A Fourier Series Approach to Analyze Pump Modulation Induced Noise in Raman Amplifiers With TDM Pumps

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***Abstract*—In this letter, Raman amplifiers (RAs) with time-division-multiplexed (TDM) pumps are analyzed using a computational cost-effective Fourier series approach. Analytical approximate formulas are derived based on this model. These formulas bring deep insight into the pump modulation induced noise (PMIN) in TDM pumped RAs. Moreover, the approach allows the analytical analysis of the TDM pumped RAs with multiple pumps while previous analytical studies mostly focused on the single pump and single signal channel case. By optimizing the pumping order of the multiple pumps, more than 3-dB reduction of the PMIN can be achieved. For short fibers, by properly choosing the modulation frequency, more than 3-dB reduction of the PMIN can be realized.**

***Index Terms*—Raman amplification, Raman scattering, time-division-multiplexed (TDM) pumps.**

I. Introduction

Raman amplification is a promising candidate for long-haul optical amplification in the next-generation optical network [1], and it has already attracted significant attention during recent years. The design issue for Raman amplifiers (RAs) focuses on gain ripple reduction and noise figure (NF) improvement. Various approaches were proposed to optimize the gain profile and NF [2], [3] of the RAs using wavelength-division-multiplexed (WDM) pumping. Unlike erbium-doped fiber amplifiers (EDFAs), the nonlinear interaction among the pumps makes it a difficult task to configure RAs, both numerically and experimentally [4]. Meanwhile, interactions between the pumps will require higher power for the pumps at shorter wavelengths, which will cause the NF spectrum to be uneven. Furthermore, using numerous pumps will cause pump–pump four-wave mixing (FWM) and degrade the performance. Time-division-multiplexed (TDM) pumped RAs were proposed to overcome these problems [5]–[10].

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Several papers have addressed the design issue of TDM pumped RAs. Winzer *et al.*, proposed the modulation frequency limit for TDM pumps and the analytical gain variation expression for the single pump and single signal channel case [6]. The same team also reported amplified spontaneous emission (ASE) enhancement [8]. Bolognini, *et al.* analyzed the double Rayleigh backscattering (DRB) noise enhancement in these amplifiers with the assistance of a time-domain model [9]. Karasek *et al.*, analyzed the transient performance of the amplifiers, based on a full-scale time-domain numerical model [10]. Despite these pioneer works, there are still demands for a more detailed model together with analytical formulas to assist the design and analysis of this kind of RA while reducing the computational time.

In this letter, we propose a Fourier series approach to analyze TDM pumped RAs, which allows us to propose a simple set of equations to study various aspects of the amplifiers, especially for TDM pumped RAs with multiple pumps while previous studies mostly focused on the single pump and single channel case. The periodical nature of the pump waveform enables us to use this approach to represent arbitrary pump wave all over the fiber length using an analytical formula. In the analysis, we pay special attention to the pump modulation induced noise (PMIN). Study will reveal that this noise is actually mitigated by the walk-off effect, which can be modeled by multiplying the relative intensity noise (RIN) transfer function [11] on the Fourier series coefficients. By carefully changing the order of the different pumps, the Fourier coefficients for the gain variation can be reduced and, therefore, the PMIN reduction of 3 dB can be achieved. Further investigation shows that by choosing the proper modulation frequency for RAs with short fibers, the PMIN can also be reduced by more than 3 dB.

II. Theoretical Model

A. Basic Mathematical Model

The well-known Raman propagation equation gives

$$\begin{aligned}\pm\frac{\partial P_s^{\pm}}{\partial z}+\frac{1}{V_s}\frac{\partial P_s^{\pm}}{\partial t} &= -\alpha_s P_s^{\pm}+\gamma_s P_s^{\mp}\\ &\quad+\sum_{j=1}^{M} C_{R,j}\Bigg[P_{p,j}P_s^{\pm}+P_{p,j}2h\nu_s\Delta\nu_s\\ &\qquad\times\left(1+\frac{1}{e^{h(\nu_s-\nu_j)/kT}-1}\right)\Bigg]\end{aligned}\tag{1}$$

where $+$ and $-$ indicate the forward and backward propagating lights, and M is the number of the pumps, V the group velocity, P_s and $P_{p,j}$ the powers of the signal wave and the jth pump propagating along the fiber, $C_{R,j}$ the Raman gain coefficient between frequency ν_s and frequency $\nu_{p,j}$, α the attenuation coefficient, γ the Raleigh scattering coefficient, h the Planck constant, T the absolute temperature, and k the Boltzman constant.

