Locking range, phase noise and power spectrum of an injection-locked semiconductor laser

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Abstract: The locking range of an optically injected semiconductor laser is discussed, including the spectral hole burning and lateral carrier diffusion effects. These effects are modelled with a gain suppression coefficient. In addition, an expression for the mean square phase jitter of an injection-locked semiconductor laser is derived when these effects are insignificant. The power spectrum of a locked laser is obtained and compared with experimental results.

1 Introduction

Injection locking is an example of the intracavity interaction of an external radiation and a lasing field. This technique represents a great interest for semiconductor laser applications. It improves the laser's free running properties by reducing the partition noise [1], suppressing the mode hopping, improving the coherence properties [2] and reducing their sensitivity to spurious feedback. In addition, the technique allows a direct modulation of the injected laser with a reduction in the frequency chirp [3–5], improves the system performances [6], the conversion of frequency to phase modulation [7] and the achievement of optical carrier recovery.

In order to avoid penalties in optical fibre systems, transient relaxation oscillations should be considered since they generate sidebands in the optical spectrum and can widen the spectral linespread of the laser. Relaxation oscillations are caused by an intrinsic resonance in the gain saturation process and by the subsequent coupled fluctuations of light intensity and carrier density. For a synchronised laser their damping is strongly dependent on the locking conditions. The latter are usually expressed in terms of a locking range whose width is strongly dependent on the phase amplitude coupling. This dependence represents the main difference with the locking conditions of microwave oscillators [8–14]. Moreover, spectral hole burning and lateral carrier diffusion, treated analytically by a gain saturation term [15], are typical features of a semiconductor laser, and their influences on the locking conditions have to be considered.

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2 Locking range: influence of spectal hole burning and lateral carrier diffusion

2.1 Theory

Two semiconductor lasers are considered: a master and a slave. The light of the former is injected into the latter. An isolator prevents the reverse coupling. The operating conditions have been extensively reported [14, 16].

The theoretical analysis is based on the single mode rate equations written to take into account the dynamic behaviour of both the semiconductor lasers. Two sets of equations concerning, respectively, the master laser and the locked laser are considered. They relate the mode intensities, the optical phase and the numbers of carrier responsible for the gains of the two lasers [5, 17], and are written:

$$\frac{dP_m}{dt} = \left(G_m - \frac{1}{\tau_{pm}}\right)P_m + R_m + F_{pm} \tag{1}$$

$$\frac{d\Phi_m}{dt} = \frac{\alpha_m}{2}\left(G_m - \frac{1}{\tau_{pm}}\right) + F_{\varphi m} \tag{2}$$

$$\frac{dN_m}{dt} = -\frac{N_m}{\tau_{em}} - G_m P_m + \frac{I_m}{e} + F_{nm} \tag{3}$$

$$\frac{dP}{dt} = \left(G - \frac{1}{\tau_p}\right)P + \frac{1}{L} v_g(P_i P)^{0.5} \cos\theta + R + F_p \tag{4}$$

$$\frac{d\Phi}{dt} = (\omega_j - \omega_0) + \frac{1}{2L} v_g\left(\frac{P_i}{P}\right)^{0.5} \sin\theta + F_\varphi \tag{5}$$

$$\frac{dN}{dt} = -\frac{N}{\tau_e} - GP + \frac{I}{e} + F_n \tag{6}$$

P and P_m are the photon numbers in the slave and master cavities, respectively. The injected photon number P_i is related to P_m through $P_i = \eta P_m$ where $\eta^{0.5}$ is the coupling coefficient between the master and slave laser fields. θ is the difference between the phase of the optical field of the master laser Φ_m and that of the slave laser Φ, that is $\theta = \Phi_m - \Phi$. N and N_m stand for the carrier numbers in each active volume. R and R_m are the spontaneous emission rates, G and G_m the gains per unit time, τ_p and τ_{pm} the photon lifetimes, τ_e and τ_{em} the spontaneous carrier lifetimes, I/e and I_m/e the carrier injection rates and α_m is the linewidth enhancement factor of the master laser [18]. ω_0 is the stationary value of the optical frequency which is equal to that of the master and ω_j the resonant frequency of the jth longitudinal mode of the slave cavity. v_g is the group velocity of the light and L the cavity length. The Langevin forces $F_p(t)$, $F_{pm}(t)$, $F_\varphi(t)$,

$F_{\varphi m}(t)$, $F_n(t)$, $F_{nm}(t)$ are noise sources accounting for fluctuations in P, P_m, Φ, Φ_m, N, N_m.

The steady state values are obtained by setting the left parts of the eqns. 1–6 equal to 0 and neglecting the Langevin forces.

We call $p(t)$, $n(t)$, $\varphi(t)$, $p_m(t)$, $n_m(t)$, $\varphi_m(t)$ the deviation of $P(t)$, $N(t)$, $\Phi(t)$, $P_m(t)$, $N_m(t)$, $\Phi_m(t)$ from their steady state values P, N, Φ, P_m, N_m and Φ_m.

