Analysis of the Phase-Amplitude Coupling Factor and Spectral Linewidth of Distributed Feedback and Composite-Cavity Semiconductor Lasers

GUANG-HUA DUAN, PHILIPPE GALLION, MEMBER, IEEE, AND GUY DEBARGE

Abstract—The method for the analysis of semiconductor lasers based on a Green's function approach is developed in a form suitable for complex-cavity structures. Besides the spontaneous emission rate, the effective phase-amplitude coupling factor can also be accurately evaluated. The application of this method to distributed feedback (DFB) and composite-cavity lasers gives interesting new results. For DFB lasers, the spontaneous emission rate is strongly dependent on both the facet reflectivities and the grating coupling coefficient. The effective phase-amplitude coupling factor depends on the wavelength detuning from the gain maximum. The calculated linewidth of DFB lasers differs considerably from previous results and gives a better agreement with reported experimental results. For composite-cavity lasers, the frequency dependence of the equivalent reflectivity has a strong impact on the phase-amplitude coupling factor and the spontaneous emission rate. Distributed Bragg reflector (DBR) lasers are investigated as an example of a composite-cavity structure. An optimum grating coupling coefficient in the Bragg region is found which minimizes the spectral linewidth. Negative detuning from the Bragg frequency results in low phase-amplitude coupling factor and narrow spectral linewidth. This method is also useful for lasers with multiple active sections.

I. Introduction

Narrow linewidth semiconductor lasers are key components in coherent optical communication and optical sensing systems [1]. To fulfill this requirement, great progress has been made in the development of such semiconductor lasers in recent years. Single or multiple section distributed feedback (DFB) and distributed Bragg reflector (DBR) lasers are of current interest, as they provide narrow linewidth, wavelength tunability, and low frequency chirp [2]–[5]. In the optimum design of these complex structure lasers, the analysis of their linewidth becomes an important subject.

Much attention has been focused on improving the theoretical description of the linewidth of semiconductor lasers. Schawlow and Townes first pointed out that the spectral linewidth-stimulated emission power product is determined by the cold cavity bandwidth, which determines the amount of spontaneous emission coupled to the lasing mode [6]. More recently, Henry has found that for semiconductor lasers, the variation of the carrier density caused by the random field amplitude fluctuation involves an additional, delayed phase change, which enhances the laser linewidth [7]. This is the well-known phase-amplitude coupling effect [8]–[10]. Thereafter, it was recognized that in addition to the material contribution, a large number of factors contribute to this effect. Some important factors are: the lateral and transverse waveguide structure [11], [12], the wavelength detuning from the gain maximum, and the coupling with the external resonator [13]–[17]. Meanwhile, it was shown that the spontaneous emission coupled to the lasing mode and in turn the linewidth, are enhanced by the output coupling, i.e., the oscillator loading, due to the low facet reflectivities in semiconductor lasers [18]–[21]. While exhibiting only a minor impact on standard Fabry–Perot (FP) lasers, the complete phase-amplitude coupling effect and the oscillator loading effect strongly influence the noise properties of complex structure lasers, and thus should be included in the analysis of their linewidth.

The first objective of this paper is to present a method for the analysis of laser linewidth, which includes simultaneously these two effects. This method follows the Green's function approach proposed by Henry for the noise calculations of the open resonator laser [19]. In this approach, the spontaneous emission noise is treated by using classical electromagnetism and the dissipation fluctuation theorem in a semiconductor at thermal equilibrium. The lasing field is the total cavity response resulting from excitation by the distributed spontaneous emission. As a factor of normalization, an explicitly spatially-independent Wronskian is introduced, which determines both the threshold condition and the dynamic resonance condition. The Wronskian is then expanded about the complex resonant frequency. The main difference between the method used in this paper and that of Henry is the consideration of the Wronskian as a function of both frequency and carrier density. By expanding the Wronskian about the mean values of these two variables, a structural dependent phase-amplitude coupling factor is introduced. Although this treatment is not basically different from that of Henry, it allows us to consider the effects of gain detuning or dispersive laser structures. This method is also efficient in describing the dynamics and high-frequency noise properties of complex structure semiconductor lasers. Compared with other recently-developed models [22], [23], it has the advantage of giving

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The authors are with the Ecole Nationale Supérieure des Télécommunications, 75634 Paris Cedex, France.
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an accurate evaluation of both the spontaneous emission rate and phase-amplitude coupling factor, and consequently, the spectral linewidth.

The second objective is the application of the proposed method to the analysis of the spectral linewidth of DFB lasers. In particular, two important features are emphasized. As is first pointed out by Wang *et al.*, in a DFB laser, the spontaneous emission rate is enhanced compared to a similar FP laser without mirror loss [21]. In this paper, this enhancement factor is studied systematically and expressed as a function of both facet reflectivities and the grating coupling coefficient. This feature is an extension of Henry's analysis, in which the FP laser and the extended cavity laser are treated in detail and the DFB laser is only mentioned [19]. Moreover, the effective phase-amplitude coupling factor is discussed in terms of the wavelength detuning from the gain peak of the semiconductor material. These two factors are then included in the linewidth evaluation. These results lead to a more complete description of the linewidth than those previously reported, which relate the total photon number of the laser cavity to facet output power [24], [25].

The third and final objective is the investigation of the spectral properties of composite-cavity lasers. The term "composite-cavity lasers" refers to a large variety of structures, including external cavity, external resonator loaded, and multisection lasers [2], [4], [5]. In our analysis, the role of the total external passive region is equivalent to a frequency-dependent effective reflectivity [15]–[17]. The spontaneous emission coupling to the lasing mode is found to be reduced by the additional filtering due to the frequency dependence of the effective reflectivity, but to be enhanced by the laser oscillator loading. A DBR laser is investigated as an example of a composite-cavity laser. It is pointed out that an optimum value of the grating coupling coefficient in the Bragg region exists, yielding the narrowest linewidth. The negative frequency detuning from the Bragg frequency gives a low effective phase-amplitude factor and thus a narrow linewidth.

This paper is organized as follows. Section II presents the Green's function method for the analysis of the spectral properties. In Section III, this method is applied in detail to various DFB laser structures. In Section IV, a general analysis is first given to composite-cavity lasers. Next, DBR lasers are investigated. Some conclusions are drawn in Section V.

II. Method for the Analysis of the Spectral Linewidth

The analysis of the spectral properties is based on classical electromagnetism, which has been widely used for the description of semiconductor lasers [18]–[20]. The lasing process is a frequency selective amplification of the spontaneous emission. It is assumed that the cavity medium is isotropic and the laser is perfectly index guided. Moreover, to concentrate our attention to the longitudinal axis, which is of major interest in complex structure lasers, transverse and lateral axes are neglected. Under these assumptions, the complex Fourier component $E_\omega(z)$ of the electric field in the laser cavity is governed by the one-dimensional scalar wave equation

$$\nabla_z^2 E_\omega(z) + k_0^2 \epsilon E_\omega(z) = F_\omega(z) \tag{2.1}$$

where $\nabla_z^2 = \partial^2/\partial z^2$ is the Laplacian operator for the longitudinal coordinate z, $k_0 = \omega/c$ is the wavenumber, c is the speed of light, both in vacuum, ϵ is the complex dielectric constant, and $F_\omega(z)$ is the frequential Langevin force term accounting for the distributed spontaneous emission. By relating the imaginary part of the refractive index with optical gain and internal loss, the complex dielectric constant is written as

$$\epsilon = \left[n + j(g - \alpha_l)/(2k_0)\right]^2. \tag{2.2}$$

Here, n is the real part of the refractive index, g is the optical gain, and α_l is the internal loss due to waveguide absorption and scattering. The dielectric constant ϵ is, in general, a function of the frequency ω, carrier density N and longitudinal coordinate z. The Langevin force is assumed to have negligible spatial and frequency correlation:

$$\langle F_\omega(z) F_{\omega'}(z')^* \rangle = 2D_{FF^*}\delta(z - z')\delta(\omega - \omega') \tag{2.3}$$

where the diffusion coefficient $2D_{FF^*}$ is obtained by the requirement that in equilibrium, dissipation by optical absorption is balanced by field fluctuations originating from spontaneous emission. This leads to [19]

$$2D_{FF^*} = \frac{4\omega^3\hbar}{c^3} n g n_{sp} \tag{2.4}$$

where $\hbar$ is Planck's constant divided by 2π, n_{sp} is the spontaneous emission factor given by [19]

$$n_{sp} = \left[1 - \exp\left(\frac{\hbar\omega - E_f}{kT}\right)\right]^{-1} \tag{2.5}$$

where E_f is the energy separation of the quasi-Fermi levels between the conduction band and the valence band, k is Boltzmann's constant, and T is the absolute temperature. By using the Green's function approach, the general solution of (2.1) is written as [19]

$$E_\omega(z) = \int_{(L)} G_\omega(z, z') F_\omega(z')\, dz' \tag{2.6}$$

where the integration is over the total cavity length. $G_\omega(z, z')$ is the Green's function given by [26]

$$G_\omega(z, z') = \frac{Z_+(z_>)Z_-(z_<)}{W(\omega, N)}. \tag{2.7}$$

$z_>$ or $z_<$ are the greater or lesser values of z and z', $Z_+(z)$ and $Z_-(z)$ are two independent solutions of the homogeneous part of (2.1), satisfying the boundary condition at the left facet and the right facet, respectively. $W(\omega, N)$ is the explicitly z-independent Wronskian of these two solutions defined as [26]

$$W(\omega, N) = \frac{\partial Z_+}{\partial z} Z_- - \frac{\partial Z_-}{\partial z} Z_+. \tag{2.8}$$

