## Optical corpuscular theory of semiconductor laser intensity noise and intensity squeezed-light generation

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Fluctuations of the stored energy and the emitted power of a semiconductor laser are derived by classical corpuscular optical theory. The fundamental noise sources are the shot noise associated with a field's conversion to emitting or absorbing atoms and the mirror loss noise. The latter is taken into account in the form of partition noise forces linked to laser facet reflection. The theory permits the description of the nonclassical states of light, and its results agree with quantum theory. For quiet pumping conditions and for a high-reflection coated Fabry–Perot laser, 50% of internal photon noise suppression is obtained, whereas nonfluctuating optical output is possible for negligible internal loss. The influence of gain suppression on amplitude squeezing is discussed. The effect of attenuation on the propagation of laser fluctuations is studied by use of the concept of optical partition noise. This leads to the invariance of a suitably defined relative intensity noise, which becomes negative for sub-Poissonian photon statistics. The setup for the intensity noise measurement with the balanced detection technique is analyzed by use of optical partition noise. © 1997 Optical Society of America [S0740-3224(97)02702-1]

## 1. INTRODUCTION

Many models have been developed in the past years to study the amplitude noise of a semiconductor laser and to describe the amplitude noise squeezing under quiet pumping conditions. Until now the generation of these nonclassical states of light was examined by use of either quantum or semiclassical descriptions. In both cases it is necessary to introduce noise forces linked to the output coupling of the laser. Most semiclassical models use the concept of vacuum fluctuations.<sup>1,2</sup> Other quantummechanical models use the Glauber function to eliminate the need for noise sources at the facets.<sup>3</sup> A recent semiclassical model introduced an additive Langevin noise force linked to vacuum fluctuations and to nonresonant loss.4 The semiclassical theory of semiconductor laser noise considers the spontaneous-emission fluctuations as the major noise source. A description of semiconductor laser noise without quantification of the optical field was recently proposed by Arnaud.<sup>5</sup> He points out that the fundamental noise source is the shot noise associated with the field's conversion to absorbing atoms. The only sources of fluctuations are the internal and the external

absorption. However, such a theory requires the existence of a feedback wave, similar to vacuum fluctuations, for transmission into the cavity of the fluctuations associated with the nonlocal, and sometimes far away, absorbing atoms.

We propose a new approach that uses a classical corpuscular description for the laser field and in which the intrinsic field fluctuations are described as the shot noise associated with photon production and absorption. The facet reflection is considered no longer as a deterministic but as a random process, by introduction of a partition noise. This analysis is developed in Section 1. The effect of linear-optical attenuation on laser beam fluctuations is treated in Section 2. It leads to the invariance of the relative intensity noise as originally introduced by Schimpe.<sup>6</sup> In Section 3 the internal and external fluctuations of a Fabry-Perot laser are derived by stochastic rate equations. It is first demonstrated that this simple self-consistent and classical model easily gives the expected results for amplitude squeezing. In Section 4 the influence of nonlinear gain on amplitude noise squeezing is examined. Finally, the optimum conditions for measurement of intensity noise with balanced dual detection are pointed out.

# 2. SEMICONDUCTOR LASER NOISE SOURCES

In classical light theory, the active medium but not the electromagnetic field is quantified. The intrinsic fluctuations of the electromagnetic field are then expressed as an additive optical noise, called spontaneous-emission noise. The field is also described as a classical optical phasor in which the phase and the amplitude are disturbed by addition of random spontaneous events.<sup>7,8</sup> Spontaneousemission fluctuations, which are usually considered the major noise mechanism, are no longer considered, as the fundamental field fluctuations are already expressed by the uncorrelated absorption and emission events, whether the field is quantified or not.<sup>5</sup> In such a theory, under quiet pump conditions the pump current injection is also no longer a noisy process.

