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# Intrinsic Noise Figure Derivation for Fiber Raman Amplifiers From Equivalent Noise Figure Measurement

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***Abstract* — A complete analysis and measurement of the noise figure of Raman amplifiers is reported. The standard noise figure definition derived from the amplified spontaneous emission approach is discussed. A new approach, based on the field fluctuations, is proposed. It takes into account the vacuum fluctuation as a minimum noise level at the amplifier input and as a contribution to the noise generation. As a result, the noise of a phase-insensitive amplifier is shown to be generated by the input noise amplification, the intrinsic amplification and attenuation mechanisms. This model enables to exactly define the noise figure as the degradation of the optical signal-to-noise ratio, which is in complete agreement with the practice of the IEEE Standard.**

***Index Terms* — Optical fiber, optical noise, noise figure, Raman amplification.**

## I. INTRODUCTION

Noise in optical amplifiers (OAs) has been analyzed by means of several theoretical frameworks. However, the appropriate definition of the noise figure (NF) of OA is still discussed [1-3].

The control of noise accumulation is a key issue for the optical amplifier cascades used in long span optical transmission systems and in optical signal processing devices. In the former, the Raman gain distribution is well-known to reduce the noise generation [4] and to smooth the signal level variation along the link, reducing system vulnerability to nonlinear effects.

The aim of this paper is to propose a classical phase-amplitude field fluctuations description of OA noise and its application to optical signal-to-noise ratio (OSNR) and noise figure derivation for distributed fiber Raman amplifier (FRA).

The second section of this paper is devoted to the classical phase-amplitude model presentation and its application to the noise propagation equation in a distributed FRA. The result leads to the noise figure derivation. Comparison with the standard amplified spontaneous emission (ASE) approach is made in the third section of the paper. The fourth and the fifth sections are respectively devoted to the formulation of the equivalent noise figure, as a function of the Raman on-off gain, and to the experimental measurement of this equivalent NF. In the last section, the model is discussed and main conclusions are outlined.

## II. AMPLIFIED FIELD FLUCTUATIONS MODEL

As usual in the domain of electrical engineering, the optical field may be represented with a two-quadrature component description [2,5,6]. The optical field is considered as the sum of a deterministic component, with complex amplitude $A.exp(j\phi)$, and of an additive optical band-limited stationary Gaussian noise $N$, with a flat spectrum, in an optical bandwidth equal to $B_0$. It is assumed that the optical noise is co-polarized with the deterministic field. An appropriate normalization, in which the optical power equals the squared field, is also assumed. As shown in Fig. 1, the classical phase-amplitude description decomposition of the amplitude noise $N$ into an in-phase $N_I$ and a quadrature (i.e. out of phase) $N_Q$ component is used.

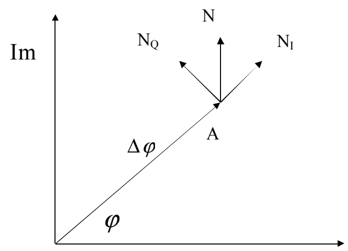


Fig. 1. Phase-amplitude representation of the optical field.

*A. Field Fluctuations*

The total instantaneous power is the squared sum of the deterministic field and of the in-phase component of the noise. Under the small noise approximation and assuming an equal noise power repartition between the in-phase and the quadrature noise components, the instantaneous optical power fluctuates around its average value $\overline{P} = A^2$ with the mean squared fluctuations:

$$\overline{(\Delta P)^2} \approx 4\, A^2\, \overline{N_I}^2 = 4\, \overline{P}\, \overline{P_I} = 2\, \overline{P}\, \overline{P_N} \qquad (1)$$

in which $\overline{P_I} = \overline{P_N}/2$ is the average power of the in-phase noise component. Observing that only the optical noise spectral components, within the spectral range $B_e$ on each side of the optical carrier frequency, produce an observable cross term within the observation (electrical) bandwidth $B_e$, the optical noise bandwidth contribution is determined by $B_o = 2B_e$ and the power fluctuations can be expressed as the well-known Schottky fluctuation relation:

$$\overline{(\Delta P)^2} = 2\,\overline{P}\,\overline{P}_N = 2\,h\nu B_e\,\overline{P} \qquad (2)$$

where $h\nu$ is the photon energy. The well-known Poisson shot noise relationship can easily be obtained from (2) for the photon number $n = P\tau/h\nu$. However, the shot noise is signal-dependent and may be not considered as an intrinsic input noise for the amplifier, but only as a noise reference level, as far as power measurements are only concerned. This noise reference level, required by any noise figure definition, is usually referred as “shot noise-limited signal”.