B. Power Evolution Under Small-Signal Assumption

As we know, the modulation on the pump can be transferred to the signal. It is, however, essentially a limiting factor for the pump modulation frequency and will only allow the backward pump scheme to be applied. We start the investigation by neglecting the ASE and DRB noise. The output signal power is, therefore, given by

$$P_s(L,t) = P_s\left(0, t-\frac{L}{V_s}\right)e^{-\alpha_s L} \times \exp\left(\sum_{j=1}^{M} C_{R,j}\int_0^L P_{p,j}\left(z, t-\frac{L-z}{V_s}\right)dz\right). \quad (2)$$

If we assume that the pump is not depleted, the jth backward pump power is given by

$$P_{p,j}(z,t) = P_{p,j}\left(L, t-\frac{L-z}{V_{p,j}}\right)e^{-\alpha_{p,j}(L-z)}. \quad (3)$$

Now, since the periodically modulated pump can be expended as Fourier series, such as

$$P_{p,j}(L,t) = \sum_{n=-\infty}^{+\infty} \pi_{j,n} e^{in\Omega t} \quad (4)$$

where $\Omega = 2\pi/T$, with T being the pump modulation period, and $\pi_{j,n}$ the nth Fourier coefficient of the jth pump, we can find

$$P_s(L,t) = P_s\left(0, t-V_s^{-1}L\right)e^{-\alpha_s L}\exp\left(\sum_{n=-\infty}^{+\infty} c_n e^{in\Omega t}\right) \quad (5)$$

with

$$c_n = \sum_{j=1}^{M} C_{R,j}\pi_{j,n} e^{-in\Omega\left(V_{p,j}^{-1}+V_s^{-1}\right)L} \times \frac{e^{in\Omega\left(V_{p,j}^{-1}+V_s^{-1}\right)L} - e^{-\alpha_{p,j}L}}{\alpha_{p,j} + in\Omega\left(V_{p,j}^{-1}+V_s^{-1}\right)}. \quad (6)$$

C. Pump Modulation Induced Noise (PMIN)

It can be seen from (5) and (6) that the pump modulation introduces periodical fluctuations on the gain which has the Fourier coefficients as the products of the initial pump Fourier coefficients and the square root of the RIN transfer function described in [11] (eq. (21) and (32), if we do not consider the phase term). This explains why the gain fluctuation greatly reduces when the modulation frequency increases, because the RIN transfer function has such behavior.

From (5), we can see the output signal gain $G_s(L,t)$ is a periodical function with the period of T, and therefore, it can be expanded into Fourier series as well. We have

$$G_s(L,t) = e^{-\alpha_s L}\exp\left(\sum_{n=-\infty}^{+\infty} c_n e^{in\Omega t}\right) = \sum_{n=-\infty}^{+\infty} d_n e^{in\Omega t} \quad (7)$$

where d_n is the nth Fourier coefficient. The pump modulation induced signal fluctuation is just like the RIN on the signal and its impact at frequency $n\Omega$ can be evaluated by $|d_n/d_0|^2$. The coefficient d_n can be either obtained numerically (by integrating the signal over the period T), or analytically with the approximation where we assume c_n is very small for $n \neq 0$ at high modulation frequency ($f \gg 10$ kHz for long fiber, which is usually fulfilled during practical applications). That is to say, we can write

$$G_s(L,t) \approx \bar{G}\left(1 + \sum_{n\neq 0} c_n e^{in\Omega t}\right) \quad (8)$$

with $\overline{G}$ being the average gain, and therefore,

$$|d_n/d_0|^2 = |c_n|^2 = \left|\sum_{j=1}^{M} C_{R,j}\pi_{j,n}\frac{1-e^{-\left[\alpha_{p,j}+in\Omega\left(V_{p,j}^{-1}+V_s^{-1}\right)\right]L}}{\alpha_{p,j}+in\Omega\left(V_{p,j}^{-1}+V_s^{-1}\right)}\right|^2 \quad (9)$$

for $n \neq 0$.

TABLE I
RELATIVE FOURIER COEFFICIENTS OF THE SIGNAL FLUCTUATION DUE TO THE PUMP MODULATION

	Equ. (9)	Temporal model
abs(d1/d0)	3.01E-03	2.95E-03
abs(d2/d0)	0.00E-06	5.36E-06
abs(d3/d0)	3.32E-04	3.31E-04
abs(d4/d0)	0.00E-06	1.35E-06
abs(d5/d0)	1.20E-04	1.19E-04

III. MODEL VERIFICATION

In order to verify our proposed model, we simulated an RA with our model [(4) or (9)] and the time domain model (i.e., the model close to (1) while neglecting the Rayleigh scattering and ASE terms and including the pump depletion and signal-signal interactions). An 80-km standard single-mode fiber with 80-μm^2 effective area is used as the gain media. A pump with 370-mW average power is launched backwardly into the fiber to amplify the 1-mW signal. The wavelengths of the pump and the signal are 1450 and 1550 nm, respectively. The pump wave is assumed to be a perfect square wave with a duty circle of 0.5. The modulation frequency is 500 kHz. The group velocity difference is calculated by the dispersive relation. Table I illustrates the relative Fourier coefficients of the power fluctuation on the signal caused by the pump modulation based on different methods; the discrepancy is within 1%. For the computation time, the time-domain method requires tens of minutes while our method can finish within 1 s.

