Increase the Number of Input for Hadamard Transform Using Two Dimensional Multimode Interference Couplers

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Abstract—In this letter, Hadamard transform based on two dimensional multimode interference (2DMMI) couplers is proposed. Due to the nature of Hadamard transform, the 2DMMI structure can increase the number of input from N to N^2 in comparison to the one dimensional case (1D). This method, which extends 1D structure to 2D, can be used to optimize all kinds of optical Hadamard transformers.

Index Terms— Hadamard transform, 2DMMI

I. INTRODUCTION

Due to the booming demand for the optical transmission bandwidth, numerous techniques have been proposed to enhance the capacity of the optical communication system. Optical code division multiple access (OCDMA) is one of the promising techniques. It allows multiple users to use the same channel simultaneously by using the orthogonal code words.

For OCDMA, the most commonly used optical component is fiber Bragg grating (FBG) [1]. FBG can reflect the optical pulse, and change one pulse into a code word via cascaded structures. However, the FBG-based coding method has the disadvantage that each FBG-based component can only generate one code word. Different code words will require different structures. In practical CDMA systems, the number of code words can be as many as 64 (For instance, the wireless CDMA systems use 64 Walsh codes in one transmission channel). This will greatly increase the cost and the difficulty to apply the FBG coding technique to practical OCDMA systems.

Another way for OCDMA encoding and decoding is to use the Hadamard transform devices to construct the OCDMA codes directly. There are several specific ways to realize the Hadamard transform all optically. One way is to use the cascaded structure of couplers or Mach-Zehnder interferometers, like what Marhic [2] and Koichi [3] have done for discrete Fourier transform (DFT). The other way is to use the LPFGs [4]. Another very important approach to realize optical

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orthogonal transforms is to use the MMI couplers [5], which have been widely utilized for orthogonal transforms, such as the 4*4 (using one MMI coupler) and the 8*8 (using cascaded MMI couplers) Hadamard transforms [6,7], the discrete cosine/sine transform [8] and the DFT [9]. The preceding studies on Hadamard transform with MMI couplers were limited to the one dimensional case and therefore either cascaded structures [6] or holograms [7] are required to increase the number of input. When the number of input is large, the structures will be a little complicated.

In this letter, we propose to increase the number of input for Hadamard transform by using the 2DMMI couplers. Due to the nature of Hadamard transform, a 2^{N} points Hadamard transform is actually an N dimensional 2*2 DFT. Therefore, if the dimension is increased from one to two, the number of the input can be increased from $2^{N/2}$ to 2^{N} . This will greatly reduce the design and the fabrication effort for the higher order Hadamard transform devices.

II. THEORY

A. Matrix theory for two dimensional MMI couplers revisited [13]

The 2DMMI coupler has been studied in [10-12] previously. To fully understand the principle of the 2D device, a matrix theory like [5] is required. It was introduced in [13] and we briefly revisit it here. The electrical field can be formulated by:

$$E \quad z, x, y = \exp -jkz$$

$$\sum E_{mn} \exp\left(j\pi z \left(\frac{m^2}{4\Lambda_x} + \frac{n^2}{4\Lambda_y}\right)\right) \sin\left(\frac{m\pi x}{W_{ex}}\right) \sin\left(\frac{n\pi y}{W_{ey}}\right)$$
(1)

where m/n refers to the number of the mode in the x/y direction, $\Lambda_{x/y} = n_{eff} W_{ex/ey}^2/\lambda_0$, with λ_0 to be the wavelength, n_{eff} the effective index. We set the coupler effective widths in the x and y direction W_{ex} W_{ey} , the length L_{MN} and the position of the input/output ports (x_p, y_k) to satisfy the following [6,10]:

$$W_{ex}: W_{ey} = \sqrt{M} : \sqrt{N}$$

$$L_{MN} = \frac{4nW_{ex}^{2}}{M\lambda_{0}} = \frac{4nW_{ey}^{2}}{N\lambda_{0}} = \frac{4\Lambda_{x}}{M} = \frac{4\Lambda_{y}}{N}$$
(2)