As the real Fourier frequency ω and the carrier density N are concerned in the solutions $Z_+(z)$ and $Z_-(z)$, the Wronskian is thus an explicit function of these two independent variables. Equation (2.6) can be rewritten as

$$E_\omega(z) = \frac{1}{W(\omega, N)} \int_{(L)} Z_+(z_>)Z_-(z_<)F_\omega(z')\,dz'. \tag{2.9}$$

The laser longitudinal mode corresponds to a zero point of the Wronskian:

$$W(\omega_0, N_{th}) = 0. \tag{2.10}$$

As the Wronskian is complex, both the lasing frequency ω_0 and the carrier density N_{th} at threshold are determined from this equation. Generally multiple solutions exist for the pair (ω_0, N_{th}). For each pair (ω_0, N_{th}), the functions $Z_+(z)$ and $Z_-(z)$ degenerate to one solution and represent the longitudinal distribution of the lasing mode. The semiconductor laser is assumed to operate only in one longitudinal mode with a field distribution denoted by $Z_+(z) = Z_-(z) = Z_0(z)$. As the Wronskian is a function of two independent variables ω and N, its expansion about the operating point can be written as

$$W = \left.\frac{\partial W}{\partial \omega}\right|_{\omega_0, N_{th}} (\omega - \omega_0) + \left.\frac{\partial W}{\partial N}\right|_{\omega_0, N_{th}} (N - N_{th}). \tag{2.11}$$

An equivalent treatment is to expand the Wronskian about a complex resonant frequency ω_c:

$$W = \left.\frac{\partial W}{\partial \omega}\right|_{\omega_c} (\omega - \omega_c) \tag{2.12}$$

where ω_c can be obtained by the requirement that the resonant condition is always satisfied, i.e.,

$$W(\omega_c, N) = 0. \tag{2.13}$$

In general, the Wronskian is not an analytical function of the pair (ω, N). The solution of the above equation yields a complex solution of ω_c for an arbitrary N. For a small deviation ΔN of N from N_{th}, the complex resonant frequency ω_c is obtained from (2.13) by using the first-order expansion in (2.11):

$$\omega_c = \omega_0 - \frac{\partial W/\partial N}{\partial W/\partial \omega}\Delta N. \tag{2.14}$$

Thus the complex resonant frequency is a linear function of the carrier density. By substituting (2.14) in (2.12) and neglecting the second-order derivative $\partial^2 W/\partial\omega^2$ at ω_0, these two expansions are found to be identical.

Henry has used the second expansion about a different complex resonant frequency, which in our notation is written as [19]

$$\omega = \omega_0 + \frac{(\alpha_H - j)}{2} v_g g_N \Delta N \tag{2.15}$$

where v_g is the group velocity, $g_N = \partial g/\partial N$ is the differential gain, and α_H is the linewidth enhancement factor defined as [7]–[10]

$$\alpha_H = -2k_0 \frac{\partial n/\partial N}{\partial g/\partial N}. \tag{2.16}$$

By this definition, α_H takes positive values in the lasing frequency range [8]. Thus the expression given in (2.15) considers simultaneously the change of gain and index with the change of carrier density. In the case of open FP resonator lasers discussed in [19], it can be proved that these two expressions of ω_c in (2.14) and (2.15) are rigorously equivalent.

For lasers with multiple active sections, the Wronskian should be expressed as a function of $W(\omega, N_1, N_2, \cdots)$, where N_i is the carrier density of the ith section. Consequently the Wronskian should be expanded as a sum of each deviation $N_i - N_{ith}$. Although in this paper the discussion is restricted to lasers with one active section, the formalism can be easily extended to lasers with multiple active sections.

By multiplying the two sides of (2.9) by $jW(\omega, N)/Z_0(z)$ and using (2.11), the rate equation for the electrical field is then obtained after the inverse Fourier transformation of (2.9):

$$\frac{d\beta_0(t)}{dt} = -j\frac{\partial W/\partial N}{\partial W/\partial \omega}(N - N_{th})\beta_0(t) + F_{\beta_0}(t) \tag{2.17}$$

where $\beta_0(t)$ represents the slowly varying envelope of the electrical field in the laser cavity:

$$\beta_0(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \beta_\omega \exp j(\omega - \omega_0)t\,d\omega \tag{2.18}$$

where $\beta_\omega = E_\omega(z)/Z_0(z)$ is an explicitly z-independent Fourier component of the electric field; $F_{\beta_0}(t)$ is the temporal Langevin force term given by

$$F_{\beta_0}(t) = \frac{j\int_{(L)} Z_0(z)F_0(z, t)\,dz}{\dfrac{\partial W}{\partial \omega}} \tag{2.19}$$

where $F_0(z, t)$ is

$$F_0(z, t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F_\omega(z) \exp j(\omega - \omega_0)t\,d\omega. \tag{2.20}$$

Equation (2.17) is the basis for the analysis of the spectral and dynamic properties of semiconductor lasers. By using (2.3) and (2.19), it can be shown that the temporal Langevin force $F_{\beta_0}(t)$ is delta correlated [19]:

$$\langle F_{\beta_0}(t)F_0^*(t')\rangle = R\delta(t - t') \tag{2.21}$$

where the spontaneous emission rate R is discussed in detail by Henry and written as [19]:

$$R = \frac{4\omega_0^2}{c^3} \frac{\int_{(L)} Z^*(z) n g n_{sp} Z(z)\, dz \int_{(L)} Z^*(z) n n_g Z(z)\, dz}{\left|\frac{\partial W}{\partial \omega}\right|^2} \tag{2.22}$$

where n_g is the group index in the laser cavity. The parameters n, g, n_{sp}, n_g in the integral are generally z-dependent for complex structure lasers.

By comparing R to the conventional result of the spontaneous emission rate $R_0 = v_g g n_{sp}$, the enhancement factor K of this rate used in the following sections is defined as

$$K = \frac{R}{R_0}. \tag{2.23}$$

Although the enhancement factor K takes the same definition as Petermann's K-factor [27], their origins are very different. The former results from the laser output power coupling—a longitudinal effect, and the latter is due to the gain-guiding mechanism—a transverse effect. As stated previously, the latter effect is not considered in our analysis.

To investigate the phase-amplitude coupling effect, the rate equation without the Langevin force term is considered. The solution of the homogeneous part of (2.17) is denoted by $A(t) = [I(t)]^{1/2} \exp(j\phi(t))$, where $I(t)$ is the intensity and $\phi(t)$ is the phase. Then the rate equation for $A(t)$ is written as

$$\frac{dA(t)}{dt} = -j\frac{\partial W/\partial N}{\partial W/\partial \omega}(N - N_{th})A(t). \tag{2.24}$$

The effective phase-amplitude coupling factor is defined as

$$\alpha_{eff} = \frac{2d\phi/dt}{d(\ln I)/dt} = \frac{\text{Im}[d(\ln A)/dt]}{\text{Re}[d(\ln A)/dt]} \tag{2.25}$$

where Re(), Im() denote the real part and the imaginary part of (), respectively. By using (2.24), the effective phase-amplitude coupling factor defined above is then found to be

$$\alpha_{eff} = -\frac{\text{Re}\left(\frac{\partial W/\partial N}{\partial W/\partial \omega}\right)}{\text{Im}\left(\frac{\partial W/\partial N}{\partial W/\partial \omega}\right)}. \tag{2.26}$$

In the above equation, both the material effect and the structural effect are taken into account in α_{eff} through the Wronskian. This result is similar to that obtained by R. Lang for multielement cavity lasers [14]. The relation between the phase-amplitude coupling factor α_{eff} and Henry's factor α_H will be discussed for DFB lasers and DBR lasers.

