Optimum conditions for soliton launching from chirped sech² pulses

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Simple and exact analytical formulas are derived for the optimum power and width of chirped sech² pulses that maximize the energy transfer to the fundamental soliton. The maximum achievable energy transfer is shown to be equal to $2/(1 + \sqrt{1 + \alpha^2})$, normalized relative to the initial pulse energy, where α is the laser phase-amplitude coupling factor. The critical values of the α factor for the fundamental soliton are given simply in terms of the normalized pulse amplitude A by $\alpha_{\rm cr} = (4A^2 - 1)^{1/2}$. © 1995 Optical Society of America

The generation of picosecond pulses required for soliton communications is often accompanied by the frequency chirping effect. This is particularly the case for gainswitched pulses, for which the frequency chirp results in pulses that are far above the Fourier-transform limit. This effect is observed in the form of a timebandwidth product (TBP) $\Delta t \Delta f$ much higher^{1,2} than the 0.315 value obtained for the unchirped sech^2 pulse. There had been some early interest in the study of soliton formation from chirped pulses,³⁻⁵ but the results obtained were only numerical and considered a linear frequency chirp, and they did not determine some specifically favorable launching conditions. On the other hand, experimental results with chirped pulses having bandwidths up to seven times the transform limit⁶ indicate that the loss of energy that is due to chirping of the initial pulse forms a dispersive nonsoliton pedestal; this can have a highly detrimental effect on soliton propagation. The purpose of this Letter is to propose simple and exact formulas for the optimum launching conditions to maximize energy transfer in the more realistic case of chirping as given by the laser's rate equations. In the case of gain-switched laser pulses, the complex electric field envelope at the output of the laser is given by^{2,7,8}

$$E(t) = \operatorname{sech}^{1+j\alpha}(t/\tau),$$

where α is the laser phase-amplitude coupling factor and τ is a time scaling parameter. To study the pulse evolution along the fiber we make use of the method of inverse scattering.^{9,10} We need to solve the following system of eigenvalue equations with appropriate boundary conditions¹⁰:

$$j \frac{\partial v_1}{\partial t} + u v_2 = \zeta v_1, \qquad (1)$$

$$j \frac{\partial v_2}{\partial t} + u^* v_1 = -\zeta v_2, \qquad (2)$$

where $u(t) = A \operatorname{sech}^{1+j\alpha}(t)$ is the initial pulse launched into the fiber at z = 0 and $\zeta = \xi + j\eta$ are the complex eigenvalues of the system of Eqs. (1) and (2). The boundary conditions are

$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \rightarrow \begin{bmatrix} \exp(-j\zeta t) \\ 0 \end{bmatrix} \quad \text{for } t \rightarrow -\infty,$$
$$\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} \rightarrow \begin{bmatrix} a(\zeta)\exp(-j\zeta t) \\ b(\zeta)\exp(j\zeta t) \end{bmatrix} \quad \text{for } t \rightarrow +\infty$$

which define the scattering coefficients $a(\zeta)$ and $b(\zeta)$. The boundary conditions are imposed by the form of Eqs. (1) and (2) and by the fact that $|u(z, t)| \to 0$ as $t \to \pm \infty$. The discrete eigenvalues are determined by the zeros of $a(\zeta)$ lying in the upper half complex plane, i.e., by

$$a(\zeta) = 0$$
, $\operatorname{Im}(\zeta) > 0$.

There is a soliton associated with each eigenvalue that is of the $\ensuremath{\mathsf{form}^{10}}$

$$u(z, t) = 2\eta \operatorname{sech}[2\eta(t - 2\xi z - t_0)]$$
$$\times \exp[-i2\xi t + i2(\xi^2 - \eta^2)z]$$

and is a particular solution of the nonlinear Schrödinger equation with no loss. The soliton amplitude is equal to 2η , the soliton velocity is equal to 2ξ , and t_0 is a real constant. It is also easily seen that the soliton energy is given by 4η and its FWHM pulse width is $1/2\eta$ normalized relative to the initial pulse width. Because of the symmetry¹⁰ of the input pulse the real part ξ of the eigenvalue ζ vanishes, meaning that chirping does not affect the velocity of soliton propagation. An analytical solution of the system of Eqs. (1) and (2) is obtained in terms of hypergeometric functions for this initial condition, and the functions that characterize the scattering problem are found, after some lengthy calculations, to be expressible in terms of gamma functions of a complex argument as

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$$a(\xi) = \frac{\Gamma[1/2 - j(\xi - \alpha/2)]\Gamma[1/2 - j(\xi + \alpha/2)]}{\Gamma[1/2 - j\xi + (A^2 - \alpha^2/4)^{1/2}]\Gamma[1/2 - j\xi - (A^2 - \alpha^2/4)^{1/2}]}, \quad (3)$$

$$b(\xi) = j2^{-j\alpha}A \frac{\Gamma[1/2 + j(\xi - \alpha/2)]\Gamma[1/2 - j(\xi + \alpha/2)]}{\Gamma[1 - (A^2 - \alpha^2/4)^{1/2} - j\alpha/2]\Gamma[1 + (A^2 - \alpha^2/4)^{1/2} - j\alpha/2]}. \quad (4)$$

