

# Nonlinear Gain and Its Influence on the Laser Dynamics in Single-Quantum-Well Lasers Operating at the First and Second Quantized States

J. Yao, P. Gallion, W. Elsässer, and G. Debarge

**Abstract**—The gain nonlinearity due to spectral-hole-burning in InGaAs/InP single-quantum-well lasers operating at the first and second quantized states has been investigated by third-order perturbation solution and by strong signal approximation of the density-matrix equations, respectively. We find, besides the higher differential gain, a smaller nonlinear gain contribution for lasing at the second quantized state as compared to the first quantized state operation; hence, these improved differential gain and nonlinear gain lead to a smaller  $K$  factor, defined as the ratio of damping rate to the square of relaxation resonance frequency, and consequently a higher maximum modulation bandwidth.

## I. INTRODUCTION

THE application of semiconductor lasers in optical communication requires their high dynamic performance. The relaxation resonance frequency  $f_R$  is a measure of useful direct modulation bandwidth. If the optical nonlinearities are not considered, there are two ways for improving  $f_R$  at a given photon density: decreasing the photon lifetime within the cold cavity or increasing the modal differential gain.

Quantum-well lasers are of potential advantage in high speed modulation application due to their higher differential gain as compared to conventional heterostructure laser [1]. However, single-quantum-well (SQW) lasers are known to exhibit a marked saturation of the optical gain at high injected carrier density because of the step-like density of state; this may lead to the fact that the expected increase in relaxation resonance frequency with decreasing photon lifetime may be compensated by a simultaneous decrease in differential gain. One possibility to overcome this crucial point is to increase the total cavity loss so much that the second energy level is populated, consequently leading to an operation at the second quantized state. A 55% increase of  $f_R$  due to a decrease

in photon lifetime accompanied by an increase in differential gain for lasing at the second quantized state as compared to the first quantized state operation has been observed [2]. For this reason, the performance of SQW lasers operating at the second quantized state has attracted our particular attention.

It is well known that optical nonlinearities play an important role in the operation of quantum-well lasers [3], [4]. The more important gain nonlinearity in this type of lasers than that in conventional heterostructure lasers imposes severe limits on the laser performance. Therefore, we found it important to discuss the gain nonlinearities for SQW lasers operating at the first and second quantized states and their influence on the laser dynamics. The physical origin of these nonlinearities is under strong discussion considering a lot of possible mechanisms such as spectral-hole-burning effect and even carrier capture into the wells. In this letter, only the spectral-hole-burning effect is considered.

## II. RESULTS

The SQW laser we used as a model structure was an  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$  laser with a quantum well thickness equal to 100 Å. The intraband relaxation times for electrons, holes, and polarization used in our calculations for the first and second quantized states were assumed to be 200, 70, and 100 fs, respectively; the optical confinement factor was 2%.

The starting point of gain modeling is the density-matrix equations. The peak gain wavelength  $\lambda_p$ , calculated by the third-order perturbation approximation of the density-matrix equations, is shown in Fig. 1 as a function of modal gain. The discontinuity occurring at a modal gain equal to  $40 \text{ cm}^{-1}$  is due to the transition from the first quantized state operation to the second quantized state one; therefore, we call this modal gain value the transition modal gain  $g_{tr}$  for our particular structure. We even see that  $\lambda_p$  decreases slightly with increasing modal gain for both quantized state operations. Lasing at the second quantized state can be induced by increasing the total cavity loss, which can be practically accomplished either by decreasing the length of cavity [5], increasing the material loss [6], or by decreasing the facet reflectivity [7].

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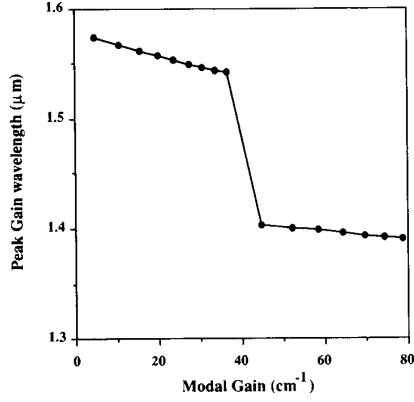


Fig. 1. Peak gain wavelength as a function of modal gain for a  $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$  single-quantum-well laser with well thickness equal to 100 Å.