Using (2.14) and (2.26), the effective phase-amplitude coupling factor α_{eff} can also be expressed as

$$\alpha_{eff} = -\frac{\text{Re}(\Delta\omega_c)}{\text{Im}(\Delta\omega_c)}. \tag{2.27}$$

Thus the α_{eff} is expressed as a ratio of variation of the real and imaginary part of the complex resonant frequency. This alternative expression for α_{eff} results in a simplification for many practical cases, and was used by Arnaud [12], Gallion, *et al.* [16], and Henry [19].

Finally, using the standard procedure [7], the linewidth of a complex structure laser is expressed by

$$\Delta v = \frac{KR_0}{4\pi I}(1 + \alpha_{eff}^2) \tag{2.28}$$

where I is the total photon number in the laser cavity. This formula relates the spontaneous emission rate and the effective phase-amplitude coupling factor with the laser structure. Its correspondence with the familar result in FP lasers is straightforward.

It should be mentioned that a field independent carrier density is used in the above formalism. This is a good approximation for semiconductor lasers near threshold. But when semiconductor lasers operate well above threshold, some nonlinear effects such as spectral hole burning, spatial hole burning, etc., arise [28], [29]. Further study by including these effects would be interesting for understanding the phenomenon of the linewidth floor [29], for instance.

However, the above formulation gives a relatively complete description of laser properties. Besides the phase-amplitude factor and the linewidth, which are the central subjects of the present paper, the other main results of this analysis are

1) the longitudinal field distribution in the laser cavity represented by Z_0;

2) the emission spectrum below threshold, $E_\omega(z)$ in (2.9);

3) possible prediction of noise properties when the laser operates as an optical amplifier [19];

4) the threshold condition given in (2.10);

5) the rate equation of the electrical field written in (2.17). The incorporation of this equation with the rate equation of the carrier density allows the prediction of the intensity and phase noise spectrum and the current modulation properties.

III. Application to Various DFB Laser Structures

A. Derivation of the Wronskian

From the analysis in Section II, it is clear that the Wronskian of the laser cavity is the key in the analysis of the phase-amplitude coupling factor and the spectral linewidth. Therefore, we begin by solving Maxwell's propagation equation without the noise driving term.

A DFB laser with a phase-shifted region is illustrated in Fig. 1. The facet field reflectivities are denoted as $\hat{\rho}_l$ and $\hat{\rho}_r$. In a semiconductor laser of this type, having the periodic variation of refractive index or gain, the Max-

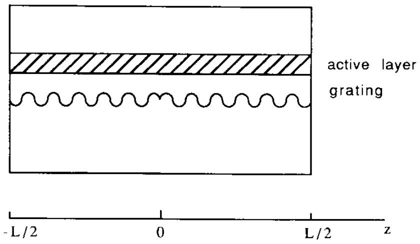

Fig. 1. Notations used for a phase-shifted DFB laser.

well's homogeneous propagation equation reduces to [30]–[33]

$$\nabla z^2 Z^{(i)}(z) + \left[K^{(i)}(z)\right]^2 Z^{(i)}(z) = 0 \qquad (3.1)$$

where $Z^{(i)}(z)$ is the field distribution for negative $z(i = 1)$ and for positive $z(i = 2)$. The square of the z-dependent propagation constant $K^{(i)}(z)$ is given by [31]

$$\left[K^{(i)}(z)\right]^2 = \beta^2 + 2j\alpha\beta + 4\kappa\beta \cos\left(2\beta_0 z + \Omega^{(i)}\right). \qquad (3.2)$$

$\beta_0 = m\pi/\Lambda$ is the Bragg wave vector, $\beta = n\omega/c$ the wavenumber, κ the coupling coefficient of the grating, $\alpha = (g - \alpha_l)/2$ the net amplitude gain, $\Omega^{(i)}$ the phase shift of the grating in the ith region at $z = 0$, and $\Omega^{(1)} = -\Omega^{(2)} = \Omega$. A conventional DFB structure is obtained by setting $\Omega = 0$. The solution $Z^{(i)}(z)$ of (3.1) is sought by the decomposition of two counterpropagating waves [30]–[33]:

$$Z^{(i)}(z) = R^{(i)}(z) \exp\left(-j\beta_0 z\right) + S^{(i)}(z) \exp\left(j\beta_0 z\right) \qquad (3.3)$$

where $R^{(i)}(z)$ and $S^{(i)}(z)$ are the forward and backward field amplitudes, respectively. By inserting the above expression into the wave equation (3.1) and using the boundary conditions, the solutions $Z_-(z)$ and $Z_+(z)$ can be found after substantial calculation. To focus on final results, detailed calculations are presented in the Appendix. Under the approximation that the coupling coefficient in DFB lasers is not very high ($\kappa L < 10$), the Wronskian in a phase-shifted DFB laser with reflecting facets is evaluated by its value at $z = 0$ and is written as (see the Appendix)

$$W(\omega, N) = 4\beta_0 e^{j\Omega} \frac{\gamma}{\kappa} \frac{\hat{\Gamma}}{\Gamma e^{2j\Omega}(1 - Qe^{\gamma L}) - \hat{\Gamma} + \Gamma Q e^{\gamma L}} \cdot F(\gamma L) \qquad (3.4)$$

where the function $F(\gamma L)$, defining the threshold condition, is given by [3], [32]

$$F(\gamma L) = (1 - \mathrm{P}e^{-\gamma L})(1 - Qe^{\gamma L})e^{2j\Omega} - \left[1 - \left(\frac{\hat{\Gamma}}{j\kappa}\right)^2 Pe^{-\gamma L}\right]\left[1 - \left(\frac{\Gamma}{j\kappa}\right)^2 Qe^{\gamma L}\right]. \qquad (3.5)$$

$\delta = \beta - \beta_0$ is the detuning from the Bragg frequency and γ is the complex propagation constant. Other parameters Γ, $\hat{\Gamma}$, P, and Q are defined in (A.4), (A.9), and (A.12), respectively.

B. Spontaneous Emission Rate Enhancement Factor

Using (2.22), (2.23), and neglecting the small variation term of the gain and index of refraction in the integration, the enhancement factor K of the spontaneous emission rate is then expressed as

$$K = \frac{1}{4}\left[\frac{\int_{-L/2}^{L/2} ZZ^* \, dz}{L}\right]^2 \frac{\kappa^2}{\alpha^2 + \delta^2} \cdot \left|\frac{\Gamma e^{2j\Omega}(1 - Qe^{\gamma L}) - \hat{\Gamma} + \Gamma Q e^{\gamma L}}{\hat{\Gamma} \dfrac{dF}{d(\gamma L)}}\right|^2. \qquad (3.6)$$

All of the parameters appearing in the above equation are defined at threshold. Under the assumption of weak grating coupling ($\kappa L < 10$), the enhancement factor K is a function of the normalized coupling coefficient κL and the facet reflectivities. In the following paragraphs, simplifications and numerical results for this general expression will be discussed for different practical cases.

1) Phase-Shifted DFB Lasers with Two AR-Coated Facets: DFB lasers with phase shift at the center are well known to give higher mode discrimination than conventional DFB lasers. DFB lasers with two AR-coated facets correspond to the maximum mode discrimination. In this case $\rho_l = \rho_r = 0$. From (A.9) and (A.12), the parameters P and Q are written as $P = \Gamma/\hat{\Gamma}$ and $Q = \hat{\Gamma}/\Gamma$. The enhancement factor K is simplified to

$$K = \frac{1}{4}\left[\frac{\int_{-L/2}^{L/2} ZZ^* \, dz}{L}\right]^2 \frac{\kappa^2}{\alpha^2 + \delta^2} \cdot \left|\frac{\Gamma e^{2j\Omega}\left(1 - \dfrac{\hat{\Gamma}}{\Gamma} e^{\gamma L}\right) - \hat{\Gamma}(1 - e^{\gamma L})}{\hat{\Gamma} \dfrac{dF}{d(\gamma L)}}\right|^2 \qquad (3.7)$$

where $F(\gamma L)$ is then written as

$$F(\gamma L) = \left[1 - \left(\frac{\Gamma}{j\kappa}\right)^2 e^{-\gamma L}\right]\left[1 - \left(\frac{\hat{\Gamma}}{j\kappa}\right)^2 e^{\gamma L}\right] e^{2j\Omega} - (1 - e^{-\gamma L})(1 - e^{\gamma L}). \qquad (3.8)$$