Within a first order approximation, we have [17]:

$$G = G_0 + G_n n + G_p p$$

$$G_m = G_{om} + G_{nm} n_m + G_{pm} p_m$$

$$\omega_j = \omega_{oj} + \omega_n n + \omega_p p \simeq \omega_{oj} + \frac{\alpha G_n n}{2} \quad (7)$$

where G_o, G_{om} and ω_{oj} are the steady state values of G, G_m and ω_j, respectively. α is the linewidth enhancement factor of the slave. $G_n = (\partial G/\partial N)$ and $G_{nm} = (\partial G_m/\partial N_m)$ are the differential gains. $G_p = (\partial G/\partial P)$ and $G_{pm} = (\partial G_m/\partial P_m)$ are the parameters standing for spectral hole burning [17]. It has been shown in Reference 15 that a narrow stripe laser with non-uniform electron density and lateral carrier diffusion is equivalent to a laser exhibiting no lateral diffusion but gain saturation instead. G_p and G_{pm} are also parameters standing for lateral carrier diffusion; $\omega_p = (\partial \omega_j/\partial P)$, expresses the relation between the resonant frequency and the number of photons. It can be neglected in eqn. 7 because the gain change resulting from spectral hole burning is nearly symmetric with the laser line [19] and the corresponding index change obtained by the Kramers-Kronig transform is nearly equal to zero. That is also the reason why G_m will be only expressed as $G_{om} + G_{nm} n_m$ in eqn. 2.

Eqns. 1–6 are linearised in terms of small deviations from the steady state values. The second order terms in n, n_m, p, p_m, φ, φ_m are neglected.

The Fourier analysis is then used to solve the linear system of equations through the use of the transform definition:

$$a(\omega) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} a(t) \exp(-i\omega t)\, dt$$

Eqns. 1–6 now become:

$$(\Gamma_{pm} + i\omega)p_m(\omega) - (G_{nm} P_m)n_m(\omega) = F_{pm}(\omega) \quad (8)$$

$$(G_{om} + P_m G_{pm})p_m(\omega) + (\Gamma_{nm} + i\omega)n_m(\omega) = F_{nm}(\omega) \quad (9)$$

$$(i\omega)\varphi_m(\omega) - \frac{\alpha_m}{2} G_{nm} n_m(\omega) = F_{\varphi m}(\omega) \quad (10)$$

$$(i\omega + \Gamma_p + k\cos\theta)p(\omega) - 2P(k\sin\theta)\varphi(\omega) - (G_n P)n(\omega) = \frac{P}{P_m}(k\cos\theta)p_m(\omega) - 2P(k\sin\theta)\varphi_m(\omega) + F_p(\omega) = F'_p(\omega) \quad (11)$$

$$(G_o + G_p P)p(\omega) + (\Gamma_n + i\omega)n(\omega) = F_n(\omega) = F'_n(\omega) \quad (12)$$

$$\frac{1}{2P}(k\sin\theta)p(\omega) + (k\cos\theta + i\omega)\varphi(\omega) - \frac{\alpha G_n}{2} n(\omega) = \frac{1}{2P_m}(k\sin\theta)p_m(\omega) + (k\cos\theta)\varphi_m(\omega) + F_\varphi(\omega) = F'_\varphi(\omega) \quad (13)$$

where:

$$\Gamma_{p(m)} = \frac{R_{(m)}}{P_{(m)}} - P_{(m)} G_{p(m)}$$

$$\Gamma_{n(m)} = G_{n(m)} P_{(m)} + \frac{1}{\tau_{e(m)}}$$

$$k = \frac{1}{2L} v_g \sqrt{(P_i/p)}$$

$F'_\varphi(\omega)$, $F'_n(\omega)$ and $F'_p(\omega)$ are the frequency domain Langevin forces for the injected laser including the master noise influence. Γ_p and Γ_n are the contributions to the damping rates of the slave, the subscript (m) stands for those of the master and k is the normalised injection rate.

Defining:

$$A = 2Pk\sin\theta$$

$$B = G_n P$$

$$C = \frac{k\sin\theta}{2P}$$

$$D = k\cos\theta$$

$$E = \frac{\alpha G_n}{2}$$

$$F = G_o + G_p P$$

$$H = \frac{1}{\tau_e} + G_n P$$

$$K = \Gamma_p + k\cos\theta$$

the Fourier transform of the phase change is then:

$$\varphi(\omega) = \frac{F'_n(\omega)[(i\omega + K)E - BC] + F'_\varphi(\omega) \times [(H + i\omega)(K + i\omega) + FB] - F'_p(\omega)[(i\omega + H)C + EF]}{L(\omega)} \quad (14)$$

where:

$$L(\omega) = (FAE + FBD + HKD + HAC) - \omega^2(H + D + K) + i\omega(FB + HD + HK + KD + AC - \omega^2) \quad (15)$$

The roots of the eqn. $L(\omega) = 0$ correspond to the complex frequencies $\omega = ix$, $iy \pm z$ where x and y represent the decay rates of the relaxation oscillations (they describe the transient behaviour necessary to saturate the locked laser gain) and z is the frequency of the relaxation oscillations.