Let us consider that the optical measurement bandwidth $B_0$ is small enough to make the hypothesis of an additive white Gaussian noise. In this case, the optical noise power can easily be expressed as a function of its spectral density $S_N$ :

$$P_N = S_N B_0 \qquad (3)$$

As a result,

$$S_N^{\min} = \frac{h\nu}{2} \qquad (4)$$

The minimum additive optical noise, which accompanies any optical field, is usually referred, in quantum electrodynamics, as the zero-point field fluctuations or the vacuum fluctuations. This noise is only observable through its cross-term product with another signal, referred as shot noise. Only vacuum fluctuations, producing shot noise in power detection, are the intrinsic signal-independent input noise.

*B. Amplified Field Fluctuations*

It is useful to model the noise propagation through an amplifier medium in order to estimate the noise power at the amplifier output and thus to express the noise figure of the considered amplifier. The amplified field fluctuations theory may use the spectral density of the optical noise to study the intensity noise propagation.

Let us consider an attenuating medium with a linear absorption coefficient $\alpha$. In such a case, the noise propagating through this medium is attenuated and must be maintained to the minimum vacuum fluctuations level, which is the minimum noise level. Thus , the evolution of the spectral density of the optical noise is expressed as:

$$\frac{dS_N}{dz} = -\alpha\left(S_N - \frac{h\nu}{2}\right) \qquad (5)$$

The fiber attenuation process does not act on the vacuum fluctuations, which are constantly maintained thanks to an attenuation noise source also known as partition noise.

Let us consider now an amplifying medium with an amplifying coefficient $\beta$. It is to be mentioned that only phase-insensitive and non-saturated amplifiers are considered. Thus, it is necessary to take into account an amplification noise source in order to satisfy the minimum uncertainty Heisenberg principle. As a result, the evolution of the spectral density of noise is expressed as:

$$\frac{dS_N}{dz} = \beta\left(S_N + \frac{h\nu}{2}\right) \qquad (6)$$

An amplifier medium is both attenuating and amplifying. As a result, it combines both (5) and (6). Thus, the evolution of the spectral density of the optical noise is expressed as:

$$\frac{dS_N}{dz} = (-\alpha + \beta)S_N + (\alpha + \beta)\frac{h\nu}{2} \qquad (7)$$

The noise at the amplifier output is generated by the amplification of the intrinsic input vacuum fluctuations, an attenuation noise source and an amplification noise source. It is to be noticed that both intrinsic attenuation and amplification added noises are required by conservation of commutator brackets in any linear systems [7].

*C. Case of Distributed Fiber Raman Amplifiers*

Fiber Raman amplification is based on the stimulated Raman scattering (SRS) [8]. FRA is usually designed employing the averaged power analysis considering forward and backward propagation directions for the pump and the signal through the optical fiber [4,8,9]. The simplest model of propagation for the pump and the signal powers is expressed by the following set of differential equations:

$$\frac{dP_S}{dz} = (-\alpha_S + C_R P_P) P_S \tag{8}$$

$$\pm \frac{dP_P}{dz} = \left( -\alpha_P - \frac{\lambda_S}{\lambda_P} C_R P_S \right) P_P \tag{9}$$

where $\alpha_P$, $\alpha_S$ represent respectively the fiber attenuation coefficients at the pump wavelength $\lambda_P$ and at the signal wavelength $\lambda_S$ respectively. The sign +/- refers to the forward and backward propagation for the pump power. $C_R$ is the Raman gain efficiency mainly depending on the fiber core area and the frequency shift between the pump and the signal. Thus, the Raman net gain $G$ includes both the fiber attenuation and the Raman gain $G_R$ also known as the Raman on-off gain.

$$G(L) = \exp\left\{ -\alpha_S L + C_R \int_0^L P_P(z) dz \right\} = G_R \, e^{\alpha_S L} \tag{10}$$

