LDPC Code Design and Performance Analysis on OOK Chi-Square-Based Optical Channels

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Abstract—Without being restrictive, optical signal propagation simulations performed in the case of a new concept of packet ring network pointed out that the Gaussian model is not sufficiently accurate to constitute a valuable model of noise-corrupted optical systems. This fact can be generalized to any common wavelengthdivision-multiplexing systems, using on-off keying transmissions where amplified spontaneous emission is the main noise source. In this letter, we propose an alternative chi-square model, which is more accurate and corrects deficiencies of the Gaussian model. In such specific channel, we design an error-correcting scheme based on low-density parity check (LDPC) codes associated to soft decoding. The performance of a chi-square-based LDPC soft decoder and a Gaussian-based one are compared, both applied to a real chi-square optical channel. We point out that the design can be done assuming an additive white Gaussian noise statistic but also that considering the real channel statistics is essential to achieve optimal performance.

Index Terms—Chi-square statistics, Gaussian statistics, low-density parity check (LDPC) codes, optical channel.

I. INTRODUCTION

I N order to support high-speed real-time applications, innovative optical network architectures are investigated. Our study focuses on an optical packet ring, where optical amplifiers in nodes have a severe impact on the signal quality.

The optical channel under study, using an on-off keying (OOK) modulation, is generally assumed to be Gaussian. However, in optically amplified transmissions and because of amplified spontaneous emission (ASE), noise statistic after photodetection is non-Gaussian and signal dependent.

We present an alternative chi-square channel model and, we point out that this model corrects deficiencies of the Gaussian one. By analyzing propagation simulations, we estimate the photocurrent distribution. We show that a chi-square statistic is in accordance with simulation results and constitutes a proper model of the optical OOK noise-corrupted channel.

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Forward error correction (FEC) using low-density parity check (LDPC) codes have been shown to be an efficient solution to compensate for the performance degradation in optical networks [1], [2]. However, no detailed analysis of an FEC design constrained by optical packet specifications and assuming a chi-square statistic has been reported so far.

Here, we present FEC design taking into account optical constraints and chi-square channel statistics. We consider at the reception a decoder based on belief propagation (BP) algorithm [3], [4]. We particularly point out that it is necessary to adapt the LDPC decoder to ensure the optimal performance.

II. NETWORK AND OPTICAL CHANNEL CHARACTERISTICS

Our study has been done in the frame of a new optical network concept: the metro-access packet ring network ECOFRAME [5], using wavelength-division-multiplexing (WDM) transmissions. Nodes of this network benefit from transparency for transient traffic, thus an optical packet is not regenerated until it is extracted from the ring. Consequently, the signal accumulates impairments from its whole path and not only from a node-tonode propagation span. This network based on the optical packet commutation does not need packet demodulation and label processing nor spatial or spectral switching. The data rate per wavelength D_{max} is 10 Gb/s. Packets have a constant duration d equal to $10 \pm 1 \,\mu s$. The structure of the optical packet imposes the LDPC dimensions but does not impact the channel statistics. So, the channel model we propose in the following is valid for various WDM packet ring networks, which uses transparent nodes and where ASE is the main noise source.

The statistical model generally assumed for noise-limited OOK optical communications on fibers is Gaussian [6]. In this model, noises are additive and the photocurrent probability density function (pdf) is Gaussian with a different variance on "1" and "0" in order to take into account beatings between noise and signal due to photodetection. However, in practice, it is easy to notice asymmetry in the photocurrent distributions [7], so symmetrical pdf and additive noises are inappropriate models. Moreover, the postphotodetection thermal noise is generally negligible and in this case the photocurrent takes only positive values. Therefore, the Gaussian model, allowing negative photocurrent values, is not appropriated. We propose an alternative model based on a simple but more accurate description of the photocurrent. We start from the description of the total electromagnetic field E in the optical domain:

$$\vec{E} = (A_i + N_{I_r} + jN_{Q_r})\vec{e_r} + (N_{I_{\theta}} + jN_{Q_{\theta}})\vec{e_{\theta}}.$$
 (1)

 \vec{e}_r and \vec{e}_{θ} refer to the polarizations, respectively, parallel and orthogonal to the signal. A_i , is the signal amplitude coding an emitted bit *i*. N_{Ip} and N_{Qp} are independent real and imaginary



Fig. 1. Simulation results: histograms of symbols (a) "0" and (b) "1."

parts of the complex Gaussian noise for each polarization referred by r or θ . Denoting R the photodetector responsivity, the photocurrent I is expressed as

$$\frac{I}{R} = (A_i + N_{I_r})^2 + N_{Q_r}^2 + N_{I_{\theta}}^2 + N_{Q_{\theta}}^2.$$
 (2)

