Analytical Design, Analysis, and Optimization of Raman Fiber Amplifiers with TDM Pumps

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***Abstract*— In this paper, we analyze Raman amplifiers (RAs) with time division multiplexed (TDM) pumps via an analytical approach. First of all, a detailed analytical solution for the forward and backward propagating waves in the TDM-pumped RAs is presented. Then, the gain of the TDM-pumped RAs is treated analytically. After revealing the nature of resonant gain enhancement, a relatively simple formula of the optimal input pump power configuration is derived via the least squares method. Afterwards, enhanced double Rayleigh backscattering noise and amplified spontaneous emission noise are analyzed and explicit analytical formulas derived. Thereafter, a simple formula to evaluate the impact of the pump modulation is proposed for the design, analysis, and optimization. The analysis of the TDM-pumped RAs highlights the optimization of the parameters of the TDM pumps, such as the pump wavelengths, pump power, and pump modulation frequency as well as the pumping order.**

***Index Terms*— Raman amplification, Raman scattering, time division multiplexed pumps.**

I. INTRODUCTION

RAMAN amplification has attracted much attention during recent years [1]–[4]. There have been numerous papers devoted to the optimization of the Raman amplifier (RA) gain profile and the noise figure (NF) spectrum. By employing a genetic algorithm [2], a matrix-based algorithm [3], or other experimentally applicable algorithms [4], very flat gain and NF spectrum can be achieved. However, these methods require either advanced numerical simulation techniques [5] that might not be applicable directly to the Raman amplification modules, or time-consuming pre-measurement which cannot be achieved rapidly during reconfiguration. Furthermore, the pump-to-pump interactions will cause four-wave mixing (FWM) and therefore degrade the system performance, especially in dispersion-shifted fibers. Also, in order to balance the pump-to-pump interactions, higher powers are required at the shorter pump wavelengths. This configuration, however, will cause the optical signal-to-noise ratio (OSNR) to be nonflat [6]. To overcome these difficulties, time division multiplexed (TDM) pumped RAs have been proposed [6]–[13]. Winzer *et al.* studied the speed requirements for TDM pump tuning both theoretically and experimentally and presented an analytical model for temporal Raman gain variation [7], [8]. The amplified spontaneous emission (ASE) noise enhancement was also reported afterwards [9] by the same team. Bolognini *et al.* discovered the resonant gain enhancement [10] due to the relative intensity noise (RIN) transfer in TDM-pumped RAs. Furthermore, the group found that the double Rayleigh backscattering (DRB) noise would be enhanced in TDM-pumped RAs and proved it numerically and experimentally [11]. Fludger *et al.* demonstrated experimentally that four TDM pumps could balance the uneven NF and improve the system performance [12]. Karasek *et al.* proposed a full-scale time-domain numerical model [13] to analyze this type of RAs. Despite these inspiring and pioneering works, there are still a few questions to be answered, such as the mechanism behind the phenomenon and whether there are analytical formulas for the design and analysis.

In this paper, we propose to analyze TDM-pumped RAs via an analytical approach. First of all, a detailed Fourier series model is illustrated and discussed. This method was proposed to analyze the pump modulation-induced noise in our previous paper [14] and verified by time-domain simulation. In this paper, we will provide a full solution of the model including the formulas for the various forward and backward propagating waves, which were not presented in the previous work. Based on this model, we will be able to study the typical features of TDM-pumped RAs.

When the modulation frequency is low, the TDM-pumped RAs will experience a resonant gain enhancement which can be explained by our model. Also, the analysis of the model will reveal that, when the modulation frequency is high, the first-order approximation of the TDM-pumped RA gain is equivalent to the gain of the RAs with continuous pumps, whose powers equal the average powers of the TDM pumps. This has already been verified by the experimental and theoretical study in [6]. Since the pump-to-pump interactions will not appear, if we neglect the signal-to-signal interactions and the pump depletion, which are reasonable and usually applied assumptions [15], we can find the optimal configuration for the pump powers to minimize the gain ripple. The mathematical method to obtain the optimal pump powers is similar to that in [15] and has already been applied in [16] and [17]. In [15], the adjustment of the power integrals to achieve a flat gain was obtained using the least squares method (LSM) and the optimal pump powers were solved through iterations. In [16], the optimal pump power integrals were obtained

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directly, but to get the pump powers iterations are still required because of the pump-to-pump interactions. Because pump-to-pump interactions are eliminated in TDM-pumped RAs, the computational cost of calculating the optimal pump powers is reduced. This was realized by Grant [17]. However, the equation derived in [17] was not in the simple matrix form as in [15] and [16], it dealt with pump path average powers rather than input pump powers. We will follow the previous works [15], [17] and derive a relatively simple formula. This will provide great feasibility for the system reconfiguration of the Raman-amplified optical network.

