Phase jitter in an injection-locked semiconductor laser

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An expression for the mean-square phase jitter of an injection-locked semiconductor laser is derived. It leads to the determination of the power spectrum of the locked laser. The theoretical results are in good agreement with experimental results reported previously [IEEE J. Quantum Electron. 24, 148 (1988)].

The injection-locking conditions of semiconductor lasers are usually expressed in terms of a locking range, which can be reduced by instabilities.[1-4] The stable locking range is bounded by two limits: below the lower limit, the optical injection results in modulating the free-running signal,[5] whereas above the upper limit, the locked laser becomes unstable, i.e., multimode. For a locked laser, the damping of the relaxation oscillations is strongly dependent on the locking conditions. The relaxation oscillations are caused by the intrinsic resonance in the gain saturation process and the resulting coupled fluctuations of light intensity and carrier density. They give rise to sidebands in the optical spectrum and may widen the spectral line spread of the laser, resulting in penalties in transmission in optical-fiber systems. With good locking conditions, they can be reduced substantially, improving the system performance.

This Letter presents a study of the phase jitter of an injection-locked semiconductor laser. Relaxation oscillation resonance is included in standard injection-locking theory. The locked laser is described by three coupled rate equations for the field amplitude, the phase, and the carrier density. An expression for the mean-square phase jitter is derived and allows one to calculate the power spectrum.

The field $E(t)$ of the slave laser, locked to the master laser frequency ω_0, is expressed as

$$E(t) = E_0 \exp\{i[\omega_0 t + \Phi + \varphi(t)]\},$$

where E_0 is a real constant, since amplitude fluctuations well above threshold are neglected, and Φ is the constant cw phase. The phase noise $\varphi(t)$ is treated under the stationarity assumption. Thus the field power spectrum is given by

$$S(\omega) = \frac{{E_0}^2}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp\left[-\frac{\langle \Delta\varphi^2(\tau)\rangle}{2} + i\tau(\omega_0 - \omega)\right] \mathrm{d}\tau, \tag{1}$$

where $\langle \Delta\varphi^2(\tau)\rangle$ is the mean-square phase jitter, expressed by

$$\langle \Delta\varphi^2(\tau)\rangle = \frac{1}{\pi} \int_{-\infty}^{+\infty} \langle |\varphi(\omega)|^2\rangle (1 - \cos\omega\tau) \mathrm{d}\omega. \tag{2}$$

The phase change $\varphi(\omega)$ in the frequency domain is evaluated by solving the rate equations for the slave and master lasers[4,6] (the subscript m refers to the master laser),

$$\frac{\mathrm{d}P_m}{\mathrm{d}t} = \left(G_m - \frac{1}{\tau_{pm}}\right)P_m + R_m + F_{pm}, \tag{3}$$

$$\frac{\mathrm{d}\Phi_m}{\mathrm{d}t} = \frac{\alpha}{2}\left(G_m - \frac{1}{\tau_{pm}}\right) + F_{\varphi m}, \tag{4}$$

$$\frac{\mathrm{d}N_m}{\mathrm{d}t} = -\frac{N_m}{\tau_{em}} - G_m P_m + \frac{I_m}{e} + F_{nm}, \tag{5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \left(G - \frac{1}{\tau_p}\right)P + \frac{1}{L} v_g (P_i P)^{0.5} \cos\theta + R + F_p, \tag{6}$$

$$\frac{\mathrm{d}\Phi}{\mathrm{d}t} = (\omega_j - \omega_0) + \frac{1}{2L} v_g \left(\frac{P_i}{P}\right)^{0.5} \sin\theta + F_\varphi, \tag{7}$$

$$\frac{\mathrm{d}N}{\mathrm{d}t} = -\frac{N}{\tau_e} - GP + \frac{I}{e} + F_n. \tag{8}$$

