Strong Signal Analysis of Optical Nonlinearities in Single-Quantum-Well and Double-Heterostructure Lasers

J. Yao, G.-H. Duan, and P. Gallion

***Abstract*—Based on the numerical strong signal solution of the density-matrix formalism, the nonlinearities of the gain and the refractive index are discussed for both bulk double-heterostructure (DH) and single-quantum-well (SQW) InGaAs lasers. The results show that for both structures the nonlinear gain is well approximated by the recently proposed analytical expression and modified two-level system approximation up to a range of photon density discussed in this letter, and the refractive index could either increase or decrease with the photon density depending on the wavelength detuning from the gain peak. The results of new analytical expression for nonlinear refractive index is in qualitative agreement with the numerical ones; however, significant quantitative difference occurs between these two model results for high photon density values. Due to the more important nonlinear gain in SQW structures, the linewidth enhancement factor increases more rapidly with increasing photon density in these structures than in DH structures.**

I. Introduction

It has been shown that the optical nonlinearities have a strong impact on the noise and dynamic performances of semiconductor lasers [1], [2]. Most previous studies on spectral-hole-burning induced optical nonlinearities are based on the third-order perturbation theory [3], which is only valid in the low output power regime. In effect, the optical nonlinearities become predominant particularly in the high output power regime; thus, it is important to discuss these properties using a strong signal analysis. Usually, for double-heterostructure (DH) semiconductor lasers, the same form of the nonlinear gain as in an homogeneously broadened two-level system is used directly and the refractive index nonlinearity is neglected [4]. Based on the strong signal theory (SST) of the density-matrix formalism, a new analytical nonlinear gain expression taking into account the energy band structure of bulk semiconductors has been proposed, and the refractive index has also been shown to have a nonnegligible nonlinear part [5]. However, the validity of these analytical expressions has not been examined, so the first purpose of its letter is to give an exact numerical solution for nonlinear gain and refractive index, and to compare these results with that obtained by the new analytical expressions.

Recently, quantum-well (QW) structures have attracted much attention owing to some improved characteristics, such as high differential gain and small linewidth enhancement factor [6]. In this type of structure the conventional conduction and valence bands are split into many subbands due to the energy quantization of electrons and holes in the direction of quantum well thickness. The three-dimensional parabolic density of states becomes two dimensional step-like. This modified density state leads to a high differential gain and thus to a potentially enhanced modulation bandwidth [6]. Unfortunately, this advantage is compensated by a high nonlinear gain in this type of structures [7]; theoretical analysis based on the small perturbation solution confirms this result [8]. But in QW structures, practically achievable output power is of the same order of magnitude as the output saturation power because of their decreased output saturation power and increased maximum output power; thus, the validity of the small perturbation theory usually used for QW structures becomes questionable. The other purpose of this letter is to analyze, for the first time, the gain and the refractive index nonlinearities in this type of structures by using the numerical strong signal solution, and these results will also be used to verify if the new analytical expressions could be extended to QW structures.

II. Theory

The starting point of our analysis is the density-matrix formalism, which gives the linear and nonlinear contributions to the optical susceptibility [3]. For lasers oscillating in a single longitudinal and transverse mode, the density-matrix equations have been solved exactly. The susceptibility χ, which represents the response of the medium to applied field including the spectral-hole-burning effect, is written as [5]

$$\chi = \frac{1}{\epsilon_0}\int_{E_0}^{\infty} \frac{R_{\text{ch}}^2 D(E_{\text{ch}})(f_c - f_v)(E - E_{\text{ch}} - i\hbar/\tau_{\text{in}})}{(E - E_{\text{ch}})^2 + (1 + P/P_s)(\hbar/\tau_{\text{in}})^2}\, dE_{\text{ch}} \tag{1}$$

where E is the photon energy, P is the photon density, R_{ch} is the dipole moment, E_{ch} is the transition energy, f_c

Manuscript received April 27, 1992; revised July 16, 1992.

The authors are with the Département Communications, Ecole Nationale Supérieure des Telecommunications, 75634 Paris Cedex 13, France.

IEEE Log Number 9203183.