IV. OPTIMIZATION OF THE PUMPING ORDER AND PUMP MODULATION FREQUENCY

From (7)–(9), we can see the signal power Fourier coefficients are associated with the pump ones. With this knowledge, we will try to reduce the signal power fluctuation by optimizing the pumping order and frequency to lower the pump power fluctuation represented by (9). We here assume all the pumps have the square wave with the same duty circle d, but differ by a delay $T * d$, where T is the time period.

As an example, we propose to optimize the pumping order of a TDM pumped RA with four pump wavelengths, which are 1420, 1430, 1450, and 1460 nm, with the average powers of 134, 127, 258, and 178 mW. The 40 signal wavelengths range from

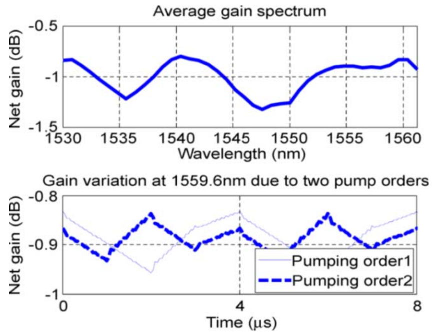

Fig. 1. Average gain spectrum and the gain variations of the two pumping orders.

TABLE II
RELATIVE FOURIER COEFFICIENTS OF THE SIGNAL FLUCTUATION USING DIFFERENT PUMPING ORDER

	pump order 1	pump order 2
abs(d1/d0)	5.64E-03	1.61E-03
abs(d2/d0)	6.05E-04	2.93E-03

1530 to 1561.2 nm and the total power is 40 mW. We select to monitor the signal wavelength of 1559.6 nm. The pump power configuration has been optimized to achieve flattened gain spectrum in C-band (Fig. 1). The modulation frequency is 250 kHz and the duty circle is 0.25. This means that in a period of 4 μs, each pump occupies 1-μs time slot. The gain media is an 80-km Corning SMF28e fiber, which eliminates the water absorption peak at 14xxnm. We use two pumping orders: the original one and the optimized one. First, the four time slots are occupied by 1420, 1430, 1450, and 1460 nm, and second, the four time slots are occupied by 1420, 1450, 1430, and 1460 nm. In Fig. 1, the gain variations of the two pumping orders are illustrated. We can see pumping order 2 produces smaller gain variation. In Table II, the relative Fourier coefficients of the signal power fluctuations using the two pumping orders also show that pumping order 2 produces 3 dB less signal fluctuation. This can be explained that the first and the second pumps provide similar gain on the signal, and so do the third and the fourth pumps. Therefore, pump order 2 can be viewed as two pumps with doubled modulation frequency and higher modulation frequency brings lower PMIN. The results are simulated by the time-domain model and have little discrepancy compared with the results by (9).

Up to now, the discussion has focused on long fibers. For short fibers, the RIN transfer function will have peaks and dips [11, Fig. 6]. As is shown in Section III, the Fourier coefficients for signal fluctuation are the products of the pump initial Fourier coefficients and the square root of the RIN transfer function. Therefore, the modulation frequency should be chosen properly to be at the dips of the RIN transfer function, which can be calculated by

$$f_N = \frac{NV}{2L} \tag{10}$$

TABLE III
RELATIVE FOURIER COEFFICIENTS OF THE SIGNAL FLUCTUATION WITH DIFFERENT PUMP MODULATION FREQUENCY

	103.5 kHz	108.5 kHz
abs(d1/d0)	2.11E-02	4.51E-02
abs(d2/d0)	2.76E-04	9.74E-04

where V is the group velocity, which is about 2.07e8 m/s while the refractive index of fibers is about 1.45. As an example, we use a 10-km DSF as the gain media. The average power is 370 mW for the pump and 1 mW for the signal. Its modulation frequency is 103.5 and 108.5 kHz, respectively, with the same duty cycle of 0.5. The pump and signal wavelengths are 1450 and 1550 nm. It is clearly shown in Table III that by choosing the modulation frequency at the dip of the RIN transfer function, the fluctuation of the signal can be reduced by at least 3 dB.

In conclusion, we have proposed an analytical model to investigate the TDM pumped RAs, which shows good agreement with the results of time-domain simulation. It gives deeper insight into the design and optimization of this promising RA pumping scheme. This model can also explain the DRB and ASE noise enhancement and optimize the pump configuration accordingly. Due to the page limit, the study will be presented elsewhere.

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