In the case of FRA, the evolution of the spectral density of the optical noise becomes:

$$\frac{dS_N}{dz} = (-\alpha_S + C_R P_P) S_N + (\alpha_S + C_R P_P) \frac{h\nu}{2} \tag{12}$$

As a result, the spectral density of the optical noise at the FRA output is expressed as:

$$S_N(L) = G\, S_N(0) + (G-1)\frac{h\nu}{2} + \alpha_S \, G \, D_{INV} \frac{h\nu}{2} \tag{13}$$

with

$$D_{INV} = \int_0^L [1/G(z)] dz \tag{14}$$

When the input signal is limited to the vacuum fluctuations, meaning shot noise limitation of an hypothetical power detection, the spectral density of the optical noise is:

$$S_N(L) = G\frac{h\nu}{2} + (G-1)\frac{h\nu}{2} + 2\,\alpha_S \, G \, D_{INV} \frac{h\nu}{2} \tag{15}$$

Vacuum fluctuations are considered as an input noise which amplification contributes to the output noise in addition to intrinsic amplifier noise.

*D. Noise Figure Derivation*

The original formulation of noise figure in the optical domain defines it as the OSNR at the input of an optical amplifier divided by the OSNR at the OA output [10,11]. In this section, concerning the amplified field fluctuations, the intrinsic input noise was defined as the vacuum fluctuations. Thus the NF is expressed as:

$$NF = \frac{S_N(L)}{Gh\nu/2} = 1 + \frac{G-1}{G} + 2\,\alpha_S \, D_{INV} \tag{16}$$

This noise figure formulation is very simple and intuitive. Moreover, it is in complete agreement with the IRE standards definition [11] adopted by the IEEE. The minimum quantum limit of 3 dB for the NF of a very high gain FRA is satisfied [12]. It is evident that the forward pumping allows to reach the lowest NF due to the minimization of the term $D_{INV}$.

## III. Amplified Spontaneous Emission Theory

One may define the OA noise figure in term of amplified spontaneous emission [1-3,5]. It is to be mentioned that, while the terminology of spontaneous emission is used, the correct terminology for FRA is spontaneous Raman scattering (SpRS). In the ASE approach, the shot noise of an hypothetical power detection is considered as a reference level for input noise. However the amplification of the corresponding input fluctuation is not explicitly considered as a contribution to output noise. As a result, the optical shot noise contribution $h\nu B_0$ is added at the input and at the output of the OA but it is not amplified. Thus the measure of the noise power added by the amplifier $P_{ASE}$ gives full information to estimate the NF.

In the case of FRA [4,9], the propagation of $P_{ASE}$ is described by the differential equation:

$$\frac{dP_{ASE}}{dz} = (-\alpha_S + C_R P_P) P_{ASE} + C_R P_P h\nu B_0 \tag{17}$$

It is still assumed that gain and noise do not vary over the optical measurement bandwidth $B_0$ and only one polarization is considered, the polarization of the signal. The first bracket terms acting on $P_{ASE}$ are the fiber attenuation and SRS. The last term is SpRS. Then the noise power at the FRA output is expressed as:

$$P_{ASE}(L) = GP_{ASE}(0) + (G-1)h\nu B_0 + 2\alpha_S G h\nu B_0 D_{INV} \tag{18}$$

and the noise generated in the amplifier is:

$$P_{ASE}(L) = (G-1)h\nu B_0 + \alpha_S G h\nu B_0 D_{INV} \quad (19)$$

The ASE derivation is a determinant average power accumulation process. The stochastic nature of ASE is only considered usually through beating noise at the power detection

The noise figure formulation is still consistent with the OSNR. Nevertheless, as pointed out before, the shot noise contribution $h\nu B_0$ must be taken into account as an input reference and heuristically added at the OA output. Thus the noise figure is expressed as:

$$NF = \frac{2\,P_{ASE} + h\nu\,B_0}{G\,h\nu\,B_0} = 1 + \frac{G-1}{G} + 2\,\alpha_S\;D_{INV} \quad (20)$$

This noise figure formulation is still in agreement with the spirit of the IEEE. Despite the noise interpretation is different from the amplified field fluctuations theory, the NF formulation obtained by the ASE approach leads to the same result as expressed in (16) or (20) after heuristic output shot noise correction.

