneous distribution of the field inside the cavity and consequently the noise emitted by the laser increases. There is consequently an optimum reflection coefficient with the greatest amount of squeezing achievable, which should be considered in optimizing the laser structure.

## V. Conclusion

A semiclassical model of quietly pumped semiconductor laser, based on a Green's function method, has been presented. Analytical formulas for internal and external amplitude and frequency noise spectra are given, taking into account various effects such as SHB, gain suppression, and facet reflectivities. A complete agreement is found with results already published for Fabry–Perot structures, which exhibit a high potential for squeezed light generation.

Structural parameters such as the facet reflectivities and SHB will be considered to calculate precisely the squeezing performances of such structures. The maximum amount of squeezing achievable with a particular realistic Fabry–Perot laser is not only given by its efficiency but is modified by the laser structure through, for example, SHB as well as the laser properties such as gain suppression. In consequence, the structures as well as the properties of the laser considered must be taken into account to precisely estimate its squeezing performances.

In the second part of this paper [20], based on these first results and this first conclusion, more complex laser structures such as DFB and DBR lasers will be studied where SHB is even more dominant.

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