It is generally admitted that the emitted photon statistic of an ideal laser operating well above threshold is Poissonian.<sup>9</sup> An intensity noise reduction below the shot-noise limit can be obtained by use of a constantcurrent pump-noise-suppressed semiconductor laser.<sup>10-12</sup> When a *pn* junction is driven by a constant-current source with a source resistance *R* much larger than the diode's differential resistance, the pump fluctuation force is identified as the thermal noise generated in the source resistance, with the spectral density given by the Johnson-Nyquist theorem

$$S_I = 4(kT/R),\tag{1}$$

where k is the Boltzmann constant and T is the temperature. High above the threshold pumping condition the semiconductor laser diode's differential resistance is less than 10  $\Omega$ , and at a frequency below the cavity bandwidth a device that directly and efficiently converts the electron into photons should be limited only by the electron noise.<sup>13</sup> Consequently, when the electron flow itself is regular, it produce a photon flow that is more regular than a Poissonnian flow.

Vacuum fluctuations are considered an additional noise field that is partly reflected and partly transmitted inside the cavity. The mirror reflectivities are considered determinants. Let us consider a laser beam reaching a mirror as shown in Fig. 1(a). f(t) represents the fluctuations of the incident beam and r and t are the field's reflection and transmission coefficients, respectively. The reflected fluctuations rf(t) add to the transmitted field, whereas an anticorrelated noise tf(t) acts as a feedback on the field inside the cavity. Because the cavity is continually driven by fluctuations anticorrelated to the emitted one, the introduced feedback is able to provide low-frequency compensation of emitted fluctuations. In the highfrequency range, because of the gain saturation time delay, the emitted fluctuations become more important because the laser reaction no longer permits significant compensation.

In our model a laser beam is considered a flux of classical particles without interaction, and the mirror loss noise is taken into account in the form of classical partition noise. To introduce this partition noise, let us consider Fig. 1(b), where  $R = r^2$  is the power reflection coefficient. When an average number  $\bar{n}$  of photons hit the mirror,  $R\bar{n}$  photons are reflected and  $\bar{n}(1-R)$  are transmitted. Owing to the corpuscular nature of light, the reflection (or transmission) is a random process and the observed photon number during a given time  $\tau$  is a random variable. For classical particle-number conservation the fluctuation of the transmitted photon number must be opposite the fluctuation of the reflected photon number. The mean-squared fluctuation induced by this stochastic process is then  $\overline{\Delta n^2} = \overline{n}R(1-R)$ . Consequently, the transmitted and reflected photon rates  $\rho_1(t)$  and  $\rho_2(t)$ , defined by  $\rho(t) = n(t)/\tau$ , obey the following relations:

$$\rho_1(t) + \rho_2(t) = \rho(t), \quad \overline{\rho_1} + \overline{\rho_2} = \overline{\rho},$$
$$\overline{\Delta\rho_1}^2 = \overline{\Delta\rho_2}^2 = \overline{\rho}R(1-R). \tag{2}$$

This partition noise induced by the beam splitting can be represented by a partition Langevin noise force as

$$\langle f_{\rho_i}(t) \rangle = 0, \qquad \langle f_{\rho_i}(t) f_{\rho_i}^*(t-\tau) \rangle = \overline{\Delta \rho_i}^2 \delta(\tau),$$

$$i \in \{1, 2\},$$
(3)

where  $\langle \rangle$  represents the ensemble average. In this representation, as for vacuum fluctuation, the anticorrelation of the reflected and transmitted powers provides a feedback signal that can correct the fluctuations that result from the partition noise. Such a noise force is to be used for the laser facet reflection and for the noise associated with the nonresonant internal loss, which can also be described by beam splitting.



Fig. 1. Vacuum fluctuations and partition noise.

