Mitigation of PDL in Coherent Optical Communications: How Close to the Fundamental Limit?

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Abstract: *Considering coherent optical transmissions with PDL disturbance, we evaluate how far from the fundamental limit (based on the outage probability) are conventional coding schemes using Polarization-Time Codes and/or LDPC codes.*

Introduction

In coherent optical communication systems, linear optical impairments such as Chromatic-Dispersion (CD) and Polarization-Mode-Dispersion (PMD), can be fully compensated by using advanced digital signal processing tools. Such an efficient compensation is actually possible since CD and PMD can be viewed as unitary operator. Contrariwise, Polarization-Dependent-Loss (PDL) is not unitary and thus remains one of the major bottlenecks in high-bit-rate optical communication schemes.

PDL being a time-varying phenomenon, from an information-theoretic point-of-view, it is convenient to analyse it using the outage probability (P_{out}) as done in[1]. As the outage probability only provides the fundamental limit of PDL-disturbed communication systems, practical coding schemes mitigating the PDL may offer different performance. Therefore, in this paper, we evaluate how far from the outage probability is the Bit-Error-Rate (BER) of the most powerful existing coding schemes relying on Polarization-Time Code (PT Code)[2] and/or Low-Density-Parity-Check code (LDPC Code). Notice that, while the use of similar combinations (Space-Time Codes + LDPC) is widespread in wireless communications, its benefit has never been analyzed to mitigate PDL in fiber optical links.

System Model

In order to completely remove the CD and PMD impairments, Orthogonal-Frequency-Division-Multiplexing (OFDM) is considered. Consequently, we assume a Polarization-Division-Multiplexing (PDM) scheme disturbed only by the PDL impairment. Since PDL is not a frequency-selective phenomenon, the received signal $\mathbf{Y}(\omega)$, in the frequency domain, can be written as follows:

$$\mathbf{Y}(\omega) = \mathbf{H}_{\text{PDL}}\mathbf{X}(\omega) + \mathbf{N}(\omega)$$

where $\mathbf{X}(\omega)$ is the transmitted signal and $\mathbf{N}(\omega)$ is the additive white Gaussian noise (AWGN) with variance N_0 per real dimension[3]. The 2×2 matrix $\mathbf{H}_{\text{PDL}}$ stands for the PDL and can be modeled in two different ways:

- $\mathbf{H}_{\text{PDL}} = \mathbf{R}_\alpha \mathbf{D}_\gamma \mathbf{R}_\beta$ where $\mathbf{R}_\theta$ is a rotation matrix of angle θ and $\mathbf{D}_\gamma$ is a 2×2 diagonal matrix whose diagonal is equal to $[\sqrt{1-\gamma}, \sqrt{1+\gamma}]$ with $\gamma \in [0,1]$. Several distribution models have been presented in the literature to characterize γ or $\Gamma = 10\log_{10}((1+\gamma)/(1-\gamma))$. Hereafter, we consider one of the most relevant models for which Γ is a Maxwellian-distributed random variable[4]. Furthermore α and β are uniformly distributed in $[0, 2\pi]$.

- $\mathbf{H}_{\text{PDL}} = \sqrt{2}\tilde{\mathbf{H}}_{\text{PDL}}/\|\tilde{\mathbf{H}}_{\text{PDL}}\|_F$ where $\|\bullet\|_F$ is the Frobenius norm and $\tilde{\mathbf{H}}_{\text{PDL}} = \prod_{\ell=1}^{N}(\mathbf{R}_{\alpha_\ell}\mathbf{D}_{\gamma_\ell}\mathbf{B}_{\phi_\ell})$ with N the number of elementary PDL slices and $\mathbf{B}_\phi$ is a birefringence diagonal matrix whose diagonal is $[e^{i\phi}, e^{-i\phi}]$, α_ℓ and ϕ_ℓ are uniformly distributed in $[0, 2\pi]$, and γ_ℓ are independent but identically distributed according to a truncated Gaussian-distribution. This phenomenological model has been partly presented in[1]. The normalization operator has been added in order to keep the same energy at the receiver side regardless of the channel realization, as in the above-mentioned theoretical model. Notice that, for each $\mathbf{H}_{\text{PDL}}$, we define an equivalent $\gamma_{\text{eq.}}$ as follows: the square condition number of $\mathbf{H}_{\text{PDL}}$ is identified to $(1+\gamma_{\text{eq.}})/(1-\gamma_{\text{eq.}})$. For a given N, we have computed a lookup table whose input is $\mathbb{E}[\Gamma_{\text{eq.}}]$ with $\Gamma_{\text{eq.}} = 10\log_{10}((1+\gamma_{\text{eq.}})/(1-\gamma_{\text{eq.}}))$ and the outputs are the two parameters of the truncated Gaussian-distribution.

With the phenomenological model, the outage probability can only be evaluated numerically. In contrast, the outage probability under a Maxwellian model assumption takes the following closed-form expression[1].

