Coherent photodetection with applications in quantum communications and cryptography

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ABSTRACT

We present an application of coherent homodyne detection to the problem of low photon number communications and cryptography. As the coherent demodulation of an optical field requires the measurement of its (non commutating) in-phase and quadrature components, we present the structure and operation of an 8-port optical hybrid comprising 2 balanced homodyne detection structures, for the simultaneous measurement of the 2 quadratures. We analyze this receiver operating with a strong local oscillator field, when the received field is in weak coherent states, with digital phase modulation: we obtain the homodyne statistics and the uncertainty product in the presence of vacuum noises from the input signal port and unused ports and discuss the increase in uncertainty due to the simultaneous measurements of the quadratures. We obtain the signal to noise ratio as well of the bit error rate performance for binary phase shift keying and discuss the departure from the standard quantum limit.

Key words: quantum communication, quantum cryptography, coherent photodetection, weak coherent states, optical phase shift keying, in-phase and quadrature measurements.

1. INTRODUCTION

Coherent reception constitutes an interesting technique in optical communications and cryptography due to its unique characteristics of operation close to the standard quantum limit (SQL), when used in balanced homodyne configurations with a strong laser local oscillator: therefore it can work with substantially lower signal-to-noise ratios (SNR) for a given post-detection bit-error-rate (BER) than the incoherent direct detection systems[1]; furthermore it is sensitive to complex amplitude modulations that are more suitable for modern photonic transmission systems[2,3].

Optical homodyne detection constitutes a continuous-variable measurement which possesses interesting characteristics concerning noise free conversion gain, requiring only room temperature standard p.i.n. photodetectors, that exhibit a high quantum efficiency, speed, dynamic range, resolution and input mode selectivity. Additionally, for densely wavelength division multiplexed transmission (DWDM), its highly selective spectral transposition into baseband constitutes an efficient demultiplexing scheme in fiber optic systems; furthermore its highly selective spatial filtering properties provide a strong background radiation rejection in free space communications and cryptography.

In four – port balanced homodyne detection (BHD) configuration, the local oscillator (LO) noise has negligible influence, and the only relevant noise is the zero-point fluctuation in the received signal field. Therefore the output noise is dominated only by vacuum fluctuation at the signal port, and a standard quantum limited (SQL) reception is attainable when a strong LO is employed. However, it measures only one field quadrature.

Now, for the reception of multilevel phase shift keying (PSK) modulations such as QPSK, 8PSK and higher[4], of utmost importance in communications and cryptography, the 2 field quadratures must be measured: structures based on multiport optical hybrids are suitable for this task, with the additional capabilities of frequency and phase tracking in the presence of channel impairments in communications and cryptography applications.

These multiport structures are extensively used in classical optical optical communications (this is with a large number of photons per observation time) especially 8-port receivers with in-phase and quadrature balanced homodyne detection schemes, in Costas loop or decision directed loop configurations[5]. They constitute an attractive solution for modern applications such as quantum communications, quantum cryptography, long distance free space communications and lidar, highly sensitive sensors and other instrumentation and scientific instrumentation[6]. These photonic systems operate

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with photon numbers substantially lower than those used in classical transmission, such as weak coherent states (WCS), and the use of multiport receivers faces diverse challenges when only very few average photons per observation time are available from the received signal

In this work, after briefly reviewing the principles of the eight port optical hybrid detection for two field quadrature measurements, we obtain the measured in-phase and quadrature signal statistics for a quantum coherent state model, which comprises the effects of the zero point fluctuations in the received field and the vacuum noises leaked in the unused ports, we derive the uncertainty relations, the signal - to - noise ratio (SNR) and the bit error rate (BER), with application in quantum communications and cryptography systems with phase shift keying (PSK) modulation.

2. COHERENT OPTICAL 8-PORT HOMODYNE FIELD DETECTION

Structures for in-phase and quadrature measurements in quantum level signals based on 8 port interferometers can be mechanized with photon counters or with homodyne detectors[7]. Figure 1 shows a general 8 - port 90° hybrid receiver structure using fiber optic couplers, based on the splitting of one of the interacting fields into its in-phase and quadrature components and their separate beating with the other optical field in two balanced homodyne detection (BHD) schemes. The local oscillator excess noise and phase noise are cancelled by these balanced configurations. This allows the simultaneous measurements of the 2 quadratures of the input field at the price of an additional noise due to the vacuum fields that leak via the unused input ports, in addition to the zero-point fluctuation in the received signal field[8, 9].