They can be calculated by solving:

$$x + 2y = 2(\Gamma + k\cos\theta) \quad (16)$$

$$2xy + y^2 + z^2 = z_{oe}^2\left(1 + G_p P\tau_p - \frac{R}{P}\tau_p - 2k\tau_p\cos\theta\right) + \Gamma_p\Gamma_n + 4\Gamma k\cos\theta + k^2 \quad (17)$$

$$x(y^2 + z^2) = k\left[z_{oe}^2\left(1 + G_p P\tau_p - \frac{R}{P}\tau_p - 2k\tau_p\cos\theta\right) \times (\alpha\sin\theta + \cos\theta) + \Gamma_p(\Gamma_n\cos\theta + k) + 2\Gamma k\right] \quad (18)$$

$$\Gamma = \frac{\Gamma_n + \Gamma_p}{2} \tag{19}$$

$$z_{oe}^2 = \frac{G_n P}{\tau_p} \tag{20}$$

An analytical solution can be obtained in the case of weak injection, i.e. $k \ll z_{oe}$, and $\Gamma \ll z_{oe}$; then the decay rates x, y and the frequency of relaxation oscillations z reduce to:

$$x = k(\alpha \sin\theta + \cos\theta) \tag{21}$$

$$y = \Gamma + k\frac{\cos\theta - \alpha\sin\theta}{2} \tag{22}$$

$$z^2 = z_{oe}^2\left(1 + G_p P\tau_p - \frac{R}{P}\tau_p - 2k\tau_p\cos\theta\right) - y^2 \tag{23}$$

Stability is achieved when, after a perturbation, the laser returns to its stationary values after an unperiodic behaviour or damped relaxation oscillations. In mathematical terms, the roots of the equation $L(\omega) = 0$ must be in the upper half of the complex frequency plane. In addition, the net gain $G - 1/\tau_p$ must be negative in order to have a forced oscillation prevailing over the amplification of the spontaneous emission [12]. The locking range can be expressed in terms of a phase detuning θ, which can be easily related to a frequency detuning $\Delta\omega$ through:

$$\Delta\omega = \omega_o - \omega_{jo} = k(\sin\theta - \alpha\cos\theta)$$

where ω_o is the optical frequency of the master and ω_{jo} the resonant frequency of the jth longitudinal mode for the free running laser.

2.2 Discussion

The parameters used in the calculations correspond to some 0.83 μm weakly index guided CSP AlGaAs Hitachi 1400 semiconductor lasers used in the experiment. The active region is 300 μm long, its volume 1.2×10^{-10} cm^{-3}. The photon lifetime is $\tau_p = n_g c^{-1}[\alpha_a + 1/L \log(1/R_{mr})]^{-1}$ where $n_g = 4.3$ is the group index, c the velocity of light, $\alpha_a = 45$ cm^{-1} the internal distributed loss and $R_{mr} = 0.31$ the mirror reflectivity. $\alpha = 5.4$ is the linewidth enhancement factor. This value originates from locking range observations [14] and linewidth measurements [20] for the same type of laser. The active region gain is evaluated by using $G = G_n(N - N_o)$ where $G_n = 5.75 \times 10^3$ s^{-1} is the differential gain and $N_o = 1.7 \times 10^8$ the carrier number at transparency. The spontaneous emission rate R is related to the gain G by $R \cong n_{sp} G$, where $n_{sp} = 2.6$ is the spontaneous emission factor. The carrier recombination rate is N/τ_e where $\tau_e = 2.2 \times 10^{-9}$ s. The bias current I is chosen to correspond to an average output power with injection P_o of 2 mW per facet, and is related to the number of intracavity photons P through $P = 2LP_o n_g/c \cdot h\nu_o \cdot \log(1/R)$. The gain suppression coefficient is $\zeta = G_p P/G$.

The stable locking range is calculated by ensuring that the roots the equation $L(\omega) = 0$ are in the upper half of the complex frequency plane. Fig. 1 shows that the stable locking range widens as the injection level increases, adding to the results of Mogensen *et al.* [11] and in agreement with Lang [9].

The spectral hole burning and the lateral carrier diffusion are phenomena enlarging the stable locking range as shown in Fig. 2 where the stable locking range has been calculated for different gain suppression coefficients ζ in the weak injection level approximation. The maximum injection level under which a quite large phase difference can be obtained increases with increasing gain suppression coefficient.

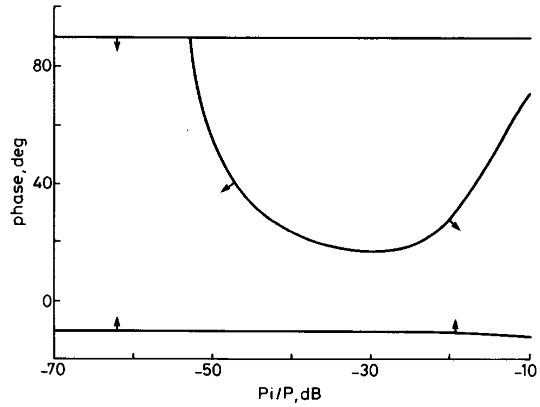

Fig. 1 *Stability range diagram*

An extension of the stability range at high injection levels, for a gain suppression coefficient $\zeta = 0.1\%$. Heads of arrows indicate the stability range

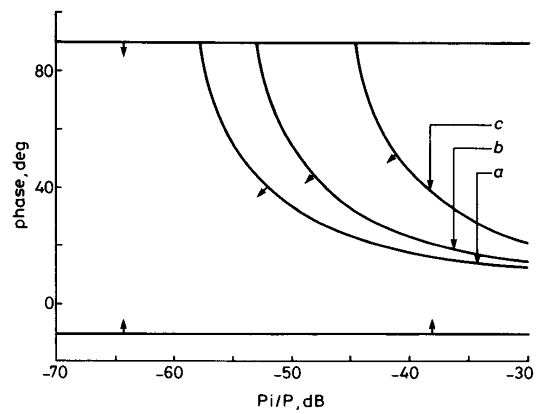

Fig. 2 *Stability range diagram*

An influence of spectral hole burning and lateral carrier diffusion on the stability range for different values of the gain suppression coefficient ζ: (*a*) $\zeta = 0\%$, (*b*) $\zeta = 0.1\%$, (*c*) $\zeta = 0.5\%$