1041-1135/92$03.00

and f_v are the occupation probabilities of electrons in the conduction band and the valence band at thermal equilibrium, E_0 is the energy separation between the fundamental levels in the conduction band and the valence band, $D(E_{ch})$ is either the step-like reduced density of states for QW structures or the parabolical reduced density of states for DH structures, P_s is the saturation photon density defined as [5]

$$P_s = \frac{2\epsilon_0 n n_g}{\hbar \omega}\left\{\frac{R_{ch}^2}{\hbar^2}\tau_{in}(\tau_c + \tau_v)\right\}^{-1} \quad (2)$$

where τ_c, τ_v, τ_{in} are the intraband relaxation times for electrons, holes and polarization respectively, and n, n_g are the effective index and group index.

From (1), Agrawal obtained new approximative analytical expressions for nonlinear gain and refractive index for DH structures [5]:

$$g = \frac{g_1}{\sqrt{1 + P/P_s}} \quad (3)$$

$$\Delta n = -\frac{g_1}{2k_0}\left(\alpha_H - \frac{\beta P/P_s}{1 + \sqrt{1 + P/P_s}}\right) \quad (4)$$

where g_1 is the linear gain, k_0 is equal to ω/c, c is the light velocity in vacuum, α_H is the linear linewidth enhancement factor, and β is defined as $\beta = 1/(g_1\tau_{in})\, dg_1/d\omega$.

The analytical expressions are obtained on the basis of some approximations such as infinite polarization relaxation time. In the following, an exact solution of (1) will be obtained by using numerical simulation for both DH and QW structures. Finally, these numerical results will be compared with analytical ones given by (3) and (4).

III. Results

In our discussion, the recombination of electrons and light holes is neglected due to the small effective mass of the latter. We assume that the following parameters have the common values in both DH and QW structures: the effective mass of electrons m_c and heavy holes m_v are equal to 0.041 m_0 and 0.424 m_0 where m_0 is the free electron mass; τ_c, τ_v, and τ_{in} are equal to 3×10^{-13} s, 7×10^{-14} s, and 1×10^{-13} s respectively; n and n_g are equal to 3.65 and 4; the temperature considered is 300 K. The band gap energy E_g(DH) and E_g(QW) are equal to 0.75 and 0.675 eV. The thickness of the active layer L_z(DH) and L_z(QW) are assumed to be 100 and 10 nm, respectively. The dipole moment is calculated by the following formula [9]:

$$R_{ch}^2 = \frac{B\left(\frac{e\hbar}{2E_{ch}}\right)^2 E_g(E_g + \Delta)}{(E_g + 2\Delta/3)m_c} \quad (5)$$

where e is the electron charge, Δ is the spin-orbit splitting assumed to be 0.33 eV for both structures, B is equal to 2/3 for DH structure and about 1 for QW structure at subband edge [9].

In the following, we will discuss the calculated results.

A. Gain Nonlinearity

The numerical results of gain as a function of normalized photon density P/P_s is depicted in Fig. 1 for a carrier density N equal to 2×10^{18} cm^{-3} and at the peak gain wavelength λ_P, which is equal to 1.55 μm for both structures. In this figure, the saturation photon density P_s(DH) and P_s(QW) are equal to 5.86×10^{16} cm^{-3} and 4.3×10^{16} cm^{-3}, corresponding to an output saturation power of 234 and 172 mW respectively, for a mode volume $V/\Gamma = 2 \times 10^{-10}$ cm^3 and a mirror loss $\alpha_m = 45$ cm^{-1} for both structures; this difference in the output saturation power originates in the different dipole moments values for the two structures. In order to demonstrate the validity of different approximate expressions for nonlinear gain used until now, the results by using the new expression (3), the modified two-level system expression $g = g_1/(1 + P/(2P_s))$ and the small signal approximation $g = g_1(1 - P/(2P_s))$ are also given in this figure. It can be seen that for both structures the numerical results of the SST are well approximated by the analytical expression (3) and the modified two-level system approximation in the range of photon density discussed. The small signal theory is only valid for low photon density values ($P/P_s < 0.1$). The enhanced gain nonlinearity in QW structures is also observed from this figure. This is in agreement with previous small perturbation theory predictions [9] and the experimental observations [7]. This stronger gain nonlinearity in QW lasers limits seriously its maximum achievable modulation bandwidth.