Taking into account the symmetry of the field distribution about the origin, $\int_{-L/2}^{L/2} Z^* Z \, dz$ is written as

$$\int_{-L/2}^{L/2} Z^*Z\,dz$$
$$= \big[|r_1^{(1)}|^2 e^{-\gamma' L/2} + |s_1^{(1)}|^2 e^{-\gamma' L/2} + |r_2^{(1)}|^2 e^{\gamma' L/2}$$
$$+ |s_2^{(1)}|^2 e^{\gamma'/2}\big] \frac{2\sinh(\gamma' L/2)}{\gamma'}$$
$$+ \big[r_1^{(1)*} r^{(1)} 2 e^{-j\gamma'' L/2} + s_1^{(1)*} s_2^{(1)} e^{-j\gamma'' L/2}$$
$$+ r_1^{(1)} r_2^{(1)*} e^{j\gamma'' L/2} + s_1^{(1)} s_2^{(1)*} e^{j\gamma'' L/2}\big]$$
$$\cdot \frac{2\sin(\gamma'' L/2)}{\gamma''} \tag{3.9}$$

where $r_1^{(1)}$, $r_2^{(1)}$, $s_1^{(1)}$, and $s_2^{(1)}$ are the field coefficients given in (A.8), the super-index * denotes the complex conjugate, and γ' and γ'' are real and imaginary parts of γ, respectively. By solving the threshold equation numerically, the enhancement factor K for the spontaneous emission rate is plotted as a function of the normalized coupling coefficient in Fig. 2, for different values of phase shift: $2\Omega = 0$, $\pi/3$, and π. It can be seen that for low values of κL, the enhancement factor K greatly exceeds unity and decreases rapidly with increasing κL. The enhancement factor is not very sensitive to the phase deviation from the optimum value $2\Omega = \pi$. In this optimum case, the enhancement factor K given by (3.7) is equivalent to the result of Wang *et al.* [21].

2) Conventional DFB Lasers with Partially-Reflecting Facets: For a conventional DFB laser with two partially-reflecting facets, the Wronskian is obtained by setting $\Omega = 0$ in (3.4):

$$W(\omega, N) = 2\beta_0 \frac{\hat{\Gamma}}{\kappa} F(\gamma L) \tag{3.10}$$

where $F(\gamma L)$ is reduced to

$$F(\gamma L) = \left[\left(\frac{\hat{\Gamma}}{j\kappa}\right)^2 - 1\right] P \exp(-\gamma L) + \left[\left(\frac{\Gamma}{j\kappa}\right)^2 - 1\right] \cdot Q \exp(\gamma L). \tag{3.11}$$

The enhancement factor K of the spontaneous emission rate of DFB lasers is simplified to

$$K = \left[\frac{\int_{-L/2}^{L/2} ZZ^*\,dz}{L}\right]^2 \frac{\hat{\Gamma}^2}{\alpha^2 + \delta^2} \left|\frac{dF}{d(\gamma L)}\right|^{-2} \tag{3.12}$$

where the integral is

$$\int_{-L/2}^{L/2} Z^*Z\,dz$$
$$= (|r_1|^2 + |s_1|^2 + |r_2|^2 + |s_2|^2) \frac{\sinh(\gamma' L)}{\gamma'}$$
$$+ (r_1^* r_2 + r_1 r_2^* + s_1^* s_2 + s_1 s_2^*) \frac{\sin(\gamma'' L)}{\gamma''}. \tag{3.13}$$

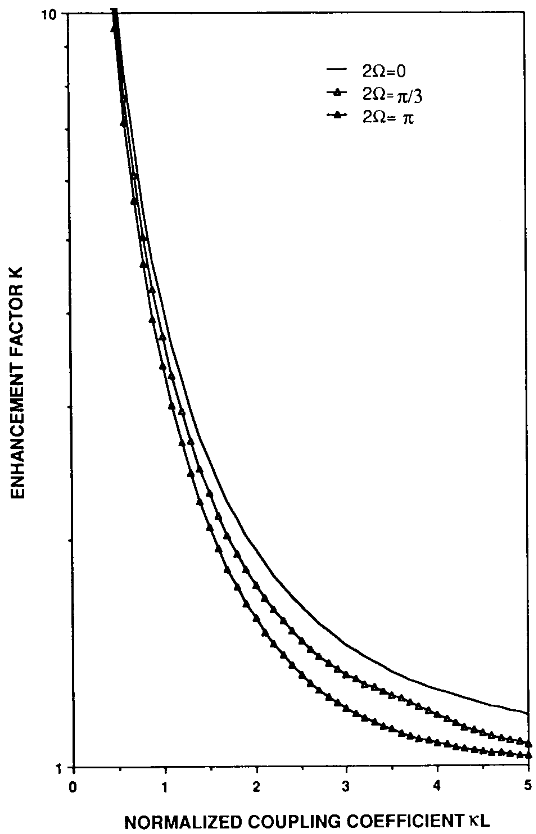

Fig. 2. The enhancement factor K of the spontaneous emission rate as a function of the normalized coupling coefficient κL for a phase-shifted DFB laser without facet reflections. The parameter is the phase shift.

The numerical results of the enhancement factor K are plotted in Fig. 3 for three cases: i) DFB laser with two AR-coated facets $\rho_l = \rho_r = 0$; ii) DFB laser with one AR-coated facet and one cleaved facet $\rho_l = 0$; $\rho_r = 0.565$; iii) DFB laser with two cleaved facets $\rho_l = \rho_r = 0.565$. For the first two cases, this enhancement factor is dramatically larger than unity for small values of κL. In the range $1 < \kappa L < 2$, the enhancement factor varies between 1.5 and 4.2, which is considerable. For the last case this factor is practically negligible. In all three cases, K decreases with the increase of the normalized coupling coefficient κL except for a discontinuity, which corresponds to the change of the lasing mode in the DFB laser. As κL tends towards zero, the spontaneous emission rate tends to the value for the FP laser with the same facet reflectivities. In fact by setting $\kappa L = 0$ in (3.12), the enhancement factor for the standard FP laser is obtained [18], [19]:

$$K = \left[\frac{(\hat{\rho}_l + \hat{\rho}_r)(1 - \hat{\rho}_l\hat{\rho}_r)}{2\hat{\rho}_l\hat{\rho}_r \ln(\hat{\rho}_l\hat{\rho}_r)}\right]^2. \tag{3.14}$$

It should be noted that since ρ_l and ρ_r for DFB lasers are generally complex, the spontaneous emission rate depends not only on their modulus, but also on their phase.

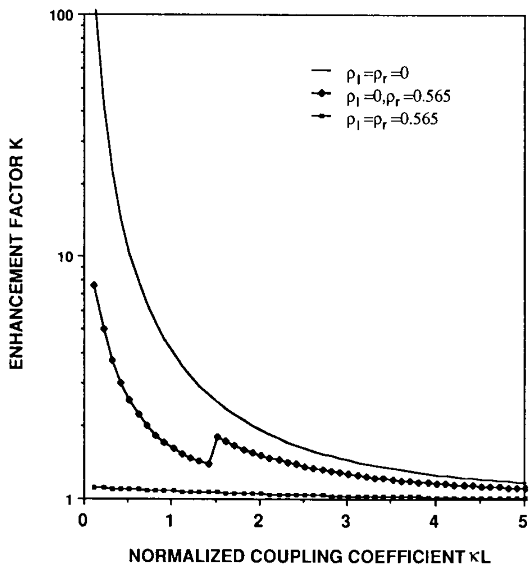

Fig. 3. The enhancement factor K of the spontaneous emission rate as a function of the normalized coupling coefficient κL for a conventional DFB laser with different facets reflectivities: i) $\rho_l = \rho_r = 0$; ii) $\rho_l = 0$, $\rho_r = 0.565$; iii) $\rho_l = \rho_r = 0.565$.

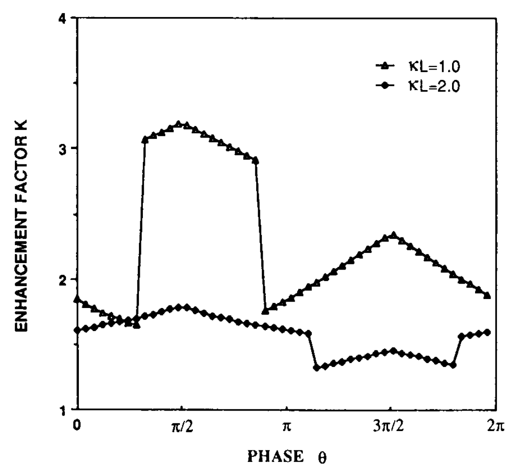

Fig. 4. The enhancement factor K of the spontaneous emission rate as a function of the phase θ of the right facet reflectivity: $\rho_l = 0$, $\rho_r = 0.565 \exp(j\theta)$. The parameter is the normalized coupling coefficient κL.

The enhancement factor is plotted as a function of the phase θ in Fig. 4, for $\rho_l = 0$ and $\rho_r = 0.565 \exp(j\theta)$. These curves present many discontinuities due to jumps of lasing modes. It can be seen that the enhancement factor varies between 1.5 and 3.0 for $\kappa L = 1.0$, and 1.3 and 1.7 for $\kappa L = 2$. For a DFB laser with two AR-coated facets, the enhancement factor given by (3.12) reduces to the form obtained by Wang *et al.* [21].