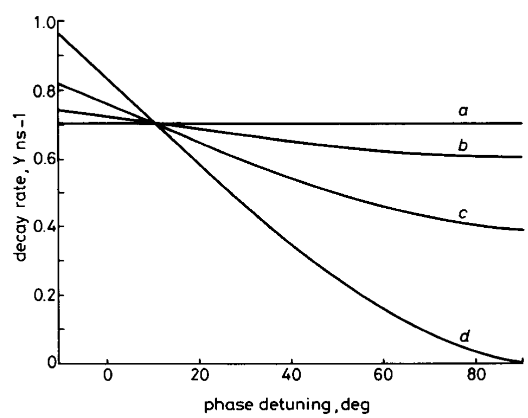

Fig. 3 *Decay rate of relaxation oscillations*

An influence of the injection rate Pi/P on the decay rate y through the stability range at an output power P_o of 2 mW and for different values of the injection rate: (*a*) $P_i/P = 0$, (*b*) $P_i/P = -70$ dB, (*c*) $P_i/P = -60$ dB, (*d*) $P_i/P = -53$ dB

For coherent communication applications using optical phase modulation, it appears that the maximum phase range is limited by stability considerations to a value slightly higher than $\pi/2$ for the value of α considered. In that particular operating region, the decay rate of relaxation oscillations y_{fr} for the free running slave laser has been compared with its injection locked value y, for an output power of 2 mW (Fig. 3). Particular attention must be paid to the working injection level: if the decay rate y decreases dramatically, the power spectrum exhibits high lateral peaks (see Section 3). This is critical for long-haul transmission in dispersive optical fibre.

3 Phase jitter in an injection locked semiconductor laser

3.1 Analytical approach

An expression for the mean square phase jitter caused by noise is derived in this Section. It shows the effects of the dynamics of the master and of the locked laser on the power spectrum. Studies on noise in injection locked semiconductor lasers, i.e. on frequency and intensity noise and on side mode suppression and relative intensity noise, have been reported elsewhere [21–23]. The present theory is based on that of Spano *et al.* [22], and includes the derivations of the phase jitter and the laser power spectra.

Following Henry [17], we neglect amplitude fluctuations above threshold. These fluctuations lead to an asymmetry in the side peak intensities resulting from a correlation between phase and amplitude fluctuations [24].

The field is then expressed as $E(t) = E_o \exp i\{\omega_o t + \varphi(t)\}$ where E_o is real. The autocorrelation function of the field $R_{EE}(\tau)$ may be written, under a stationary phase noise assumption:

$$R_{EE}(\tau) = E_o^2 \langle \exp i\, \Delta\varphi(t, \tau) \rangle \exp i\omega_o \tau \qquad (24)$$

where

$$\Delta\varphi(t, \tau) = \varphi(t + \tau) - \varphi(t) \qquad (25)$$

is the phase jitter.

As $\Delta\varphi(t, \tau)$ is known to have a Gaussian distribution [25, 26], it can be shown [27] that:

$$\langle \exp i\, \Delta\varphi(t, \tau) \rangle = \exp\left(- \frac{\langle \Delta\varphi^2(\tau) \rangle}{2} \right) \qquad (26)$$

thus the spectrum may be written as:

$$S(\omega) = \frac{E_o^2}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} \exp\left\{ - \frac{\langle \Delta\varphi^2(\tau) \rangle}{2} + i\tau(\omega_o - \omega) \right\} d\tau \qquad (27)$$

In order to evaluate $\langle \Delta\varphi^2(\tau) \rangle$, $\Delta\varphi(t, \tau)$ is expressed by its Fourier transform:

$$\Delta\varphi(t, \tau) = \frac{1}{\sqrt{(2\pi)}} \int_{-\infty}^{+\infty} \varphi(\omega) \exp(i\omega t)[\exp(i\omega\tau) - 1]\, d\omega \qquad (28)$$

$\varphi(\omega)$ depends on the Langevin forces which describe the random phase, intensity, and current fluctuations and are modelled by Diracs functions [17] as:

$$\langle \varphi(\omega)\varphi^*(\omega) \rangle = \langle |\varphi(\omega)|^2 \rangle \delta(\omega - \omega') \qquad (29)$$

by squaring eqn. 28 we obtain:

$$\langle \Delta\varphi^2(\tau) \rangle = \frac{1}{\pi} \int_{-\infty}^{+\infty} \langle |\varphi(\omega)|^2 \rangle (1 - \cos\omega\tau)\, d\omega \qquad (30)$$