After proposing the pump power optimization method, we will analyze other amplifier parameters. Pump-modulation-induced noise or gain fluctuation is the key feature that prevents the application of the forward pumping scheme in this kind of RAs. This effect has been thoroughly studied in [14]. We will briefly review the contents and move to two similarly important features that deteriorate the performance, i.e., the DRB noise and the ASE noise enhancement. They have not been explained analytically by the existing models and we will analyze the phenomena via the Fourier series approach, which gives the analytical expressions and provides a deeper insight into the phenomena. It is worth mentioning that in this paper the two phenomena are analyzed separately. However, during the system performance evaluation, the DRB noise and the ASE noise contribution should not be simply added but properly weighted [18], [19]. Afterwards, a simple formula that evaluates the impact of the pump modulation will be proposed and used to optimize the pumping order of the TDM pumps.

II. Theoretical Model

Although the well-known Raman propagation equation and the pump-undepleted approximate formula have been given in [14], to ensure the integrity of the whole theory they will be reintroduced. Then the full solution will be provided without neglecting the DRB noise and the ASE noise terms.

A. The Well-Known Raman Propagation Equation

The Raman propagation equation is given by

$$\pm\frac{\partial P_s^{\pm}}{\partial z}+\frac{1}{V_s}\frac{\partial P_s^{\pm}}{\partial t}=-\alpha_s P_s^{\pm}+\gamma_s P_s^{\mp}+\sum_{j=1}^{M}C_{R,j}\left[P_{p,j}P_s^{\pm}+P_{p,j}2h\upsilon_s\Delta\upsilon_s n_{sp,j}\right] \quad (1)$$

$$n_{sp,j}=\left(1+\frac{1}{e^{h(\upsilon_{p,j}-\upsilon_s)/kT}-1}\right)$$

where $+$ and $-$ indicate the forward and backward propagation lights, M is the number of pumps, V_s is the group velocity, P_s and $P_{p,j}$ are the powers of the signal wave and the jth pump wave propagating along the fiber, $C_{R,j}$ is the Raman gain coefficient between frequency υ_s and frequency $\upsilon_{p,j}$, α is the attenuation coefficient, γ is the Raleigh scattering coefficient, h is the Planck constant, T is the absolute temperature, and k is the Boltzman constant. In this model, the signal-to-signal interactions are neglected.

B. Pump Evolution Under Undepletion Assumption

In this model, we assume that the pump is undepleted and the pump periodic waveform can be expanded as the Fourier series

$$P_{p,j}(z,t)=P_{p,j}\left(L,t-V_{p,j}^{-1}(L-z)\right)\exp\left[-\alpha_{p,j}(L-z)\right]$$
$$P_{p,j}(L,t)=\sum_{n=-\infty}^{+\infty}\pi_{j,n}e^{in\Omega t},\ g(z,t)=\sum_{j=1}^{M}C_{R,j}P_{p,j}(z,t) \quad (2)$$

where $\Omega=\frac{2\pi}{T}$, T stands for the modulation period of the pump wave, $\pi_{j,n}$ is the nth Fourier coefficient of the jth pump which is associated with the wave form of the pumps, L is the Raman fiber length, and $g(z,t)$ is the temporally varying gain provided by the pumps.

C. Forward Gain and Backward Gain for the Coupled Equations

Equation (1) actually consists of two coupled equations: i.e., the equation for the forward signal power, the forward ASE noise power, and the power of the backward propagating waves due to Rayleigh scattering (including the DRB noise and the Rayleigh scattering of the backward ASE noise); and the equation for the backward ASE power and the Rayleigh scattering noise power. The forward propagating power can be expressed as follows:

$$\begin{aligned}P_s^{+}(z,t)=&\,P_s^{+}\left(0,t-V_s^{-1}z\right)\times\exp\left(\int_0^z\left[g\left(x',t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\\&+\gamma_R\int_0^z P_s^{-}\left(x,t-\frac{z-x}{V_s}\right)\times\exp\left(\int_x^z\left[g\left(x',t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x\\&+\int_0^z\sum_{j=1}^{M}C_{R,j}\left[P_{p,j}\left(z,t-\frac{z-x}{V_s}\right)2h\upsilon_s\Delta\upsilon_s n_{sp,j}\right]\times\exp\left(\int_x^z\left[g\left(x',t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x.\end{aligned} \quad (3)$$

And the backward propagating power can be expressed by

$$\begin{aligned} P_s^-(z,t) &= \gamma_R \int_z^L P_s^+\left(x, t+\frac{z-x}{V_s}\right) \\ &\quad \times \exp\left(\int_z^x \left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x \\ &\quad + \int_z^L \sum_{j=1}^{M} C_{R,j}\left[P_{p,j}\left(z, t+\frac{z-x}{V_s}\right) 2h\upsilon_s \Delta\upsilon_s n_{sp,j}\right] \\ &\quad \times \exp\left(\int_z^x \left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x \end{aligned} \tag{4}$$