Next, we calculate the nonlinear gain by using the third-order perturbation solution of the density-matrix equations. The nonlinear coefficient  $\epsilon$  is introduced via

$$g = g_l + g_{nl} = g_l(1 - \epsilon P) \quad (1)$$

where  $g_l$  is the linear gain,  $g_{nl}$  is the nonlinear gain, and  $P$  is the photon density. The  $\epsilon$  value as a function of modal gain is shown in Fig. 2. It can be seen that  $\epsilon$  decreases with increasing modal gain. The discontinuity occurring at  $g_{tr}$  is not very evident; we think that this is because of the discontinuity occurring at the same time for the linear and nonlinear gain, so that their ratio does not exhibit an evident discontinuity. We can also see in this figure that the  $\epsilon$  value for lasing at the second quantized state is smaller than that for the first quantized state operation, this is in agreement with experimental observation in [8]. This result shows the advantage of the second quantized state operation of SQW lasers concerning the gain nonlinearity.

Because (1) is only valid for small photon densities, we have used the strong signal approximation [9] of the density-matrix equations to confirm the above result. The nonlinear gain obtained by this approximation is written as

$$g = \frac{g_l}{\sqrt{1 + P/P_s}} \quad (2)$$

where  $P_s$  is the saturation photon density, defined as

$$P_s = \frac{2\epsilon_0 n n_g}{\hbar \omega} \left\{ \frac{R_{ch}^2}{\hbar^2} \tau_{in} (\tau_c + \tau_v) \right\}^{-1} \quad (3)$$

where  $\epsilon_0$  is the dielectric constant,  $\omega$  is the laser emission frequency,  $n$ ,  $n_g$  are the effective index and group index,  $\tau_c$ ,  $\tau_v$ ,  $\tau_{in}$  are the intraband relaxation times for electrons, holes, and polarization, respectively, and  $R_{ch}$  is the dipole moment, written as [10]

$$R_{ch} = \left( \frac{e}{2\omega} \right)^2 \frac{E_g(E_g + \Delta)}{(E_g + 2\Delta/3)m_c} \quad (4)$$

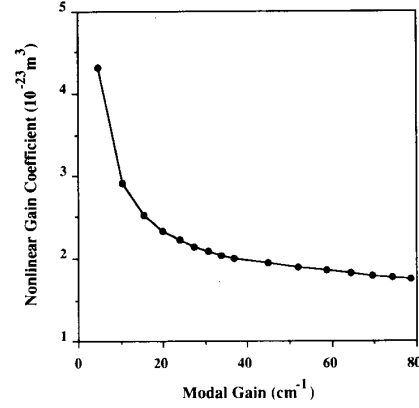


Fig. 2. Nonlinear gain coefficient as a function of modal gain.

for quantum-well lasers where  $e$  is the electron charge,  $E_g$  is the band gap energy,  $\Delta$  is the spin-orbit splitting, and  $m_c$  is the effective mass of electrons. Substituting (4) into (3), one obtains that  $P_s$  is proportional to the laser frequency. Using the result of Fig. 1 and assuming the same values of intraband relaxation times for the first and second quantized states, we find that the saturation photon density increases with increasing modal gain, and the gain nonlinearity is smaller for the second quantized state operation than that for the first quantized state one. This is in agreement with the result obtained by the third-order perturbation method.

When  $P$  is small, (2) can be approximated as  $g = g_l(1 - P/(2P_s))$ . Comparing this equation with equation (1), one obtains  $\epsilon \approx 1/(2P_s)$ .

