Decoding algorithm: Let $f(x)$ represent the sum of the transmitted codeword $v(x)$ and the error polynomial $e(x)$. The affine transform is applied to $f(x)$, thus generating

$$f(y) = v(y) + e(y)$$

(*a*) Syndrome polynomial $S(z)$ calculation:

$$S(z) = \sum_{i=0}^{d-2} S_i z^i \qquad \text{where } S_i = f(\alpha^{ir}) \quad 0 \le i \le (d-2)$$

Since $f(y) = v(y) + e(y)$, then it follows that

$$S_i = v(\alpha^{ir}) + e(\alpha^{ir}) = e(\alpha^{ir})$$

because $v(\alpha^{ir}) = 0$. Notice that $S_j = E_j$, $0 \le j \le (d-2)$ and that $E = (E_0, E_1, E_2, \ldots, E_j, \ldots, E_{n-1})$ represents the FFFT of the error vector.

(*b*) Error locator polynomial $L(z)$ calculation: Apply the Euclidean algorithm to the pair of polynomials z^{2t} and $S(z)$. Notice that $S(z)$ is of degree

$$d - 2 = 2t + 1 - 2 = 2t - 1$$

Stop when the degree of the partial-remainder polynomial becomes less than t (Reference 6). The location of the errors in the received word is indicated by the exponents of the reciprocals of the roots of $L(z)$.

(*c*) Determine the vector E, associated with the polynomial $E(z)$, which represents the FFFT of the error vector e, associated to the polynomial $e(x)$, by recursive extension[6] using $L(z)$ and $S(z)$.

(*d*) Calculate the inverse FFFT of $E(z)$ to find $e(y)$.

$$e = (e_0, e_1, e_2, \ldots, e_{n-1})$$

$$e_i = \frac{1}{n(\text{mod } p)} \sum_{j=0}^{n-1} E_j \alpha^{-rij}$$

However $n(\text{mod } p) = (p-1)/r$. Therefore it follows that

$$e_i = \sum_{j=0}^{n-1} E_j \alpha^{-rij} \,.\, r/(p-1)$$

(*e*) Apply the inverse affine transform to find $e(x)$.

(*f*) Perform the correction of the errors by subtracting $e(x)$ from $f(x)$, i.e.

$$v(x) = f(x) - e(x)$$

Conclusion: In this paper it has been shown that multilevel pseudocyclic codes can be decoded by an algebraic procedure. The use of the affine transformation was the key step in extending the applicability of algebraic decoding techniques to pseudocyclic codes. As a result of the affine transformation, a cyclic code over $GF(q^m)$ was obtained. Once a strictly cyclic code is obtained, the application of classical algebraic-decoding methods is immediate. The procedure described offers an interesting alternative to the decoding of megacyclic codes,[7] introduced by Berlekamp. Although other decoding procedures are known, e.g. exhaustive search, information sets, error-trapping, etc.,[3,6,8] algebraic decoding techniques for block codes are less dependent on code parameters when codes with $t \ge 2$ are used.

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ANALYSIS OF SPONTANEOUS EMISSION RATE OF DISTRIBUTED FEEDBACK SEMICONDUCTOR LASERS

Indexing term: Semiconductor lasers

The spontaneous emission rate of distributed feedback semiconductor lasers with partially reflecting facets is analysed using the Green function approach. It is found that this rate depends on both the facet reflectivities and the coupling coefficient. The linewidth of DFB lasers is evaluated by including this dependence.

Introduction: The linewidth of a semiconductor laser is of great importance for its application to coherent optical communication systems. Using Henry's theory, the linewidth of a semiconductor laser can be written as[1]

$$\Delta v = \frac{R}{\text{r}\pi I}(1 + \alpha_H^2) \tag{1}$$

where R is the spontaneous emission rate (s^{-1}), I the total photon number in the laser cavity and α_H the linewidth enhancement factor. For distributed feedback lasers the linewidth has been expressed in a more practical form by the relation between the photon number and the output power.[2] On the other hand, the spontaneous emission rate determined by the bandwidth of the optical resonator has to be calculated. In a previous letter by Wang *et al.* this rate was evaluated for DFB lasers without facet reflectivities using the generalised Petermann factor.[3,4] In this letter the spontaneous emission rate and linewidth are investigated based on the Green function approach.[5] The influence of both the facet reflectivities and the coupling coefficient on the spontaneous emission rate is pointed out. The linewidth of the DFB is evaluated by considering this effect.