where

$$\begin{aligned} & g\left(z, t \pm V_s^{-1}(L-z)\right) \\ &\quad = \sum_{n=-\infty}^{+\infty} g_n^{\mp}(z)\, e^{in\Omega t} \\ & g_n^{\pm}(z) = \sum_j C_{R,j}\pi_{j,n} e^{in\Omega\left(V_{P,j}^{-1} \pm V_s^{-1}\right)z} \\ &\qquad \times e^{-\alpha_{p,j}(L-z)} e^{-in\Omega\left(V_{P,j}^{-1} \pm V_s^{-1}\right)L} \\ & \int_{x_1}^{x_2} g_n^{\pm}(z)\,\mathrm{d}z = \sum_j C_{R,j}\pi_{j,n} e^{-\left(in\Omega\left(V_{P,j}^{-1}\pm V_s^{-1}\right)+\alpha_{p,j}\right)L} \\ &\quad \times \frac{e^{\left(in\Omega\left(V_{P,j}^{-1}\pm V_s^{-1}\right)+\alpha_{p,j}\right)x_2} - e^{\left(in\Omega\left(V_{P,j}^{-1}\pm V_s^{-1}\right)+\alpha_{p,j}\right)x_1}}{\alpha_{p,j} + in\Omega\left(V_{P,j}^{-1} \pm V_s^{-1}\right)}. \end{aligned} \tag{5}$$

It is clearly indicated that the forward propagating power is composed of three terms and the backward propagating power is composed of two terms, as stated above.

III. First-Order Fourier Coefficient for Signal Gain (Average Gain)

A. Resonant Gain Enhancement for Low Modulation Frequency

In [10], the resonant gain enhancement occurs when the RIN on the pump is strong. Similarly, the pump modulation will also enhance the gain. For the signal gain analysis, we only consider (3) and neglect the DRB noise and the ASE noise terms. Assuming that the input signal is monochromatic with constant power of P_{s0} and using the definition of Bessel functions

$$e^{\alpha\cos\phi} = \sum_{n=-\infty}^{+\infty} I_n(\alpha)\, e^{in\phi} \tag{6}$$

where I_n is the modified Bessel function, we have

$$\begin{aligned} & P_s(L,t) \\ &\quad = P_{s0}\exp\left(\sum_{n=-\infty}^{+\infty}\int_0^L g_n^+(z)\,dz e^{in\Omega t}\right) \\ &\quad = P_{s0}\overline{G}\prod_{n=-\infty,n\neq 0}^{+\infty}\left(\sum_{m=-\infty}^{+\infty} I_m\left[2\left|\int_0^L g_n^+(z)\,dz\right|\right] e^{imn(\Omega t+\varphi_n)}\right) \\ & \overline{G} = \exp\int_0^L \left[g_0^+(z)-\alpha_s\right]\mathrm{d}z \\ &\quad = \exp(-\alpha_s L)\exp\left(\sum_j C_{R,j}\pi_{j,0}\alpha_{p,j}^{-1}\left(1-\exp\left(-\alpha_{p,j}L\right)\right)\right) \\ & \varphi_n = \arg\left(\int_0^L g_n^+(z)\,dz\right) \end{aligned} \tag{7a}$$

where $\overline{G}$ represents the gain of the RAs with continuous pumps whose powers equal the average powers of the TDM pumps. Due to the periodic nature of the gain, the output signal power can also be expanded as

$$P_s(L,t) = P_{s0}\left(\sum_{n=-\infty}^{\infty} d_{+n}(L,0)\exp(jn\Omega t)\right) \tag{7b}$$

where $d_{+n}(L,0)$ is the coefficient of the Fourier series resulting from the term $\exp\left(\sum_{n=-\infty}^{+\infty}\int_0^L g_n^+(z)dz e^{in\Omega t}\right)$. Comparing (7a) with (7b), we have

$$\begin{aligned} & d_{+0}(L,0) \\ & = \overline{G}\prod_{n=-\infty,n\neq 0}^{+\infty} I_0\left[2\left|\int_0^L g_n^+(z)\,dz\right|\right] \\ &\quad + \overline{G}\left(\sum_{m=-\infty,m\neq 0}^{+\infty} I_m\left(2\left|\int_0^L g_1^+(z)\,dz\right|\right) I_{-2m}\left(2\left|\int_0^L g_2^+(z)\,dz\right|\right)\right) \\ &\quad \times \prod_{n=-\infty,n\neq 0,1,2}^{+\infty} I_0\left[2\left|\int_0^L g_n^+(z)\,dz\right|\right] + \cdots \end{aligned} \tag{8}$$

where

$$d_{+0}(L,0) = \frac{1}{T}\int_0^T \exp\left(\int_0^L\left[g\left(z, t-\frac{L-z}{V_s}\right)-\alpha_s\right]\mathrm{d}z\right)dt$$

stands for the "DC," or average gain of the TDM-pumped RAs.