For simplification, new parameters are defined as follows:

$$A' = \frac{P}{P_m} k \cos\theta$$

$$B' = -2Pk \sin\theta$$

$$C' = \frac{k}{2P_m} \sin\theta$$

$$D' = k \cos\theta$$

$$a = (KE - BC) + i\omega E$$

$$b = (HK + FB - \omega^2) + i\omega(H + K)$$

$$c = (HC + EF) + i\omega C$$

$$d = (C'EF + C'HK - A'EF - A'HC - C'\omega^2) + i\omega(C'H + C'K - A'C)$$

$$e = (D'EF + D'HK - B'EF - B'HC - D'\omega^2) + i\omega(D'H + D'K - B'C)$$

Leading to:

$$L(\omega)L^*(\omega)\langle |\varphi(\omega)|^2 \rangle = aa^* \langle F_n(\omega)F_n^*(\omega) \rangle + bb^* \langle F_\varphi(\omega)F_\varphi^*(\omega) \rangle + cc^* \langle F_p(\omega)F_p^*(\omega) \rangle - 2\Re e(ac^* \langle F_n(\omega)F_p^*(\omega) \rangle) + dd^* \langle p_m(\omega)p_m^*(\omega) \rangle + ee^* \langle \varphi_m(\omega)\varphi_m^*(\omega) \rangle + \Re e(ed^* \langle p_m(\omega)\varphi_m^*(\omega) \rangle) \qquad (31)$$

where $\Re e(x)$ is the real part of the variable x. $F_n(\omega)$, $F_\varphi(\omega)$, $F_p(\omega)$ are the frequency domain Langevin forces characterising noise in the locked laser, and $p_m(\omega)$ and $\varphi_m(\omega)$ intensity and phase linked to the Langevin forces characterising noise in the master laser.

In the following analysis, the influence of a gain dependence on the number of photons is neglected. Consequently, the calculation of the mean square phase jitter is simplified greatly. This has obviously an effect on the quantitative results since the damping has been modified. However, the general behaviour is not significantly altered. The linewidth enhancement factors of the slave and master lasers are assumed to be similar as the same type of laser diodes are used.

The diffusion coefficients for the Langevin forces in the master laser can be written [17]:

$$D_{pmpm} = R_m P_m \qquad D_{pmnm} = -R_m P_m$$

$$D_{\varphi m \varphi m} = \frac{R_m}{4P_m} \qquad D_{nmnm} = R_m P_m + \frac{N_m}{\tau_{em}}$$

whereas those of the locked laser are given by:

$$D_{pp} = RP \qquad D_{pn} = -RP$$

$$D_{\varphi\varphi} = \frac{R}{4P} \qquad D_{nn} = RP + \frac{N}{\tau_e}$$

where N/τ_e and N_m/τ_{em} are the carrier recombination rates, R and R_m the spontaneous emission rates.

Now $\langle \Delta\varphi^2(\tau) \rangle$ can be evaluated by contour integration, providing that $\lim_{|\omega| \to \infty} \omega \langle |\varphi(\omega)|^2 \rangle = 0$, i.e. that $\langle \Delta\varphi^2(\tau) \rangle$ converges. Therefore:

$$\langle \Delta\varphi^2(\tau) \rangle = \Re e\{2i(\textstyle\sum c_i - \sum d_i)\} \qquad (32)$$

where

$$c_i = \text{Res}\ \{\langle|\varphi(\omega)|^2\rangle\}$$

and

$$d_i = \text{Res}\ \{\langle|\varphi(\omega)|^2\rangle \exp(i\omega\tau)\}$$

$\Re e(x)$ is required because the cosine function in eqn. 30 is not bounded in any half plane. The summation is extended over all the poles ω_i located in the upper half of the complex frequency plane. c_i and d_i are the residues of $\langle|\varphi(\omega)|^2\rangle$ and $\langle|\varphi(\omega)|^2\rangle \exp(i\omega\tau)$, respectively, at the pole ω_i.

The double pole at the frequency $\omega = 0$ is handled by setting $\omega = \lim_{\varepsilon\to 0}(\omega \pm i\varepsilon)$, leading to:

$$2i\ \text{Res}\ (\omega = 0) = \frac{R_m(1+\alpha^2)\tau}{2P_m} \tag{33}$$

The simple pole at the frequency $\omega = ix$ gives:

$$2i\ \text{Res}\ (\omega = ix) = \frac{(1+\alpha^2)}{2x}\left(\frac{R}{P} - \frac{R_m}{P_m}\right)[1 - \exp(-\tau x)] \tag{34}$$

The two poles at frequencies $\omega = iy_m \pm z_m$ issued from the zeros of $\Delta(\omega)$ for the master laser (see Appendix) are obtained in the same way as $L(\omega)$, and give:

$$2i\ \text{Res}\ (\omega = iy_m \pm z_m) = \frac{R_m k^2[(1+\alpha^2)\sin\theta z_{oe}^2 - (\alpha\cos\theta + \sin\theta)z_{om}^2]^2}{4P_m y_m z_{om}^2[(z_{om}^2 - z_{oe}^2)^2 + 4z_{om}^2 y^2]} \times [1 - \exp(-\tau y_m)\cos(\tau z_m)] \tag{35}$$

where

$$z_{om}^2 = z_m^2 + y_m^2$$

The two poles at frequencies $\omega = iy \pm z$ issued from the zeros of $L(\omega)$ give:

$$2i\ \text{Res}\ (\omega = iy \pm z) = R\alpha^2\left\{1 + \frac{k^2[z^2 + y^2 - z_{om}^2\sin\theta(\alpha\cos\theta + \sin\theta)]^2 + z_{om}^4\sin^2\theta(\cos\theta - \alpha\sin\theta)^2]}{4Py\{z_{om}^2[(z^2 + y^2 - z_{om}^2)^2 + 4(y^2 + z^2)y_m^2]\}}\right\} \times [1 - \exp(-\tau y)\cos(z\tau)] \tag{36}$$

We now define the lower and the upper limit of the stability range as the phase detuning for which the decay rate x in eqn. 21 is zero and the phase detuning for which the decay rate y in eqn. 22 is zero, respectively.

