Low-Complexity PDL-Resilient Signaling Design

Arnaud Dumenil, Elie Awwad, Cyril Méasson
Nokia Bell Labs France, Route de Villejust, F-91620 Nozay
arnaud.dumenil@nokia-bell-labs.com

Abstract: Building upon recent works on PDL-resilience, we show how to derive optimal and practically-efficient low-complexity multidimensional signaling. Analytic arguments provide optimized unitary transforms of multiplexed square QAM as a function of the PDL.

OCIS codes: 060.2330, 060.4230.

1. Next-Generation Optical Systems, Polarization-Multiplexed Model, and PDL-Resilient Signaling

Polarization Dependent Loss (PDL) is a non-unitary impairment expected to significantly impact next-generation optical technologies [1, 2]. For example, current WSSs experience up to 0.6dB of PDL, which translates onto an average PDL exceeding 2dB for typical links including 16 spans with two WSSs per node. In practice, in order to guarantee a 10^{-5} system outage probability, one optimizes the worst-case performance given by 3 times the average PDL, e.g., 6dB. PDL-resilience can be obtained from specific modulations. Single-carrier *Spatially Balanced* (SB) signaling [2] has been proposed in order to universally increase the information rate. It improves upon previously investigated Silver code [3] that appears to be prohibitively complex. SB signaling exploits subchannel independence which could as well implemented in the spectral or temporal domain. The principle consists in averaging or *balancing* mutual information profiles. In [2], it is numerically achieved using a rotation of angle $\pi/4$. In this work, we use the fact that the mutual information is well-described by the associated minimum Euclidean distance for typical SNRs. This enables us to maximize the minimum distance and therefore to obtain analytic design criteria of the SB signaling. We see that the angle $\pi/4$ is not the optimal angle over the full range of PDL. It is however satisfying when addressing current systems. The method importantly extends to the design of bit-parametrized coding and multi-dimensional modulation schemes (involving, e.g., MDL).

Linear Model: The PDL channel can be simplified as a 2×2-MIMO polarization-multiplexed system $Y = HX + Z$, where X and Y are the respective channel input and output, Z is a non-correlated AWGN complex-valued noise, H factorizes as the block-random $H_{\gamma,\alpha} = D_\gamma R_\alpha$ with $D_\gamma = \mathrm{diag}\left\{\sqrt{1+\gamma}, \sqrt{1-\gamma}\right\}$ and R_α the rotation matrix with $\alpha \in [0, 2\pi)$. PDL is given by $\lambda = (1+\gamma)/(1-\gamma)$ or $\Lambda = 10\log_{10}(\lambda)$ in dB. It is shown in [2] that, in addition to the capacity limiting $\gamma > 0$, the discrete-input capacity may further deteriorate as a function of α.

PDL-Resilience: The previous observation led us in [2] to construct SB signaling that leverages the performance of square M-QAM modulation by improving the worst-case capacity. A key construction ingredient is to consider two independent channels $\Re Y = DR_\alpha \Re X + \Re Z$ and $\Im Y = DR_\alpha \Im X + \Im Z$, where $\Re A$ and $\Im A$ represent the respective real and imaginary part of A. Then, SB encoding consists in rotating real and imaginary parts of the M-QAM^2 constellation to get an offset angle η between the two rotated M-QAM-like square lattices. In [2], $\eta = \pi/4$ is found based on numerical evaluation of the information rate of the respective complex parts.

2. Optimization of PDL-Resilient Signaling

We now build further upon [2] and provide a general framework to analyze the optimal η in practical cases.

Euclidean Distance Analysis: The simplified PDL channel model enables the study on either the real or the imaginary part. For a given PDL value and for any $A_{ij} \in M$-PAM$^2 = \{A_{ij} = (2i-1, 2j-1)^T | i, j \in \mathbb{Z}, -M/2 \le i, j \le M/2\}$, let the corresponding symbol after multiplicative PDL-impairment be $\tilde{A}_{ij} \in \mathscr{C}(\lambda, \alpha) =$