Analysis: This is based on the Green function method, which has been used for Fabry–Perot semiconductor lasers and external cavity lasers. Suppose that the DFB laser is perfectly index-guided and that all transverse effects are neglected; then the spontaneous emission rate can be written as[5]

$$R = \frac{4\omega_0^2}{c^3} \frac{\langle Z^* g_0 n_{sp} Z\rangle\langle Z^* n_0 n_g Z\rangle}{\left|\dfrac{dW}{d\omega}\right|^2} \tag{2}$$

where ω_0 is the lasing frequency, c the velocity of light in a vacuum, g_0 the threshold gain in the laser cavity, n_g the group index, n_0 the refractive index and n_{sp} the spontaneous emission factor; $\langle . \rangle$ denotes the integration over the total longitudinal axis of the laser cavity. $Z(z)$ is the solution of the wave equation without the spontaneous polarisation term and W is the Wronskian of two such solutions, satisfying the boundary conditions at the left and right facets, respectively. In a DFB laser having a periodic variation in index or gain, the solution of the wave equation $Z(z)$ has the form[6,7]

$$Z(z) = R(z)\exp(-i\beta_0 z) + S(z)\exp(i\beta_0 z) \quad (3)$$

where $R(z)$ and $S(z)$ are the forward and backward field amplitudes respectively. Inserting $Z(z)$ in the wave equation and using the boundary conditions at the left and right facets, the Wronskian is obtained after substantial calculations (the detailed analysis will be published elsewhere):

$$W(\omega) = 4\beta_0 e^{i\Omega}\frac{\gamma}{\kappa}\frac{1}{\left(\rho_l - \frac{\hat{\Gamma}}{i\kappa}\right)\left(\rho_r - \frac{\Gamma}{i\kappa}\right)}F(\gamma l) \quad (4)$$

where $\beta_0 = m\pi/\Lambda$ is the Bragg wave vector, Ω the phase of the variable index, l the length of the laser cavity and κ the coupling coefficient. $\rho_{l,r}$ are related to the facet reflectivities $\hat{\rho}_l$, $\hat{\rho}_r$ by $\rho_l = \hat{\rho}_l \exp(-2i\beta_0 l + i\Omega)$ and $\rho_r \exp(-2i\beta_0 l - i\Omega)$. γ is the complex propagation constant: $\gamma^2 = \kappa^2 + (\alpha - i\delta)^2$, where α is the net amplitude gain and δ the detuning from the Bragg frequency. Γ and $\hat{\Gamma}$ are defined as $\Gamma = \gamma + \alpha - i\delta$ and $\hat{\Gamma} = -\gamma + \alpha - i\delta$. Finally, $F(\gamma l) =$ describes the threshold condition:

$$F = \left(1 - \rho_l\frac{\hat{\Gamma}}{i\kappa}\right)\left(1 - \rho_r\frac{\hat{\Gamma}}{i\kappa}\right) - \left(\rho_l - \frac{\hat{\Gamma}}{i\kappa}\right)\left(\rho_r - \frac{\hat{\Gamma}}{i\kappa}\right)\exp(2\gamma l) \quad (5)$$

Using eqns. 2–5, one obtains the correction factor for the spontaneous emission rate of DFB lasers:

$$\frac{R}{R_0} = \frac{1}{4}\frac{\langle ZZ^*\rangle^2}{l^2}\frac{\kappa^2}{\alpha^2 + \delta^2}\left|\left(\rho_l - \frac{\hat{\Gamma}}{i\kappa}\right)\left(\rho_r - \frac{\Gamma}{i\kappa}\right)\right|^2\left|\frac{dF}{d(\gamma l)}\right|^{-2} \quad (6)$$

where $R_0 = g_0 v_g n_{sp}$ is the spontaneous emission rate for the Fabry–Perot laser with highly reflecting facets. Note that all the parameters appearing in the above equation are defined at threshold.