Eqns. 33 and 34 lead to the static power spectrum. As the phase detuning approaches the upper limit of the stability range the master imposes its linewidth on the slave laser, whereas by detuning towards the lower limit, the linewidth becomes that of the free running laser [28].

Eqn. 35 takes into account dynamics associated with the master laser. It appears that a reduction in the effect of the dynamics of the master laser, for any arbitrary phase detuning in the stability range, requires an increase of the average output power P_m of the master laser or an operation at low injection level. When the mode intensities of the slave and master lasers are equal, the dynamics of the master laser have no effect on the mean phase square jitter for a phase detuning $\theta = \tan^{-1}(1/\alpha)$; but when the upper limit is neared, the contribution of the master dynamics to the mean square phase jitter increases, leading to enhanced sidebands in the power spectrum.

Eqn. 36 accounts for dynamics associated with the locked laser. When the phase detuning approaches the lower limit of the stability range, the mean square phase jitter undergoes strongly damped oscillations at the relaxation frequency z, whereas they are weakly damped near the upper limit. For small values of the normalised injection rate k in relation to the decay rate Γ, the phase jitter of the locked laser approaches that of the free running laser.

The mean square phase jitter is then obtained by summing up the terms of eqns. 33–36.

3.2 Numerical results

The parameters chosen for the calculations are those used in Section 2.2. The total cavity loss rates, the carrier lifetimes and the differential gains are assumed to be identical for the slave and master lasers because attention is paid to the effects of the phase detuning and the injection level on the power spectrum. Fig. 4 shows the mean

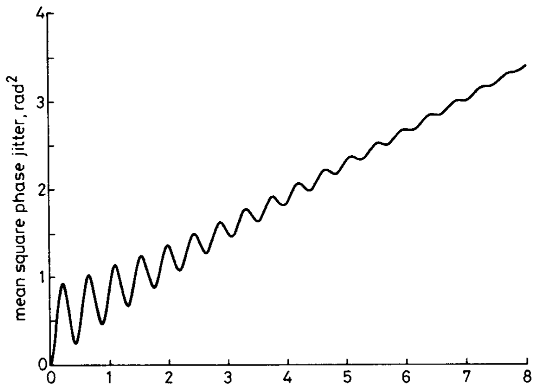

Fig. 4 *Mean square phase jitter $\langle\Delta\varphi^2(\tau)\rangle$ of the free-running laser*

square phase jitter of the free running slave laser for an output power of 2 mW. The mean square phase jitters of the locked laser for the same output power is shown in Fig. 5 and Fig. 6 for different values of the phase detuning θ. The output power of the master laser is 3 mW at

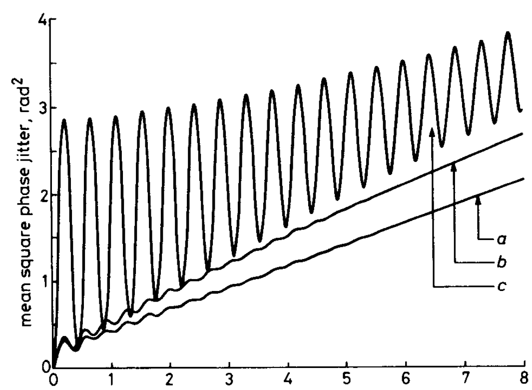

Fig. 5 *Mean square phase jitter $\langle\Delta\varphi^2(\tau)\rangle$ of the locked laser*
Injection rate $P_i/P = -30$ dB for different values of phase detunings: (*a*) $\theta = -10°$, (*b*) $\theta = 0°$, (*c*) $\theta = 12°$

injection levels of -30 dB in Fig. 5 and -60 dB in Fig. 6. The behaviour of the locked laser can be understood by comparing the decay rate y in eqn. 22 with the decay rate Γ in the same equation: when θ lies in the interval $[-\pi/2 + \tan^{-1}\alpha,\ \pi/2 - \tan^{-1}\alpha]$, i.e. $[-10.5°,\ 10.5°]$ with $\alpha = 5.4$, the damping of relaxation oscillations for the locked laser is stronger than that for the free running

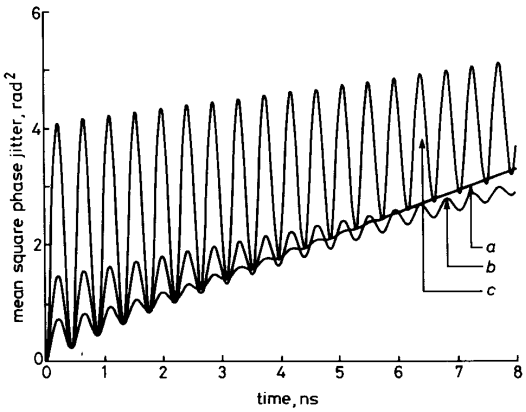

Fig. 6 *Mean square phase jitter $\langle \Delta\varphi^2(\tau) \rangle$ of the locked laser*
Injection rate $P_i/P = -60$ dB for different values of phase detunings: (*a*) $\theta = -10°$, (*b*) $\theta = 40°$, (*c*) $\theta = 90°$

laser. It is the case in Fig. 5 with curves *a* and *b* and in Fig. 6 with curve *a*. In Fig. 5 curve *c*, the phase detuning approaches the upper limit of the stable locking range: the decay rate y, given by eqn. 22 decreases towards a zero value. It results in the vanishing of the damping of the relaxation oscillations. At the upper limit of the stable locking range, the locked laser becomes unstable, i.e. multimode [11]. Under a critical injection level, the locked laser is stable on its whole locking range. At the value $\theta = \pi - \tan^{-1}\alpha$, the decay rate y has a minimum value. When the phase detuning θ increases towards this value but remains in the locking range, the mean square phase jitter shows enhanced oscillations, as shown in Fig. 6 with curves *b* and *c*.