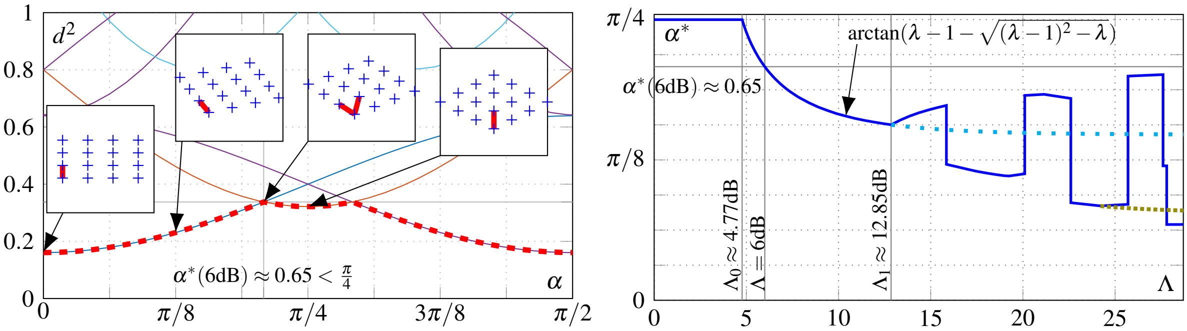
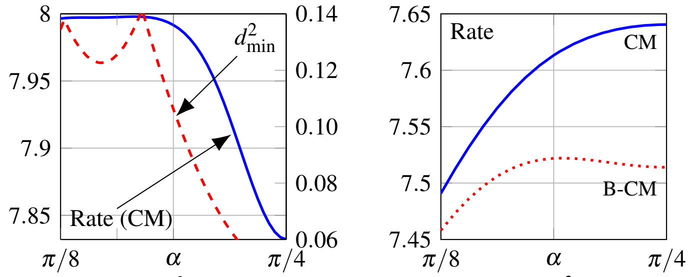


Fig. 1: Left: Square distances d^2 in $\mathscr{C}(\lambda,\alpha)$ as a function of the angle α for $\Lambda = 6$dB. The lower envelop indicates $d^2_{\min}$. It achieves its max in $\alpha^* \approx 0.65$. Right: Optimal angle α^* as a function of Λ. For 4-PAM^2 and 2-PAM^2 α^* is lower-bounded by the respective dashed asymptotes.



CM	Λ	2dB	4dB	6dB	15dB
	α^* $[d_{\min}]$	$\pi/4$	$\pi/4$	0.65	0.53
	α^* $[I(X;Y)]$	$\pi/4$	$\pi/4$	$\pi/4$	0.52

B-CM	Λ	2dB	4dB	6dB	15dB
	α^*	$\pi/4$	$\pi/4$	0.6	0.53

Fig. 2: CM, B-CM, and $d^2_{\min}$ variation for $\alpha \in [\pi/8, \pi/4]$ and 16-QAM^2 at 0.9 coding rate, 15dB PDL (left, SNR=23.1dB) and for 6dB PDL (middle, SNR=15dB). Values for optimal SB angle are reported in the table (right) for CM and B-CM information rates (0.9 coding rate).