C. Effective Phase-Amplitude Coupling Factor

Under the assumption of negligible frequency dependence of the facet reflectivities $\hat{\rho}_l$ and $\hat{\rho}_r$, we have from (3.4)

$$\frac{\partial W}{\partial N} = \frac{dW}{d\gamma}\frac{\partial \gamma}{\partial N} \tag{3.15a}$$

$$\frac{\partial W}{\partial \omega} = \frac{dW}{d\gamma}\frac{\partial \gamma}{\partial \omega}. \tag{3.15b}$$

The above assumption is only valid for solitary DFB lasers without external optical feedback. Using (2.26) and (A.5), the effective phase-amplitude coupling factor for a DFB laser is then written as

$$\alpha_{\text{eff}} = \frac{\alpha_H - G_\omega/2}{1 + \alpha_H G_\omega/2} \tag{3.16}$$

where $G_\omega = v_g \partial g/\partial\omega$. For FP lasers, the α_{eff} is reduced to the α_H because the lasing frequency is always centered at the gain peak. For DFB lasers, a detuning-dependent phase-amplitude coupling factor is introduced. In general, $G_\omega << 1$, so (3.16) can be approximated by

$$\alpha_{\text{eff}} \doteq \alpha_H - (1 + \alpha_H^2)G_\omega/2. \tag{3.17}$$

The correction term is enhanced by $(1 + \alpha_H^2)$, which results in a large difference between α_{eff} and α_H for high values of Henry's parameter α_H.

Semiconductor materials are generally considered to have a parabolic gain profile which can be expressed as

$$g(\omega) = g(\omega_p)\left[1 - \frac{1}{2}\left(\frac{\omega - \omega_p}{\delta\omega}\right)^2\right] \tag{3.18}$$

where $g(\omega_p)$ is the gain peak, ω_p is the corresponding frequency, and $\delta\omega$ is the half-width of the gain band. It is more practical to use the wavelength λ as the parameter. By using the relation $\omega = 2\pi c/\lambda$, and (3.17) and (3.18), α_{eff} is then related to the wavelength detuning by

$$\alpha_{\text{eff}}(\lambda) \doteq \alpha_H(\lambda) - \frac{1}{2}\left[1 + \alpha_H^2(\lambda)\right]\frac{g(\lambda_p)}{2\pi n_g}\left(\frac{\lambda_p}{\Delta\lambda}\right)^2 \cdot (\lambda - \lambda_p) \tag{3.19}$$

where $g(\lambda_p)$ is the gain peak, λ_p is the corresponding wavelength, and $\Delta\lambda$ is the half-width of the gain band in wavelength. By using the theoretical model presented in [10] and [34] for Henry's material factor α_H and (3.19), α_H and α_{eff} are plotted as a function of wavelength for an InGaAsP semiconductor laser in Fig. 5. It is supposed that the peak gain is a linear function of carrier density: $g(\lambda_p) = g_N(N - N_0)$. The numerical values used in the calculation are listed in Table I.

From Fig. 5, it can be seen that there is a rapid increase in α_H for longer wavelengths [8]–[10]. In addition, an increase in α_H appears for shorter wavelengths [34]. In fact, to compensate for the decrease in gain due to wavelength detuning from the gain peak, a large value of carrier density is required, which results in a larger value of α_H. This effect does not appear in the curves presented in [8]–[10], which are plotted for a given carrier density value. It is clear that the effect of detuning tends to compensate the change of the material factor α_H. Coinciding with pre-

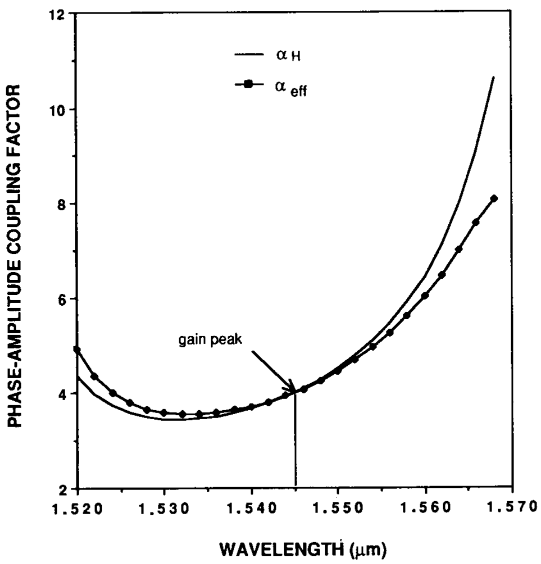

Fig. 5. The effective and material phase-amplitude coupling factor α_{eff} and α_H as a function of lasing wavelength.

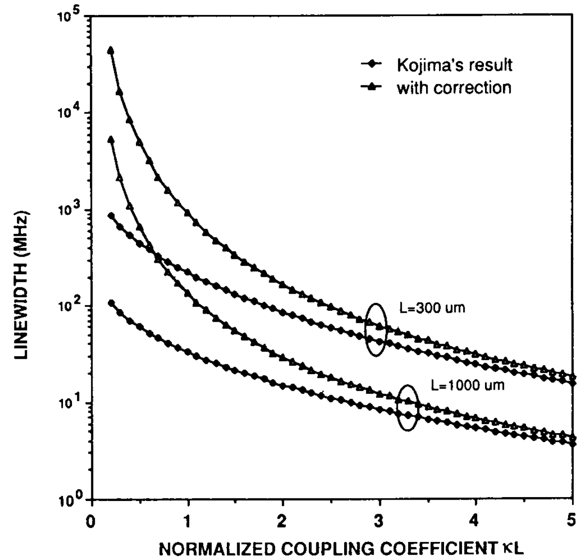

Fig. 6. The linewidth of a DFB laser with AR-coated facets as a function of the normalized coupling coefficient κL for two lengths of laser cavity: i) $L = 300$ μm; ii) $L = 1000$ μm.

TABLE I
DFB LASER PARAMETERS

Parameters		Numerical Values
Cavity Length	L	300 μm
Group Index	n_g	4.33
Differential Gain	g_N	1.5×10^{-16} cm^2
Carrier Density at Transparency	N_0	1.0×10^{18} cm^{-3}
Peak Gain Wavelength	λ_p	1.545 μm
Peak Gain	$g(\lambda_p)$	85 cm^{-1}
Half Gain Bandwidth	$\Delta\lambda$	20 nm
Spontaneous Emission Factor	n_{sp}	2.7
Photon Energy	$h\nu$	0.78 eV
Internal Loss	α_0	45 cm^{-1}

vious experimental results [35], negative detuning up to approximately 10 nm from the gain peak decreases α_H and α_{eff}. However, further negative detuning causes a large α_{eff} and thus a large linewidth.

D. Linewidth Calculation and Comparison to Experimental Results

Relating the total photon number to the facet output power, the linewidth can be written as [24], [25]

$$\Delta\nu = \frac{2\alpha K v_g^2 h\nu g n_{\text{sp}}}{4\pi(P_l + P_r)}(1 + \alpha_{\text{eff}}^2) \tag{3.20}$$

where P_l, P_r are the output powers from the left and right facets, respectively. The term 2α, representing the equivalent mirror loss in the DFB lasers, can be obtained by solving the threshold equation. The term $g = \alpha_l + 2\alpha$ is the total threshold gain. The facet output powers P_l and P_r are related by

$$\frac{P_l}{P_r} = \frac{|S^{(1)}(-L/2)|^2}{|R^{(2)}(L/2)|^2}\,\frac{1 - |\rho_l|^2}{1 - |\rho_r|^2}. \tag{3.21}$$

Taking into account the field distribution (A.8) in the Appendix, the linewidth can be expressed as a function of left or right facet output power.

The linewidth given in (3.20) differs from the previous results [24], [25] in two aspects. The enhancement factor K of the spontaneous emission rate is included, and an effective phase-amplitude coupling factor α_{eff} is used, instead of Henry's material factor α_H. To show quantitatively the differences, the linewidth of a DFB laser without facet reflections is compared with the results of Kojima [24] in Fig. 6 for a 0.8 μm GaAs DFB laser. The decrease of linewidth with the increase in κL is more rapid for small values of κL than is predicted in [24].

A recent letter by Ogita *et al.* [35] has reported measurements of the linewidth of a DFB laser with one AR-coated facet and one cleaved facet for κL in the range of 0.8–1.0. The measured linewidth was found to be two times larger than that predicted by conventional theory. A large value of α_H was used to fit the experimental results. The Green's function approach leads, in this case, to an enhancement factor of 2.0–1.7 in the range $0.8 < \kappa L < 1.0$, as is shown in Fig. 3 for $\rho_l = 0$, $\rho_r = 0.565$. This is in good agreement with Ogita's experimental results. However, systematic verification of the theoretical predictions is very difficult because parameter control in the fabrication of DFB lasers is not yet sufficiently precise.