With the above discussion, we can see that the nonlinear gain can be approximately written as

$$g = g_l / \sqrt{1 + 2\epsilon P} \quad (5)$$

and the relaxation resonance frequency including the gain nonlinearity is given by

$$f_R = \frac{1}{2\pi} \left\{ \frac{\Gamma v_g \frac{dg}{dN} P (1 + \epsilon P)}{\tau_p (1 + 2\epsilon P)^2} \right\}^{1/2} \quad (6)$$

where  $\tau_p$  is the photon lifetime,  $\Gamma$  is the optical confinement factor,  $v_g$  is the group velocity, and  $dg/dN$  is the differential gain. To compare the contribution of the differential gain and the nonlinear effect to  $f_R$  for a SQW laser operating at the first and second quantized states, we have chosen two modal gain values: 28  $\text{cm}^{-1}$  and 59  $\text{cm}^{-1}$ , corresponding respectively to the first and second quantized state operations. The calculated  $f_R$  including or without the gain nonlinearity of these two operation points is shown in Fig. 3 as a function of output power. For the second quantized state operation, the enhancement of  $f_R$  within the linear gain model is due to a 52.5% decrease in

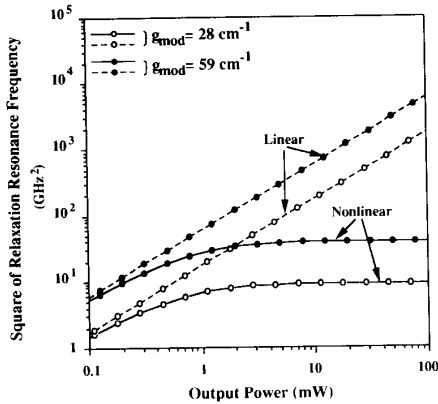


Fig. 3. Relaxation resonance frequency as a function of output power for modal gain equal to 28 and 59 cm<sup>-1</sup>. Linear: without gain nonlinearity; Nonlinear: including gain nonlinearity.

photon lifetime and a 78% increase in differential gain as compared to the first quantized state operation; within the nonlinear gain model, the small  $\epsilon$  value leads only to a small decrease of  $f_R$  at a given output power.

The maximum 3 dB modulation bandwidth  $f_{\max}$  can be expressed by the  $K$  factor [11]:

$$f_{\max} = 2\pi\sqrt{2}/K \quad (7)$$

where  $K$  is defined as the ratio of damping rate to the square of relaxation resonance frequency. If we use equation (5) as the nonlinear gain model, the  $K$  factor can be approximatively written as

$$K = 4\pi^2 \left( \tau_p + \frac{\epsilon}{v_g dg/dN} \right). \quad (8)$$

From this equation, we can see that any decrease in photon lifetime and nonlinear coefficient  $\epsilon$  or increase in differential gain will cause a decrease of the  $K$  factor and consequently lead to an increase of the maximum 3 dB modulation bandwidth. The above calculated results show that the second quantized state operation fulfills all these three possibilities for improving the laser modulation bandwidth. The calculated  $K$  factor as a function of modal gain is depicted in Fig. 4. It can be seen that the  $K$  factor decreases with increasing modal gain and is smaller for the second quantized state operation than that for the first quantized state one. For a modal gain equal to 28 cm<sup>-1</sup> and 59 cm<sup>-1</sup>, corresponding to the first and second quantized state operations, the  $K$  factor is equal to 0.312 and 0.148 ns, respectively; this means a 115% increase in maximum modulation frequency for lasing at the second quantized state as compared to the first quantized state operation.

### III. CONCLUSION

A detailed analysis of gain nonlinearity due to the spectral-hole-burning in single quantum-well lasers operating at the first and second quantized states has been

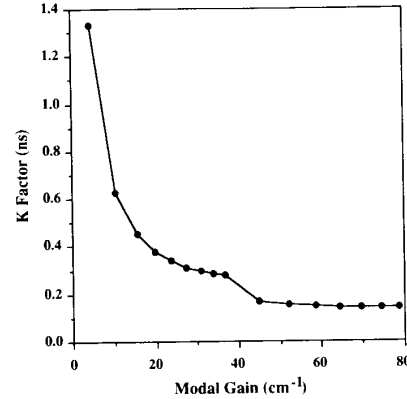


Fig. 4. The  $K$  factor as a function of modal gain.

presented in this letter. The results obtained by the third-order perturbation solution and the strong signal approximation of density-matrix equations have shown a less important gain nonlinearity for lasing at the second quantized state than that for the first quantized state operation. With a higher differential gain at the same time, SQW lasers operating at the second quantized state show particular advantage for high speed modulation application.

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