RATE OPTIMIZATION USING SO(4) TRANSFORMS FOR PDL MITIGATION

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Abstract

SO(4)-based polarization codes are described in order to increase PDL-resilience in coherent transmissions. A 2-parameter optimization allows to maximize the minimal information rate achieved by discrete Pol-Mux QAM transmissions. The new optimal polarization code shows an increased capacity compared to the recently proposed Spatially-Balanced scheme.

1. Introduction

Among linear channel impairments in coherent optical fiber transmission networks, Polarization Dependent Loss (PDL) is a non-unitary impairment expected to be key for the design of next-generation systems [1]. Indeed, while EDFA amplifiers exhibit low PDL values, newly deployed Wavelength Selective Switches (WSSs) offering sharp filtering can experience up to 0.6 dB of PDL. This translates onto an average PDL exceeding 2 dB for typical operational links (16 nodes, two WSSs per node).

As shown in [2], PDL-resilience can be achieved by designing multidimensional unitary modulation schemes. 4D (I and Q components over two polarization states) single-carrier Spatially Balanced (SB) signaling [2], 8D (previous 4 dimensions over two consecutive timeslots) Silver [3] and number-theory-based [4] codes have been previously proposed among other schemes to increase the information rate. Through all investigated PDL-mitigating modulations obtained through unitary transformations, it has been observed that joint modulation of information over multiple dimensions is key to provide certain diversity or orientation gains that enhance the worst-case performance. Moreover, for the practical case of a single Pol-Mux slot, pioneer works in [5] exploit the physics of coherent lightwave systems to exemplify the use of 4D rotations for unitary optical 2×2 MIMO schemes. However, the definition and construction of an *optimal* modulation designed over a given set of dimensions was still lacking for non-unitary optical schemes. We qualify as optimal a scheme that maximizes the worst-case mutual information (MI), or equivalently throughput, over all possible unitary schemes. In this paper, we design and test PDL-resilient signaling over 4 dimensions (Ix, Qx, Iy & Qy, x and y being two orthogonal polarization states) by analyzing all possible transformations in the special orthogonal group SO(4). We compare the obtained performance to the uncoded as well as to the recently introduced SB schemes on QPSK and 16QAM modulation formats. Hence this work builds upon previous work on polarization coding to construct optimal and practical PDL-resilient signaling.

Finally notice that, as described in [5], it is remarkable that the DSP implementation of non-physical degrees of freedom enabled by left-isoclinic 4D transformations permits to improve the performance of lightwave systems.

2. Channel Model and SO(4) Transforms

PDL effect can be modelled through the following complex-valued 2×2 MIMO channel $\boldsymbol{Y} = \boldsymbol{H}\boldsymbol{X} + \boldsymbol{Z}$, where $\boldsymbol{X} = (X_1, X_2)^T$ and $\boldsymbol{Y} = (Y_1, Y_2)^T$ are, respectively, the polarization-multiplexed input and output, with X_1 and X_2 modulated symbols, $\boldsymbol{H} = \boldsymbol{H}(\gamma, \alpha, \beta)$ is the channel matrix, and $\boldsymbol{Z}$ an additive white Gaussian noise. The random PDL matrix consists of $\boldsymbol{H} = \boldsymbol{D}_{\boldsymbol{\gamma}}\boldsymbol{R}_{\boldsymbol{\alpha}}\boldsymbol{B}_{\boldsymbol{\beta}}$ where $\boldsymbol{D}_{\boldsymbol{\gamma}} = \mathrm{diag}\{\sqrt{1+\gamma}, \sqrt{1-\gamma}\}$ with gain imbalance $\Gamma = 10\log_{10}((1+\gamma)/(1-\gamma))$ in dB, $\boldsymbol{R}_{\boldsymbol{\alpha}}$ is a real-valued rotation matrix representing the incident angle $\alpha = [0, 2\pi)$ and $\boldsymbol{B}_{\boldsymbol{\beta}} = \mathrm{diag}\{e^{i\beta}, e^{-i\beta}\}$ is the phase birefringence matrix with $\beta = [0, 2\pi)$. A frequency flat model is assumed (negligible PMD) with lumped noise at the receiver side.