B. Refractive Index Nonlinearity

The numerical and analytical results for refractive index are given as a function of normalized photon density P/P_s in Fig. 2 for the DH structure and in Fig. 3 for the QW structure. The carrier density is assumed to be 2×10^{18} cm^{-3} and the parameter is the wavelength detuning $\Delta\lambda(= \lambda - \lambda_p)$ from the peak gain wavelength. It can be seen that the refractive index increases for $\Delta\lambda > 0$ and decreases for $\Delta\lambda < 0$, and at the peak gain wavelength, it remains almost constant. We can qualitatively explain these refractive index nonlinearities as follows: the nonlinear gain and nonlinear refractive index are related by a modified Kramers–Kronig relation, which can be obtained from (1). At the peak gain wavelength, the spectral hole in the gain spectra is symmetrical and its contribution to nonlinear refractive index is about zero; at wavelength detuned from the gain peak, the spectral hole is not symmetrical and leads a nonlinear part of the refractive index; the shape of the spectral hole, which is different for positive and negative wavelength detuning, determines the sign of the nonlinear refractive index. For the DH structure, the nonlinear index is nearly symmetric against the wavelength detuning, and for the QW structure, an asymmetric behavior is observed; this symmetry or asymmetry

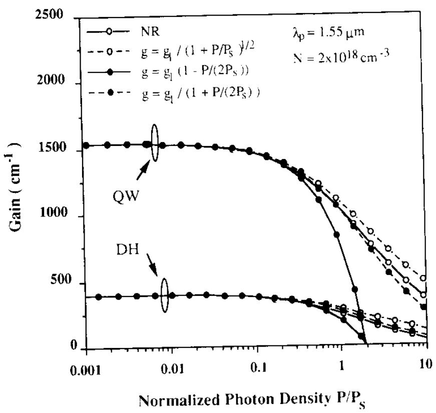

Fig. 1. The gain at the peak gain wavelength as a function of normalized photon density obtained by different models for the DH structure and the QW structure. NR: numerical results.

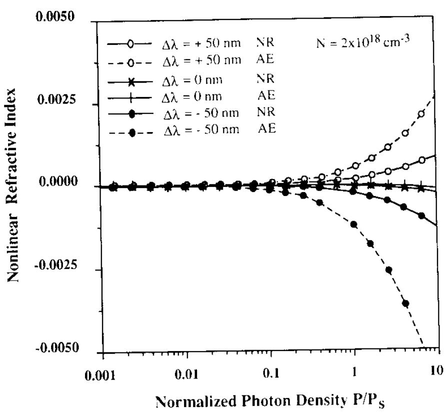

Fig. 2. The refractive index as a function of normalized photon density for different wavelength detunings for the DH structure. NR: numerical results, AE: results obtained by the analytical expression (4).

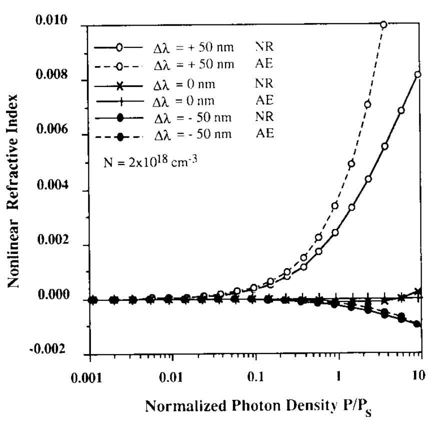

Fig. 3. The refractive index as a function of normalized photon density for different wavelength detunings for the QW structure. Symbols and abbreviations have the same definitions as in Fig. 2.

could be well accounted by the shape of linear gain spectra in these structures. The results of the analytical expression (4) are in qualitative agreement with the numerical ones for both structures. However, for the DH structure, the analytical expression appears to give a more important refractive index nonlinearity than the numerical calculation of SST for wavelengths detuned from the gain peak for high photon density ($P/P_s > 1$); for the SQW structure, there is a good agreement between these two results for the negative detuning in the range of photon density discussed, and for the positive detuning, as found in the DH structure, the analytical results give a more important nonlinear refractive index than the numerical ones for $P/P_s > 1$. The small contribution of refractive index change with photon density in the usual power range ($P/P_s < 1.0$) is expected to give negligible effects on laser dynamics and noise.