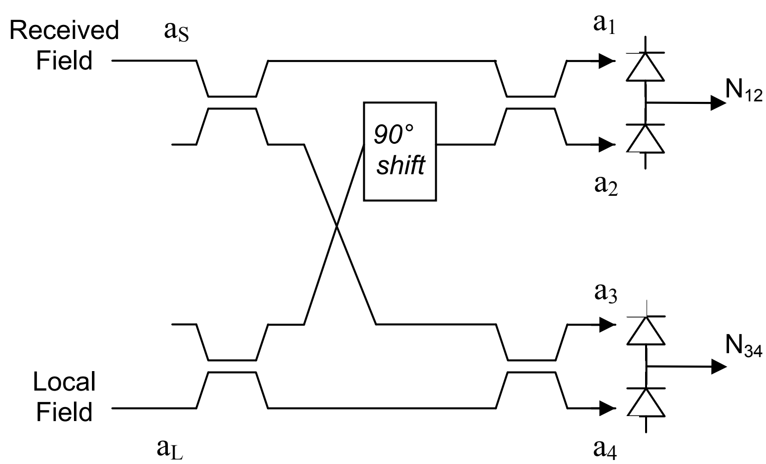

Figure 1. Eight - port homodyne detection of a signal field, based on a 90° optical hybrid: the local oscillator is splitted and phase shifted by 90° to beat simultaneously with the in-phase (upper branch) and quadrature (lower branch) components of the signal phasor at 2 balanced homodyne detectors.

For an incoming signal and a local oscillator in a single spatial mode, we use the description in terms of the photon annihilation operators $\hat{a}_S$ and $\hat{a}_L$, for the signal and local oscillator fields, respectively, such as the Hermitian field operators are written as:

$$\hat{E}_S(t) = \sqrt{\frac{h\nu}{T}}\left[c\hat{a}_S(t) + c^*\hat{a}_S{}^+(t)\right] \text{ and } \hat{E}_L(t) = \sqrt{\frac{h\nu}{T}}\left[c\hat{a}_L(t) + c^*\hat{a}_L{}^+(t)\right]$$

Where $\hat{a}_S{}^+$ and $\hat{a}_L{}^+$ are the respective adjoint operators; the time duration T is the observation time which is much longer than the optical period but much smaller than the coherence time of the optical source; h is the Planck's constant; $c = \exp[j(2\pi\nu t + \phi(t))]$: as we are dealing with homodyne detection, we assume identical optical frequencies ν and phases $\phi(t)$ for the signal and local fields.

For the receiver in figure 1, assuming lossless and perfectly balanced couplers and photodetectors with unit quantum efficiency, the $1/\sqrt{2}$ splitting of the signal and local oscillator fields produces the photon operators at the four outputs shown in table 1, with their relative phases.

Channel	In-phase		Quadrature	
Photon annihilation operators	$\hat{a}_3 = \frac{1}{\sqrt{2}}(-j\hat{a}_S - j\hat{a}_L)$	$\hat{a}_4 = \frac{1}{\sqrt{2}}(-\hat{a}_S + \hat{a}_L)$	$\hat{a}_1 = \frac{1}{\sqrt{2}}(\hat{a}_S - j\hat{a}_L)$	$\hat{a}_2 = \frac{1}{\sqrt{2}}(-j\hat{a}_S + \hat{a}_L)$
Relative optical phases	0	π	$\frac{3\pi}{2}$	$\frac{\pi}{2}$
Electron number difference operators	$\hat{N}_{34} = \frac{1}{2}(\hat{a}_S^{+}\hat{a}_L + \hat{a}_L^{+}\hat{a}_S)$		$\hat{N}_{12} = \frac{1}{2j}(\hat{a}_S^{+}\hat{a}_L - \hat{a}_L^{+}\hat{a}_S)$	

Table 1. Operators resulting from the coherent superposition of the received signal and local oscillator fields in the 8-port hybrid, their relative optical phases and the electron number difference count operators.

The annihilation operators are found to commute, allowing the simultaneous measurement of the 2 signal quadratures, at the price of the introduction of vacuum noises entering at the unused ports, in agreement with the uncertainty principle[7, 8]. Electron number difference count operators obtained after balanced photodetection are also indicated in table 1.

3. IN-PHASE AND QUADRATURE UNCERTAINTY FOR QUANTUM COHERENT STATES

Let's assume that the signal and local oscillator fields are in Glauber's coherent states, with photon numbers N_S and N_L respectively; for the received signal this corresponds well with a typical stabilized laser with constant envelope modulation, as in PSK and higher orders. In a scalar analysis the annihilation operators are expressed in terms of their (statistically independent) in-phase (I) and quadrature (Q) Hermitian components[10]:

$$\hat{a}_S = \hat{a}_{SI} + j\hat{a}_{SQ} \text{ and } \quad \hat{a}_{\mathrm{L}} = \hat{a}_{LI} + j\hat{a}_{LQ}$$

we can separate the classical and quantum contributions for the 2 quadratures of the signal in the form:

$$\hat{a}_{SI} = \langle \hat{a}_{SI} \rangle + \Delta\hat{a}_{SI} \text{ and } \hat{a}_{SQ} = \langle \hat{a}_{SQ} \rangle + \Delta\hat{a}_{SQ}$$

And similarly for the local oscillator

$$\hat{a}_{LI} = \langle \hat{a}_{LI} \rangle + \Delta\hat{a}_{LI} \text{ and } \hat{a}_{LQ} = \langle \hat{a}_{LQ} \rangle + \Delta\hat{a}_{LQ}$$

Where the brackets $\langle . \rangle$ indicate infinite time averaging.