P and P_m are the photon numbers in the slave and master laser cavities, respectively. P_i is the injected photon number and is related to P_m through $P_i = \eta P_m$, where $\sqrt{\eta}$ is the coupling coefficient between the master and slave laser electric fields. θ is the phase detuning, i.e., the difference between the phase of the optical field of the master laser Φ_m and that of the slave laser Φ. N and N_m denote the carrier numbers in each active volume, R and R_m are the spontaneous emission rates, and G and G_m are the gains per unit time. ω_0 is the stationary value of the optical frequency that is equal to that of the master, and ω_j is the resonant frequency of the jth longitudinal mode of the slave cavity. The Langevin forces $F_p(t)$, $F_{pm}(t)$, $F_\varphi(t)$, $F_{\varphi m}(t)$, $F_n(t)$, and $F_{nm}(t)$ are noise sources accounting for fluctuations in P, P_m, Φ, Φ_m, N, and N_m, respectively. In the following analysis, the linewidth enhancement factors of both lasers are assumed identical, because the same type of diode was used experimentally for the master and slave lasers.[4]

We call $P(t)$, $n(t)$, $\varphi(t)$, $p_m(t)$, $n_m(t)$, and $\varphi_m(t)$ the deviations of $P(t)$, $N(t)$, $\Phi(t)$, $P_m(t)$, $N_m(t)$, and $\Phi_m(t)$ from their steady-state values P, N, Φ, P_m, N_m, and Φ_m. Within a first-order approximation,[6] we have

$$G_{(m)} = G_{0(m)} + G_{n(m)} n_{(m)}, \tag{9}$$

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$$\omega_j \simeq \omega_{0j} + \frac{\alpha G_n n}{2}. \quad (10)$$

Equations (3)–(8) are linearized and solved by Fourier analysis, leading to the expression of the phase change $\varphi(\omega)$,

$$\varphi(\omega) = \frac{F_n'(\omega)[(i\omega + K)E - BC] + F_\varphi'(\omega)[(H + i\omega)(K + i\omega) + FB] - F_p'(\omega)[(i\omega + H)C + EF]}{L(\omega)}, \quad (11)$$

where

$$L(\omega) = FAE + FBD + HKD + HAC - \omega^2(H + D + K) + i\omega(FB + HD + HK + KD + AC - \omega^2),$$

$$A = 2Pk\sin\theta, \quad B = G_n P, \quad C = \frac{k\sin\theta}{2P},$$

$$D = k\cos\theta, \quad E = \frac{\alpha G_n}{2}, \quad F = G_0,$$

$$H = \frac{1}{\tau_e} + G_n P, \quad K = \frac{R}{P} + k\cos\theta,$$

$$k = \frac{1}{2L} v_g (P_i/P)^{0.5},$$

$$F_n'(\omega) = F_n(\omega),$$

$$F_p'(\omega) = \frac{P}{P_m} k\cos\theta\, p_m(\omega) - 2Pk\sin\theta\, \varphi_m(\omega) + F_p(\omega),$$

$$F_\varphi'(\omega) = \frac{k}{2P_m}\sin\theta\, p_m(\omega) + k\cos\theta\, \varphi_m(\omega) + F_\varphi(\omega).$$

$F_\varphi'(\omega)$, $F_n'(\omega)$, and $F_p'(\omega)$ are the frequency-domain Langevin forces for the injected laser including the influence of the master laser noise, and k is a normalized injection rate. The term $\langle|\varphi(\omega)|^2\rangle$ in Eq. (2) is then calculated by using the diffusion coefficient for the Langevin forces in the master laser, which are written[6]

$$D_{pmpm} = R_m P_m, \quad D_{pmnm} = -R_m P_m,$$

$$D_{nmnm} = R_m P_m + \frac{N_m}{\tau_{em}}, \quad D_{\varphi m\varphi m} = \frac{R_m}{4P_m}.$$

