# Mode Selection and Larger Set Equalization for Mode-Multiplexed Fiber Transmission Systems 

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#### Abstract

We study the performance of six-mode and ten-mode fiber-based SDM systems impaired by mode-dependent-loss when selecting an appropriate set of modes for multiplexing at the transmitter, and detecting more modes at the receiver.


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## 1. Introduction

The last two decades have known an exponential growth in the demand for network bandwidth. This growth was mainly caused by the build-out of the Internet and the growing traffic generated by an increasing number of users [1]. Since frequency, time, phase, polarization have already been employed in order to satisfy the demand for bandwidth, space remains the last degree of freedom that can be used in an optical transmission system to increase its capacity [2]. Spatial division multiplexing (SDM) can be realized through multi-mode fibers (MMFs) that allow the propagation of more than one mode in a single core or multi-core fibers (MCFs) where each core can be single-mode or multi-mode. However, in all schemes, cross-talk is inevitable especially if the cores are close in MCF or if the differential modal group delay (DMGD) is close to zero in MMF. In both cases, it is compulsory to use multiple-input multiple-output (MIMO) approaches (already used in wireless communications) to recover the signals at the receiver. Moreover, in MMF, propagating signals are also affected by a non-unitary cross-talk known as mode dependent loss (MDL) arising from fiber imperfections (splices, connectors) and optical components (amplifiers, multiplexers).

While unitary modal cross-talk can be mitigated by using MIMO equalization at the receiver, the accumulated MDL introduces penalties and reduces the capacity of the mode-division-multiplexed (MDM) system. In [3,4], a two-mode transmission involving only the $L P_{11}$ modes propagating at the same velocity is performed on a 3-mode fiber with a large DMGD. At the receiver, all modes are detected in order to retrieve the energy coupled in the $L P_{01}$ at imperfect multiplexers/de-multiplexers. In our work, we investigate the performance of an MDL-impaired system based on lowDMGD MMFs with a larger core when an appropriate set of modes is selected for multiplexing according to an energy criterion. Then, we enhance the system performance by detecting a larger number of modes at the receiver side. We show through numerical simulations that the $3 \times 6 \mathrm{MIMO}$ (resp. $6 \times 10$ ) based on a 6-mode fiber (resp. 10-mode fiber) gives better performance than the $3 \times 3$ MIMO (resp. $6 \times 6$ ) based on the 3-mode fiber (resp. 6-mode fiber).

## 2. MDM Channel Model

We consider a spliced MMF-based MDM transmission system supporting $M$ modes. To focus on the impact of the MDL on the system performance, polarization cross-talk is not considered and we neglect fiber nonlinearity. Modal dispersion is also not considered since it does not reduce the capacity of the system and can be managed using OFDM format with suitable cyclic prefix. The resulting MIMO channel is given by:

$$
\begin{equation*}
\mathbf{Y}_{M \times 1}=\mathbf{H}_{M \times M} \mathbf{X}_{M \times 1}+\mathbf{N}_{M \times l}=\sqrt{L} \prod_{k=1}^{K}\left(\mathbf{T}_{\mathbf{k}} \mathbf{C}_{\mathbf{k}}\right) \mathbf{X}_{M \times 1}+\mathbf{N}_{M \times l} \tag{1}
\end{equation*}
$$

where $\mathbf{X}_{M \times 1}$ (resp. $\mathbf{Y}_{M \times 1}$ ) is the transmitted (resp. the received) symbol vector, $\mathbf{N}_{M \times 1}$ is an additive white noise vector with components assumed to be Gaussian with zero mean and variance $2 N_{0}$ per complex dimension.

The matrix $\mathbf{H}_{M \times M}$ represents the spliced MMF composed of $K$ fiber sections. The factor $L$ represents the modeaveraged propagation loss. $\mathbf{T}_{k}$ is a diagonal matrix with random phase entries $\exp \left(i \phi_{m}\right)$ and $\phi_{m} \in[0,2 \pi]$. $\mathbf{C}_{k}$ represents the modal coupling at a fiber misalignment or connector. Its coefficients $c_{i, j}$ are computed using an overlap integral of the electrical field distribution $E_{i}^{k-1}(x, y)$ of the $i$-th mode before the splice and the electrical field distribution $E_{j}^{k}(x+\Delta x, y+\Delta y)$ of the $j$-th mode after the splice, $\Delta x$ and $\Delta y$ being the misalignments in $x$ and $y$ directions with zero mean and standard deviation $\sigma_{x, y}$ as in [5].

## 3. Mode Selection

In the following, we consider the previous MDM spliced channel model and we select a set of $M_{T}<M$ modes to multiplex the data at the transmitter. We detect $M_{R} \leq M$ modes at the receiver as shown in Fig. 1. The new MIMO system is given by:

$$
\begin{equation*}
\mathbf{Y}_{M_{R} \times l}=\mathbf{H}_{M_{R} \times M_{T}} \mathbf{X}_{M_{T} \times l}+\mathbf{N}_{M_{R} \times l} \tag{2}
\end{equation*}
$$

where the channel matrix $\mathbf{H}_{M_{R} \times M_{T}}$ spans a subspace of $\mathbf{H}_{M \times M}$. In wireless environments, transmit antenna selection was proved to gain rate and reduce hardware cost. Different algorithms were proposed to find the set of optimal antennas without an exhaustive search over all possible combinations [6-8]. In our work, we apply the same concept in the MDM system. Our selection criterion consists in choosing modes having the maximal average received energy after propagating in the MDL-impaired channel. At the transmitter, we excite all modes with a unit energy $E_{s}=1$ and we compute the average received energy per mode at the receiver side.