#### 3. LINEAR-OPTICAL ATTENUATION

To measure the intensity noise of a semiconductor laser, one needs to consider carefully partly collected light and attenuation problems. These phenomena introduce a photon random killing and therefore corrupt the photon statistics. Because attenuation is a stochastic process, the transmission probability for a slice of thickness dz is defined as  $(1 - \alpha dz)$  and the absorption probability as  $\alpha dz$ , where  $\alpha$  is the linear absorption coefficient of the medium (Fig. 2). Thus during a given observation time  $\tau$  the average number of transmitted photons  $\overline{n(z + dz)}$  is  $(1 - \alpha dz)\overline{n(z)}$ . In such conditions the mean square induced by this partition process is expressed as  $\alpha dz(1 - \alpha dz)\overline{n(z)}$ . The mean square of the transmitted photon number is

$$\overline{\Delta n^2(z+\mathrm{d}z)} = (1-\alpha\mathrm{d}z)^2 \overline{\Delta n^2(z)} + \alpha\mathrm{d}z(1-\alpha\mathrm{d}z)\overline{n(z)}.$$
(4)

The first term describes the attenuation of the incoming light fluctuations, and the second term arises from the partition noise introduced by the stochastic nature of the attenuation. Let us now consider the photon rate  $\rho(z, t) = n(z, t)/\tau$ . The double-sided noise spectrum associated with  $\rho(z + dz, t)$  is written as

$$S_{\rho}(\omega, z + dz) = (1 - \alpha dz)^{2} S_{\rho}(\omega, z)$$
  
+  $\alpha dz (1 - \alpha dz) \overline{\rho(z)}.$  (5)

The second term on the right-hand side of Eq. (4) is additional white noise, and Eq. (5) shows that propagation of fluctuation inside a linear attenuator induces noise whitening.

The Fano factor F(z), usually used to characterize photon beam fluctuation, is defined as

$$\overline{\Delta n^2(z)} = F(z)\overline{n(z)},\tag{6}$$

where F(z) is equal to, more than, and less than 1 for Poissonian, super-Poissonian, and sub-Poissonian fluctuations, respectively. Substituting Eq. (6) into Eq. (5), we find that after a standard integration

$$F(z) = F(0)\exp(-\alpha z) + 1 - \exp(-\alpha z).$$
(7)

During propagation, F(z) increases for sub-Poissonian fluctuation and decreases for super-Poissonian fluctuation but in both cases converges to 1, showing that attenuation turns any initial fluctuation into pure shot noise.



Fig. 2. Noise propagation in an absorbing medium.

From the above analysis it is easy to see that there is a space-invariant constant ratio in the fluctuation propagation:

$$\operatorname{RIN}(\omega) = 2 \frac{S_{\rho}(\omega, z) - \rho(z)}{\overline{\rho(z)^2}}$$
$$= 2 \frac{S_{\rho}(\omega, z + dz) - \overline{\rho(z + dz)}}{\overline{\rho(z + dz)^2}}.$$
(8)

This ratio remains unchanged after linear attenuation, even if it is negative, and characterizes intrinsic optical source properties. It represents the so-called relative intensity noise (RIN), defined as the difference between the photonic spectral density and the average photon rate that corresponds to the shot-noise level.<sup>14</sup> An additional factor of 2 has been included because we changed from a double-sided to the more common single-sided noise spectrum. The RIN describes the normalized relative fluctuations of the optical source, and it becomes negative for sub-Poissonian fluctuation. From a corpuscular point of view in which the optical source is characterized by its statistical photon distribution, RIN is also a measure of temporal correlations between photons.

## 4. STOCHASTIC RATE EQUATION FOR A SEMICONDUCTOR LASER

For sake of simplicity, the laser studied here is a Fabry– Perot laser with a perfect mirror at one end and a mirror with a reflectivity close to 1 at the other end. The laser is assumed to be homogeneous enough that we can consider single-mode operation and describe it only by the internal photon number P and the minority carrier number N inside the active medium.

Under these assumptions, the evolution equations for N(t) and P(t) are linked by the standard rate equations

$$\frac{\partial P}{\partial t} = \left(G - \frac{1}{\tau_P}\right)P + F_P(t), \qquad (9a)$$

$$\frac{\partial N}{\partial t} = \frac{I}{e} - GP - \frac{N}{\tau_E} + F_N(t),$$
 (9b)

with

$$G = A(N - N_0). \tag{9c}$$

*G* is the net gain per unit of time, *A* is the differential gain,  $\tau_P$  is the cold-cavity photon lifetime,  $N_0$  is the transparency carrier number, *I* is the pump current,  $\tau_E$  is the spontaneous carrier lifetime, and *e* is the electron charge.  $F_N(t)$  and  $F_P(t)$  are the Langevin noise forces that affect, respectively, the photon-number and the carrier-number dynamics.