whereas those of the locked laser are given by

$$D_{pp} = RP, \quad D_{pn} = -RP,$$

$$D_{nn} = RP + \frac{N}{\tau_e}, \quad D_{\varphi\varphi} = \frac{R}{4P}.$$

The mean-square phase jitter $\langle\Delta\varphi^2(\tau)\rangle$ is evaluated by contour integration, closing the contour in the upper half of the complex frequency plane. The contour encloses several poles at complex frequencies $\omega = 0, ix, iy \pm z$, and $iy_m \pm z_m$. The real part of ω denotes the relaxation oscillation frequency, and the imaginary part of ω denotes the damping rate. If we neglect the high-frequency master laser phase noise contribution, the mean-square phase jitter of the locked laser is written as

$$\langle\Delta\varphi^2(\tau)\rangle = \frac{R_m(1+\alpha^2)\tau}{2P_m} + \frac{(1+\alpha^2)}{2x}\left(\frac{R}{P} - \frac{R_m}{P_m}\right)(1 - \exp - \tau x) + R\alpha^2\left(1 + \frac{k^2\{[z^2 + y^2 - z_{om}^2 \sin\theta(\alpha\cos\theta + \sin\theta)]^2 + z_{om}^4 \sin^2\theta(\cos\theta - \alpha\sin\theta)^2\}}{4Py\{z_{om}^2[(z^2 + y^2 - z_{om}^2)^2 + 4(y^2 + z^2)y_m^2]\}}\right)[1 - \exp(-\tau y)\cos(z\tau)], \quad (12)$$

where

$$x = k(\alpha\sin\theta + \cos\theta),$$

$$y = \frac{G_n P}{2} + \frac{1}{2\tau_2} + \frac{k}{2}(\cos\theta - \alpha\sin\theta),$$

$$z^2 = z_{oe}^2\left(1 - \frac{R}{P}\tau_p - 2k\tau_p\cos\theta\right) - y^2,$$

$$y_m = \frac{G_{nm}P_m}{2} + \frac{1}{2\tau_{em}},$$

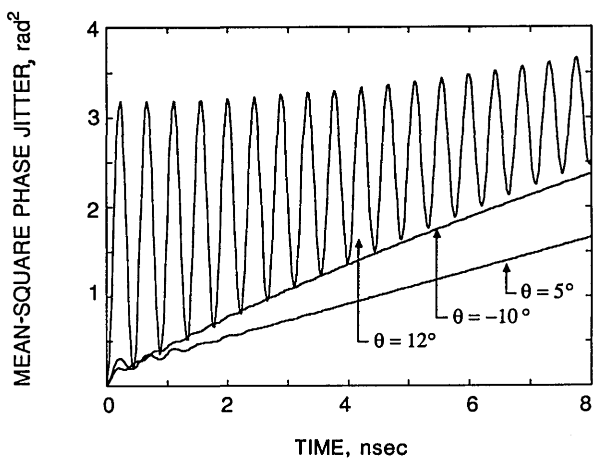

Fig. 1. Mean-square phase jitter $\langle\Delta\varphi^2(\tau)\rangle$ of the locked laser: $\langle\Delta\varphi^2(\tau)\rangle$ versus time τ at an output power of 2 mW for the locked laser, an output power of 4 mW for the master laser, and an injection level P_i/P of −30 dB for different values of the phase detuning θ.

Table 1. Laser Parameters Used

λ_0	Wavelength	830 nm
L	Cavity length	300 μm
v_g	Group velocity of light	6.9×10^7 msec^{-1}
α	Linewidth enhancement factor	5.4
G_n	Differential gain	5.75×10^3 sec^{-1}
τ_p	Photon lifetime	1.7×10^{-12} sec
τ_e	Spontaneous carrier lifetime	2.2×10^{-9} sec

