Linewidth Rebroadening Due to Nonlinear Gain and Index Induced by Carrier Heating in Strained Quantum-Well Lasers

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***Abstract*—The carrier heating has been recently recognized as one of the main origins of nonlinear gain, in particular in strained quantum-well lasers. The asymmetry of this effect introduces a nonlinear refractive index. The joint effects of nonlinear gain and nonlinear refractive index that are both due to carrier heating together with the spatial-hole-burning give rise to an increase in the carrier density and in the linewidth enhancement factor. These effects can explain the linewidth rebroadening at high power in phase-shifted single-mode DFB lasers.**

I. Introduction

In the early calculations of the phase noise in single-mode semiconductor lasers, a linear increase of the linewidth with the inverse of the emitted optical power has been expected [1]. In the experimental point of view, a linewidth floor with a rebroadening for increasing power is usually observed in single-mode semiconductor lasers [2]. It has been previously demonstrated that the nonlinear gain and the spatial nonuniformities can affect moderately the linewidth through the increase of the effective linewidth enhancement factor [4], [5]. An attempt has been made by Agrawal *et al.* [6] to include the nonlinear refractive index introduced by spectral-hole-burning (SHB). As explained by Vahala *et al.*, another mechanism, the occupation fluctuation noise, gives a power independant contribution to the linewidth, which determines a linewidth floor [7]. But this calculated floor is very low for intraband relaxation time of about 0.1 ps, which is the case at room temperature in quaternary materials. To the best of our knowledge, the two main explanations for the linewidth rebroadening are a reduction of the side-mode suppression ratio that arises with an instability of the laser mode due to spatial hole burning [8] and the $1/f$ noise [9].

Recently, the carrier heating (CH) has been identified as one of the main sources of nonlinear gain particularly in new materials such as strained quantum wells [10], [11]. As the SHB is nearly symmetric around the lasing wavelength, its influence on the refractive index at the lasing wavelength is very small. On the opposite, the effect of CH is strongly asymetric with respect to the lasing wavelength, thus it results in a nonlinear phase-amplitude coupling factor due to CH, α_{ch}. This coefficient has been studied by Kikuchi *et al.* [12] and values for α_{ch} between 5 and 20 depending on the considered wavelength have been predicted. The purpose of the present paper is to investigate the effect of the CH on the linewidth of DFB semiconductor lasers.

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II. Theoretical Analysis

We will first focus on the case of a laser with a uniform optical intensity distribution along the cavity like a Fabry–Perot with highly reflective facets. The material gain, g, is usually expressed as

$$g = \frac{g_d(N - N_0)}{1 + \epsilon S}, \tag{1}$$

where N and N_0 are the carrier density and the carrier density at transparency, g_d is the differential gain, S the photon density and ϵ the total gain suppression factor. The analysis of Tromborg *et al.* [4] gives an expression of the effective linewidth enhancement factor in this case

$$\alpha_{\text{eff}}^{\text{fp}} = \frac{\alpha_h + q\alpha_s}{1 + q} \tag{2}$$

with

$$q = \frac{\frac{-\partial g}{\partial S}}{\tau_r \frac{\partial g}{\partial N}\frac{\partial R_{\text{st}}}{\partial S}}, \tag{3}$$

where α_h is the linear phase-amplitude coupling coefficient and α_s is the nonlinear phase-amplitude coefficient which relates the variation of index and gain with the optical power. In this expression, the CH is represented by a nonzero value of α_s. τ_r and R_{st} are the total carrier lifetime and the stimulated emission rate. The calculation results show that the nonlinear gain alone forces the effective linewidth enhancement factor to decrease with increasing output power. On the opposite, a rapid increase of this parameter is observed, due to the nonlinear refractive index, which leads to a saturation of the linewidth for very high output power. However, no linewidth rebroadening has been observed in such lasers for reasonable emission powers.

