

# Polarization Dependent Loss: Fundamental Limits and How to Approach Them

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**Abstract:** PDL impairs Pol-Mux systems and limits their capacity. Significant gains are achieved when signaling and channel equalization are properly addressed. Compared with mismatched processing, 0.9dB gain is measured for error-free transmission at PDL=6dB.

**OCIS codes:** 060.2330 Fiber optics communications, 060.4230 Multiplexing.

## 1. Introduction

Polarization dependent loss (PDL) mitigation is of major interest for polarization-multiplexed coherent optical systems. Depending on the optical components inserted in the link as well as the dynamics of the channel, large values of PDL may occur leading to a possible system outage [1, 2]. For instance, wavelength selective switches (WSS) in modern optical networks can show typical PDL values of 0.5 dB per unit. Unlike polarization mode dispersion (PMD) and unitary polarization rotation which can be perfectly equalized using a multiple-input multiple-output (MIMO) linear equalizer, PDL causes an SNR imbalance between the two polarization tributaries that limits the overall performance of the system [1]. Until now, SNR margins are added when designing the optical link in order to absorb the penalties induced by PDL. Recently, enhanced equalization methods and new signaling schemes such as Polarization-Time codes [3, 4] or Pairwise coding [5] have been suggested to reduce these penalties. In this work, we present the fundamental limits enforced by polarization dependent loss on practical systems that use M-QAM modulation formats, and illustrate it using 16-QAM. The results given in terms of constrained capacity curves and bit-error-rate performance underline that significant gains can be achieved if we carefully design the receiver and the coding/signaling scheme to address PDL mitigation. On the one hand, we show that the Silver-coded 16-QAM capacity exhibits interesting shaping gains compared to conventional 16-QAM. On the other hand, we use state-of-the art (spatially-coupled) LDPC codes to approach these capacities. The observed gains draw on the use of a  $2 \times 2$  non-unitary memoryless MIMO channel model with additive white Gaussian noise.

## 2. Achievable Information Rate

To get insight into PDL, we use in this paper a simple memoryless  $2 \times 2$  MIMO model that neglects dispersive effects (assuming that the chromatic dispersion and PMD have already been compensated) and only considers the remaining non-unitary aspect. The channel outputs are two-dimensional complex-valued vectors obtained from  $\mathbf{Y} = \mathbf{H}\mathbf{X} + \mathbf{Z}$ , where  $\mathbf{X}$  is the bi-dimensional (polarization-multiplexed) channel input,  $\mathbf{Z}$  is the additive white Gaussian noise modeled as circularly-symmetric Gaussian with zero mean and unit covariance matrix, and  $\mathbf{H}$  is a particular instance of the PDL random matrix. More precisely, we have  $\mathbf{H}(\alpha, \gamma) = \mathbf{R}_{-\alpha} \mathbf{D}_{\gamma} \mathbf{R}_{\alpha}$  where  $\mathbf{R}_{\alpha}$  is a real-valued rotation matrix with angle  $\alpha \in [0, 2\pi)$  and  $\mathbf{D}_{\gamma}$  is a real-valued rotation matrix of eigenvalues  $\{\sqrt{1+\gamma}, \sqrt{1-\gamma}\}$  with gain imbalance  $\gamma$ . The PDL capacity can be computed by entropy maximization [6]. For a Gaussian input  $\mathbf{X}$  with covariance matrix  $\mathbf{Q} = \frac{\rho}{2} \mathbf{I}$  where  $\rho$  is the total SNR, we get  $C = C(\gamma, \rho) = 2 \log(1 + \rho/2) + \log(1 - \gamma^2 \frac{\rho^2}{(2+\rho)^2})$  where the first term is the standard capacity of a  $2 \times 2$  MIMO Gaussian channel (raw diversity) and the second (negative) term defines the PDL-induced rate loss. This quantity, which is independent of  $\alpha$ , is depicted in Fig. 1(a) as a function of the SNR per polarization for PDL =  $10 \log_{10}[(1+\gamma)/(1-\gamma)] = 6$ dB. We see, for instance, that a PDL of 6dB translates into a loss of 0.6 bit/s/Hz in capacity at SNR=10dB, which may imply a transmission outage in practice. Consider now that channel inputs are restricted to discrete signals such as a 16-QAM constellation. First, we observe a dependency of the information rate on the rotation angle as shown in Fig. 1(a) where only the two extreme-case angles 0 and  $\pi/4$  are represented (all other values yield capacities in between these two curves). Depending on the available SNR budget, this can result in important penalties. Multidimensional schemes such as Polarization-Time coding [3] or Pairwise coding [5] are