It can be seen that

$$d_{+0}(L,0) > \overline{G}\prod_{n=-\infty,n\neq 0}^{+\infty} I_0\left[2\left|\int_0^L g_n^+(z)\,dz\right|\right] > \overline{G}. \tag{9}$$

Therefore, it is demonstrated that TDM pumps provide higher gain than continuous pumps with the same average powers. Therefore, the gain is enhanced because of the periodic nature of the pump waves. When the modulation frequency is high enough (>10 kHz), $\int_0^L g_n^+(z)dz \approx 0$ for all $n \neq 0$. Since $I_0(0) = 1$ and $I_m(0) = 0$ for $m \neq 0$, we have

$$d_{+0}\left(L, 0\right) \approx \overline{G}. \tag{10}$$

This is a common practice for the TDM-pumped RAs, which means that the average amplifier gain is determined by the average pump powers when the modulation frequency is high enough (>10 kHz).

B. Optimal Pump Power Configuration Using LSM

As stated in Section I, the LSM has already been applied to the optimal pump power configuration problem [15], [17]. Grant [17] has derived a formula similar to that derived in this paper. However, it dealt with the pump path average powers and was not in an explicit form. A further simplified equation is derived below.

For TDM-pumped RAs, if we neglect the signal-to-signal interactions and the pump-to-signal depletion, the gain can be written as

$$\begin{aligned} &Gain_k(dB)\\ &\quad= 10\log_{10}(e)\ln\left(\overline{G}_k\right)\\ &\quad= 10\log_{10}(e)\left(-\alpha_{s,k}L + \left(\sum_j \pi_{j,0} C_{R,j,k} L_{eff,j}\right)\right)\\ &L_{eff,j} = \alpha_{p,j}^{-1}\left(1-\exp\left(-\alpha_{p,j}L\right)\right)\\ &for\ k = 1,\ldots,N \end{aligned} \tag{11}$$

where k stands for the kth signal and N is the signal channel number. We can rewrite the above equation in the matrix form

$$\begin{aligned} &\mathbf{Gain}_{dB} = 10\log_{10}(e)\left(-\boldsymbol{\alpha}_{\mathbf{s}}L + \mathbf{C}\boldsymbol{\pi}_0\right)\\ &\mathbf{Gain}_{dB} = \begin{pmatrix} Gain_1 \\ \vdots \\ Gain_N \end{pmatrix}\\ &\mathbf{C} = \begin{pmatrix} C_{R,1,1}L_{eff,1} & \ldots & C_{R,M,1}L_{eff,M} \\ \vdots & \ddots & \vdots \\ C_{R,1,N}L_{eff,1} & \ldots & C_{R,M,N}L_{eff,M} \end{pmatrix}\\ &\boldsymbol{\pi}_0 = \begin{pmatrix} \pi_{1,0} \\ \vdots \\ \pi_{M,0} \end{pmatrix} \quad \boldsymbol{\alpha}_{\mathrm{s}} = \begin{pmatrix} \alpha_{s,1} \\ \vdots \\ \alpha_{s,N} \end{pmatrix}. \end{aligned} \tag{12}$$

Now, assuming that we have a gain target of T, we have the optimal input pump average powers $\boldsymbol{\pi}_0$ to minimize $|\mathbf{Gain} - \mathbf{T}|^2$ according to the LSM

$$\pi_0 = \left(\mathbf{C^T C}\right)^{-1}\mathbf{C^T}\left(\mathbf{T}/\left(10\log_{10}(e)\right) + \alpha_{\mathbf{s}}L\right). \tag{13}$$

To verify the effectiveness of (13), we apply it to an example: an RA with an 80-km Corning SMF28e fiber, which eliminates the water absorption peak. Four pumps are used, and the wavelengths are 1420, 1430, 1450, and 1460 nm. The signal wavelengths range from 1525 to 1564 nm, with 100 GHz channel spacing. The Raman gain coefficient used in the simulation is the Corning non-dispersion shifted fiber gain coefficient mentioned in [20] (which used a continuous pump during the measurement).

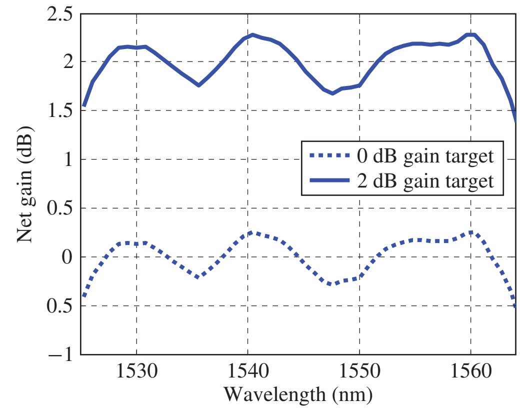

Fig. 1. Net gain spectrum of the optimized RA with the SMF28e fiber.

First, we set the target net gain to be 0 dB, and we obtain the optimal average pump powers of 134, 129, 252, and 186 mW. Then we reset the target gain to be 2 dB, and the optimal pump powers change to 150, 145, 281, and 216 mW. Fig. 1 shows the C-band gain spectrum that has been optimized with the targeted net gain of 0 and 2 dB. Fig. 1 is obtained via (13) directly and it is verified by the time-domain simulation. In the time-domain simulation, the parameters of the TDM pumps are as follows: The duty cycle for the TDM pumps is 0.25, the pulse shape is rectangular, and the modulation frequency is 250 kHz.