Let us now consider the case of DFB lasers. The basis of the model we used is presented in [5], and the method used for the calculation of the side-mode-suppression ratio is presented in [13]. The nonlinear gain is divided into two parts

$$\epsilon = \epsilon_{\text{shb}} + \epsilon_{\text{ch}}, \tag{4}$$

one describing the effect of SHB, ϵ_{shb} and the other for the CH, ϵ_{ch}. Considering (1), it supposes that $\epsilon_{\text{ch}}\epsilon_{\text{shb}}/(\epsilon_{\text{ch}} + \epsilon_{\text{shb}})S \ll 1$, which means that the gain suppression due to the weakest of the two considered processes is no more than a few %. We use the usual definitions for the nonlinear phase-amplitude

TABLE I
PARAMETER VALUES FOR THE LASER SIMULATION

Symbol	Parameter	Value	Unit
A	Surface recombination	0	s^{-1}
B	Radiative recombination	8×10^{-11}	$cm^3.s^{-1}$
C	Auger recombination	1×10^{-29}	$cm^6.s^{-1}$
g_d	Differential gain	8×10^{-16}	cm^2
α_0	Total losses at transparency	15	cm^{-1}
β_{sp}	Spontaneous emission factor	1×10^{-5}	
N_0	Carrier density at transparency	1×10^{18}	cm^{-3}
L	Laser length	450	μm
κ	Grating coupling coefficient	33	cm^{-1}
w	Active layer width	1.5	μm
d	Active layer thickness	0.06	μm
Γ	Confinement factor	0.08	
α_d	Differential losses	3.5×10^{-17}	cm^2
n_d	Differential index	-1.5×10^{-20}	cm^3
ϵ	Total nonlinear gain coefficient	16×10^{-17}	cm^3
ϵ_{ch}	Nonlinear gain coefficient for CH	14×10^{-17}	cm^3
ϵ_{shb}	Nonlinear gain coefficient for SHB	2×10^{-17}	cm^3
α_{ch}	Nonlinear phase-amplitude coupling for CH	10	
α_{shb}	Nonlinear phase-amplitude coupling for SHB	0	
$\lvert r_l \rvert$	Left reflectivity	0.56	
$\lvert r_r \rvert$	Right reflectivity	0.56	

coupling coefficient for the CH and the SHB respectively [12]

$$\alpha_{\mathrm{ch}} = -2k\frac{\partial n}{\partial T}\Big/\frac{\partial g}{\partial T} \quad \text{and} \quad \alpha_{\mathrm{shb}} = -2k\frac{\partial n}{\partial S}\Big/\frac{\partial g}{\partial S}, \tag{5}$$

where k is the wave vector, n the refractive index and T the carrier temperature. Neglecting the contributions of the carrier shot noise and of the cross correlation between carrier shot-noise and spontaneous emission, the effective linewidth enhancement factor is used to calculate the linewidth by using the following [5]:

$$\Delta\nu = \frac{R_{\mathrm{sp}}}{4\pi SV/\Gamma}\left(1 + \left(\alpha_{\mathrm{eff}}^{\mathrm{dfb}}\right)^2\right). \tag{6}$$

In this equation, $R_{\mathrm{sp}} = \beta_{\mathrm{sp}} BN^2$ represents the spontaneous emission rate coupled into the lasing mode, V is the volume of the active section and Γ is the mode confinement factor. B and β_{sp} are given in Table I and are kept constant during the modeling. The linewidth enhancement factor, $\alpha_{\mathrm{eff}}^{\mathrm{dfb}}$, is calculated in [5]. Its expression can be written as

$$\alpha_{\mathrm{eff}}^{\mathrm{dfb}} = \frac{\frac{\partial G_{\mathrm{nl}}}{\partial P}_i/2 + \int_L \left[W_{N_r} - \frac{\partial G_{\mathrm{nl}}}{\partial N}_i/2\right]\tau_r \frac{\partial R_{\mathrm{st}}}{\partial S}|E|^2 \frac{dz}{L}}{-\frac{\partial G_{\mathrm{nl}}}{\partial P}_r/2 + \int_L \left[W_{N_i} + \frac{\partial G_{\mathrm{nl}}}{\partial N}_r/2\right]\tau_r \frac{\partial R_{\mathrm{st}}}{\partial S}|E|^2 \frac{dz}{L}}. \tag{7}$$