To obtain insight into the mode selection criterion, we consider gradient-index multi-mode fibers (GI-MMFs) with a parabolic profile as in [5] and a numerical aperture $N A=0.205$. The wavelength is fixed to $\lambda=1550 \mu \mathrm{~m}$. $K=300$ fiber sections are concatenated with random Gaussian misalignments $\Delta x, \Delta y$ of zero mean and a standard deviation (std) $\sigma_{x, y}=0.26 \mu \mathrm{~m}$. We simulate $10^{5}$ realizations of the $6 \times 6$ channel based on a 6 -mode fiber of core radius $8.7 \mu \mathrm{~m}$ and the $10 \times 10$ channel based on a 10 -mode fiber of core radius $11 \mu \mathrm{~m}$. In Fig. 2 a and 2 b , we plot the probability distribution functions (PDF) of the average received energies per mode. We notice that for the 6-mode fiber (resp. 10-mode fiber), modes with the highest received energies are $\left\{L P_{01}, L P_{11 a}, L P_{11 b}\right\}$ (resp. $\left\{L P_{01}, L P_{11 a}, L P_{11 b}, L P_{21 a}, L P_{21 b}, L P_{02}\right\}$ ).


Fig. 1: MDM transmission system.

(a) 6-mode fiber

(b) 10-mode fiber

Fig. 2: PDFs of the average received energies per mode with a misalignment std $\sigma_{x, y}=0.26 \mu \mathrm{~m}$.

## 4. Simulation Results

In this section, we test the benefits of mode selection with larger set equalization using fibers supporting $M=\{3,6,10\}$ modes. The fiber parameters are the same as in the previous section, and the core radius of the 3-mode fiber is $6 \mu \mathrm{~m}$. Misalignments with two std are considered: $\sigma_{x, y}=\{0.26 \mu \mathrm{~m}, 0.35 \mu \mathrm{~m}\}$. We compare the performance in term of biterror rate (BER) curves versus the signal to noise ratio $\mathrm{SNR}=E_{S} / 2 N_{0}$. The modulated symbols are taken from a 4-QAM constellation. At the receiver, a maximum-likelihood (ML) decoding searches for the symbol that minimizes the quadratic distance with the received symbol.

In Fig. 3a and 3b, we compare a 3-mode fiber to a 6 -mode fiber with the selection of 3 modes $\left(M_{T}=3\right)$ according to the criterion mentioned above, at the same spectral efficiency $\mathrm{SE}=6 \mathrm{bits} / \mathrm{s}$ and the same average received energy normalized with respect to the 3 -mode system. We observe that the $3 \times 3$ channel in the 6 -mode fiber performs worse than the $3 \times 3$ channel on the 3 -mode fiber. This is due to important power leakage in the other three unexcited modes in the 6-mode fiber as shown in Fig. 2a. To retrieve this power loss, we increase the number of detected modes $\left(M_{R}=6\right)$. From the triangle-marked curves, we notice that at $\mathrm{BER}=10^{-2}$, the $3 \times 6$ channel has an SNR gain of 0.5 dB (resp. 1.9 dB ) compared to the $3 \times 3$ channel in the 3-mode fiber for $\sigma_{x, y}=0.26 \mu \mathrm{~m}$ (resp. $\sigma_{x, y}=0.35 \mu \mathrm{~m}$ ).

In Fig. 4 a and 4 b, we compare a 6 -mode fiber to a 10 -mode fiber with the selection of 6 modes $\left(M_{T}=6\right)$, at the same spectral efficiency $\mathrm{SE}=12 \mathrm{bits} / \mathrm{s}$ and the same average received energy normalized with respect to the 6 -mode systems . At BER $=10^{-2}$, we notice that for $\sigma_{x, y}=0.26 \mu \mathrm{~m}$, the $6 \times 6$ channel based on the 10 -mode fiber is only 0.4 dB degraded compared to the $6 \times 6$ channel based on the 6 -mode fiber. On the other side, for $\sigma_{x, y}=0.35 \mu \mathrm{~m}$, the former channel has a gain of 0.8 dB compared to the latter. This gain is explained by a lower power leakage in the unexcited modes that suffer further losses. When we detect all modes in the 10 -mode fiber ( $M_{R}=10$ ), a gain of 1.5 dB (resp. 2.5 dB ) is obtained for $\sigma_{x, y}=0.26 \mu \mathrm{~m}$ (resp. $\sigma_{x, y}=0.35 \mu \mathrm{~m}$ ) compared to the $6 \times 6$ channel on the 6-mode fiber.

The above results show that mode selection is an efficient technique that can be used in MDM systems based on low-DMGD MMFs with a large number of modes. An additionnal gain can be obtained by increasing the number of detected modes at the receiver side to gain back the power leakage in unselected modes and therefore improve the performance of the system.


Fig. 3: Performance of 3-mode fiber and 6-mode fiber using $M_{T}$ transmit modes and $M_{R}$ receiver side modes, $\mathrm{SE}=6 \mathrm{bits} / \mathrm{s}$


Fig. 4: Performance of 6-mode fiber and 10-mode fiber using $M_{T}$ transmit modes and $M_{R}$ receiver side modes, $\mathrm{SE}=12 \mathrm{bits} / \mathrm{s}$

## 5. Conclusion

In this paper, we have shown that mode selection is a high-potential solution that allows the use of large-core MMFs instead of small-core MMFs and reach almost the same performance in MDM systems impaired by MDL generated at imperfect splices. To further enhance the performance, we increase the number of detected modes at the receiver to gain back the power leakage in unselected modes. Hence, it is possible to start deploying MMFs supporting a greater number of modes, and use for the moment low-complexity MIMO schemes while having an infrastructure prepared to welcome future MIMO solutions in which all modes can be used for multiplexing.

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