IV. Impact of the Modulted Pump

A. Enhanced DRB Noise

The pump modulation will not only cause the signal to fluctuate [14] and enhance the average gain but also enhance the DRB noise. To investigate the DRB noise in RAs with TDM pumps, we have to take a close look at (3) and (4). In order to consider the "pure" DRB noise, we neglect the ASE noise term in (3) and (4), which is also done in [11]. And therefore, (3) and (4) can be combined to have the DRB noise as

$$\begin{aligned} P_{DRB}^+(z,t) &= \gamma_R \int_0^z P_{RB}^-\left(x, t - \frac{z-x}{V_s}\right)\\ &\qquad \times \exp\left(\int_x^z \left[g\left(x', t - \frac{z-x'}{V_s}\right) - \alpha_s\right]\mathrm{d}x'\right) dx\\ P_{RB}^-(z,t) &= \gamma_R \int_z^L P_s^+\left(x'', t + \frac{z-x''}{V_s}\right)\\ &\qquad \times \exp\left(\int_z^{x''} \left[g\left(x', t + \frac{z-x'}{V_s}\right) - \alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x''\\ P_s^+(z,t) &= P_{s0}\exp\left(\int_0^z \left[g\left(x', t - \frac{z-x'}{V_s}\right) - \alpha_s\right]\mathrm{d}x'\right). \end{aligned} \tag{14}$$

As explained in [7] and [14], the signal power variation is quite small compared with to the average power in practical applications. Therefore, it can be neglected during the calculation of the DRB noise [11]. Under this assumption, and assuming that the input signal is monochromatic with a constant power P_{s0}, (14) can be simplified to

$$P_{DRB}^{+}(z,t) \approx \gamma_R \int_0^z P_{RB}^{-}\left(x, t-\frac{z-x}{V_s}\right) d_{+0}(z,x)\ \mathrm{d}x$$
$$P_{RB}^{-}(z,t) \approx \gamma_R \int_z^L P_{s0}\, d_{+0}\left(x'',0\right) \times \exp\left(\int_z^{x''}\left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x''$$
$$d_{+0}(z,x) = \left\langle \exp\left(\int_x^z\left[g\left(x', t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\right\rangle. \tag{15}$$

Here $\langle\rangle$ denotes the average over time. What we are mostly concerned about is the average power for the DRB noise [11], which will be one of the main sources that degrade the performance

$$\left\langle P_{DRB}^{+}(L,t)\right\rangle \approx \gamma_R^2 P_{s0} \int_0^L \int_x^L d_{+0}(L,x)\, d_{+0}\left(x'',0\right) d_{-0}\left(x'',x\right) \mathrm{d}x''\ \mathrm{d}x$$
$$d_{-0}\left(x'',x\right) = \left\langle \exp\left(\int_x^{x''}\left[g\left(x', t+\frac{x-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\right\rangle. \tag{16}$$

The signal to DRB noise ratio $OSNR_{\mathrm{DRB}}$ is defined as

$$OSNR_{\mathrm{DRB}} = \frac{P_{s0} d_{+0}(L,0)}{\left\langle P_{DRB}^{+}(L,t)\right\rangle} \approx \frac{1}{\gamma_R^2 \int_0^L \int_x^L d_{+0}\left(x'',x\right) d_{-0}\left(x'',x\right) \mathrm{d}x''\ \mathrm{d}x}. \tag{17}$$

The DRB noise enhancement can be fully explained by (17). As discussed in the previous section, the resonant gain enhancement effect will enable the TDM pumps to provide higher gain on the DRB noise in comparison to the gain provided by the continuous pumps with the same average powers, especially when the wave is co-propagating with the pump waves. And therefore, we have $d_{-0}(x'',x) \gg \overline{G}(x'',x)$, which is the main cause of the DRB noise enhancement effect.

When the modulation frequency is very high ($f \gg$ 10 MHz), (17) reduces to

$$OSNR_{\mathrm{DRB}} = \frac{1}{\gamma_R^2 \int_0^L \int_x^L G^2\left(x'',x\right) \mathrm{d}x''\ \mathrm{d}x} \tag{18}$$

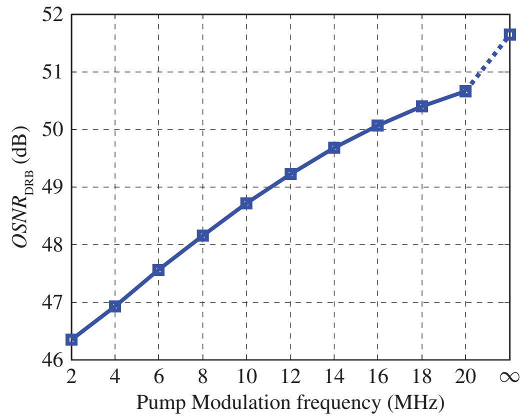

Fig. 2. Impact of the modulation frequency on $OSNR_{\mathrm{DRB}}$ in a TDM-pumped RA with four pumps.

where $G(x'',x)$ is the amplifier gain from x to x'' in continuously pumped RAs. Equ. (18) is in accordance with published results [21], [22].