The subscript $_i$ and $_r$ represent the imaginary and real part of a complex number respectively. In this equation, P is the spatially averaged photon density, E is the electrical field longitudinal distribution. The effective nonlinear gain and the W_N coefficient are

$$G_{\mathrm{nl}} = v_g \frac{\int_L E^2 \epsilon S g dz}{\int_L E^2 dz} \tag{8}$$

$$W_N = \frac{n_{\mathrm{eff}}\Gamma g_d(\jmath - \alpha_h)E^2 L}{2\int_L n_{\mathrm{eff}} E^2 dz}, \tag{9}$$

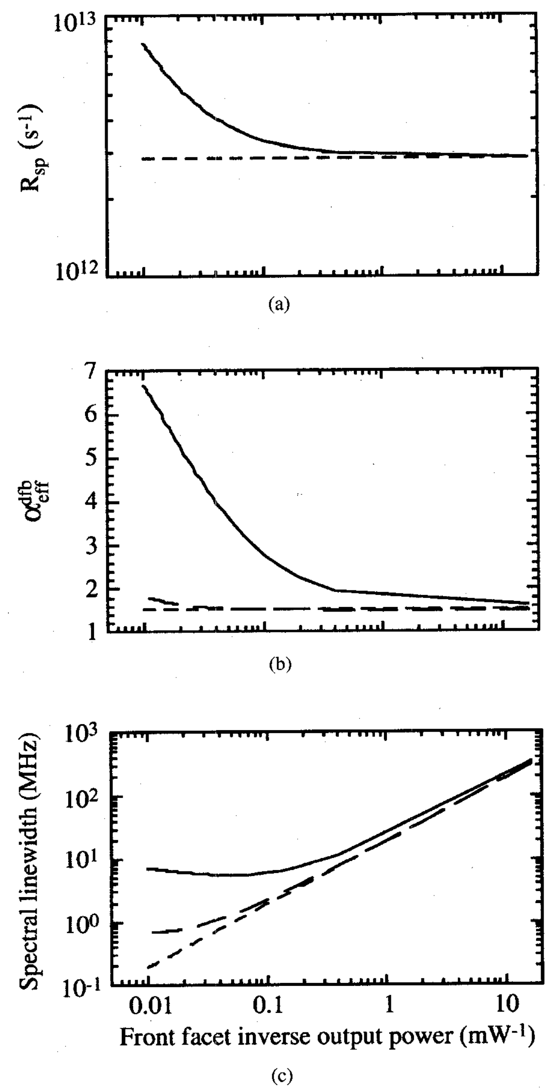

Fig. 1. Evolution of the spontaneous emission factor (a), the effective line-width enhancement factor (b) and the linewidth (c) versus the inverse of the output power for a phase-shifted DFBlaser. Without nonlinear gain and index (- - -), without nonlinear index (– –) and with these two effects (—).

where n_{eff} is the effective index, v_g the group speed and L the laser length. In order to simplify the notation, the z dependence of the different parameters are not written here.

The device we modeled is a strained multiple quantum-well phase-shifted DFB laser. The $\lambda/4$ phase shift is obtained with a variation of the active layer width. All the parameters used for the simulation are presented in Table I. The high value of the nonlinear gain coefficient, which depends strongly on the structure, is chosen by considering previous measurements performed on this type of lasers [11].