The Wigner function for the received coherent state field is[11]:

$$W(\hat{a}_{SI}, \hat{a}_{SQ}) = \frac{1}{\pi}\exp\left[-\left(a_{SI} - \langle a_{SI} \rangle\right)^2 - \left(a_{SQ} - \langle a_{SQ} \rangle\right)^2\right]$$

Now, in our eight port receiver with the 90° optical hybrid with a strong local oscillator $N_L >> N_S$ and ideal components, i.e. lossless couplers and unit quantum efficiency photodetectors, the joint probability distribution between the in-phase N_{34} and quadrature N_{12} outputs is related to the (antinormally ordered) Q function; this is, a Wigner function convoluted with the vacuum which, for a received field in coherent state, has the form[12,13]:

$$Q(\hat{a}_{SI}, \hat{a}_{SQ}) = \frac{1}{2\pi}\exp\left[-\frac{1}{2}\left(a_{SI} - \langle a_{SI} \rangle\right)^2 - \frac{1}{2}\left(a_{SQ} - \langle a_{SQ} \rangle\right)^2\right]$$

Computing the marginal probability densities from the Q function, respectively:

$$p(N_{12})=\sqrt{\frac{1}{2\pi}}\exp\left[-\frac{1}{2}\left(\hat{a}_{SQ}-\langle\hat{a}_{SQ}\rangle\right)^2\right] \text{ and } p(N_{34})=\sqrt{\frac{1}{2\pi}}\exp\left[-\frac{1}{2}\left(\hat{a}_{SI}-\langle\hat{a}_{SI}\rangle\right)^2\right]$$

having a common standard deviation of 1.

Therefore the simultaneous measurement of the two non commutating quadratures is possible, but at the price of detecting unavoidably the leaked vacuum modes from unused optical ports in figure 1; the uncertainty product is then:

$$\left\langle \Delta\hat{a}_{SI}{}^2\right\rangle^{1/2}\left\langle \Delta\hat{a}_{SQ}{}^2\right\rangle^{1/2}\geq 1$$

this causes additional noise in the measurement as compared to the minimum uncertainty for non simultaneous measurements:

$$\left\langle \Delta\hat{a}_{SI}{}^2\right\rangle_{ns}^{1/2}\left\langle \Delta\hat{a}_{SQ}{}^2\right\rangle_{ns}^{1/2}\geq \frac{1}{4}$$

Where the subscript $_{ns}$ stands for non – simultaneous.

4. SIGNAL – TO- NOISE RATIO AND BIT ERROR RATE ANALYSIS

For a signal and local oscillator fields in quantum coherent states with photon numbers N_S and N_L respectively, from table 1we obtain the following electron number difference count operators for the in – phase and quadrature outputs, respectively:

$$\hat{N}_{12}=\hat{a}_{SI}\hat{a}_{LQ}-\hat{a}_{SQ}\hat{a}_{LI} \text{ and } \hat{N}_{34}=\hat{a}_{SI}\hat{a}_{LI}+\hat{a}_{SQ}\hat{a}_{LQ}$$

To perform the noise analysis, let's consider the in - phase channel N_{34} output in figure 1, with a strong local oscillator $N_L >> N_S$, the dominant term at this output is:

$$\hat{N}_{34}=\langle\hat{a}_{LQ}\rangle\left(\langle\hat{a}_{SI}\rangle+\Delta\hat{a}_{SI}\right)$$

Consisting of the quadrature $\langle\hat{a}_{SI}\rangle$ and its additional quantum noise $\Delta\hat{a}_{SI}$ amplified by the deterministic part of the quadrature local oscillator component, both local oscillator quantum and excess noises are cancelled as a noise free mixing gain: however in this 8 – port receiver the input signal quantum noise is no longer the only noise limitation, and we have to consider the vacuum noise entering at unused ports, resulting in additional uncertainty.