IV. APPLICATION TO COMPOSITE-CAVITY LASERS

A. General Analysis of Composite-Cavity Lasers

Composite-cavity semiconductor lasers are promising candidates for achieving narrow linewidth, reduced chirp, flat FM response, and wavelength tunability [2], [4], [5]. In this type of laser structure, the active region is loaded by a separate passive region with a higher Q than a simple FP resonator whose phase and resonant frequency are electrically controlled. The functions of amplification and mode selection are then separated between these two different regions.

Composite-cavity lasers are usually described as a Fabry–Perot cavity in which the external passive resonator is taken into account by a complex frequency-dependent effective reflectivity $r_{\text{eff}}(\omega) = r(\omega) \exp(j\phi(\omega))$ [15]–[17]. The solutions of the homogeneous part of the propagation equation in the active FP region are [19]

$$Z_{0-} = r_1 \exp(-j\beta z) + \exp(j\beta z) \tag{4.1a}$$

$$Z_{0+} = r_1 \exp(-j\beta z) + r_1 r_{\text{eff}}(\omega) \exp(-2j\beta L) \cdot \exp(j\beta z). \tag{4.1b}$$

The complex propagation constant β is given by

$$\beta = k_0 \epsilon^{1/2} = k_0 n + j(g - \alpha_l)/2 \tag{4.2}$$

where n is the refractive index, g is the optical gain, and α_l is the internal loss, all in the active region. By using (4.1a) and (4.1b), the Wronskian for the composite-cavity laser is then written as [19]

$$W(\omega, N) = 2j\beta r_1 [r_1 r_{\text{eff}}(\omega) \exp(-2j\beta L) - 1]. \tag{4.3}$$

The angular lasing frequency and the threshold gain are obtained by solving the equation $W(\omega_0, N_{\text{th}}) = 0$:

$$\omega_0 = \frac{c}{2nL} [2m\pi - \phi(\omega_0)] \tag{4.4a}$$

$$g = \alpha_l - \frac{1}{L} \ln [r_1 r(\omega_0)] \tag{4.4b}$$

where m is an integer. The effective phase-amplitude coupling factor is obtained by using (2.26) and (4.3):

$$\alpha_{\text{eff}} = \frac{\alpha_H(1 + A) - (B + G_\omega/2)}{1 + A + \alpha_H(B + G_\omega/2)}. \tag{4.5}$$

Here, α_H is the phase-amplitude coupling factor in the active region, and A and B are defined as

$$A = \frac{1}{\tau} \frac{d\phi(\omega)}{d\omega}\bigg|_{\omega_0}, \quad B = \frac{1}{\tau} \frac{d \ln r(\omega)}{d\omega}\bigg|_{\omega_0} \tag{4.6}$$

which differ slightly from the definition of Kazarinov and Henry in that the factor α_H is eliminated in B [15]. τ is the photon round-trip time in the active region. It is clear that this effective phase-amplitude coupling factor depends strongly on A and B, and that it could be greatly different from Henry's factor. This suggests that by an appropriate design of the passive section, it is possible to achieve a high α_{eff} for frequency modulation with suppressed intensity modulation, or a low α_{eff} for intensity modulation with reduced chirp. It is noted that this result agrees with that previously obtained by Vahala *et al.* [13], Gallion *et al.* [16], and Tromberg *et al.* [23].

By using (2.22), (4.1), and (4.3), the enhancement factor K of the spontaneous emission rate for composite-cavity lasers is found to be

$$K = \frac{K_0}{(1 + A)^2 + (B + G_\omega/2)^2}. \tag{4.7}$$

The denominator in the above equation shows the reduction of the spontaneous emission, due to the frequency dependence of the effective reflectivity and the gain. The numerator K_0 indicates the enhancement of the spontaneous emission rate due to the output coupling and is given by

$$K_0 = \left[\frac{[r_1 + r(\omega_0)][1 - r_1 r(\omega_0)]}{2r_1 r(\omega_0) \ln [r_1 r(\omega_0)]} \right]^2. \tag{4.8}$$

The relation between the output power from the left facet and the total photon number is written as [19]

$$P_l = \hbar\omega_0 v_g \alpha_m I \frac{r(\omega_0)(1 - r_1^2)}{[r_1 + r(\omega_0)][1 - r_1 r(\omega_0)]}. \tag{4.9}$$

The mirror loss α_m is related to facet reflectivities by $\alpha_m = -(1/L) \ln [r_1 r(\omega_0)]$. By using (2.28), (4.5), (4.7), and (4.9), the linewidth of a composite-cavity laser is then obtained:

$$\Delta v = \frac{v_g^2 \hbar\omega_0 \alpha_m g n_{\text{sp}} K_0}{4\pi P_l} \frac{(1 + \alpha_H^2)}{[1 + A + \alpha_H(B + G_\omega/2)]^2} \cdot \frac{r(\omega_0)(1 - r_1^2)}{[r_1 + r(\omega_0)][1 - r_1 r(\omega_0)]}. \tag{4.10}$$

This linewidth expression differs from previous results [15] by considering the oscillator loading K_0 and gain detuning G_ω. The influence of the oscillator loading may be strong for composite-cavity lasers with low facet reflectivities, and should not, generally, be neglected.

B. Results for DBR Lasers

The notations used for a DBR laser are represented in Fig. 7. The power coupling efficiency between these two regions is denoted by C_0. By using coupled-wave theory, the effective reflectivity $r_{\text{eff}}(\omega)$ is given by [33]

$$r_{\text{eff}} = \frac{-j\kappa C_0}{\gamma \coth(\gamma L_b) + (\alpha_1 - j\delta)} \tag{4.11}$$

where α_1 is the residual amplitude amplification coefficient, κ is the grating coupling coefficient, δ is the detuning from the Bragg frequency, γ is the complex propagation constant, and L_b is the length, all within the Bragg region.

When the DBR laser oscillates at the Bragg frequency, the mode discrimination is maximum. The lasing frequency corresponds to the maximum of the effective reflectivity, which results in $B = 0$. The effective phase-amplitude coupling factor is then simplified to

$$\alpha_{\text{eff}} = \frac{\alpha_H(1 + A) - G_\omega/2}{(1 + A) + \alpha_H G_\omega/2}. \tag{4.12}$$

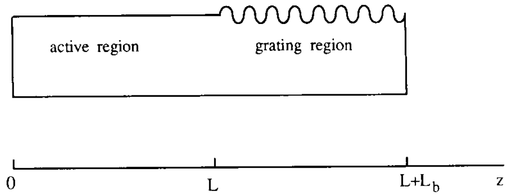

Fig. 7. Notations used for a DBR laser.

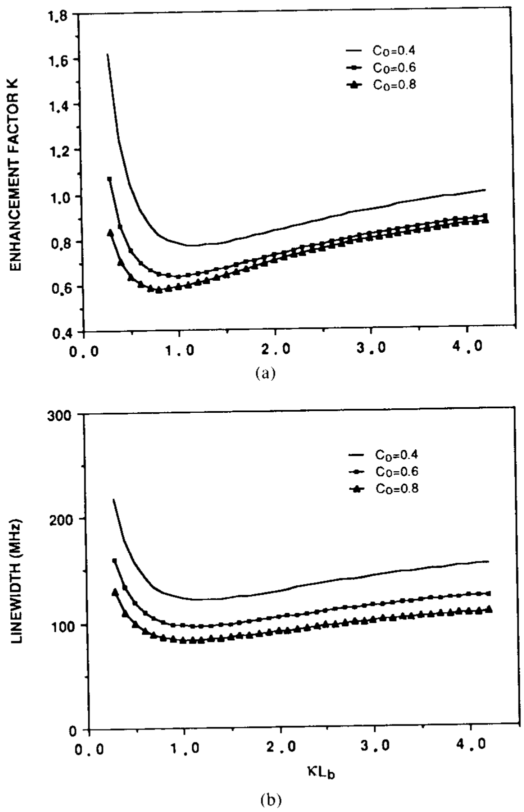

Fig. 8. (a) The enhancement factor K of the spontaneous emission rate and (b) the linewidth as a function of the normalized coupling coefficient κL_b for a DBR laser. The parameter is the power coupling efficiency C_0.

In the range 1.0–2.0 of the normalized coupling coefficient κL_b, the value of A varies between 0.1 and 0.3. When the DBR laser is detuned from the gain maximum, for any given wavelength its α_{eff} is located between α_H and α_{eff} for a DFB laser.