$\{\tilde{A}_{ij} = H_{\gamma,\alpha} A_{ij} | A_{ij} \in M\text{-PAM}^2\}$. Let $d_{\min}(\mathscr{C}(\lambda,\alpha))$ define the minimum distance associated with the codebook $\mathscr{C}(\lambda,\alpha)$. We are interested in the angle $\alpha^*(\lambda) = \operatorname{argmax}_\alpha d_{\min}(\mathscr{C}(\lambda,\alpha))$ that defines the diversity phase offset for SB signaling. For symmetric square constellations, it is sufficient to consider $\alpha \in [0,\pi/4]$ by $\pi/2$-periodicity and evenness. While the presented concepts extend to any order QAM, Fig. 1 (left) illustrates them using the particular case of 4-PAM^2. Notice that this case is also found in [4] in the context of space-time codes and wireless communications. In contrast, our work originates from optical communications with polarization and mode multiplexing. Hence, it extends [4] and generalizes to any square lattice scaling. We demonstrate that different types of regime are to be considered for large PDL/MDL levels. Geometric considerations show indeed that, when the imbalance increases, the distance profile evolves and the minimum distance continuously decreases due to α-directional scaling of the lattice. In the general case of square two-dimensional lattices M-PAM^2 and small imbalance, the sides of the fundamental polytope give the minimum distance and $\alpha^* = \pi/4$. The first transition corresponds to the equality between $d^2(\tilde{A}_{ij}, \tilde{A}_{i+1,j+1}) = (1+\gamma)(2\cos\alpha - 2\sin\alpha)^2 + (1-\gamma)(2\cos\alpha + 2\sin\alpha)^2$ and $d^2(\tilde{A}_{ij}, \tilde{A}_{i+1,j}) = 4(1+\gamma)\sin^2\alpha + 4(1-\gamma)^2\cos^2\alpha$. It appears at $\lambda_0 = 3$ or $\Lambda_0 \approx 4.77$dB. For $\lambda < \lambda_0$, $\alpha^*(\lambda) = \pi/4$. Above λ_0, $\pi/4$ is no longer optimal in terms of Euclidean distance. While $d_{\min}$ is continuous in λ, $\alpha^* = \alpha^*(\lambda)$ is a piece-wise continuous function that is fully analytically characterized and presents discontinuous phase transitions. For example, assuming $M > 2$, the second transition appears at $\lambda_1 = (29 + 8\sqrt{13})/3$ or $\Lambda_1 \approx 12.85$dB and $\alpha^*(\lambda)$ becomes $\alpha^*(\lambda) = \arctan(\lambda - 1 - \sqrt{(\lambda-1)^2 - \lambda})$ for $\lambda_0 < \lambda < \lambda_1$. The function α^* is represented up to $\Lambda = 28.7$dB in Fig. 1 (right). The values λ_k at subsequent α^* transitions are obtained from the equality of corresponding distances in $\mathscr{C}(\lambda,\alpha)$. Notice that additional geometric observations follow such as $\alpha^* \to \arctan(1/M)$.

Information Rate Maximization: We will now further refine the distance-based observations for optimizing SB signaling by studying the actual information rate for PDL-impaired systems. Recall that fundamental communication limits are measured by the mutual information. Let us call *coded-modulation* (CM) rate the information rate $I(X;Y)$ achieved for a discrete input symbol alphabet. Let us further call *bit-decoding* (B-CM) rate the information rate associated with a bit-labeled alphabet and (mismatched) bit-parametrized decoding. For sufficiently large SNR, the information rate becomes a function of $d_{\min}$. As shown in Fig. 2 (left) the CM rate is *not* always maximal for $\alpha = \pi/4$. This is directly explained by our $d_{\min}$ investigations. Moreover the case of B-CM rate is even more insightful. When using Gray mapping, at high spectral efficiency, the non-optimality of $\pi/4$ becomes critical. This is exemplified in Fig. 2 (middle and right table) where the SB signaling is further numerically optimized using information rates. As expected, for finite SNR, this slightly differs from Euclidean distance results.

3. Conclusion

For channels presenting a gain imbalance between signal tributaries, special attention should be paid in order to raise the capacity associated with discrete imbalance-resilient modulation. Euclidean distance considerations demonstrate that the optimal SB angle is not universally equal to $\pi/4$ but depends on the imbalance value, the SNR, and/or the modulation order. For practical targeted PDL-impaired systems however, provided that the PDL is $\Lambda < 4.77$dB, SB signaling with $\pi/4$ as in [2] appears to be satisfying even for large SNR. This is confirmed using information rate consideration. Interestingly, the same method applies to the operational B-CM metric.

References

1. C. Xie and L. F. Mollenauer, "Performance Degradation Induced by Polarization-Dependent Loss in Optical Fiber Transmission Systems With and Without Polarization-Mode Dispersion," J. Lightwave Technol. 21, pp. 1953-, 2003.
2. A. Dumenil, E. Awwad and C. Méasson, "Low-Complexity Polarization Coding for PDL-Resilience," 2018 European Conference on Optical Communication (ECOC), Rome, 2018, pp. 1-3.
3. S. Mumtaz, G. Rekaya-Ben and Y. Jaouën, "Space-time codes for optical fiber communication with polarization multiplexing," 2010 IEEE International Conference on Communications (ICC), pp. 1-5, May 2010.
4. S. K. Mohammed, E. Viterbo, Y. Hong and A. Chockalingam, "MIMO Precoding With X- and Y-Codes," in IEEE Transactions on Information Theory, vol. 57, no. 6, pp. 3542-3566, June 2011.