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$$x_{i} = \left(i - \frac{1}{2}\right) \frac{W_{ex}}{M} \quad i = 1 \cdots M$$

$$y_{k} = \left(k - \frac{1}{2}\right) \frac{W_{ey}}{N} \quad k = 1 \cdots N$$
(3)

where M/N is the number of the input and output ports in the x/y direction. The output matrix and the input matrix have the relationship of:

$$\mathbf{A}_{\rm out} = \mathbf{M}_{\rm v} \mathbf{A}_{\rm in} \mathbf{M}_{\rm u} \tag{4}$$

where $\mathbf{M}_{u}/\mathbf{M}_{v}$ is the transfer matrix for the *x/y* direction. According to Ref. [5-7, 13], they can be calculated by:

$$\mathbf{M}_{\mathbf{v}} = \mathbf{V}\mathbf{B}_{\mathbf{v}}\mathbf{V}^{\mathrm{T}}$$

$$\mathbf{M}_{\mathbf{v}} = \mathbf{U}^{\mathrm{T}}\mathbf{B}_{\mathbf{v}}\mathbf{U}$$
(5)

V and U^T are the unitary matrixes which reflect the relationship between the external field and the internal field, and their elements can be found in Ref [6, 13].

Matrix \mathbf{B}_{v} and \mathbf{B}_{u} are diagonal matrixes indicating the phase difference introduced by different propagation modes, and the diagonal elements of the two matrixes are [6, 13]:

$$b_{\nu,ii} = \exp\left(j\frac{i^2\pi}{M}\right)$$

$$b_{u,kk} = \exp\left(j\frac{k^2\pi}{N}\right)$$
(6)

B. Realization of Hadamard transform on 2DMMI couplers The *m*th order Hadamard transform has the matrix of [14]

$$H_{m} = \frac{1}{\sqrt{2}} \begin{pmatrix} H_{m-1} & H_{m-1} \\ H_{m-1} & -H_{m-1} \end{pmatrix}$$

= $H_{1} \otimes H_{m-1} = H_{2} \otimes H_{m-2} = \dots = H_{m/2} \otimes H_{m/2}$ (7)
 $H_{1} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$

where \otimes denotes the matrix Kronecker product. According to [15], if the matrix representation of the input and output in Eq. (4) is rewritten in the vector form, we have

$$\operatorname{vec} \mathbf{A}_{\operatorname{out}} = \mathbf{M}_{u}^{\mathrm{T}} \otimes \mathbf{M}_{v} \operatorname{vec} \mathbf{A}_{\operatorname{in}}$$
(8)

where **vec** denotes the operation that piles up the columns of a matrix into one vector. From Eq. (7-8), it can be seen that if we can realize the m/2th order Hadamard transform in one dimension, the mth order Hadamard transform can be realized provided that similar structures are realized in two dimensions.

For example, we propose to realize the 4*4 Hadamard transform in this approach. Atma Ram Gupta et. al. [6] has already proposed and realized the 2*2 Hadamard transform in one dimension using a MMI coupler and a few phase shifters. Hence, if similar structures are used in the two dimensional case, the 4*4 Hadamard transform can be realized. It is analyzed as follows : if we place the phase shifter of 0, and $3/2\pi$ at the input and output ports, the one dimensional 2*2 coupler has the transfer matrix of [6]

$$\mathbf{M} = \mathbf{D}_{out} \vee \mathbf{B} \vee \mathbf{D}_{in}$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(j\frac{3\pi}{2}\right) \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \exp\left(j\frac{\pi}{2}\right) & 0 \\ 0 & \exp\left(j2\pi\right) \end{pmatrix} \quad (9)$$

$$* \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(j\frac{3\pi}{2}\right) \end{pmatrix}$$

$$= e^{-j\frac{\pi}{4}} \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

VDV/TD

where $\mathbf{D}_{out}/\mathbf{D}_{in}$ indicate the contribution of the phase shifters at the input/output ports. Matrix in Eq. (9) is the 2*2 Hadamard transform matrix 1/sqrt(2)*[1 1; -1 1] while ignoring the common phase. Therefore, if we place the phase shifters of 0, $3/2\pi$, $3/2\pi$, 3π , at the two dimensional MMI coupler input and output ports, we have the matrix of the 4*4 Hadamard transform if the common phase term is discarded:

III. SIMULATION RESULTS AND DISCUSSION

We perform the simulation via 3D beam propagation method (3DBPM) on the proposed 2D device discussed in the previous section, i. e. a 4*4 2DMMI coupler. Phase shifters 0, $3/2\pi$, $3/2\pi$, 3π are placed at the input/output ports to realize the 4*4 Hadamard transform. The waveguide has the core index of 3.3989 and the cladding index of 3.1645 which is the same as Ref. [12]. The waveguide has a rectangle shape with the width of 32µ m and the height of 32µ m. The calculated effective width and height mentioned in Eq. (2) is 32.37μ m. The calculated length of the device is 4603μ m. The input and the output waveguides have the cylindrical shape with diameter of 2.6µ m. The position of the input and output waveguides are designed according to eq. (3). The launching signal wavelength is 1.55µ m. The optical schematic of the device is depicted in Fig. 1.

We launch the input vectors $(1\ 0\ 0\ 0)^{T}$, $(1\ 1\ 0\ 0)^{T}$, $(1\ 1\ 1\ 0)^{T}$ and $(1\ 1\ 1\ 1)^{T}$ into the MMI coupler. The numerically calculated output vectors have the amplitudes of $(0.46\ 0.45\ 0.45\ 0.45\ 0.45)^{T}$, $(0.91\ 0.01\ 0.90\ 0.01)^{T}$, $(1.36\ 0.45\ 0.45\ 0.45\ 0.46)^{T}$ and $(1.81\ 0.01\ 0.05\ 0.01)^{T}$ which are obtained after the wave amplitudes are stabilized within the single mode waveguides at the output ports. The amplitudes of the ideal Hadamard transform output vectors are $(0.50\ 0.50\ 0.50\ 0.50\ 0.50\ 0.7\ (1.00\ 0.00\ 1.00\ 0.00)^{T}$, $(1.50\ 0.5$ (7). Comparing the 3DBPM numerical simulation results with the theoretical predictions, the effectiveness and the accuracy of the proposed method are verified. The output field amplitudes with the different input vectors are plotted in fig. 2.



Fig 1. Optical schematic of the device



Fig 2. The output field amplitudes of the different input vectors (a) the input vector is $(1\ 0\ 0\ 0)$; (b) the input vector is $(1\ 1\ 0\ 0)$; (c) the input vector is $(1\ 1\ 1\ 0)$; (d) the input vector is $(1\ 1\ 1\ 1)$

We also investigated the fabrication error tolerance of the device by slightly altering the width and the effective index of the device. We assume that the input vector is $(1 \ 0 \ 0 \ 0)$. The output amplitude varies about 15% when the width varies 100nm. When the index has the change of 0.001, the output amplitude varies about 3%. The impact of other parameter variation can be deduced from the above example. The error tolerance is much worse compared with the one dimensional case [6], this is due to the fact that the fabrication error impacts the results in the x and the y directions simultaneously and the total error on final results adds up. Although the fabrication precision requirement is high, it is however, achievable with current technology [16]. Moreover, the thermal/electrical tuning of the index can compensate the fabrication error [17].

It is worth mentioning that the example analyzed above is the extension of the one dimensional 2*2 Hadamard transform device [6] to two dimensions. The number of input is increased from 2 to 2^2 . The extension can be further applied to other devices, such as 4*4 and 8*8 Hadamard transform devices proposed in Ref. [6] and [7]. The configuration of the device should be extended as the above example does, which means that if cascaded structures [6] or holograms [7] are used in one dimension, the same structure or techniques should be applied

to the two dimensional waveguide. It is expected 16*16 and 64*64 Hadamard transform devices can be realized in this way. This will greatly reduce the design and fabrication effort.

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IV. CONCLUSION

We have proposed to realize the Hadamard transform on a 2DMMI coupler. The simulation results match the theoretical prediction precisely. The approach demonstrates the possibility to achieve the higher order Hadamard transform using simple 2D structures and provides a possible solution for all optical CDMA systems. It is worth mentioning that although the discussion is based on the MMI structure, the method of extension of 1D structure to 2D can be generalized for other devices, such as the LPFGs based Hadamard transform devices.

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