We simulated a TDM-pumped RA with (17) to obtain the optical signal to DRB noise ratio $OSNR_{\mathrm{DRB}}$. The pump configuration is the same as in the example introduced in Section III-B. The pumps are located at 1420, 1430, 1450, and 1460 nm with the average powers of 134, 129, 252, and 186 mW. The gain medium is the 80-km Corning SMF28e fiber. The Rayleigh scattering coefficient is from [11], with the value of 4×10^{-5}/km. The dispersion coefficient is also from [11], with the value of 17 ps/nm/km. These values are given at 1550 nm. The zero-dispersion wavelength is 1300 nm. The pumping order is 1420, 1430, 1450, and 1460 nm. The pumping duty cycle is 0.25 and the shape of the pump pulses is rectangular. The ON–OFF gain for the C-band signal is around 16 dB.

First, we take a look at the impact of the modulation frequency upon the DRB noise. The signal wavelength is 1550 nm. The modulation frequency changes from 2 to 20 MHz and the $OSNR_{\mathrm{DRB}}$ resulting from the continuous pumps (the modulation frequency ∞) are plotted in Fig. 2. It can be seen from Fig. 2 that $OSNR_{\mathrm{DRB}}$ increases as the modulation frequency increases. We can also see a significant DRB noise enhancement for the low pump modulation frequency configuration, which is about 5 dB higher than the high pump modulation frequency (20 MHz) configuration.

Second, the pump modulation frequency is fixed at 8 MHz. The signal wavelengths range from 1525 to 1564 nm. The signal to $OSNR_{\mathrm{DRB}}$ spectrum is illustrated in Fig. 3. It can be seen that $OSNR_{\mathrm{DRB}}$ is higher at short wavelengths. This is because the short wavelength signal gain is provided by the pumps at 1420, 1430, 1450, and 1460 nm, while the long wavelength signal gain is mainly provided by the pumps at 1450 and 1460 nm (the pumps at 1420 and 1430 nm have very small gain on the 1560-nm signal). The long wavelength signal has higher gain variation because the gain is low during the two continuous time slots but gets higher during the other two continuous time slots.

It is worth mentioning that, in [7], only 10 kHz pump modulation frequency was needed to ensure the amplifier

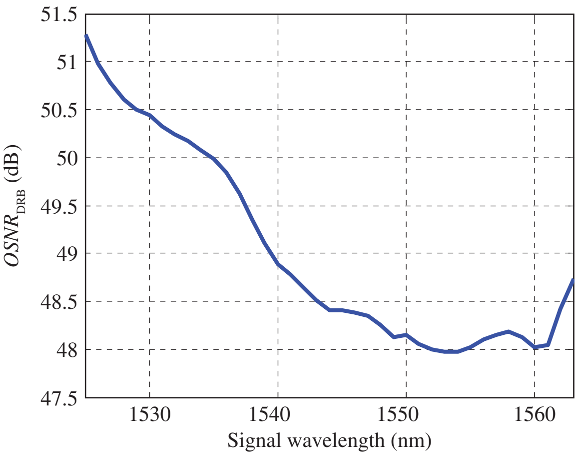

Fig. 3. $OSNR_{\mathrm{DRB}}$ spectrum in a TDM-pumped RA with four pumps. The pump modulation frequency is 8 MHz.

performance. However, a modulation frequency of more than a few megahertz is required in order to reduce the DRB noise as demonstrated in [11] and this paper. This is because, in [7] the main problem is the pump-modulation-induced gain variation and the frequency limit is determined by the 3-dB corner frequency of the RIN transfer function for the counterpumping scheme [14]; however, for the enhanced DRB noise the main cause is that the backward Rayleigh scattering wave is greatly amplified by the backward pump waves (because they share the same propagation direction), so the frequency limit is determined by the 3-dB corner frequency of the RIN transfer function for the co-pumping scheme. As stated in [23], the 3-dB corner frequency of the RIN transfer function is a few kilohertz for the counterpumping scheme and a few megahertz for the co-pumping scheme in single mode fiber. Therefore, to suppress the DRB noise enhancement more efficiently, a higher modulation frequency of a few megahertz is required.

It is hard to achieve the high modulation frequency by modulating the pump drive current. One alternative approach is to use Mach–Zehnder interferometers to modulate the pumps [11] and combine the pumps afterwards.