C. Linewidth Enhancement Factor

The linewidth enhancement factor α describes the amplitude-phase coupling via the coupling between real and imaginary part of the susceptibility. Due to the gain and the refractive index nonlinearities, the factor α will depend on the photon density. This dependence analysed by the numerical strong signal solution for both DH and SQW structures is given Fig. 4. The α value is smaller for the SQW structure than that for the DH structure for low photon densities. With the increase of photon density, the factor α increases for both structures mainly due to the gain nonlinearity; however, this increase is more rapid in the SQW structure than in the DH structure due to the more important gain nonlinearity in the former structure. For very high photon density ($P/P_s > 11$), the factor α(SQW) could be even larger than α(DH).

IV. Conclusion

In this letter, the numerical results of the strong signal solution for nonlinear gain are given and compared to that of the new analytical expression, the modified two-level system expression and the small signal approximation for the DH and the SQW structures. The new proposed analytical expression (3) and the modified two-level system expression for nonlinear gain has been proven to be good approximations for both structures in the range of photon density discussed. The numerical results for nonlinear refractive index show that the refractive index remains almost constant at the peak gain wavelength and can either increase or decrease with the photon density depending on the wavelength detuning from the gain peak. These results are in qualitative agreement with that of the analytical expression. However, for the DH structure, the analytical expression appears to give a more important refractive index nonlinearity than the numerical calculation of SST for wavelengths detuned from the gain peak; for the QW structure, there is a good agreement between these two results for the negative wavelength detuning, and for the positive wavelength detuning, as found in the DH structure, a more important refractive

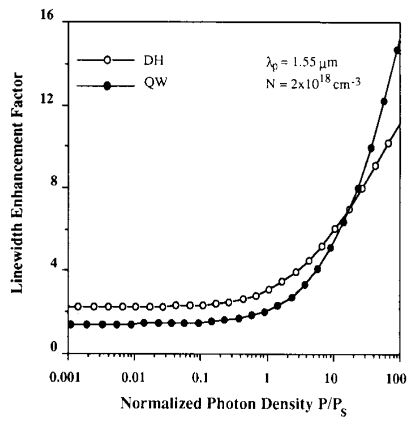

Fig. 4. Linewidth enhancement factor at the peak gain wavelength as a function of normalized photon density for the DH and the QW structures.

index nonlinearity is given by the analytical expression as compared with numerical results. Finally, the dependence of the α factor on the photon density is studied by including nonlinear gain and refractive index. It has been found that the factor α in QW structures increases more rapidly with photon density than in DH structures mainly due to the more important gain nonlinearity in the former structures.

Acknowledgment

The authors acknowledge G. Debarge, ENST and W. Elsässer, Philipps-University Marburg for useful discussions.

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FM Mode-Locking at 2.85 GHz Using a Microwave Resonant Optical Modulator

K. J. Weingarten, A. A. Godil, and Martin Gifford

***Abstract*—We harmonically mode locked a diode-pumped 1053 nm Nd : YLF laser using a microwave resonant optical modulator (MROM), achieving pulsewidths of 4.5 ps at a repetition rate of 2.85 GHz and 4 ps at 237.5 MHz. The laser produced average output powers greater than 400 mW.**

MODE-LOCKING of diode-pumped solid-state lasers has been a very active research area for the last several years. Active mode-locking at gigahertz repetition rates with diode-pumping has been demonstrated by several groups using both amplitude and phase modulators [1]–[4]. Mode-locking at a 20 GHz repetition rate was demonstrated with a new type of resonant phase modulator termed the dielectric resonator/optical modulator (DROM) [4]. We have used a similar device termed a microwave resonant optical modulator (MROM) to demonstrate harmonic mode-locking [5] in a diode-pumped Nd : YLF laser at a repetition rate of 2.85 GHz, which is very near the drive frequency of linear accelerators such as the Stanford Linear Accelerator (SLAC). Such a laser, suitably amplified and frequency converted to the UV, can be used to drive a photocathode as a high brightness source of electrons for seeding the accelerator. Other applications include optical communications and photonic switching.

Manuscript received June 26, 1992. This work was supported in part by an SBIR Phase II Contract DE-FG03-90ER81042 from the Department of Energy.

K. J. Weingarten and M. Gifford are with Lightwave Electronics, Mountain View, CA 94043.

A. A. Godil is with Edward L. Ginzton Laboratory Stanford University, Stanford, CA 94305.

IEEE Log Number 9203563.

1041-1135/92$03.00