clearly becomes more directive in the axial direction as both the beamwidth and backward radiation decrease substantially.

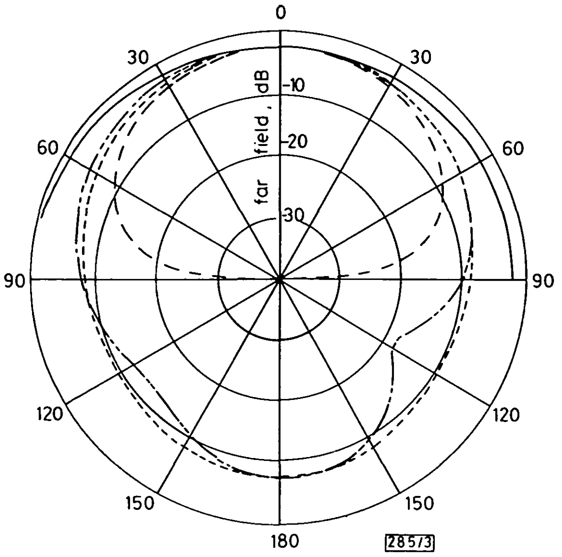

Fig. 3 *Radiation patterns of a dielectric resonator (same data as in Fig. 2) placed on (a) an infinite ground plane and on (b) a ground plane of diameter $d_g = d$*

a ——— *E*-plane pattern - - - - *H*-plane pattern
b — — — *E*-plane pattern ······· *H*-plane pattern

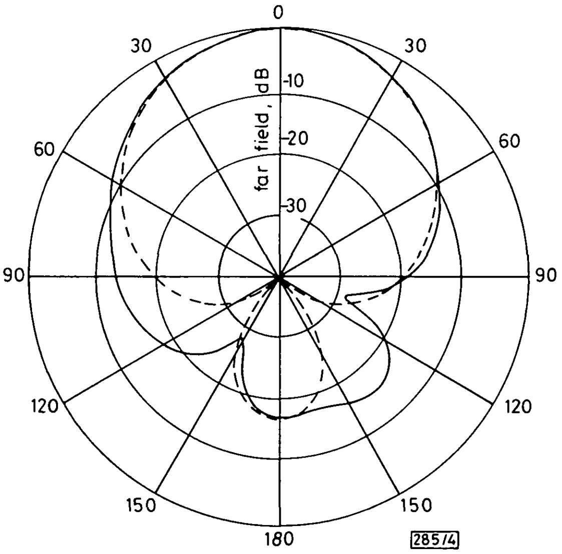

Fig. 4 *Radiation patterns of a dielectric resonator (same data as in Fig. 2) placed inside a topless cylindrical cavity of diameter $d_c = 2{\cdot}94$ cm and height $h_c = 1{\cdot}18$ cm*

——— *E*-plane pattern - - - - *H*-plane pattern

Conclusions: The radiation characteristics of the cylindrical dielectric resonator antenna are predicted more accurately using a numerical method. It is found that this antenna is characterised by high backward radiation and asymmetrical radiation patterns. The symmetry of the radiation patterns can be improved by decreasing the ground plane diameter, while the backlobe level is reduced by placing the dielectric resonator inside a topless cylindrical cavity.

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RELATIONSHIP BETWEEN LINEWIDTH AND CHIRP REDUCTIONS IN GAIN-DETUNED COMPOSITE-CAVITY SEMICONDUCTOR LASERS

Indexing term: Semiconductor lasers

Detuned gain and composite cavity frequency response are two structural sources of amplitude-phase coupling which both need to be considered in addition to the material Henry factor. The spontaneous emission rate dependence on the composite cavity bandwidth is also taken into account to calculate the linewidth and chirp reduction factors.

Introduction: Coherent optical communication systems or those with high data rates require semiconductor laser transmitters with dynamic single longitudinal mode operation and chirp reduced modulation. The use of semiconductor lasers as local oscillators induces low linewidth and tuning requirements. Line narrowing, chirp reduction and frequency tuning have been widely investigated by the use of an hybrid or monolithic composite cavity. In such a laser structure the active region is loaded by a separate passive region, with higher Q than a simple Fabry–Perot resonator, whose phase and/or frequency is electrically or mechanically controlled. The composite cavity frequency response is a structural source of amplitude-phase coupling whose effect on linewidth and chirp has been previously investigated[1–2] and observed.[3] However, the effect of the gain detuning also needs to be considered[4] as well as the composite cavity bandwidth which determines the spontaneous emission rate in the lasing mode.