In this case, the photocurrent is the sum of four squared variables of variance σ^2 . Its pdf is given by

$$p_I(I) = \frac{1}{2a\sqrt{a}}\sqrt{\frac{I}{\delta_i}} e^{-\frac{1}{2}\left(\frac{I}{a} + \delta_i\right)} I_1\left(\sqrt{\frac{I}{a}}\delta_i\right)$$
(3)

where we define $a = R\sigma^2$ and $\delta_i = A_i^2/\sigma^2$. $I_1(\beta)$ is the first kind modified Bessel function of order 1. The distribution (3) is a noncentral chi-square of order 4. This model intrinsically takes into account the beating process. It corrects the problem of negative photocurrent of the Gaussian model, and it brings out the photocurrent distribution asymmetry.

Previous works propose chi-square models [8] based on a linear expansion of noises. They deal with filtering leading to high-order chi-square models and to validation of the Gaussian approximation. However, the hypothesis established for these models is not verified for the channel under study.

In order to evaluate channel statistics resulting from the optical signal propagation over the specific network, we have developed a simulator of optical WDM signal propagation, taking into account noises added by optical amplifiers, fiber dispersion, and nonlinear effects such as self-phase modulation (SPM) and cross-phase modulation (XPM).

Our simulations of propagation are performed in a case where nonlinear effects are not the main cause of distortion, thus they can be considered as a small perturbation of our model. The network is composed of 10-20 nodes with a large range of noise factor (from 6 to 12 dB) separated by spans ranging from 50 to 80 km. We have simulated 4-10 WDM channels around 1550 nm and separated by 100 to 200 GHz. We have chosen "flat" dispersion maps in order to have the same cumulated dispersion at each node. We assume that the ASE generated by optical amplifiers at each node is Gaussian. Our simulator provides samples of the photocurrent from which typical simulation results are presented by histograms on Fig. 1. These histograms reveal an asymmetry of the photocurrent distributions, which is particularly evident on symbols "0." This is in accordance with the chi-square model since (3) is more asymmetrical for low values of the noncentrality parameter δ_i and tends to a Gaussian for high δ_i .

Fig. 2 shows fittings of the previous histograms by the chisquare distribution (3) and the Gaussian one. In this example, the mean squared error (mse) of the Gaussian fitting is three times higher than the chi-square one and it can be up to five times



Fig. 2. Gaussian and chi-square fitting of histograms of (a) "0" and (b) "1."



Fig. 3. FEC frame specifications.

higher depending on the parameters of the simulation (noise factors, extinction rate, etc). As a remark, our simulator estimates the best parameters a and δ_i . We notice that a is slightly different for "0" and "1." Actually, this parameter refers to the variance of the optical noise (ASE) and it should be identical for "1" and "0." This small difference is linked to the impact of nonlinear effects inducing a small perturbation.

Nevertheless, the photodetection process keeps the photocurrent distribution asymmetrical even with nonlinear distortions. That is why our model remains a more accurate channel description than the Gaussian one.

III. FEC DESIGN AND PERFORMANCE EVALUATION

Although previous works have focused on modeling optical channel statistics [8], none has used it to improve performances of soft decoding algorithms applied to optical systems that generally assume Gaussian distributions [1], [2]. In this letter, we design LDPC codes and associate soft decoders, taking into account the optical channel specificity.

In addition, we consider the packet duration constraint $(10 \pm 1 \ \mu s)$ for the LDPC code design. The total number of bits in the optical packet $N_{\rm bitmax}$ is set to 100 000 bits. Each packet is split into an integer number of FEC frames, and its length $N_{\rm bitmax}$ is kept constant with stuffing bits (Fig. 3).

We focus in this letter on a balanced incomplete block design LDPC (BIBD-LDPC) construction [9], which is deterministic. This method, where all rows of the parity check matrix are constructed from vector permutations, simplifies implementation and avoids short length girths, ensuring the decoder algorithm convergence. Two parameters ns and q (a prime number) define the LDPC(N, K) dimensions:

$$N = N_{\text{bit frame}} = qns;$$
 $K = N_{\text{inf frame}} = q(ns - 1).$

We aim at reducing the bit-error rate (BER) from 10^{-5} to 10^{-12} . This corresponds to a minimal net coding gain (NCG) of 4.5 dB (defined as the SNR difference for which the uncoded BER is equal to 10^{-12} and the one for which the uncoded BER equals 10^{-5}). This NCG value theoretically determined using [10] has been verified by extrapolating simulation results.