## IV. Equivalent & Intrinsic Noise Figures

Usually, it is common to express the noise figure of fiber Raman amplifier as an equivalent noise figure $NF_{eq}$. As shown in Fig. 2, $NF_{eq}$ represents the noise figure that would experiment a discrete pre-amplifier of gain $G_R$ and generating the same noise than the considered distributed FRA.

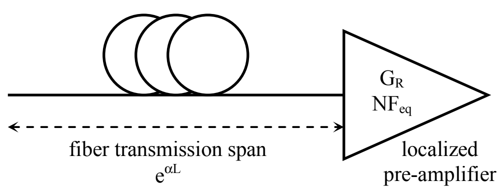


Fig. 2. Representation of a distributed FRA as an optical transmission span with a pre-amplifier.

The equivalent noise figure expression is derived from the noise figure cascading formula also known as the Friis formulation [8].

$$NF_{eq} = NF\,e^{-\alpha_S L} = \frac{1}{G_R} + \frac{2\,P_{ASE}}{G_R h\nu\,B_0} \quad (21)$$

where $NF$ is the intrinsic noise figure (20) of the distributed fiber amplifier. Thus, the equivalent noise figure may be simply estimated by measuring the Raman on-off gain $G_R$ and the amplified spontaneous emission power $P_{ASE}$. It is to be mentioned that the value of $NF_{eq}$ can be negative as a function of the Raman gain distribution. This aspect is consistent with the fact that distributed amplification, such as FRA, is preferable in terms of noise generation to a pre-amplifier solution.

## V. Experimental Results

### A. Experimental Set-Up

The experimental set-up is depicted in Fig. 3. The test bed is a realistic distributed FRA scheme. A constant 100 km fiber length was used for each fiber under test. The fiber was counter-pumped with a CW Raman fiber laser (RFL) at 1455 nm. The random polarized RFL provides a polarization independent gain.

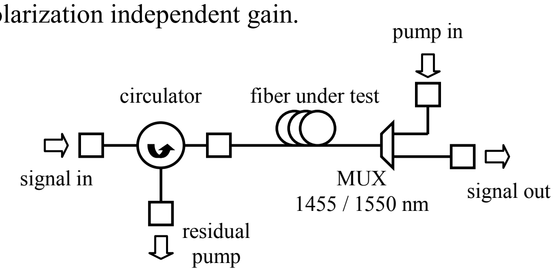


Fig. 3. Experimental set-up. The fibers under test is a 100 km long fibers spans.

A tunable wavelength laser set at –5 dBm was used as the input signal to measure the Raman small-signal dependence on wavelength as shown in Fig. 4. The Raman on-off gain $G_R$ measurement method [13,14] was chosen to cancel out any inaccuracy related to spectrally dependent attenuation of components and fiber itself. The fiber Raman efficiency $C_R$ was derived from $G_R$ using the relation:

$$C_R = \frac{\log(G_R)}{P_{pump}\,L_{eff}} \quad \text{with} \quad Leff = \frac{1 - e^{-\alpha_P L}}{\alpha_P} \quad (22)$$

$P_{pump}$ and $L_{eff}$ are the launched pump power and the effective length at the pump wavelength of the fiber under test, respectively.

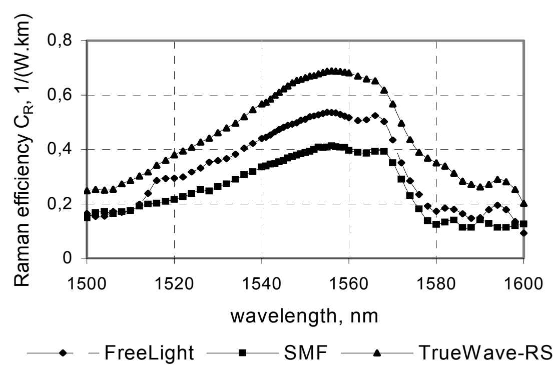


Fig. 4. Measured Raman gain efficiency spectra of Freelight[TM], SMF, and Truewave-RS[TM] fibers in the C-band. The pump wavelength is 1455 nm.

The Raman gain efficiency is dependent on the fiber under test. This dependence is inversely proportional to the value of the fiber core area $A_{eff}$ as summarized in Table I.