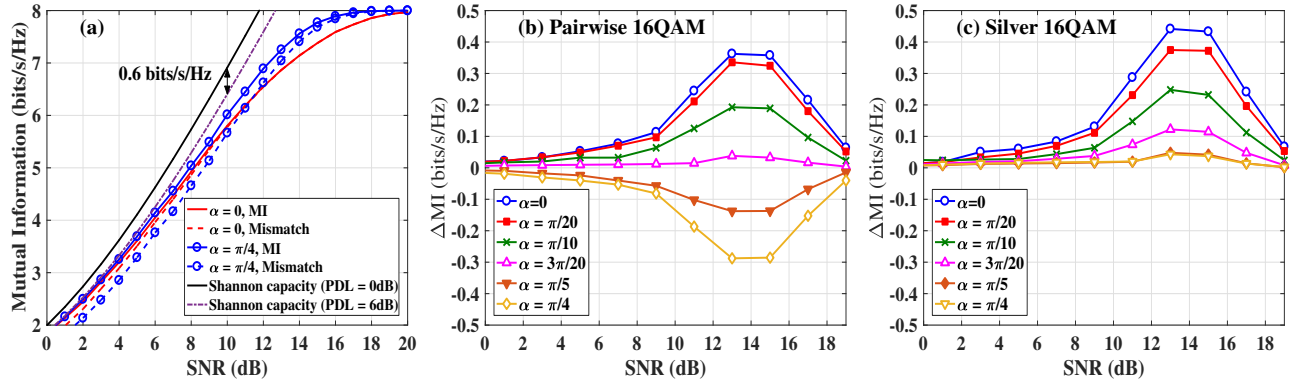


Fig. 1. (a) Shannon capacity and mutual information (MI) for 16QAM at PDL= 6dB. (b-c) Mutual information gains for Pairwise- & Silver-coded 16QAM compared to conventional 16QAM, for PDL=6dB and  $[\alpha = 0 : \pi/20 : \pi/4]$ .

helpful in this case as they can minimize this dependency, or even almost remove as with the Silver code [3]. Fig. 1(b-c) show the capacity gains that can be obtained respectively with Pairwise coding and Silver coding. We see that the Silver code exhibits gains for all rotation angles while Pairwise coding is only beneficial for selected ones. These performance gains are obtained at the expense of an increased decoding complexity of the new constellations [3, 5]. Second, a perhaps more subtle phenomenon appears when we plot the mismatch information rate, in dashed lines in Fig. 1(a), corresponding to a conventional receiver performing equalization and demapping followed by forward error-correction, as represented in solid line in Fig. 2. Namely, consider conventional 16-QAM modulation along with a Gray mapping, a significant loss is induced when a conventional processing is applied in presence of PDL with an angle different than 0 mod  $\pi/2$ . This loss can be seen on the mismatch capacity for  $\pi/4$  (dashed line with circles) in Fig. 1(a) and enlarged views of these curves in Fig. 2(b-c). This mismatch loss can be viewed as a generalization of the BICM loss in the one-dimensional case to a multidimensional MIMO case [7].

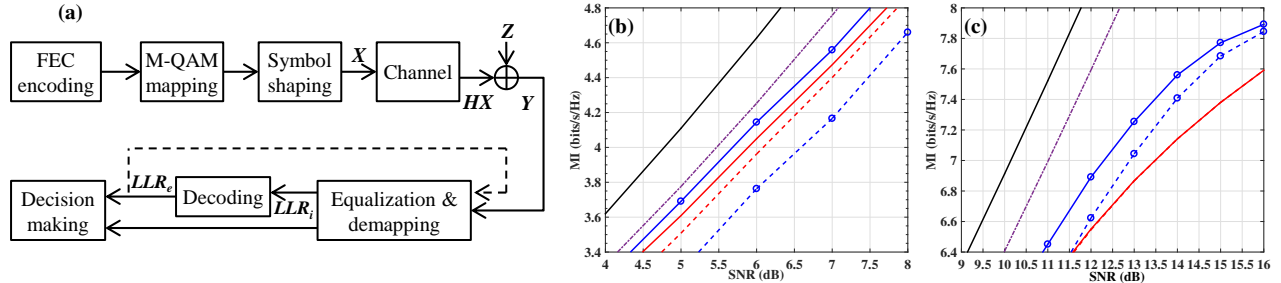


Fig. 2. (a) Transmission chain: conventional (solid line) and joint processing (dashed line). (b-c) Enlarged views of mutual information curves in Fig. 1(a) around code rates of 0.5 (b) and 0.9 (c).