#### A. Optical Losses

The cold-cavity photon lifetime results from mirror loss and nonresonant absorption, as follows:

$$\frac{1}{\tau_P} = \frac{1}{\tau_M} + \frac{1}{\tau_D},\tag{10}$$

where

$$rac{1}{ au_M} = -rac{\ln R}{ au_{ ext{RT}}} pprox rac{1-R}{ au_{ ext{RT}}}$$

expresses mirror loss,

$$rac{1}{ au_D} = rac{c}{n_g} lpha_D$$

expresses nonresonant absorption, and

$$\tau_{\rm RT} = \frac{2 \, L n_g}{c}$$

is the round-trip time of light inside the cavity. R is the power reflection coefficient, L is the cavity length,  $\alpha_D$  is the nonresonant attenuation per unit of length, and  $n_g$  is the effective refractive group index.

The steady-state solution of Eq. (9a) gives the relation

$$\bar{G} = 1/\tau_P, \qquad (11)$$

where G is the result of an excess of stimulated emission, producing S photons per unit of time and per stimulating photon, on the stimulated absorption of  $A_S$  photons under the same conditions. G can also be written as

$$G = S - A_S. \tag{12}$$

The inversion population factor  $n_{\rm SP}$ , sometimes referred to as the spontaneous-emission factor, is defined as

$$n_{\rm SP} = \frac{S}{(\bar{S} - \bar{A}_S)} = \frac{S}{\bar{G}} = \bar{S} \tau_P.$$
(13)

It follows easily that

$$\bar{A} = (n_{\rm SP} - 1)\bar{G}.$$
 (14)

### **B.** Photon Rate Evolution Equation

When a photon rate  $P(t)/\tau_{\rm RT}$  reaches the mirror, the transmitted photon rate is  $(1 - R)P(t)/\tau_{\rm RT} + f_M(t)$  and the reflected one is  $RP(t)/\tau_{\rm RT} - f_M(t)$ , where  $f_M(t)$  denotes the Langevin partition noise force linked to the facet reflection. It is necessary also to introduce a new evolution equation for  $\rho(t)$ , the photon rate emitted by the laser facet:

$$\rho(t) = \frac{(1-R)P(t)}{\tau_{\rm RT}} + f_M(t) = \frac{P(t)}{\tau_M} + f_M(t),$$
(15)

where P(t) depends on  $f_M(t)$ . Equation (15) defines the feedback mechanism that can lead to external photon noise smaller than direct reproduction of internal photon noise.

The Langevin noise forces  $F_N(t)$  and  $F_P(t)$  can be expanded in the following forms:

$$F_P(t) = f_S(t) - f_D(t) - f_M(t) - f_A(t), \qquad (16)$$

$$F_N(t) = f_I(t) + f_A(t) - f_S(t) - f_E(t), \qquad (17)$$

where  $f_S(t)$ ,  $f_A(t)$ ,  $f_D(t)$ ,  $f_I(t)$ , and  $f_E(t)$  are uncorrelated shot-noise Langevin noise forces linked to stimulated emission, resonant absorption, nonresonant absorption, injected-current noise, and spontaneous recombinations, respectively.