Complex composite cavity frequency change: The composite structure can be described as a Fabry–Perot cavity in which the external passive resonator is taken into account by a complex, frequency-dependent equivalent facet reflectivity $r_{eq}(\omega) = r(\omega) \exp[-j\phi_r(\omega)]$.[2,5] Denoting by L the active region length, N the carrier density, k the wave number and r_1 the second facet reflectivity, the standard laser equation is

$$f(\omega, N) = r_1 r(\omega) \exp[-jk(\omega, N)L] - 1 = 0 \tag{1}$$

where $k(\omega, N)$ is related to the refractive index $n(\omega, N)$, the optical gain $g(\omega, N)$ and the dielectric loss per unit length α_A by

$$k(\omega, N) = \frac{\omega}{c} n(\omega, N) + \frac{j}{2}[g(\omega, N) - \alpha_A] \tag{2}$$

Because eqn. 1 is not an analytical function of the two variables ω and N, the general solution $\omega(N) = \omega'(N) + j\omega''(N)$ is complex and its imaginary part vanishes at the lasing frequency. With the Henry factor denoted by $\alpha = -2(\omega/c)\, \partial_N n/\partial_N g$ (where ∂_N is $\partial/\partial N$), differentiating of eqn. 1 leads to the real and imaginary part of the complex cavity frequency change:

$$\Delta\omega' = \frac{-\partial_\omega \hat{g} + 2\alpha\, \partial_\omega \hat{k}}{(\partial_\omega \hat{g})^2 + (2\, \partial_\omega \hat{k})^2} \partial_N g\, \Delta N \tag{3}$$

$$\Delta\omega'' = -\frac{\alpha\, \partial_\omega \hat{g} + 2\, \partial_\omega \hat{k}}{(\partial_\omega \hat{g})^2 + (2\, \partial_\omega \hat{k})^2} \cdot \partial_N g\, \Delta N \tag{4}$$

where $\hat{g} = g + (1/L) \ln r(\omega)$ is the optical gain per unit length after distribution of the mirror losses, and $\hat{k} = \omega n/c + \phi_r(\omega)/2L$ the real part of the wave number after distribution of the facet phase shift.

Chirp and linewidth reduction factors: Eqn. 3 expresses the frequency chirping of the carrier-density-modulated laser. By denoting $\Delta\omega_0'$ the frequency chirp at the maximum of the gain curve and without frequency dependence of the equivalent facet reflectivity (i.e. $\partial_\omega \hat{g} = 0$ and $\partial_\omega \hat{k} = n_g/c$, where n_g is the group index), the chirp reduction factor of the gain-detuned composite-cavity laser is

$$\frac{\Delta\omega'}{\Delta\omega_0'} = \frac{2m_g}{c} \frac{-\frac{1}{\alpha}\partial_\omega \hat{g} + 2\,\partial_\omega \hat{k}}{(\partial_\omega \hat{g})^2 + (2\,\partial_\omega \hat{k})^2} \tag{5}$$

As shown by Arnaud, the relevant amplitude-phase coupling factor is the ratio $\alpha' = -\Delta\omega'/\Delta\omega''$ between the real and imaginary parts of the complex cavity frequency.[6] The linewidth enhancement factor $(1 + \alpha^2)$ introduced by Henry[7] is here replaced by

$$(1 + \alpha'^2) = (1 + \alpha^2)\frac{(\partial_\omega \hat{g})^2 + (2\,\partial_\omega \hat{k})^2}{(\alpha\partial_\omega \hat{g} + 2\,\partial_\omega \hat{k})^2} \tag{6}$$

Eqn. 6 expresses the correction of material linewidth enhancement factor $(1 + \alpha^2)$ by a multiplicative factor with a structural origin. This corrective factor is found formally identical to those introduced by Arnaud and Furuya to take into account the transverse waveguide structure.[6,8]