### 3. Practical Validation

In the last few years different coding schemes with remarkable asymptotic behavior such as threshold saturation [8] or channel polarization [9] have been proved to achieve the ultimate communication limits for AWGN channels while bounding the complexity per information unit. Modern optical communication systems typically use iterative codes such as LDPC codes to approach channel capacity [8, 10]. Hence, we will also build upon probabilistic graphical models to exemplify the observations of the previous section. As an illustration, we choose a half-rate (3,6)-regular spatially coupled binary LDPC graph (with lifting depth 2000 and minimal coupling window [11]) for error-correction. Channel equalization and constellation demapping are performed with the optimal local sum-product rules derived from the standard factor graph framework. Operationally, intrinsic log-likelihood ratios  $LLR_i$  are fed into the decoder that implements the belief propagation algorithm in floating point. Two architectures represented in Fig. 2 are then compared: a conventional processing from which a decision is made after a single pass through the equalizer-demapper-decoder chain (solid line), and a joint processing performed by  $\ell$  loops from the decoder that produces extrinsic LLRs  $LLR_e$  fed to the equalizer-demapper block before making the final decision (dashed line). Extrinsic LLRs reflect the probability of a bit calculated thanks to the probability of other received bits and the structure of the code that link them. Note that using this joint-processing scheduling is one way to approach capacity using the

framework of factor graphs. It is an arbitrary choice to graphically separate decoding from equalization/detection.

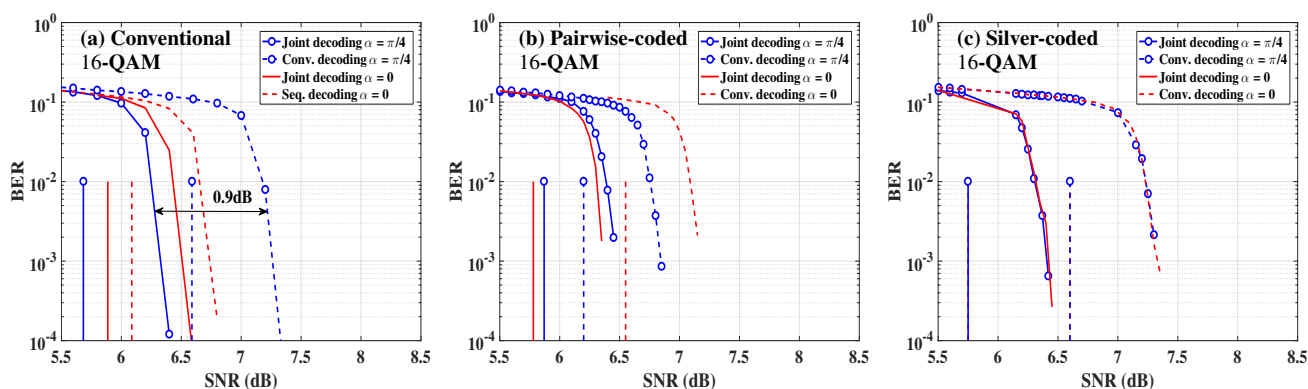


Fig. 3. Capacity limits & BER curves (using spatially-coupled LDPC) for: (a) conventional 16-QAM, (b) Pairwise-coded 16-QAM, (c) Silver-coded 16-QAM. Simulation parameters: PDL=6dB,  $\alpha = 0$  (no marker) and  $\alpha = \pi/4$  (circle markers).

For illustration, simulation results are presented in Fig.3 using 16-QAM modulation with a spatially-coupled LDPC with blocklength 80000 and  $\ell = 9$  demapping iterations. Capacity limits drawn as vertical thresholds are reported from Fig. 1 at half of the maximum information rate. First, from Fig.3(a), we see that the joint processing enables significant performance gains (0.9dB at best when  $\alpha = \pi/4$ ) when compared to the corresponding conventional architecture. Such gains can be understood as both *equalization* and *demapping* gains. Beware that the worst-case scenario can be  $\alpha = \pi/4$ , and not the aligned case  $\alpha = 0$ , when a conventional processing is used. Second, as expected from MI curves, further gains are seen when using Silver-coded or Pairwise-coded signaling. The mismatch capacities drawn as dashed vertical thresholds are computed from simulated mutual information curves as in the case of conventional 16-QAM. Finally, it is important to note that mismatch and maximum information rates might be ordered differently depending on the underlying code rate. For a code rate of 0.5, we see in Fig.2(b) that the mismatched system at  $\alpha = \pi/4$  performs roughly 0.5dB worse than at  $\alpha = 0$ . This behavior could however be potentially reversed for a higher coding rate as shown in Fig.2(c) and, hence, has to be carefully studied for practical transceiver design.

#### 4. Conclusion

This work gives practical guidelines on how to design an optical transceiver that carefully takes into account the particular (multidimensional) nature of polarization-multiplexed optical channel impaired by PDL. We saw that signaling (i.e., discrete multidimensional mapping properly shaped and tailored to the non-unitary MIMO PDL model) and algorithmic choices (e.g., conventional versus joint processing) have to be addressed and balanced in order to be fully adapted to the channel. This main lesson was exemplified in the case of a 6dB PDL when losses induced by mismatched processing are found to be of the order of the unit in dB.

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