These Langevin forces are characterized by their correlation functions, defined by

$$\langle f_i(t)f_j^*(t-\tau)\rangle = 2D_{ij}\delta(\tau),$$
  
(ij)  $\in (M, S, P, A, I, N),$  (18)

where  $D_{ij}$  are the diffusion coefficients. The different processes are uncorrelated; we have

$$D_{ii} = 0, \quad i \neq j. \tag{19}$$

If we consider a number  $\rho$  of independent events per unit of time, the double-sided fluctuation density  $S_{\rho\rho}(\omega)$  is equal to its mean  $\bar{\rho}$ , so the autocorrelation function is given by  $R_{\rho\rho}(\tau) = \bar{\rho} \delta(\tau)$ . From this the diffusion coefficient can be easily obtained for a normal pump:

$$2D_{MM} = \bar{P}/t_M,$$

$$2D_{DD} = \bar{P}/t_D,$$

$$2D_{SS} = \bar{P}\bar{S} = n_{sp}\bar{P}/t_P,$$

$$2D_{AA} = \bar{P}\overline{A_S} = (n_{sp} - 1)\bar{P}/t_P,$$

$$2D_{II} = \bar{I}/e,$$

$$2D_{EE} = \bar{N}/t_E,$$

$$2D_{PP} = 2n_{SP}\bar{P}/\tau_P,$$

$$2D_{NN} = 2n_{SP}\bar{P}/\tau_P + 2\bar{N}/\tau_E,$$

$$2D_{PN} = -(2n_{SP} - 1)\bar{P}/\tau_P,$$
(20)

where X is the time-averaged value of the parameter X.

For quiet pumping the noise linked to the pump disappears, and the diffusion coefficients for P and N become

$$2D_{PP} = 2n_{SP}P/\tau_P,$$
  

$$2D_{NN} = (2n_{SP} - 1)\bar{P}/\tau_P + \bar{N}/\tau_E,$$
  

$$2D_{PN} = -(2n_{SP} - 1)\bar{P}/\tau_P.$$
(21)

#### C. Internal Photon-Noise Spectra

After linearization of Eq. (8) around the steady-state value and a Fourier-transform analysis, the fluctuations of the internal photon number and carrier number are obtained:

$$\begin{bmatrix} \delta \hat{P}(\omega) \\ \delta \hat{N}(\omega) \end{bmatrix} = \Delta^{-1}(\omega) \begin{bmatrix} j\omega + 2/\tau_R & \omega_{R0}^2 \tau_P \\ -(1/\tau_P) & j\omega \end{bmatrix} \begin{bmatrix} \hat{F}_P(\omega) \\ \hat{F}_N(\omega) \end{bmatrix},$$
(22)

where

$$\begin{split} \omega_{R0}^{2} &= A\bar{P}/\tau_{P}, \\ 2/\tau_{R} &= A\bar{P} + 1/\tau_{E}, \\ \Delta(\omega) &= \omega_{R0}^{2} - \omega^{2} + 2j\omega/\tau_{R}, \end{split}$$

where  $\hat{X}(\omega)$  is the Fourier transformation of X(t) and  $\omega_{R0}$  and  $\tau_R$  are the angular relaxation frequency and the associated time constant, respectively.

The internal photon noise spectrum is given by

$$S_{PP}(\omega) = 2[(\omega^{2} + 4/\tau_{R}^{2})D_{PP}(\omega) + \omega_{R0}^{4}\tau_{P}^{2}D_{NN}(\omega) + 4(\omega_{R0}^{2}\tau_{P}/\tau_{R})D_{PN}(\omega)]|\Delta(\omega)|^{-2}.$$
 (23)

For high pumping  $D_{EE}$  can be neglected, and  $2/\tau_R \approx \omega_{R0}^2 \tau_P = A\bar{P}$ .  $S_{PP}(\omega)$  can also be approximated by

$$S_{PP}(\omega) = 2 \frac{\bar{P}}{\tau_P} \frac{(A\bar{P})^2 + \omega^2 n_{SP}}{|\Delta(\omega)|^2}.$$
 (24)

In the low-frequency range  $S_{PP}(0) = 2\tau_P \bar{P}$ . This result shows that under such conditions the multiplicative factor  $n_{SP}$  disappears with the correlation action of the Langevin forces  $F_N(t)$  and  $F_P(t)$ .  $S_{PP}(0)$  appears to be the result of the shot noise associated with the stimulated mechanisms and with the loss mechanisms; it can be written as

$$\frac{S_{PP}(0)}{\tau_P{}^2} = \bar{G}\bar{P} + \frac{\bar{P}}{\tau_P} = 2D_{PP} + 2D_{NN} + 4D_{PN}.$$
(25)

For quiet pumping under the same conditions the laser's internal noise decreases, as expected, with the emitted power, and a maximum reduction of 50% is obtained<sup>1-4</sup>:

$$[S_{PP}(0)]_{\text{quiet pump}} = 1/2[S_{PP}(0)]_{\text{normal pump}}.$$
 (26)

This result is also shown in Fig. 3, which shows the internal intensity noise spectrum versus frequency for different pumping currents when the laser is quietly pumped.