In [6] it is explained that, in addition to the impairment due to $\Gamma > 0$, when using discrete modulation such as polarization-multiplexed M-QAM (denoted M-QAM² thereafter), the MI between $\boldsymbol{X}$ and $\boldsymbol{Y}$ strongly depends on the incident angle α and, to a less extent, on β. For a given Γ value, the design of PDL-resilient modulations consists in reducing the dependencies on the incident state of polarization (SOP). More precisely, in our channel model, the SOP is a matrix defined by $\boldsymbol{R}_{\boldsymbol{\alpha}}\boldsymbol{B}_{\boldsymbol{\beta}} \in \mathrm{SU}(2)$, where $\mathrm{SU}(2) = \{\boldsymbol{U} \in \mathbb{C}^{2\times 2}, \det(\boldsymbol{U}) = 1\}$ is the Special Unitary group of order 2 [7]. Transformations in SU(2) that include all possible 2×2 unitary SOP rotations implemented through optical components cannot enhance the performance over the randomly oriented channel $\boldsymbol{H}$. Indeed, if the M-QAM² modulation worst rate is met for a SOP $\boldsymbol{R}_{\boldsymbol{\alpha_0}}\boldsymbol{B}_{\boldsymbol{\beta_0}}$, an M-QAM² modulation rotated by $\boldsymbol{V} \in \mathrm{SU}(2)$ will encounter the same worst rate for the SOP $\boldsymbol{R}_{\boldsymbol{\alpha_0}}\boldsymbol{B}_{\boldsymbol{\beta_0}}\boldsymbol{V}^{-1} \in \mathrm{SU}(2)$ leaving it unchanged.

An improvement can be obtained through averaging if the SOP is rotated over several time or frequency slots, e.g., using a Silver code [3] that introduces additional degrees of freedom. However, in this work, we limit ourselves to the 4 symbol dimensions for complexity constraints. Hence, we investigate orthogonal transformations in $\mathbb{R}^4$, and more precisely the

Special Orthogonal group of order 4, SO(4) $= \{\boldsymbol{G} \in \mathbb{R}^{4\times4}, \det(\boldsymbol{G}) = 1\}$. The canonical decomposition of a matrix $\boldsymbol{G} \in$ SO(4) is done through the product of a 4×4 left- and right-isoclinic matrices $\boldsymbol{G} = \boldsymbol{G_L G_R}$ with 3 degrees of freedom each [8]. This product commutes as shown in [5] or directly writing the two matrices parametrized with Hopf coordinates. The representation of SU(2) in SO(4) is a subset of the latter [5]; SU(2) matrices purely correspond to a right-isoclinic matrix $\boldsymbol{G_R}$. Therefore, it is sufficient to examine transformations with left-isoclinic only matrices that can potentially offer more robustness to PDL. These transformations applied over an M-QAM² symbol vector $\boldsymbol{X}$ can be written as $e^{i\theta}\boldsymbol{f}_{\eta,\nu}(\boldsymbol{X})$ with

$$\boldsymbol{f}_{\eta,\nu}(\boldsymbol{X}) = \left(\cos(\eta)\boldsymbol{X} + \sin(\eta)e^{i\nu}\begin{bmatrix}0 & -1\\ 1 & 0\end{bmatrix}\boldsymbol{X}^*\right) \quad (1)$$

where $\boldsymbol{X}^*$ is the conjugate of $\boldsymbol{X}$ and $e^{i\theta}$ a scalar phase common to both polarization tributaries and that cannot offer any PDL resilience. The formula (1) is analogous to the one mentioned in [5]. Hence, optimizing the two parameters η and ν is enough to construct our modulation scheme. In this work, we want to define the optimal SO(4) unitary transform in terms of maximization of the worst MI over all possible channel SOPs. Hence, for the channel model

$$\boldsymbol{Y} = \boldsymbol{D_\gamma R_\alpha B_\beta f}_{\eta,\nu}(\boldsymbol{X}) + \boldsymbol{Z} \quad (2)$$

we want to find the pair that increases the most the worst MI $(\hat{\eta}, \hat{\nu}) = \mathrm{argmax}_{\eta,\nu}\left(\min_{\alpha,\beta} I(\boldsymbol{X};\boldsymbol{Y})\right)$, where $I(\boldsymbol{X};\boldsymbol{Y})$ is the MI between $\boldsymbol{X}$ and $\boldsymbol{Y}$.