The enhancement factor K and the linewidth are plotted against κL_b for three values of the coupling efficiency $C_0 = 0.4$, 0.6, and 0.8 in Fig. 8. The parameters used in the calculation are listed in Table II. It is noted that an optimum value of κL_b exists, which gives the lowest enhancement factor and the narrowest linewidth. This effect can be easily understood; in the range of low values of κL_b, the modulus of the effective reflectivity is small, which gives rise to high mirror loss, high threshold gain and high spontaneous emission rate due to strong output coupling.

TABLE II
DBR LASER PARAMETERS[a]

Parameters		Numerical Values
Active Section Length	L	300 μm
Bragg Section Length	L_b	300 μm
Coupling Coefficient Between these Two Sections	C_0	$0 < C_0 < 1$
Residual Amplification Coefficient	α_1	3.5 cm^{-1}
Cleaved Facet Reflectivity	r_1	0.565
Cleaved Facet Output Power	P	1.0 mW
Phase-Amplitude Coupling Factor	α_H	4.0

[a]Other parameters take the same values as for the DFB laser.

With increasing normalized coupling coefficient κL_b, the modulus of the effective reflectivity increases rapidly, resulting in a decrease of K_0, thus tending to reduce the enhancement factor K. At the same time, the frequency dependence of the effective reflectivity decreases, tending to increase the enhancement factor K. The opposition of these two effects leads to a minimum enhancement factor, and the narrowest linewidth in the range 0.5–1.0 of κL_b.

Another case of interest is that of the DBR laser detuned from the Bragg frequency. Fig. 9 shows the enhancement factor K, the effective phase-amplitude coupling factor, and the linewidth as a function of the detuning parameter δL_b, in the vicinity of the Bragg frequency; C_0 is assumed to be 0.8. A negative frequency detuning reduces the phase-amplitude coupling factor α_{eff} and the enhancement factor K, and thus leads to a reduction of linewidth. This is due to the fact that in the negative detuning region, the effective reflectivity is strongly dependent on the lasing frequency and thus gives high positive values for A and B. However, for a DBR laser detuned from the Bragg frequency, the mode discrimination decreases.

V. CONCLUSION

A method for the analysis of the spectral linewidth of semiconductor lasers is presented which simultaneously takes into account the structure dependent spontaneous emission rate and the phase-amplitude coupling factor. This method is particularly useful for lasers with complex structures, which are beginning to appear in coherent optical communication systems.

The proposed method has been applied to phase-shifted and conventional DFB lasers with partially-reflecting facets. Numerical examples show that the enhancement factor of the spontaneous emission rate is significant for a DFB laser with one or two AR-coated facets, especially for low grating coupling coefficients. The effect of wavelength detuning on the phase-amplitude coupling is to partially compensate the change of the material contribution. The evaluated linewidth is in good agreement with Ogita's measurements.

This method has also been applied to composite-cavity lasers. It is pointed out that the frequency dependence of the passive section strongly affects both the effective phase-amplitude coupling factor and the spontaneous emission rate. As an example, DBR lasers are discussed

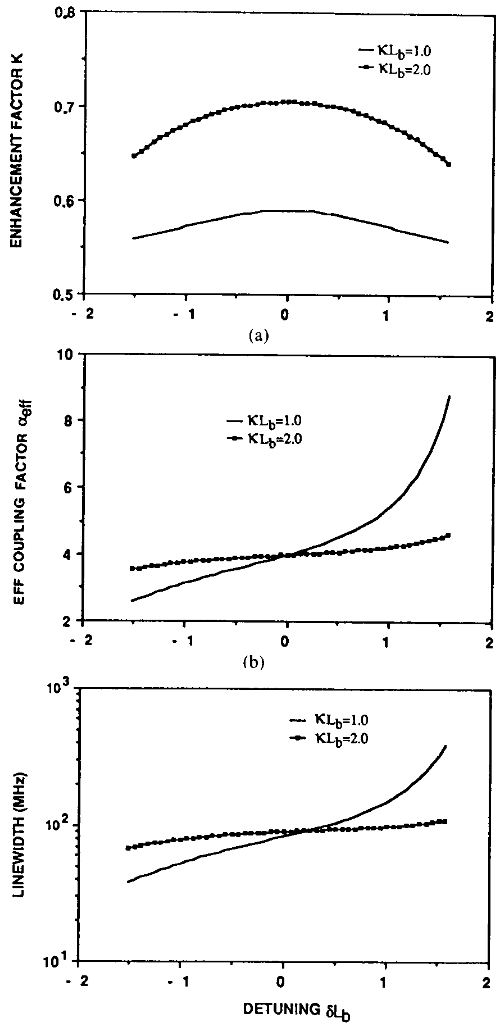

Fig. 9. (a) The enhancement factor K of the spontaneous emission rate, (b) the effective phase-amplitude coupling factor α_{eff}, and (c) the linewidth as a function of the detuning δL_b for a DBR laser. The parameter is the normalized coupling coefficient κL_b.

in detail. An optimal value of the grating coupling coefficient in the Bragg region is found, which minimizes the spectral linewidth. The phase-amplitude coupling factor and the linewidth depend strongly on the detuning from the Bragg wavelength, but compared with DFB lasers are less sensitive to the detuning from the gain maximum. In general, it appears that the DBR structure has more possibilities for obtaining narrow linewidth compared to the DFB structure.

Further improvement of the method would result from the inclusion of the gain saturation effect in the analysis. This would be helpful in the understanding of laser behavior in the high-power regime. In view of the current trends in semiconductor laser development, further application of this method to multisection DFB or DBR lasers would be of great interest.

Appendix
Derivation of the Wronskian of the DFB Laser

By inserting $Z^{(i)}(z)$ in (3.1) and neglecting second-order terms, the coupled-wave equations for $R^{(i)}$ and $S^{(i)}$ are obtained:

$$\frac{dR^{(i)}}{dz} = (\alpha - j\delta)R^{(i)} - j\kappa \exp(-j\Omega^{(i)})S^{(i)} \quad \text{(A.1a)}$$

$$\frac{dS^{(i)}}{dz} = -(\alpha - j\delta)S^{(i)} + j\kappa \exp(j\Omega^{(i)})R^{(i)} \quad \text{(A.1b)}$$

where $\delta = \beta - \beta_0$ is the detuning from the Bragg frequency. The solution of this coupled equation has the form

$$R^{(i)}(z) = r_1^{(i)} \exp(\gamma z) + r_2^{(i)} \exp(-\gamma z) \quad \text{(A.2a)}$$

$$S^{(i)}(z) = s_1^{(i)} \exp(\gamma z) + s_2^{(i)} \exp(-\gamma z) \quad \text{(A.2b)}$$

where γ is the complex propagation constant to be determined. Inserting this solution in (A.1a) and (A.1b), one obtains

$$\hat{\Gamma} r_1^{(i)} = j\kappa e^{-j\Omega^{(i)}} s_1^{(i)} \quad \text{(A.3a)}$$

$$\Gamma r_2^{(i)} = j\kappa e^{-j\Omega^{(i)}} e_2^{(i)}. \quad \text{(A.3b)}$$

Γ and $\hat{\Gamma}$ are defined as

$$\Gamma = \gamma + \alpha - j\delta \quad \text{(A.4a)}$$

$$\hat{\Gamma} = -\gamma + \alpha - j\delta \quad \text{(A.4b)}$$

and the squared complex propagation constant γ^2 is given by

$$\gamma^2 = \kappa^2 + (\alpha - j\delta)^2. \quad \text{(A.5)}$$

The continuity condition at the phase-shifted point $z = 0$ is written as

$$r_1^{(1)} + r_2^{(1)} = r_1^{(2)} + r_2^{(2)} \quad \text{(A.6a)}$$

$$s_1^{(1)} + s_2^{(1)} = s_1^{(2)} + S_2^{(2)}. \quad \text{(A.6b)}$$

Finally, the boundary condition at the left facet is written as

$$R_-^{(1)}(-L/2) \exp(j\beta_0 L/2) = \hat{\rho}_l S_-^{(1)}(-L/2) \exp(-j\beta_0 L/2). \quad \text{(A.7)}$$