#### D. External Photon Noise

In the frequency domain the evolution equation of the emitted photon rate  $\rho(t)$  becomes

$$\delta \hat{\rho}(\omega) = \frac{\delta P(\omega)}{\tau_M} + \hat{f}_M(\omega). \tag{27}$$

The double-sided noise spectrum is as follows:

$$S_{\rho\rho}(\omega) = \frac{S_{PP}(\omega)}{\tau_M^2} + [1 - 4\omega_{R0}^2/\tau_M \tau_R |\Delta|^2] 2D_{MM}.$$
(28)

For a normal pump and for high emitted power at low frequency, the emitted beam is affected only by shot noise:

$$S_{\rho\rho}(0) = P/\tau_M = \bar{\rho}.$$
 (29)



Fig. 3. Shot-noise-normalized internal amplitude noise of a quietly pumped high-reflection-coated Fabry–Perot laser for different pump levels.  $r = I/I_{th} - 1$ , R = 0.7.



Fig. 4. Normalized external intensity noise of a normally pumped high-reflection-coated Fabry-Perot laser for various values of the pumping rate.

Figure 4 shows the external intensity noise normalized by the shot-noise level versus frequency for different pumping currents when the laser is normally pumped. It shows that the noise of the emitted beam at low frequency is at the level of the shot noise for high pumping current.

When the laser is quietly pumped under the same conditions the output fluctuation spectral density is given by

$$S_{\rho\rho}(0) = \bar{\rho}(1 - \tau_P / \tau_M) = \bar{\rho} \tau_P / \tau_D.$$
(30)

Strong noise suppression can be obtained for low internal loss.

Figure 5 shows the external photon noise spectrum normalized by the shot-noise level, versus frequency, for various values of internal loss, when the laser is quietly pumped. As expected, at low frequency the noise suppression increases with the decrease in internal loss. As a consequence, a laser without internal loss ( $\tau_P \approx \tau_M$ ) can provide complete external squeezing.<sup>1</sup> For a homogeneous laser and for given loss the squeezing rate increases when the mirror loss increases, that is to say, when the reflectivity decreases. Figure 6 shows that the obtainable squeezing level depends strongly on R. A low-reflectivity coating achieves the best external photon noise suppression<sup>15</sup> for a given internal loss level.

## 5. INFLUENCE OF GAIN SUPPRESSION ON AMPLITUDE NOISE SQUEEZING

Gain suppression has several origins,<sup>16</sup> primarily spectral hole burning,<sup>7</sup> carrier heating,<sup>18</sup> and spatial hole burning.<sup>19</sup> This phenomenon modifies the static properties with the saturation of the emitted and the dynamic powers.<sup>20,21</sup> This effect can be represented phenomenologically by the dependence of the gain on the photon density inside the cavity:

$$G = A(N - N_0)(1 - \varepsilon \overline{P}), \qquad (31)$$

where is  $\varepsilon$  is the gain suppression coefficient. The factor  $\varepsilon \overline{P}$  in a bulk laser has a maximum value of 0.2–0.3 and is strongly enhanced in quantum-well laser structures.