3. Optimization over all SOPs

In this section, we find the values of η and ν that increase the most the worst MI over all channel SOPs. Firstly, note that in the case $\nu = 0$, Eq. (1) can be rewritten as

$$\boldsymbol{f}_{\eta,\nu=0}(\boldsymbol{X}) = \boldsymbol{R}_{-\eta}\mathbf{Re}(\boldsymbol{X}) + i\boldsymbol{R}_{\eta}\mathbf{Im}(\boldsymbol{X}) \quad (3)$$

where $\mathbf{Re}(\boldsymbol{X})$ and $\mathbf{Im}(\boldsymbol{X})$ are $\boldsymbol{X}$ real and imaginary parts, respectively. This particular encoding leads to an offset angle of 2η between the two complex parts of $\boldsymbol{X}$. In [2], it is shown with MI considerations that the optimal value is $2\eta = \pi/4$, which motivated the construction of the SB signaling. Now, we consider all possible (η, ν) pair and conduct a numerical evaluation of $I(\boldsymbol{X};\boldsymbol{Y})$ by means of Monte-Carlo simulations.

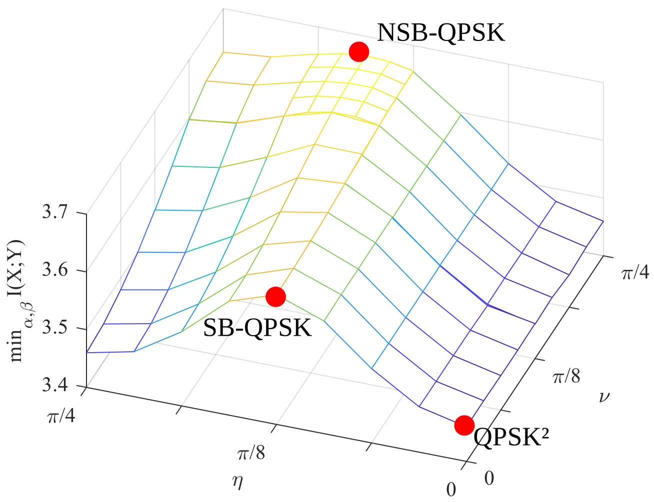

Fig. 1 Minimum value of $I(X;Y)$ as a function of η and ν encoding QPSK² symbols at SNR=8 dB for a 6 dB PDL channel.

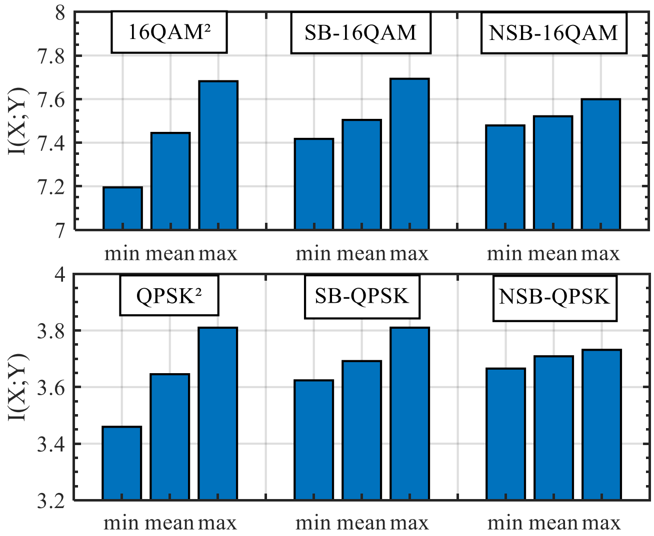

Fig. 2 Minimum, mean, and maximum value of $I(X;Y)$ in a 6 dB-PDL channel using 3 different schemes based on 16QAM² (top, SNR = 15 dB) and on QPSK² (bottom, SNR = 8 dB).