To evaluate the effective NCG in the coded case, we have designed LDPC code over the additive white Gaussian noise (AWGN) channel before analyzing computer simulations results over both channels. From computer simulations over an AWGN channel we have obtained that all codes with $ns \leq 46$ have an NCG higher than 6.5 dB, i.e., 2 dB higher than the requirement.

The available data rate is thus the specification being taken into account for the LDPC code design. It is calculated with

$$D = \frac{K \times (\text{Frame/packet})}{d}.$$
 (4)

The number of information bits per frame K increases with ns, which should be as high as possible. However, D also depends on the packet duration d and on the number of frames per packet. For each tested code, we searched the optimal d value maximizing D and among them, the LDPC(6176, 5983) (q = 193, ns = 32) maximizes the data rate (9.69 Gb/s).

We now investigate the LDPC codes over an optical ring network having a chi-square statistic of order 4, as described in part II. We consider that the amplitude A_1 is normalized to 1, A_0 is equal to 0, and that the responsivity R is equal to 1. The real A_0 values differ, but the ratio A_1^2/A_0^2 is generally high, so similar results are obtained and no generality is lost. In simulations, a chi-square variable, for which each of the four squared variables follows a normal law \mathcal{N} ($m_i = A_i/2, \sigma^2$) distribution is affected to the emitted bit. The performances are evaluated as a function of the Q factor defined by

$$Q = \frac{(I_1 - I_0)}{(\sigma_1 + \sigma_0)} \tag{5}$$

where I_1 , I_0 , σ_1 , and σ_0 are the means and standard deviation of the photocurrent for "1" and "0." In the investigated case, Qis directly linked to the optical noise variance σ^2 by

$$Q = 1/\left(2\sqrt{2}\sigma^2(1+\sqrt{1+\frac{1}{2\sigma^2}})\right).$$
 (6)

A key parameter of the soft decoding algorithm is the calculation of logarithmic likelihood ratio [4] for each incident bit y at the decoder input (x being the emitted bit):

LLR =
$$\ln\left(\frac{p(x=0|y)}{p(x=1|y)}\right)$$
. (7)

The right evaluation of this parameter is a determining factor of the FEC performance. It is consequently important to have accurate channel knowledge to correctly evaluate the probabilities in (7). In the case of AWGN channel of variance σ^2 , the LLR are simplified to LLR = $2y/\sigma^2$ [4]. However, for a noisy optical channel modeled by a chi-square distribution with $A_0 = 0$, we have developed the LLR expression from (3):

$$LLR = \frac{A_1^2}{2\sigma^2} + \ln\left(\frac{\sqrt{y}A_1^2}{\left[2\sigma^2 I_1\left(\frac{A_1^2}{\sigma^2}\sqrt{y}\right)\right]}\right).$$
 (8)

BER results of the coded chain with several LDPC codes of different lengths but of same redundancy (5.6%) are reported on Fig. 4(a). As expected the BER enhancement increases with the code length for a given redundancy, illustrating the tradeoff between the performance and the complexity.

Let us now focus on Fig. 4(b), representing the BERs (with circles) obtained with the 3% redundant LDPC(6176, 5983). The dashed line curves are obtained considering an AWGN channel with classical LLR calculation [4] for the coded case.



Fig. 4. BERs for several LDPC codes of 5.6% redundancy (a) and with the LDPC(6176, 5983) (b).

The solid lines refer to the chi-square channel. In the coded case (circles), the soft decoder assumes chi-square statistics via the use of (8). The two curves in the coded case have the same shape and are drifted along the Q axis as in the uncoded case. Therefore, by extrapolation, the BIBD-LDPC correction capacity and thus the effective NCG are identical in AWGN and chi-square cases. However, if the decoder assumes a Gaussian distribution while the optical channel has a chi-square statistic, performances are strongly degraded. This is illustrated by the solid curve marked with squares in Fig. 4(b). In such a scheme, a BER of 10^{-4} is reached at a minimal Q value of 11 dB, whereas it is reached at 8 dB with the accurate LLR calculation (8). This Q value difference between the two cases (3 dB at 10^{-4}) increases as the targeted BER is low.

IV. CONCLUSION

Physical simulations of the optical network under study reveal that a chi-square model accurately describes the optical channel statistics. Computer simulation results have shown that the effective NCG are identical over AWGN and chi-square channel. However, the analysis permits us to conclude that even if the AWGN model can be an appropriate channel representation for designing the LDPC FEC dimension, the soft decoder requires the chi-square statistics in order to achieve the best possible performance.

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