Let's for the moment assume a perfectly phase matched local oscillator, separating the classical and quantum contributions for the in - phase electron number operator in the form:

$$\hat{N}_{34}=\left\langle\hat{N}_{34}\right\rangle+\Delta\hat{N}_{34}$$

Assuming that signal and local are coherent states, we obtain the square of the averaged electron number, corresponding to the average postdetection electric power, and the averaged square of the electron number fluctuations, respectively:

$$\left\langle\hat{N}_{34}\right\rangle^2=N_L\mathrm{N_S} \text{ and } \left\langle\left(\Delta\hat{N}_{34}\right)^2\right\rangle=N_L/2$$

The latter corresponding to the postdetection electric noise power, which is the well known Poisson fluctuation relationship.

A similar analysis for the quadrature port yields:

$$\hat{N}_{12}=\left\langle\hat{N}_{12}\right\rangle+\Delta\hat{N}_{12} \text{ with } \left\langle\hat{N}_{12}\right\rangle^2=N_L\mathrm{N_S} \text{ and } \left\langle\left(\Delta\hat{N}_{12}\right)^2\right\rangle=N_L/2$$

Therefore for a perfectly phase matched local oscillator we obtain the following signal - to - noise ratios (SNR) at the in – phase and quadrature outputs:

$$SNR_{12}=SNR_{34}=2N_S$$

which is half the ratio obtained in non simultaneous measurements, i.e. with a four port (BHD):

$$SNR_{BHD}=4N_S$$

Let's consider the case of optical binary phase-shift keying (BPSK) in which 2 equally probable modulated binary symbols are represented by 2 antipodal phase states in the signal field, and a constant envelope modulation is used to minimize the signal overlap; when a strong LO field is used with perfect phase alignment, the bit error rate (BER) is given by[14]:

$$BER(N_S) = \frac{1}{2} erfc\left(\sqrt{\frac{SNR}{2}}\right)$$

Where

$$erfc[x] = \left(2/\sqrt{\pi}\right)\int_x^\infty \exp(-t^2)dt$$

is the complementary error function.
Thus for our 8 – port receiver with simultaneous quadrature measurement, the bit error rate is finally:

$$BER(N_S) = \frac{1}{2} erfc\left(\sqrt{N_s}\right)$$

Figure 2 is a plot of the bit error rate as a function of the signal photon number, it shows also the standard quantum limit in the measurement of a single quadrature for which the signal – to – noise ratio is $4N_S$ [15].

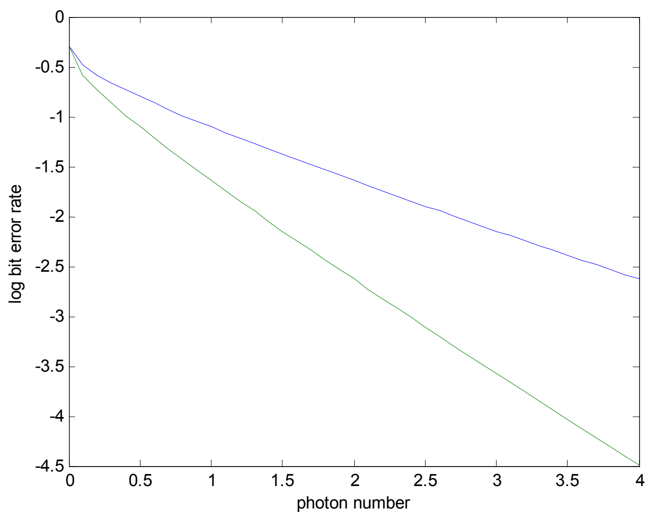

Figure 2. Bit error rate for binary phase shift keying modulation (blue trace) with simultaneous measurement of 2 field quadratures. Standard quantum limit is shown for comparison (green trace).

5. CONCLUSION

The simultaneous measurement of the two quadratures of optical fields is important in the reception of quantum level optical carriers such as those encountered in telecommunications applications and quantum cryptography. In general this task is performed by 8- port optical hybrids with two balanced detectors for the simultaneous detection of those non commutating operators. In these schemes additional uncertainty is introduced due to the vacuum fields that leak through the unused ports.
Our application was in the detection of weak coherent states signals with phase modulations formats that are the most suitable for communications and cryptography; we obtained the uncertainty product for this kind of receivers, as well as the signal – to – noise ratio and the bit error rate for binary transmission, and discussed the departure from the minimum uncertainty product and from the standard quantum limit.

Although in this work we considered only binary phase shift keying modulation, this analysis can readily be adapted to higher order modulations, of great interest in quantum communications due to the spectral efficiency, and also in cryptography for multilevel protocols for quantum key distribution.
Finally the simultaneous measurement of the two quadratures of an optical signal in low photon number levels is important in a diversity of fields, other than communications and cryptography, such as in coherent optical sensing and instrumentation, coherent lidar and scientific tomography.

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