In the limit case that $\alpha = 0$ when the pulse is unchirped Eqs. (3) and (4) simplify to those given in Ref. 10. When the principle of analytic continuation is applied to Eqs. (3) and (4) the zeros of $a(\zeta)$ that lie in the upper half-plane of the complex variable ζ are found to coincide with the poles of the gamma function at the denominator of $a(\zeta)$:

$$\zeta_r = j \eta_r = j[(A^2 - \alpha^2/4)^{1/2} - r + 1/2], \qquad (5)$$

where r is a positive integer and the fundamental soliton corresponds to r = 1. Because of the chirping a part of the initial pulse energy goes into a dispersive tail, decaying as $1/\sqrt{z}$, which is deleterious to the propagation of the soliton pulse. The ratio of the energy going into the soliton component divided by the total energy of the input pulse is given by¹⁰

$$R = 4 \sum_{r=1}^{N} \eta_r / 2A^2, \qquad (6)$$

where R is the quantity to maximize. In the general case that N-order solitons exist R can easily be shown to be equal to

$$\frac{2N(A^2 - \alpha^2/4)^{1/2} - N^2}{A^2},$$
(7)

where $N = \text{Int}[(A^2 - \alpha^2/4)^{1/2} + 1/2]$ and Int denotes the integer part. Expression (7) is plotted in Fig. 1 for various α factors depicting higher-order soliton propagation. When r = 1, only the fundamental soliton exists (this is the usual situation in soliton communication systems), and Eqs. (5) and (6) then yield

$$R = \frac{2[(A^2 - \alpha^2/4)^{1/2} - 1/2]}{A^2} \cdot$$
(8)

On differentiating Eq. (8) and putting the result equal to zero we obtain the optimum normalized pulse amplitude as

$$A_{\rm opt} = \left(\frac{1 + \alpha^2 + \sqrt{1 + \alpha^2}}{2}\right)^{1/2}$$
(9)

and the corresponding maximum energy transfer ratio

$$R_{\max} = \frac{2}{1 + \sqrt{1 + \alpha^2}} \,. \tag{10}$$

This is our main result. However, a fundamental soliton is formed as long as $(1/2)\sqrt{1 + \alpha^2} \le A < (1/2)\sqrt{9 + \alpha^2}$, leading to the well-known result for un-

chirped pulses when $\alpha = 0$ that $1/2 \leq A < 3/2$. Moreover, for $\alpha = 1$ and $\alpha = 2$ one obtains $\sqrt{2}/2 \le A < \sqrt{10}/2$ and $\sqrt{5}/2 \le A < \sqrt{13}/2$, respectively, for the region of existence of the fundamental soliton, and this is also clearly seen in Fig. 1. The discontinuities present in Fig. 1 correspond to transition in order-2 and order-3 soliton states. On the other hand, there are critical chirp parameters corresponding to vanishing eigenvalues and therefore to zero energy transfer to the fundamental soliton component. They are simply given from Eq. (5) by $\alpha_{cr} = \sqrt{4A^2 - 1}$. Furthermore, the asymptotic soliton width for infinite distance, $z \rightarrow \infty$, is given by $1/2\eta$ relative to the initial pulse width. Some examples of energy maximization are shown in Table 1. The TBP $\Delta t \Delta f$ is calculated by use of Eq. (7) of Ref. 2. It is seen that the initial pulse width should be chosen to be different from the final pulse width for optimum results. For example, let us assume that we want to launch $\Delta t = 20$ ps FWHM solitons into dispersion-shifted fiber having the following typical parameters: |D| = 2.5 ps/(km nm), $A_{\text{eff}} = 2.5 \times 10^{-7} \text{ cm}^2$, and $n_2 = 3.2 \times 10^{-16} \text{ cm}^2/\text{W},$ at $\lambda = 1.55 \ \mu m$. Using transform-limited, i.e., unchirped, pulses, we would need a peak power given by

$$P_{N=1} = 0.776 \, rac{\lambda^3}{c \, \pi^2 n_2} \, rac{|D|}{\Delta t^2} \, A_{
m eff} \, ,$$

which yields 4.77 mW. On the contrary, if we dispose of a laser source with a TBP of 0.85, we can calculate from Table 1 that $\Delta t_{opt} = 20 \times 1.80 = 36$ ps and



Fig. 1. Normalized energy transfer to the soliton part of a pulse as a function of the chirped pulse's normalized amplitude for various chirp factors.

Table 1. Optimum Launching Conditions andMaximum Normalized Energy Transfer to the
Soliton from Chirped sech2 Pulses

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α	$\Delta t \Delta f$	$R_{ m max}$ (%)	A_{opt}	$\Delta t_{ m opt}$
0.5	0.39	94.4	1.09	1.12
1.0	0.59	82.8	1.31	1.41
1.5	0.85	71.4	1.59	1.80

 $P_{\text{opt}} = (P_{N=1}/1.80^2)1.59^2 = 3.72 \text{ mW}$, yielding a maximum energy transfer to the fundamental soliton of 71.4%. In other words, we launch a wider and more powerful (relative to its N = 1 power) pulse that is asymptotically compressed to the 20-ps width. However, because these results are exact only asymptotically as $z \to \infty$ and in the zero-loss case, they should be applied with care because pulses may broaden or narrow significantly before they reach their asymptotic width.³⁻⁵ On the other hand, neglecting the chirped nature of the pulse and launching a 20-ps pulse with 4.77-mW peak power and 0.85 TBP, one would obtain asymptotically according to Eqs. (5) and (8) a soliton pulse with 62-ps FWHM carrying only 32% of the initial pulse energy. This is less than half of the energy transfer achieved with the optimum conditions and demonstrates the effectiveness of our approach.

In conclusion, we have presented simple and exact analytical expressions for the optimum launching of solitons out of chirped sech² pulses, using a realistic model for the laser's frequency chirp. It was shown that the pulse power and width should be chosen differently from those of an ideal unchirped pulse to maximize energy transfer to the soliton component and therefore to minimize at the same time the effect of the dispersive tail. Critical chirp parameters were also derived beyond which no soliton formation is possible.

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