Power spectra are calculated by using a fast Fourier transform algorithm. Figs. 7, 8 and 9 represent power

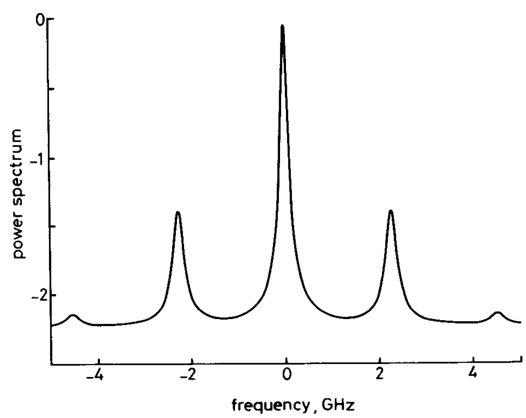

Fig. 7 *Power spectrum of the free-running laser*

spectra corresponding to the mean square phase jitters displayed in Figs. 4, 5 and 6, respectively. The influence of the different components of the mean square phase jitters, given by eqns. 33–36, is illustrated in these figures: the modification of the linewidth, the peaks at the relaxation frequency of the master laser and at the relaxation frequency of the locked laser. The damping of the relaxation frequency peaks varies strongly with the injection level and the phase detuning. In Fig. 10, the output

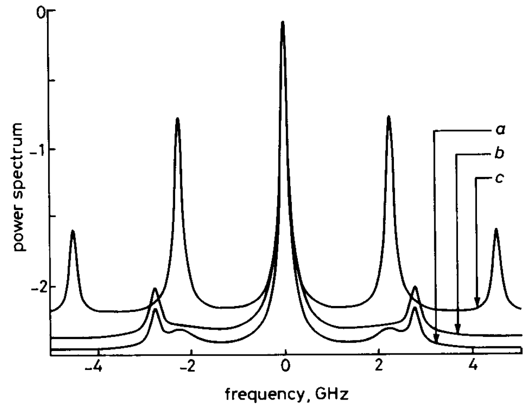

Fig. 8 *Power spectra of the locked laser*
Injection rate $P_i/P = -30$ dB for different values of phase detunings: (*a*) $\theta = 0°$, (*b*) $\theta = -10°$, (*c*) $\theta = 12°$

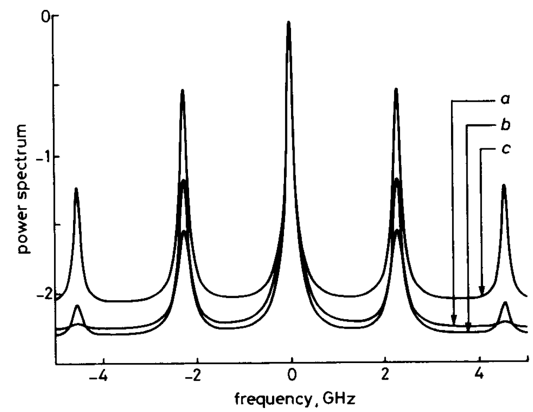

Fig. 9 *Power spectra of the locked laser*
Injection rate $P_i/P = -60$ dB for different values of phase detunings: (*a*) $\theta = -10°$, (*b*) $\theta = 40°$, (*c*) $\theta = 90°$

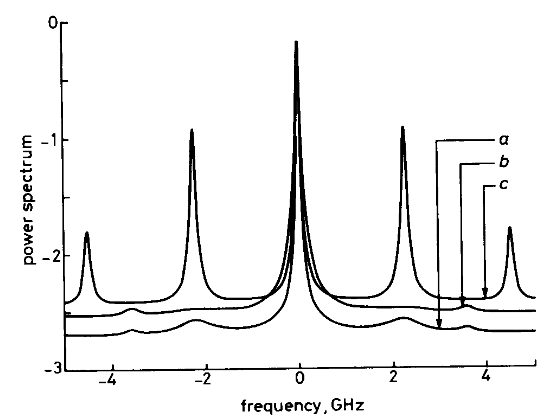

Fig. 10 *Power spectra of the locked laser*
Injection rate $P_i/P = -30$ dB for different values of phase detunings: (*a*) $\theta = 0°$, (*b*) $\theta = -10°$, (*c*) $\theta = 12°$

power of the master laser has been increased, when compared with Fig. 8. This results in a reduction of the damping time and an increase in the frequency of the relaxation oscillations of the master laser, as shown in the power spectra of the locked laser in curves *a* and *b*. But the increase in the output power of the master laser does not influence the linespread of the locked laser when the phase detuning approaches the upper limit of the stable locking range.