TABLE I
PHYSICAL PARAMETERS USED IN MODEL

| | Freelight | SMF | Truewave RS |
|---|---|---|---|
| $\alpha_S$ (dB/km) | 0.2 | 0.2 | 0.2 |
| $\alpha_P$ (dB/km) | 0.260 | 0.263 | 0.256 |
| $C_R$ ($W^{-1}.km^{-1}$) | 0.54 | 0.42 | 0.69 |
| $A_{eff}$ (µm²) | 68 | 80 | 55 |

*B. Experimental Measurement Results*

It has been shown that the amplified field fluctuations model leads to the same NF estimation than the standard ASE approach. Experimental equivalent noise figure measures have been conducted in order to consolidate the simulations. The measurement method used is the common optical measure of the amplified spontaneous emission power with an optical spectrum analyzer. From (21), measurement of $NF_{eq}$ requires $P_{ASE}$, $G_R$ and $\lambda_S$ to be determined. Thus the use of an optical spectrum analyzer is well adapted to these measures. Precautions have been taken to separate the amplified laser source spontaneous emission from the ASE generated by the amplifier [15].

Precautions have also been taken to ensure the small-signal regime. Thus, the input signal power has been limited to –5 dBm. The Raman on-off gain has been limited to 30 dB in order to avoid amplified multi-path interferences such as double Rayleigh backscattering [16].

The measurement accuracy was limited only by the power stability of the RFL (i.e. <1%) and by the relative accuracy of the used optical spectrum analyzer and power-meters.

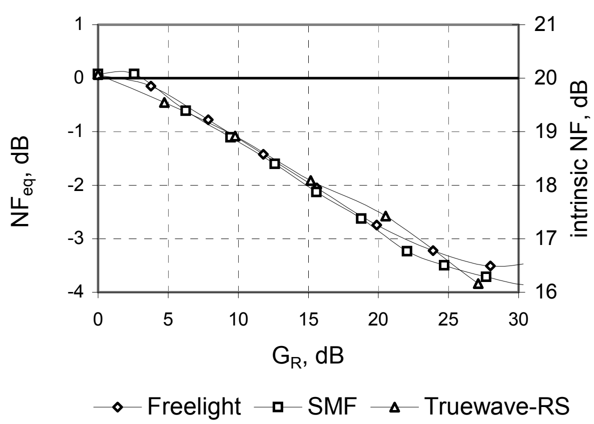


Fig. 5. Experimental measurement of the equivalent NF and intrinsic NF derivation as a function of the Raman on-off gain for different types of transmission fiber under test.

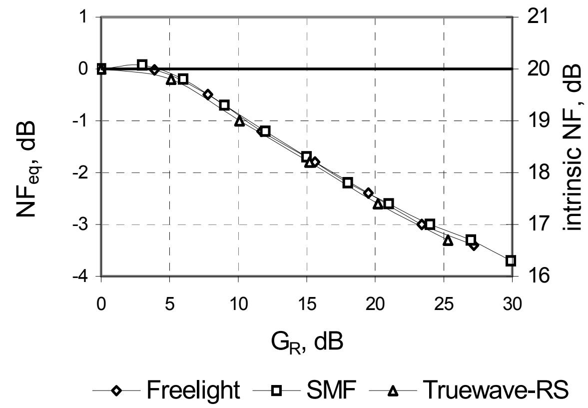


Fig. 6. Simulation results of the equivalent NF and intrinsic NF as a function of the Raman on-off gain for different types of transmission fiber under test.

The comparison of the measurement of $NF_{eq}$ by the experiment, as shown in Fig. 5, and the formulation (21) of $NF_{eq}$, as it can been seen in Fig. 6, are in a very good agreement. In the case of the three types of fiber that are Freelight, SMF (Single Mode Fiber) and Truewave-RS, the Raman on-off gain $G_R$ is the significant value which determines the value of $NF_{eq}$. It is due to the fact that, the gain distribution is similar for each fiber under test due to a pump loss coefficient $\alpha_P$ relatively constant as figured in Table I. It is to be mentioned that for a value of $G_R$ higher than 5 dB, $NF_{eq}$ reaches a negative value. The negative value of the equivalent noise figure corresponds to an intrinsic noise figure better than the 20 dB fiber attenuation. In fact, the fiber attenuation is the noise figure of the passive attenuating fiber.