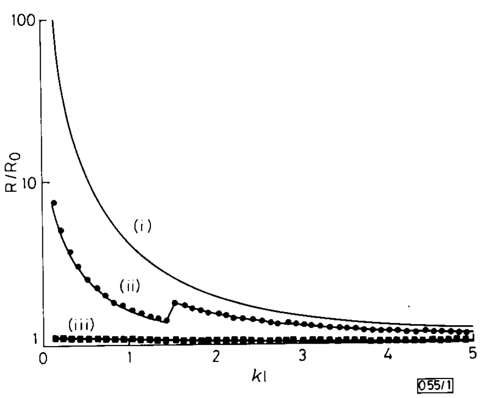

Fig. 1 *Correction factor R/R_0 for spontaneous emission rate as function of coupling coefficient κl for DFB laser with different facet reflectivities*

(i) $\rho_l = \rho_r = 0$; (ii) $\rho_l = 0$, $\rho_r = 0.565$; (iii) $\rho_l = \rho_r = 0.565$

Results: The correction factor for the spontaneous emission rate is plotted in Fig. 1 for three cases: (i) a DFB laser with two AR-coated facets, $\rho_l = \rho_r = 0$; (ii) a DFB laser with one AR-coated facet and one cleaved facet, $\rho_l = 0$; $\rho_r = 0.565$; and (iii) a DFB laser with two cleaved facets, $\rho_l = \rho_r = 0.565$. For the first two cases this correction factor is dramatically larger than unity for small values of κl. In the moderate range of κl, i.e. $1 < \kappa l < 2$, the correction factor varies between 1·5 and 4·2, which is considerable. For example with $\kappa l = 1$, this correction factor is 4·2 and 1·8 for the first two cases, respectively. For the last case this correction factor is practically negligible. As κl tends towards zero the spontaneous emission rate tends to the value for the Fabry–Perot laser with the same facet reflectivities. In fact, this can be shown analytically by setting $\kappa l = 0$ in eqn. 6. In all three cases the correction factor decreases with the increase of the coupling coefficient except at one point, which corresponds to the change of the lasing mode in the DFB laser.

It should be noted that the ρ_l and ρ_r of DFB lasers are generally complex, and that the spontaneous emission rate depends not only on their modulus, but also on their phase. This behaviour will also be discussed elsewhere.

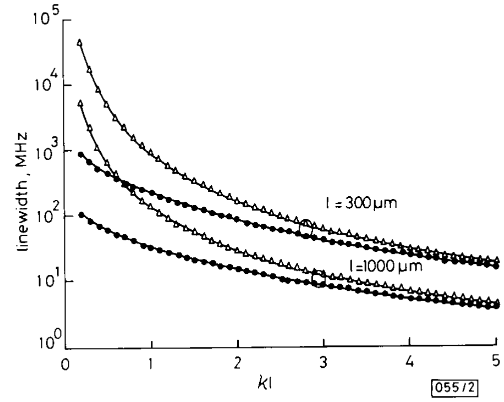

Fig. 2 *Linewidth of DFB laser with AR-coated facets as function of coupling coefficient κl for two lengths of laser cavity, $l = 300$ and $1000\,\mu m$*

● Kojima's result △ with correction factor

For a DFB laser with two AR-coated facets the correction factor given by eqn. 6 changes to the form of Wang *et al.*[3] In this case the linewidth of the DFB laser, including the correction of the spontaneous emission rate, is compared with the results of Kojima *et al.*[2] in Fig. 2. The parameters used in the numerical calculation are $v_g = c/4{\cdot}33$, $h\nu = 1{\cdot}42\,\text{eV}$, $n_{sp} = 2{\cdot}7$, $\alpha_H = 5{\cdot}4$, $\alpha_0 = 45\,\text{cm}^{-1}$ and $g_0 = \alpha_0 + 2\alpha$. The decrease of linewidth with the increase in κl is more rapid for small values of κl than that predicted by Kojima *et al.*[2]

Conclusion: We have studied the spontaneous emission rate of DFB lasers using the Green function approach. An analytical result is given, which considers simultaneously the influence of the reflecting facets and the optical gratings. The numerical example shows that the correction to this rate is significant for a DFB laser with one or two AR-coated facets, especially for low coupling coefficient. The linewidth of the DFB laser is evaluated by taking into account this correction factor.

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