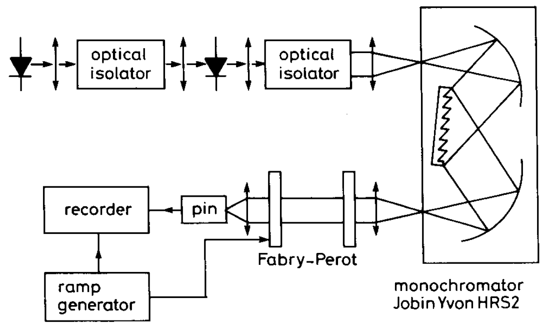

Fig. 11 *Experimental setup*

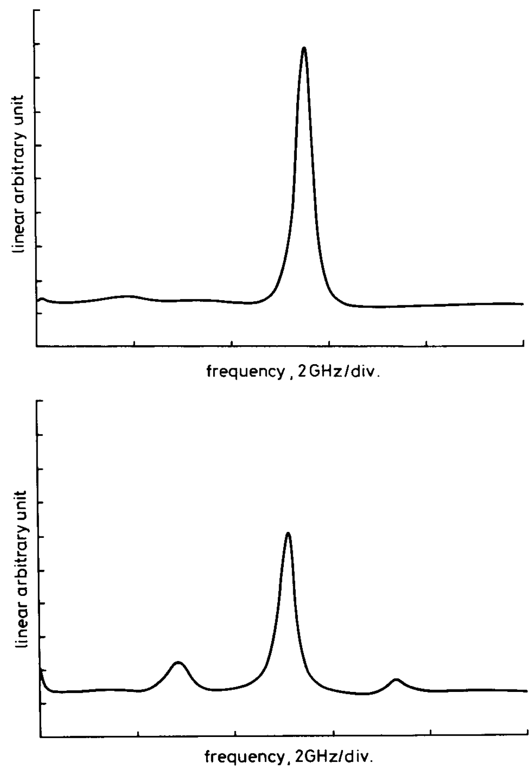

Fig. 12 *Experimental power spectra of the locked laser*
Operating point is moved from (*a*) lower limit, (*b*) to upper limit of locking range

3.3 Experimental results

The experimental measurements were carried out using two CSP-AlGaAs lasers (Hitachi 1400), thermally stabilised within a few hundredths of degrees by thermoelectric elements. The oscillator threshold currents are 53 mA for the master laser and 51 mA for the slave laser. Fig. 11 represents the experimental setup. The output beam passes through a spectrometer in order to filter the spontaneous emission and eliminate sidemode effects in the Fabry Perot interferometer. The locking of the slave laser is performed by tuning the optical frequency of either the master or slave laser. The tuning is performed by changing the temperatures or the bias currents of the lasers. The injection level is determined by the method described by Kobayashi and Kimura [29].

Before the locking occurs, for phase detunings under the lower limit of the locking range, the optical injection induces a modulation in the free-running signal [30]. When the locking occurs, the operating point is moved through the locking range by adjusting the bias current of the slave laser. It results in a slight variation of the relaxation frequency of the locked laser. At the upper limit of the stable locking range, the locked laser becomes unstable, i.e. the laser becomes multimode and the damping of the relaxation oscillations disappears. Fig. 12 shows power spectra as the phase detuning is scanned from the lower limit to the upper limit of the stable locking range. The amount of current adjustment necessary to tune through the stable locking range is 0.3 mA. As the locking range is small enough to be scanned with little change in the output power of the slave laser, the injection level is nearly constant at -33 dB. The bias currents are 59 mA for the slave laser, and 67 mA for the master laser. The master laser exhibits sidebands at a higher frequency than the slave laser because it is operated at a higher bias current. These observations qualitatively agree with the theoretical results, except for the asymmetry arising from amplitude fluctuations, not considered in our model.

4 Conclusion

A detailed study of stability of an optically injected semiconductor laser has been carried out, considering phenomena such as spectral hole burning and lateral carrier diffusion. It has been shown that they can result in an increased stable locking range. An expression for the mean square phase jitter of an injection-locked semiconductor laser has been derived when these effects are insignificant. The expression shows that the noise behaviour of the locked laser is dramatically affected by the injection level and the phase detuning between the slave and master laser fields. Power spectra are then calculated and show good agreement with experimental results.

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6 Appendix

Expressions of the mean square intensity, phase and crossed fluctuations pertaining to the master laser:

$$\Delta\Delta^*\langle p_m p_m^*\rangle = 2(\Gamma_{nm}^2 + \omega^2)D_{pmpm} + 4\Gamma_{nm} G_{nm} P_m D_{nmpm} + 2G_{nm}^2 P_m^2 D_{nmnm}$$

$$\Delta\Delta^*\omega^2\langle \varphi_m \varphi_m^*\rangle = 2((G_{nm} P_m G_m - \omega^2)^2 + (\Gamma_{nm}\omega)^2)D_{\varphi m\varphi m} + \frac{\alpha_m^2}{2}\omega^2 G_{nm}^2 D_{nmnm} + \frac{\alpha_m^2}{2} G_{nm}^2 G_m^2 D_{pmpm}$$

$$\Delta\Delta^* i\omega\langle p_m \varphi_m^*\rangle = -\alpha_m G_{nm}(\omega^2 D_{nmpm} - G_m \Gamma_{nm} D_{pmpm} - G_{nm} G_m P_m D_{nmnm}) + i\omega\alpha_m G_{nm}(\Gamma_{nm} D_{nmpm} + G_m D_{pmpm} + G_{nm} P_m D_{nm} n_{nm})$$

$$\Delta = G_{nm} G_m P_m - \omega^2 - i\omega\Gamma_{nm}$$

the roots of Δ are $iy_m \pm z_m$:

$$y_m = \frac{\Gamma_{nm}}{2}$$

$$z_m = \sqrt{(G_m G_{nm} P_m - y_m^2)}$$