Taking gain suppression into account leads to the following modifications of the fluctuations of the internal photon number:

$$\delta \hat{P}(\omega) = [(j\omega + 2/\tau_{R\varepsilon})\hat{F}_P(\omega) + \{\tau_P / [1 + K(\bar{P})]\}\omega_{R0}^{-2}\hat{F}_N(\omega)]\Delta_{\varepsilon}^{-1}(\omega),$$
(32)

with

$$K(\bar{P}) = \varepsilon \bar{P}/(1 - \varepsilon \bar{P}),$$

$$\frac{2}{\tau_{R\varepsilon}} = \frac{2}{\tau_R} - A\varepsilon \bar{P}^2,$$

$$\Delta_{\varepsilon}(\omega) = \omega_{R0}^2 + 2K(\bar{P})/(\tau_{R\varepsilon}\tau_P) - \omega^2$$

$$- j\omega[2/\tau_{R\varepsilon} + K(\bar{P})/\tau_P].$$
(33)

As expected, the relaxation frequency without damping is reduced and saturates at high emitted power. For high emitted power the internal amplitude noise at low frequency is given by



Fig. 5. Normalized external intensity noise for high-power-level, quiet pumping conditions and various values of internal loss. R = 0.7.



Fig. 6. Normalized external intensity noise at low frequency, for quiet pumping conditions, as a function of the mirror reflectivity and for various values of the internal loss.



Fig. 7. Shot-noise-normalized external noise at low frequency as a function of  $\varepsilon \overline{P}$  for high emitted power. R = 0.7,  $\alpha_D = 4 \text{ cm}^{-1}$ .

$$[S_{PP_{\varepsilon}}(0)]_{\text{quiet pump}} = \frac{1}{2}[S_{PP_{\varepsilon}}(0)]_{\text{normal pump}}$$
$$= \bar{P}\tau_{P}\frac{(1-\varepsilon\bar{P})^{2}}{(1+\varepsilon\bar{P})^{2}}.$$
(34)

As for the linear gain approximation, the internal noise is reduced by a factor of 2 at high power when the laser is quietly pumped, but whatever the conditions of pumping are, the internal photon noise is squeezed and strongly reduced.<sup>4</sup> For external photon noise under normal pumping conditions the situation is quite different. The external noise is given for a high emitted power and at low frequency by

$$S_{\rho\rho_{\varepsilon}}(0) \approx \overline{
ho} \left( 1 - 4\varepsilon \overline{P} \; \frac{\tau_P}{\tau_M} \right) \quad \text{for small } \varepsilon \overline{P}.$$
 (35)

The external noise is squeezed at high emitted power even if the laser is normally pumped. This result is in complete agreement with previously published results.<sup>4,22</sup> Under quiet pumping conditions the influence of gain suppression is the opposite. In this case the shot-noisenormalized external noise at high emitted power and low frequency is given by

$$\frac{S_{\rho\rho_{\varepsilon}}(0)}{\overline{\rho}} \approx \frac{S_{\rho\rho}(0)}{\overline{\rho}} + 2\frac{\tau_{P}}{\tau_{M}} (\varepsilon \overline{P})^{2} \quad \text{for small } \varepsilon \overline{P}.$$
(36)

The laser's external noise increases in the presence of gain suppression.<sup>4,14,22</sup> This result is explained by the decrease in the correlation between the internal fluctuations and the partition-noise forces at the laser facets and by the reduction in internal noise. Figure 7 shows the shot-noise-normalized external noise as a function of  $\varepsilon \bar{P}$  when the laser is pumped high above threshold. As the gain suppression coefficient increases, the achievable amount of squeezing is reduced, and a strong limitation appears. Because of the influence of gain suppression, quantum-well lasers that exhibit higher gain suppression coefficients may suffer a reduction of their squeezing performance.