$\hat{\rho}_l$ is the field reflectivity at the left facet. These equations give the relative field distribution in the laser cavity with one degree of freedom. For the sake of simplicity, we choose $r_{1-}^{(1)} = 1$. The different coefficients are then determined as

$$r_{1-}^{(1)} = 1; \qquad r_{2-}^{(1)} = -\left(\frac{\hat{\Gamma}}{j\kappa}\right)^2 P \exp(-\gamma L)$$

$$s_{1-}^{(1)} = \frac{\hat{\Gamma}}{j\kappa} \exp(j\Omega); \qquad s_{2-}^{(1)} = \frac{\Gamma}{j\kappa} \exp(j\Omega) r_{2-}^{(1)}$$

$$r_{1-}^{(2)} = \frac{1}{2\gamma}\left[-\hat{\Gamma}(1 + Pe^{-\gamma L})e^{2j\Omega} + \Gamma + \hat{\Gamma}Pe^{-\gamma L})\right];$$

$$r_{2-}^{(2)} = \frac{1}{2\gamma}\left[\hat{\Gamma}(1 + Pe^{-\gamma L})e^{2j\Omega} + \hat{\Gamma}\left(1 + \frac{\hat{\Gamma}}{\Gamma} Pe^{-\gamma L}\right)\right]$$

$$s_{1-}^{(2)} = \frac{\hat{\Gamma}}{j\kappa}\exp(-j\Omega)r_{2-}^{(2)}; \qquad s_{2-}^{(2)} = \frac{\Gamma}{j\kappa}\exp(-j\Omega)r_{2-}^{(2)} \tag{A.8}$$

where P is defined as

$$P = \frac{\rho_l - \dfrac{\Gamma}{j\kappa}}{\rho_l - \dfrac{\hat{\Gamma}}{j\kappa}} \tag{A.9}$$

with ρ_l related to the facet reflectivity $\hat{\rho}_l$ by $\rho_l = \hat{\rho}_l \exp(-2j\beta_0 L + i\Omega)$.

The boundary condition at the right facet is written as

$$\hat{\rho}_r R_+^{(2)}(L/2)\exp(-j\beta_0 L/2) = S_+^{(2)}(L/2)\exp(j\beta_0 L/2) \tag{A.10}$$

where $\hat{\rho}_r$ is the field reflectivity at the right facet of the DFB lasers. By choosing $r_{1+}^{(1)} = 1$ and combining (A.3), (A.6), and (A.10), the coefficients of the solution for Z_+ are then obtained:

$$r_{1+}^{(1)} = 1; \qquad r_{2+}^{(1)} = -\frac{\hat{\Gamma}e^{2j\Omega}(1 - Qe^{\gamma L}) - \hat{\Gamma} + \Gamma Qe^{\gamma L}}{\Gamma e^{2j\Omega}(1 - Qe^{\gamma L}) - \hat{\Gamma} + \Gamma Qe^{\gamma L}}$$

$$s_{1+}^{(1)} = \frac{\hat{\Gamma}}{j\kappa}\exp(j\Omega); \qquad s_{2+}^{(1)} = \frac{\Gamma}{j\kappa}\exp(j\Omega)r_{2+}^{(1)}$$

$$r_{1+}^{(2)} = \frac{1 + r_{2+}^{(1)}}{1 - Qe^{\gamma L}}; \qquad r_{2+}^{(2)} = -Qe^{\gamma L}r_{1+}^{(2)}$$

$$s_{1+}^{(2)} = \frac{\hat{\Gamma}}{j\kappa}\exp(-j\Omega)r_{1+}^{(2)}; \qquad s_{2+}^{(2)} = \frac{\Gamma}{j\kappa}\exp(-j\Omega)r_{2+}^{(2)} \tag{A.11}$$

where Q is defined as

$$Q = \frac{\rho_l - \dfrac{\hat{\Gamma}}{j\kappa}}{\rho_l - \dfrac{\Gamma}{j\kappa}} \tag{A.12}$$

where ρ_r is related to the facet reflectivity $\hat{\rho}_r$ by $\rho_r = \hat{\rho}_r \exp(-2j\beta_0 L - i\Omega)$.

It should be noted that because of the approximation made in deriving (A.1a) and (A.1b), the obtained solution is not rigorously exact, and that the Wronskian is not exactly explicitly z-independent. As an approximation, the Wronskian is estimated by its value at $z = 0$. By differentiating (A.2) and using (2.7), we have

$$W(\omega, N) = W_{RR} + W_{SS} + W_{RS} + W_{SR} + 2j\beta_0(S_- R_+ - S_+ R_-) \tag{A.13}$$

where W_{RR}, W_{SS}, W_{RS}, W_{SR} are defined as

$$W_{RR} = R'_- R_+ - R_- R'_+ = 2\gamma(r_{1-}r_{2+} - r_{1+}r_{2-}) \tag{A.14a}$$

$$W_{SS} = S'_- S_+ - S_- S'_+ = 2\gamma(s_{1-}s_{2+} - s_{1+}s_{2-}) \tag{A.14b}$$

$$W_{RS} = R'_- S_+ - R_- S'_+ = 2\gamma(r_{1-}s_{2+} - r_{2+}s_{1-}) \tag{A.14c}$$

$$W_{SR} = S'_- R_+ - S_- R'_+ = 2\gamma(s_{1+}r_{2-} - s_{2-}r_{1+}). \tag{A.14d}$$

Due to the continuity condition at $z = 0$, (A.6a), and (A.6b), the superscripts are not necessary in the above equations. Then the Wronskian can be split into two parts:

$$W(\omega, N) = W_1 + W_2 \tag{A.15}$$

where W_1 and W_2 are written as

$$W_1 = W_{RR} + W_{SS} + W_{RS} + W_{SR} \tag{A.16a}$$

$$W_2 = 2j\beta_0[(r_{1-} + r_{2-})(s_{1+})(s_{2+}) - (s_{1-} + s_{2-})(r_{1+} + r_{2+})]. \tag{A.16b}$$

For a DFB laser with a moderate coupling coefficient, the complex propagation constant γ is much less than β_0. The first part of the Wronskian can then be neglected. In this condition, the Wronskian is approximated as

$$W(\omega, N) = 2j\beta_0[(r_{1-} + r_{2-})(s_{1+} + s_{2+}) - (s_{1-} + s_{2-})(r_{1+} + r_{2+})]. \tag{A.17}$$

Using (A.8) and (A.11), the Wronskian of a DFB laser is then found to be

$$W(\omega, N) = 4\beta_0 e^{j\Omega}\frac{\gamma}{\kappa}\frac{\hat{\Gamma}}{\Gamma e^{2j\Omega}(1 - Qe^{\gamma L}) - \hat{\Gamma} + \Gamma Qe^{\gamma L}} \cdot F(\gamma L) \tag{A.18}$$

where $F(\gamma L)$ gives the threshold condition [3], [32]:

$$F(\gamma L) = (1 - Pe^{-\gamma L})(1 - Qe^{\gamma L})e^{2j\Omega} - \left[1 - \left(\frac{\hat{\Gamma}}{j\kappa}\right)^2 Pe^{-\gamma L}\right]\left[1 - \left(\frac{\Gamma}{j\kappa}\right)^2 Qe^{\gamma L}\right]. \tag{A.19}$$

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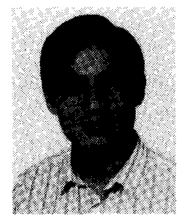


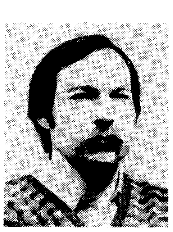
Guang-Hua Duan was born in Huangmei, China, on January 23, 1964. He received the B.E. degree in 1983 from Xidian University, Xian, China and the M.E. degree in 1987 from the Ecole Nationale Supérieure des Télécommunications (Telecom Paris), France, both in applied physics.

From 1983 to 1985, he was engaged in research on optical fiber measurements at the Guilin Institute of Optical Communications, Guilin, China. He is currently working towards the Doctorate degree at Telecom Paris. His research concerns semiconductor lasers and coherent optical communication systems.

Philippe Gallion (M'82) was born March 30, 1950, in Saint-Dizier, France. He received the Maîtrise and the Doctorat de troisième cycle from the University of Reims in 1972 and 1975, respectively, and the Doctorat d'Etat from the University of Montpellier in 1986. His initial research involved optical signal processing and electron microscopy.

He joined the Optoelectronics Group at Telecom Paris in 1978, where he is presently a Professor and Head of the Communications Department. His teaching activities include electromagnetism, Fourier optics, quantum electronics, and optical communications, in addition to being responsible for the Master's program in the field of components and devices for telecommunications. His present research includes noise, modulation, tunability, and the optical injection of semiconductor lasers with applications to coherent lightwave systems.

Dr. Gallion is a member of the Optical Society of America.



Guy Debarge was born in Clermont-Ferrand, France, on January 31, 1948. He received the Maîtrise de Physique in 1971 and the Doctorat de 3° cycle in 1976 from the University of Clermont-Ferrand.

After conducting research on organic semiconductors at the University of Clermont-Ferrand, he joined the Ecole Nationale Supérieure des Télécommunications (ENST), Paris, France, in 1983. His research has concerned coherent optical communications, coherence properties of semiconductor lasers, and polarization properties of optical fibers.