B. Enhanced ASE Noise

Similarly, to investigate the ASE noise enhancement, we neglect the DRB noise. The ASE noise is composed of two terms, the backward ASE noise reflected by the Rayliegh scattering and the forward ASE noise

$$
\begin{aligned}
&P_{ASE}^{+}(z,t)\\
&= \gamma_R \int_0^z P_{ASE}^{-}\left(x, t-\frac{z-x}{V_s}\right)\\
&\quad\times \exp\left(\int_x^z \left[g\left(x', t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x\\
&\quad+\int_0^z \sum_{j=1}^{M} C_{R,j}\left[P_{p,j}\left(x, t-\frac{z-x}{V_s}\right)2h\upsilon_s\Delta\upsilon_s n_{sp,j}\right]\\
&\quad\times \exp\left(\int_x^z \left[g\left(x', t-\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\ \mathrm{d}x\\
&P_{ASE}^{-}(z,t)\\
&= \int_z^L \sum_{j=1}^{M} C_{R,j}\left[P_{p,j}\left(x, t+\frac{z-x}{V_s}\right)2h\upsilon_s\Delta\upsilon_s n_{sp,j}\right]\\
&\quad\times \exp\left(\int_z^x \left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\ \mathrm{d}x.
\end{aligned}
\tag{19}
$$

Equation (19) can be used to calculate the ASE noise by taking the time average on the equation. For demonstration of the enhanced ASE noise, we consider the one-pump case like in [9]. Therefore, (19) can be simplified to

$$
\begin{aligned}
&P_{ASE}^{+}(z,t)\\
&\approx \gamma_R \int_0^z P_{ASE}^{-}\left(x, t-\frac{z-x}{V_s}\right) d_{+0}(z,x)\ \mathrm{d}x\\
&\quad+\int_0^z C_R P_p\left(x, t-\frac{z-x}{V_s}\right)2h\upsilon_s\Delta\upsilon_s n_{sp} d_{+0}(z,x)\ \mathrm{d}x\\
&P_{ASE}^{-}(z,t)\\
&\approx 2h\upsilon_s\Delta\upsilon_s n_{sp}\left(\exp\left(\int_z^L \left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)-1\right)\\
&+2h\upsilon_s\Delta\upsilon_s n_{sp}\int_z^L \alpha_s \exp\left(\int_z^x \left[g\left(x', t+\frac{z-x'}{V_s}\right)-\alpha_s\right]\mathrm{d}x'\right)\mathrm{d}x.
\end{aligned}
\tag{20}
$$

Taking the time average on (20), we have

$$
\begin{aligned}
\left\langle P_{ASE}^{+}(L,t)\right\rangle &\approx 2h\upsilon_s\Delta\upsilon_s n_{sp}\gamma_R\\
&\times \int_0^L \Bigg[\left(d_{-0}(L,x)-1\right)+\int_x^L \alpha_s d_{-0}\left(x'',x\right)\ \mathrm{d}x''\\
&+\frac{1}{\gamma_R} C_R \pi_0 \exp\left[-\alpha_p(L-x)\right]\Bigg] d_{+0}(L,x)\ \mathrm{d}x.
\end{aligned}
\tag{21}
$$

We performed simulation with (21) to illustrate the ASE enhancement due to the application of the TDM pumps. The fiber used is the same as the one used for the DRB noise simulation. The pump wavelength is 1450 nm and the average pump power is 420 mW. The pump is modulated with a 2-MHz rectangular pulse with the duty cycle of 0.25. The ASE spectrum resolution bandwidth is 1 nm. The ASE power spectral densities induced by the TDM pump and the CW pump are illustrated in Fig. 4.

The reason for ASE enhancement is that the gain for the backward propagating ASE wave which co-propagates with

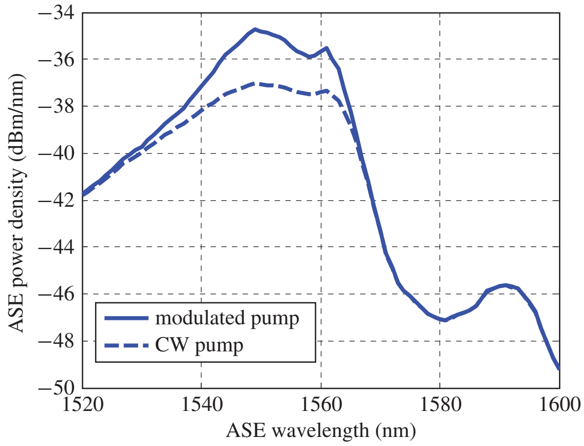

Fig. 4. ASE power spectral densities at the output of the amplifiers with a TDM pump and a CW pump.