## VI. DISCUSSION

A new interpretation of noise generation has been presented. It allows a clearer interpretation of OA output noise as due to:

- amplification of input field fluctuations with a lower value limited to vacuum fluctuations
- intrinsic fiber attenuation noise linked to the conservation of the vacuum fluctuations
- unavoidable amplification noise generated by the amplifier

Moreover the amplified field fluctuations model allows a very simple and rigorous noise figure definition as the degradation of the OSNR and it does not deal with questionable considerations about the shot noise. In fact, the ASE approach failed to give the exact result since the shot noise is to be heuristically added at the OA output.

The new model is in a complete agreement with the IEEE Standards and convergent with the standard ASE. It is also to be mentioned that the cascading formula remains rigorously valid.

The noise figure of FRA has been formulated both with the model of the field fluctuations and with the ASE approach. This formulation is in a very good agreement with the ASE power measurement method.

In the case of the small-signal regime, standard ASE approach gives correct results, despite the inconsistencies analyzed above. However, the validity of this approach is limited to the case of a pure coherent input signal. This is a limitation that can be easily overcome by the approach of the amplified field fluctuations.

## REFERENCES


[1] H.A. Haus, “The Noise Figure of Optical Amplifiers,” *IEEE Photon. Technol. Lett*., vol. 10, pp. 1602-1604, 1998.

[2] E. Desurvire, *Erbium-Doped Fiber Amplifiers-Principle and Applications*. New York: Wiley, 1994.

[3] S. Donati, and G. Giuliani,, “Noise in an Optical Amplifier: Formulation of a New Semiclassical Model,” *IEEE J. Quantum Electron*., vol. 33, pp. 1481-1488, 1997.

[4] V.E. Perlin, and G. Winful, “Optimizing the Noise Performance of Broad-Band WDM Systems with Distributed Raman Amplification,” *IEEE Photon. Technol. Lett*., vol. 14, pp. 1199-1201, 2002.

[5] P. Gallion, “Classical phase-amplitude description of optical amplifier noise.” Proceedings of SPIE. vol. 5260*, Applications of Photonic Technology* 6, pp 96-102, SPIE Bellingham, WA, 2003.

[6] P. Gallion, “A classical corpuscular approach to optical noise,” Invited paper, Conference on Optical Amplifier and their Applications, *OAA'99*, paper WB2, Nara (Japan), June 1999.

[7] H.A. Haus, “From classical to quantum noise,” *JOSA-B*, 12-11, pp. 2019-2036, Nov 1995.

[8] G.P. Agrawal, *Nonlinear Fiber Optics*, second ed., Academic Press, New York, 1995.

[9] H. Kidorf, K. Rottwitt, M. Nissov, M. Ma, and E. Rabarijaona, “Pump Interactions in a 100-nm Bandwidth Raman Amplifier,” *IEEE Photon. Technol. Lett*., vol. 11, pp. 530-532, 1999.

[10] H.T. Friis, “Noise figure of radio receivers,” *Proc. IRE*, vol. 32, pp. 419-422, 1944.

[11] H.A. Haus, “IRE standards on methods of measuring noise in linear twoports, 1959,” *Proc. IRE*, vol. 48, pp. 60-74, 1960.

[12] H. Heffner, “ The fundamental noise limit of linear amplifiers ”, *Proc. IRE*, pp. 1604-1608, 1962.

[13] C.R.S. Fludger, A. Maroney, N. Jolley, and R. Mears, “An analysis of the improvements in OSNR from distributed Raman amplifiers using modern transmission fibers,” *OFC'00*, Baltimore, OSA, FF2, pp. 100-102, 2000.

[14] D. Hamoir, N. Torabi, A. Bergonzo, S. Borne, and D. Bayart, “Raman spectra of line fibers measured over 30THz,” Symposium on optical fiber measurements, Boulder, *NIST* (National Institute of Standards and Technology), paper 019, 2000.

[15] D.M. Baney, P. Gallion, and R.S. Tucker, “Theory and measurement techniques for the noise figure of optical amplifiers,” Invited Paper, *Optical Fiber Technology*, vol. 6, pp. 122-154, 2000.

[16] R.-J. Essiambre et al., “Design of bidirectionnaly pumped fiber amplifiers generating double Rayleigh backscattering,” *IEEE Photon. Technol. Lett*., vol. 14, pp. 914-916, 2002.