III. RESULTS

The Fig. 1(a) presents the variation of R_{sp} against the inverse of the output optical power in two different cases. First, the nonlinear gain is not taken into account and secondly the nonlinear gain is taken into account. The nonlinear index

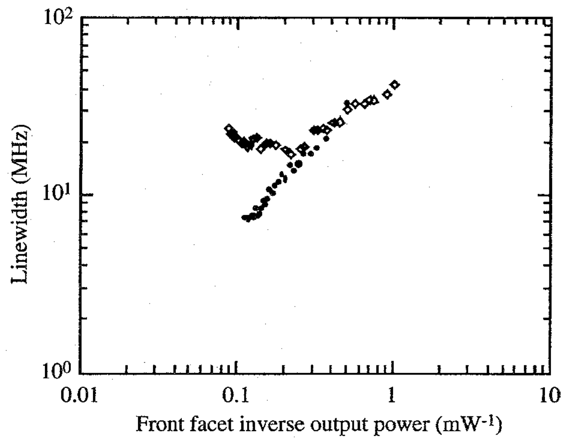

Fig. 2. Measured evolution of linewidth versus the inverse of the output power for two phase-shifted DFB lasers: with a strong CH (◇) and with a weak CH (•).

is not considered as he has negligeable effect on R_{sp}. The increase at high power of R_{sp} is due to an enhancement of the carrier density because of both nonlinear gain and spatial hole burning. Fig. 1(b) shows the increase of α_{eff}^{dfb} due, for the main part, to the nonlinear refractive index. As it can be seen from the (6), the increase of both R_{sp} and α_{eff}^{dfb} should result in a saturation of the linewidth. The Fig. 1(c) presents the linewidth plotted versus the inverse of optical output power. For the imaginary case of a DFB laser without gain suppression, the linewidth increases linearly with the inverse of the optical power as expected. The curve for a laser with a symmetric gain compression exhibits a saturation for high optical powers which is due mostly to the increase of carrier density in the laser. This results in an increase of the spontaneous emission rate coupled into the lasing mode. This behavior is different to the one in the uniform case, where the increase of the spontaneous emission rate is partly compensated by the reduction of the effective linewidth enhancement factor. Thus, as a result of α_{eff}^{fp} change, the linewidth decreases faster with the power in the uniform lasers with nonlinear gain than without. The third curve of Fig. 1(c) presents the linewidth of the laser with a strong nonlinear effective index. It exhibits a saturation with a rebroadening at high power. The side-mode suppression ratio of this laser has also been calculated. The laser shows in all three cases a high single-mode behavior with a side-mode suppression ratio higher than 40 dB for an output power greater than 5 mW. A saturation of the side-mode suppression ratio at high power due to spontaneous emission increase is observed, but even considering this, the side-mode suppression ratio is higher than 45 dB for a 100-mW output power. This means that the contribution of the side modes to the linewidth saturation is negligible in our component. The minimum calculated linewidth occurs at an output power of 19 mW and the calculated linewidth floor is about 5.5 MHz. The CH becomes the main contribution to the linewidth for output powers greater than 6 mW. This value depends strongly on the strength of the nonlinear index. In particular the extremely low linewidth obtained by Okai *et al.* [3] may be understood by taking this effect into account. We have made some measurements of the linewidth of a strained quantum-well phase-shifted DFB laser which exhibits strong CH and of a bulk phase-shifted DFB laser with low CH [11]. As shown on Fig. 2, a saturation with strong rebroadening is observed for output powers higher than 5 mW in the case of strong CH. Whereas, in a laser with low CH, no saturation is observed. However, these experiments constitute a first step and large scale investigations would be necessary to give a confirmation of our theory.

IV. Conclusion

We have shown the major role of CH in determining the linewidth of semiconductor single-mode DFB lasers. In particular, the linewidth rebroadening at high power has been attributed to the joint action of the nonlinear gain and refractive index due to CH together with the spatial-hole-burning. It is expected that CH plays also a major role in the frequency modulation properties of semiconductor lasers.

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