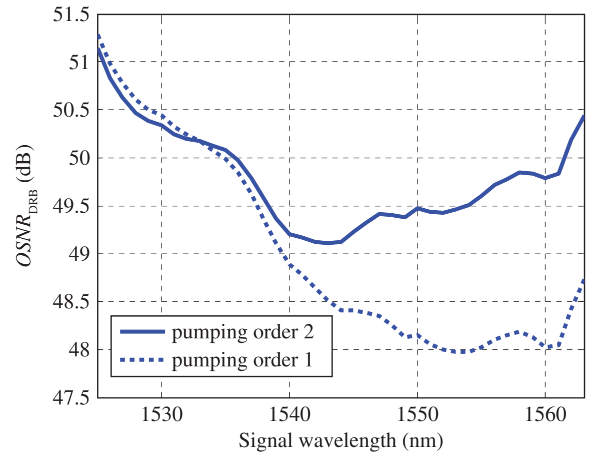

Fig. 5. $OSNR_{\mathrm{DRB}}$ spectrum in the TDM-pumped RAs with four pumps with different pumping orders.

the pump is resonantly enhanced. This will be more severe when the zero-dispersion wavelength is between the pump and the signal wavelengths. This is because, when the zero-dispersion wavelength is close to the signal and pump wavelengths, the dispersion at the signal and pump wavelengths will be relatively low and this will reduce the value of $\left(V_{P,j}^{-1} - V_s^{-1}\right)$, weaken the walk-off effect, and finally increase the backward propagation gain $g\left(z, t + V_s^{-1}(L - Z)\right)$ in (20).

It is worth noting that an explicit formula like (21) can also be derived for the multiple pump case if the spontaneous emission factor n_{sp} is assumed to be same for different pumps.

C. Simple Formula to Evaluate the Impact of Pump Modulation

To evaluate the pump modulation impact, the key problem is to evaluate the Fourier coefficients of the signal gain. Assuming that all the pumps have the same waveform, the Fourier coefficients of the gain can be written as

$$\int_{x_1}^{x_2} g_n^{\pm}(z)\,\mathrm{d}z \approx f_n\left(\sum_j C_{R,j} P_{j,L} \exp\left(jdn\,(i2\pi)\right)\right) e^{-\left(in\Omega\left(V_P^{-1} \pm V_s^{-1}\right)+\alpha_p\right)L} \times \frac{e^{\left(in\Omega\left(V_P^{-1} \pm V_s^{-1}\right)+\alpha_p\right)x_2} - e^{\left(in\Omega\left(V_P^{-1} \pm V_s^{-1}\right)+\alpha_p\right)x_1}}{\alpha_p + in\Omega\left(V_P^{-1} \pm V_s^{-1}\right)} \tag{22}$$

where $P_{j,L}$ is the average pump power, f_n is the normalized Fourier coefficient, and d is the duty cycle. j stands for the jth pump, which occupies the jth time slot. According to (22), to minimize the gain fluctuation at point z, we have the following approaches:

1) increase the modulation frequency;
2) find the dips of the RIN transfer function [23] to minimize the Fourier coefficients of the forward/backward gain, as was done in [14];
3) optimize the pumping order to minimize

$$\sum_j C_{R,j} P_{j,L} \exp\left(jdn\,(i2\pi)\right). \tag{23}$$

As discussed earlier, the gain fluctuation is mainly contributed by the first-order Fourier components, because the higher order coefficients are smaller due to the walk-off effect. Therefore, our optimization target will be the minimization of (23) when $n = 1$.

We use the RA introduced in Section IV-A as an example. Based on (23), it is found that, if the pumping order is changed from 1420, 1430, 1450, and 1460 nm (pumping order 1) to 1420, 1450, 1430, and 1460 nm (pumping order 2), we will be able to reduce the first-order Fourier coefficient of the gain for most of the signal wavelengths at the C band. The simulation is performed afterwards and the results are illustrated in Fig. 5. It shows that $OSNR_{\mathrm{DRB}}$ can be increased significantly for most of the signal wavelengths by choosing the proper pumping order. As discussed in Section IV-A, this phenomenon can be explained as follows. If we choose pump 3 and pump 4 to occupy the second and the fourth time slots instead of the third and the fourth time slots in the period, since the signals gain provided by the pumps at 1420 and 1430 nm are similar, the gain modulation frequency is doubled compared to the case where these two pumps are placed in the first and the second time slot. Therefore, the DRB noise enhancement is reduced. Similarly, positioning the pumps at 1450 and 1460 nm in the proper time slots will reduce the DRB noise enhancement effect as well. The almost unchanged $OSNR_{\mathrm{DRB}}$ at short wavelengths (1525–1532 nm) results from the fact that the gains of these wavelengths provided by the four pumps are similar and the change of the pumping order does little to help reduce the first-order Fourier coefficient of the gain at these wavelengths.

V. Conclusion

In conclusion, we have demonstrated an analytical approach to study TDM-pumped RAs. Resonant gain enhancement and ASE/DRB noise enhancement are explained. Optimal configuration for the pump powers and the pump modulation parameters are proposed. It is worth mentioning that the analysis of the noise performance in this paper considered the average OSNR or average noise power at the fiber output; however, in order to evaluate the system performance, the instantaneous OSNR is required. The instantaneous behavior of the signal, such as modulation format, should be taken into account during the analysis. We hope to extend the model on these lines in future research.

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