In the linear gain approximation, the cold composite cavity bandwidth $\Delta\omega_c$ is twice the change of the imaginary part of the complex cavity frequency, given by eqn. 4, with a gain change g equal to the loss:[9]

$$\Delta\omega_c = 2g\frac{\alpha\,\partial_\omega \hat{g} + 2\,\partial_\omega \hat{k}}{(\partial_\omega \hat{g})^2 + (2\,\partial_\omega \hat{k})^2} \tag{7}$$

By considering the proportionality of the adiabatic expression of the static laser linewidth $\Delta\nu$ to the square of the cold cavity bandwidth the linewidth reduction factor can be written as

$$\frac{\Delta\nu}{\Delta\nu_0} = \frac{(2m_g/c)^2}{(\partial_\omega \hat{g})^2 + (2\,\partial_\omega \hat{k})^2} \tag{8}$$

By adding the usual rate equation for the carrier density N to the system eqns. 1 and 2 and by using standard Fourier analysis, the relaxation oscillation frequency ω_R is found to be

$$\omega_R = \omega_{RO}\frac{2m_g}{c}\frac{\alpha\,\partial_\omega \hat{g} + 2\,\partial_\omega \hat{k}}{(\partial_\omega \hat{g})^2 + (2\,\partial_\omega \hat{k})^2} \tag{9}$$

where ω_{RO} is the relaxation oscillation frequency at the gain maximum, with no frequency dependence of the facet reflectivity. This result is in agreement with the work of Vahala and Yariv.[1,4] The damping time constant of the relaxation remains unchanged. A more detailed analysis of dynamics in such a laser will be published elsewhere.

Discussion and conclusion: Strong chirp reduction and line narrowing are obtained when g(or k) is a rapidly varying function of the frequency. This occurs on the edge of the resonance frequency curve of a high-Q external resonator such as a distributed Bragg reflector. However, just at the maximum of the external resonator reflectivity and near the maximum of the gain curve, $\partial_\omega g$ is close to zero and the linewidth and chirp reduction are only dependent on $k(\omega)$. It is easy to show that, in this case, the linewidth reduction factor is the square of the chirp reduction factor.[2] This effect has been recently observed by Olsson.[3] It appears also from eqn. 8 that the linewidth reduction factor of the composite cavity is dependent on its gain detuning. This effect needs to be considered in addition to the Henry factor dependence on the lasing frequency and the linewidth change observed for tunable lasers.[10]

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ACCURACY OF STANDARD PATCH ANTENNA MODELS

Indexing terms: Antennas, Microstrip antennas, Antenna theory

The validity of the leaky cavity and transmission-line models to predict the far zone fields from a microstrip patch antenna is investigated. The prediction of this radiation requires a knowledge of the total current on the antenna. Since the approximate techniques utilise only the current on the bottom surface of the antenna, the relative importance of top and bottom surface currents are examined.

Introduction: Leaky cavity and transmission-line models are commonly used[1–3] to analyse the performance of microstrip patch antennas. These models use only the current on the underside of the patch and not the total current. At the frequencies of interest, the skin depth of the patch being usually much smaller than its thickness, different current distributions will be excited on both sides of the patch. To accurately predict the radiated fields of the antenna a knowledge of the total current is necessary. The relative importance of the top current, and hence the validity of the conventional models have been examined by numerically solving a two-dimensional problem.

Formulation: Consider the two-dimensional antenna structure shown in Fig. 1. An infinite sheet current flows in the $\hat{z}$ direction from the ground plane to the metallic strip at a position $x = x_0$, where $-L/2 < x_0 < +L/2$, exciting the structure. The

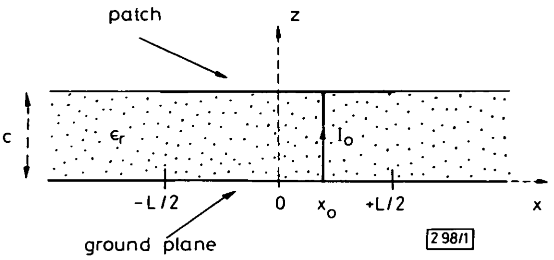

Fig. 1 *Two-dimensional microstrip antenna*