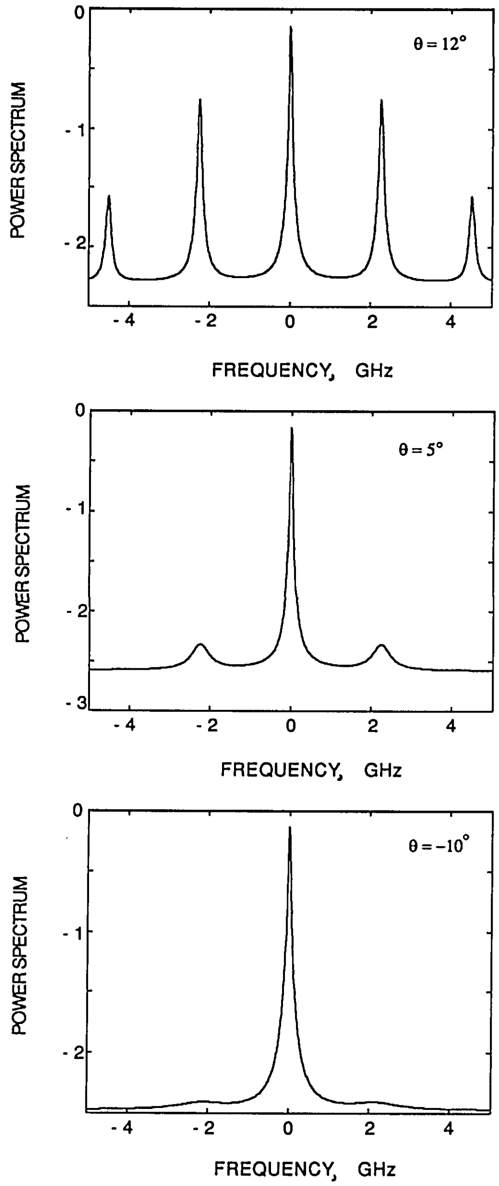

Fig. 2. Power spectra: logarithmic relative intensity versus the frequency for an output power of the locked laser of 2 mW, an output power of the master laser of 4 mW, and an injection level P_i/P of −30 dB for different values of the phase detuning θ.

$$z_{oe}^2 = \frac{G_n P}{\tau_p}, \qquad z_{om}^2 = \frac{G_{nm} P_m}{\tau_{pm}}.$$

This expression is valid for injection rates smaller than 10^{-2}, which is the case with the reported experimental injection rates.[4,7] Nonlinear gain effects are not included in the model, thus our results may underestimate the damping of the phase jitter. Figure 1 shows the mean-square phase jitter of an injection-locked laser when it is operated through the stable locking range. The values for the parameters used are given in Table 1. In the calculations, the total cavity loss rates, the carrier lifetimes, and the differential gains are taken to be identical for both lasers. We focus our attention on the influence of the phase detuning θ on the phase jitter. The damping of the mean-square phase jitter oscillation and, consequently, that of the relaxation oscillation is increased dramatically toward the lower limit of the stable locking range ($\theta = -10°$). However, this damping is strongly reduced at the upper limit of the stable locking range ($\theta = 12°$), where the locked laser becomes unstable: the mean-square phase jitter exhibits a strong oscillation, as shown in Fig. 1. Figure 2 represents the corresponding power spectra evaluated with a fast-Fourier-transform algorithm. The first two terms in Eq. (12) cause a modification of the laser linewidth, while the third term results in the appearance of secondary peaks at the relaxation frequency of the locked laser. The relaxation frequency peaks are reduced at the lower limit of the stable locking range but strongly enhanced at the upper limit. This is in good agreement with the experimental results reported earlier.[4,7] Phase detuning values correspond to experimental values of detuning. The theory also qualitatively agrees with the data when reduced to the unlocked case. The asymmetry in the sidebands does not appear in our model because the amplitude fluctuations have been neglected.

In conclusion, an expression for the mean-square phase jitter of an injection-locked semiconductor laser has been derived. This expression may be useful for evaluating the performance of a system that includes an injection-locked semiconductor laser. Theoretical calculations of the power spectrum of the locked laser field are in good qualitative agreement with the experimental results reported earlier.

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