We restrict to the channel model (2), we take a PDL value of 6 dB and use a SOP grid with step $\pi/64$. In Fig. 1, $\min_{\alpha,\beta} I(\boldsymbol{X};\boldsymbol{Y})$ is plotted when using QPSK² encoded with the function $\boldsymbol{f}$ defined in (1), at an SNR of 8 dB (corresponding to a coding rate of about 0.85). A coarse (η,ν) grid with a $\pi/32$ step is used for $(\eta,\nu) \in [0,\pi/4] \times [0,\pi/4]$ and a finer step of $\pi/64$ is considered for $(\eta,\nu) \in [\pi/8, 3\pi/16] \times [3\pi/16, \pi/4]$. We restrict the study to this range because the worst MI profile is found to be $\pi/2$-periodic, as well as even with respect to each variable η and ν. The marked point at $(\eta,\nu) = (\pi/8, 0)$ corresponds to the SB worst MI. The point at (0,0) is the one of the uncoded QPSK² scheme. We notice that for η around $\pi/8$, non-zero ν values lead to an enhanced worst-case MI when compared to the SB signaling. Consequently, the worst capacity of the PDL channel after the encoding $\boldsymbol{f}$ can be further increased. With the considered set of parameters (SNR, PDL, modulation format), we find a unique optimal pair $(\hat{\eta},\hat{\nu}) \approx (5\pi/32, \pi/4) \approx (0.49, 0.79)$ in the considered range $[0,\pi/4] \times [0,\pi/4]$ with a $\pi/64$ precision. Other solutions follow from the periodicity outside this range. The worst MI for $(\hat{\eta},\hat{\nu})$ is about 0.21 bits per channel use higher than for the uncoded scheme, representing a 0.04 bits per channel use increase compared to the SB signaling. In other words, using the SB-QPSK captured only 80% of the gain that could be achieved over all possible SO(4) transforms.

In the general case where $(\hat{\eta},\hat{\nu})$ is optimal for a given operation point, we will call the signaling associated with this pair the new Spatially Balanced (NSB) signaling. Most of the gain in the encoding $\boldsymbol{f}$ comes from the balancing parameter η, and an additional gain is captured when the complex rotation with parameter ν is used. In Fig. 2, the minimum, mean, and maximum mutual information are plotted taken over all SOPs of the uncoded scheme, the SB signaling (defined from (3) with $\eta = \pi/8$), as well as the NSB signaling. The simulated parameters are still a PDL of 6 dB, an SNR of 15 dB for 16QAM² and 8 dB for QPSK² (coding rate 0.85-0.9), and the same SOP grid precision as in Fig. 1. Similarly, the NSB signaling is a good symbol encoder of a 16-QAM² modulation. As expected, we observe that the minimum MI value for the

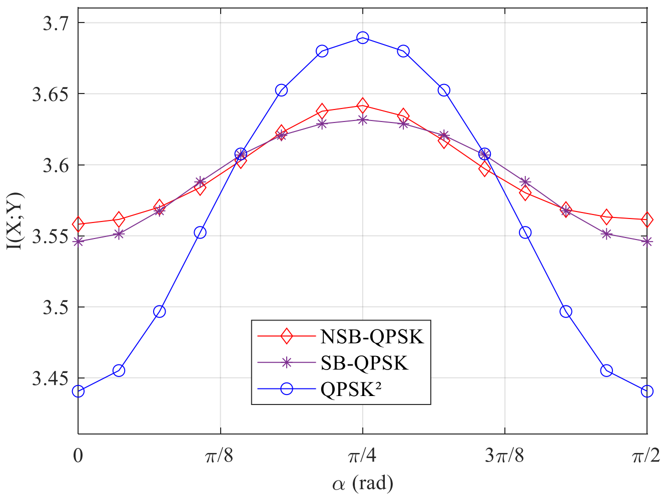

Fig. 3 MI as a function of α at SNR=8dB for a 6 dB-PDL channel. Optical link includes phase noise recovered from pilot symbols; MMSE filtering is performed after channel learning on CAZAC sequences for a total proportion of 2.3% of pilots.