## 6. MEASUREMENT OF INTENSITY NOISE

The RIN cannot be obtained in a single manipulation, and dual balanced detection is certainly a more convenient tool for its measurement.<sup>23,24</sup> Figure 8 shows a classic balanced dual-detector setup. In the following analysis, multiple photon states are not considered. The laser output photon flow is divided into the two arms by a linear and lossless beam splitter. Photons are converted into photoelectrons with two identical and ideal photodiodes. Under this condition the photoelectron statistics on a detector directly reproduce the photon statistics. Both optical and electrical path lengths are assumed to be matched. On both arms, electrical fluctuation and dc components are split with a T bias. A hybrid junction subtracts or adds the fluctuations. Let T be the beam splitter's power transmission coefficient. Then the mean reflected and transmitted photon rates are written as

$$\bar{\rho}_1 = (1 - T)\bar{\rho}, \qquad \bar{\rho}_2 = T\bar{\rho}. \tag{37}$$

The noise in both arms is determined by the attenuated incident fluctuation and the partition noise introduced by beam partition. The spectral densities associated with  $\rho_1(t)$  and  $\rho_2(t)$  are linked to the spectral density associated with  $\rho(t)$  by

$$S_{\rho_1}(\omega) = \underbrace{(1-T)^2 S_{\rho}(\omega)}_{\text{reflection of initial fluctuation}} + \underbrace{T(1-T)\overline{\rho}}_{\text{added partition noise}},$$
(38)

$$S_{\rho_2}(\omega) = \underbrace{T^2 S_{\rho}(\omega)}_{+} + \underbrace{T(1-T)\overline{\rho}}_{-}.$$

transmission of initial fluctuation added partition noise

The spectral densities associated with  $\rho_1(t) - \rho_2(t)$  and  $\rho_1(t) + \rho_2(t)$  are the following:

$$S_{\rho_1+\rho_2}(\omega) = S_{\rho}(\omega), \tag{40}$$

$$S_{\rho_1-\rho_2}(\omega) = (2T-1)^2 S_{\rho}(\omega) + 4T(1-T)\bar{\rho}. \quad (41)$$

 $S_{\rho_1+\rho_2}(\omega)$  is always equal to the initial fluctuations, whereas for a 50-50 beam splitter  $S_{\rho_1-\rho_2}(\omega)$  gives the shot-noise level  $\bar{\rho}$  that corresponds to the laser output. This result is independent of the incident fluctuation statistics. Indeed,  $S_{\rho_1-\rho_2}(\omega)$  has an extremum for T= 1/2, and, as shown in Fig. 9, this extremum is a maxi-



Fig. 8. Balanced detection scheme.



Fig. 9. Difference spectrum  $S_{p_1-p_2}(\omega)$  as a function of the transmission coefficient T for different  $S_p(\omega)$ .

mum when  $S_{\rho}(\omega)$  is less than  $\bar{\rho}$  (sub-Poissonian fluctuation) and is a minimum when  $S_{\rho}(\omega)$  is greater than  $\bar{\rho}$ (super-Poissonian fluctuation). For sub-Poissonnian fluctuation a slight deviation of the beam splitter's coefficient from 50% leads to a shot-noise level underestimate and for super-Poissonian fluctuation leads to a shot-noise level overestimate. In both cases this error increases when the laser noise deviates from the shot-noise level. The characterization of squeezed light consequently requires a high splitting accuracy, which increases with the degree of squeezing. Under these optimum operating conditions the experimental setup allows us to measure both laser noise and the laser shot-noise level by using the same setup and without any adjustment to the detectors or the laser. Then the RIN is directly accessible, with the relation

$$\operatorname{RIN}(\omega) = 2 \frac{S_{\rho_1 + \rho_2}(\omega) - S_{\rho_1 - \rho_2}(\omega)}{(\overline{\rho_1} + \overline{\rho_2})^2}.$$
 (42)

## 7. CONCLUSION

(39)

Intrinsic field fluctuations are described as the shot noise or the partition noise associated with the production or absorption of photons. Photons are considered classical particles, and a self-consistent model enables us to study the intensity noise of semiconductor lasers and the generation of nonclassical states of light under normal and quiet pump conditions. The results, which are easily obtained, are in complete agreement with the previously published quantum-mechanical and semiclassical models. The influence of gain suppression on the laser's external noise for normal and quiet pumping has been discussed. Using optical partition noise, we have demonstrated that the conveniently defined RIN is independent of linear attenuation and is negative for sub-Poissonian fluctuations. Optimal conditions for RIN measurement with a dual balanced detector have also been pointed out.

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