NSB-16QAM is the highest of the three comparable schemes, it is increased by up to 0.28 bits per channel use compared to the standard 16-QAM² at this operating point. The same conclusions hold for the QPSK²-based schemes, as already discussed in the Fig. 1 results. Interestingly, the NSB MI variations are reduced whereas the uncoded scheme has a more anisotropic distribution of MI. Besides the fact that the two considered modulation QPSK² and 16QAM² are different, the NSB-16QAM MI gain is not the same because the operating coding rate slightly differs from the one used to draw Fig. 1.

4. Practical Simulation

The signaling exhibited in the previous section is designed considering only the PDL effect in an optical link. However, in practice, additional optical effects impair the capacity of the channel. We consider here carrier phase noise of the lasers used at the transmitter and receiver side. As already mentioned in [2], the conventionally used 2×2 Constant Modulus Algorithm (CMA) equalization cannot be used with our signaling because the assumption that the two polarizations carry independent information is no longer valid. Therefore, we choose a pilot-based approach to address the two aforementioned effects. Namely, we equalize the channel with a Minimum Mean Square Error (MMSE) equalizer and use Constant-Amplitude Zero-Autocorrelation (CAZAC) sequences to learn the channel [9]. To remove the phase noise due to the lasers, we employ pilots as well in the form of arbitrary QPSK² symbols known by the receiver and periodically inserted within data symbols. As the inverse coding function $\boldsymbol{f}^{-1}$ mixes the I/Q phase rotation of the two polarizations, it is necessary to perform the carrier phase correction prior to the symbol un-coding.
In Fig. 3, $I(\boldsymbol{X};\boldsymbol{Y})$ is plotted as a function of α for an emulated optical channel at 32 Gbaud including phase noise of a laser having a spectral linewidth of 200 kHz at both the transmitter and the receiver side. We simulated 1000 channels with a PDL of 6 dB on which we consider SOPs $(\alpha,\beta) \in [0,\pi/4] \times [0,\pi/4]$ with a $\pi/28$ step for each realization and averaged the results over the β dimension. We used a blocklength of 32000 information bits modulated as either QPSK², SB-QPSK or NSB-QPSK symbols. In such a configuration, an SNR of 8 dB corresponds to operate at a coding rate of about 0.9. The pilots are added as 9 repeated CAZAC sequences of length 4 at the frame beginning as well as 30 sequences of 5 QPSK² symbols periodically distributed in the rest of the frame. It leads to a proportion of pilots of 2.3%. This proportion is chosen to be the smallest while resulting in a BER close to the one with a genie-aided signal processing (same order of magnitude). Observe that, as expected, the NSB-QPSK MI is increased, here by 0.12 bits per channel use, a slightly higher value than if the SB-QPSK were to be used. This simulation shows that the NSB signaling have interest in practice as the worst-case capacity is effectively increased compared to the conventional polarization-multiplexed QPSK.

5. Conclusion

We showed that the PDL channel capacity is significantly improved when a specifically designed modulation is used. While polarization-multiplexed M-QAM suffer from channel alignment, a simple and physical unitary complex transform cannot protect it universally on all channel SOPs. Non-physical unitary 4D transforms offer more robustness and enable more resilience by balancing the different dimensions of the signal, while not adding any degrees of freedom. A two-parameter only optimization is sufficient to find the optimal 4D transformation. The optimal pair $(\hat{\eta},\hat{\nu})$ depends on the PDL value of the channel. For instance, on a channel with 6 dB of PDL, the numerically found solution $(\hat{\eta},\hat{\nu}) \approx (5\pi/32,\pi/4)$ appears to significantly increase the PDL channel capacity. This new signaling capacity has been showed to be effectively higher than the one of the previously proposed SB signaling, and higher than the one of the uncoded scheme.

6. References

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