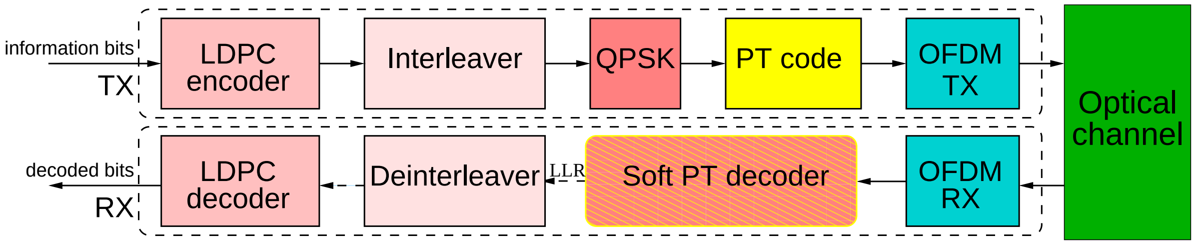

Fig. 1. Communication scheme

$$P_{\text{out}} = \begin{cases} 2Q\left(\frac{T_{R,\rho}}{\sigma}\right) + \frac{\sqrt{2}}{\sqrt{\pi}\sigma} T_{R,\rho} e^{\frac{-T_{R,\rho}^2}{2\sigma^2}}, \text{ if } \rho < \frac{2^R - 1}{2} \\ 0, \text{ elsewhere} \end{cases}$$

where $T_{R,\rho} = (20/\log 10)\text{artanh}(\sqrt{f_{R,\rho}})$ with $f_{R,\rho} = 1 - (2^R - 1 - 2\rho)/\rho^2$, ρ being RE_b/N_0, R the spectral efficiency and E_b the bit energy. Finally $Q(x)$ stands for the Gaussian tail function and σ is defined as in[4].

As the outage probabilities only provide fundamental limits, we propose to compare them to the BER offered by the simulated systems generically described in Fig. 1. We consider a PDM-based OFDM transmission scheme. The channel transfer matrix $\mathbf{H}_{\text{PDL}}$ is assumed to be perfectly known at the receiver side.

At the transmitter side, the bit stream is first coded through one of the most powerful Forward Error Correcting codes, namely, an LDPC code. Then in order to spread the consecutive bits in time and polarization, and so to be more robust to burst error, we insert an interleaver. Afterwards, these interleaved bits are transformed into symbols (PSK or QAM). In[2], it has been shown that PT coding dramatically improves the performance over PDL-disturbed systems. Therefore, a Polarization-Time code is also carried out. As PDL is not frequency-selective, the way the PT codeword is spread out on different subcarriers does not impact the final performance.

Although PDL is a time-varying phenomenon, it actually varies slowly compared to the symbol duration. Therefore we assume the PDL matrix is invariant over the LDPC codeword length.

At the receiver side, we only consider *soft* decoding. More precisely, the output of each step is the so-called LLR (Log-Likelihood ratio) instead of a hard decision, which is only made at the final step, i.e. after the LDPC decoding (via the Sum-Product Algorithm[5]). The hard decoding approach has been omitted since its performance is much poorer than the soft one.

Numerical results

We consider QPSK modulation. The LPDC code is the Quasi-Cyclic Progressive Edge Growth algorithm with the rate $r_{\text{LDPC}} = 3/4$ designed in[6]. The PT code is either the Silver or the Golden, and thus is full-rate. Since the channel bandwidth is assumed to be 50GHz (actually, ~40GHz as ~ 10GHz is dropped for guard interval), an information bit rate of at least 100Gb/s is ensured regardless of any simulated coding scheme.

In the simulations, we consider various coding schemes: no code (QPSK only), no LDPC code (Silver or Golden only), no PT code (LDPC only), PT (Silver or Golden)+LDPC. Notice that two outage probabilities have to be taken into account: on the one hand, $R = 4$ bits/s/Hz when no LDPC is considered; on the other hand, $R_{\text{LDPC}} = 3$ bits/s/Hz when the considered LDPC is carried out.

In Fig. 2 (resp. Fig. 3), the outage probabilities and BER versus SNR are plotted assuming the Maxwellian model (resp. the phenomenological model). The mean PDL, defined as $\mathbb{E}[\Gamma]$ and inherently expressed in dB, is set at 3 dB. Firstly, we observe that Silver Code is slightly better than the Golden Code for both Maxwellian and Phenomenological models. This confirms a remark done in[2] for other PDL random models. An interesting observation is that the gains offered by the PT code and the LDPC code are cumulative. Consequently, to mitigate PDL, both techniques have to be used and optimized simultaneously. Finally, given a BER of 10^{-6}, the best simulated configuration (actually, the Silver Code coupled with a soft LDPC decoding) leads only to a SNR loss of 1.5 dB compared to the fundamental limit. So, when the mean PDL has a reasonable value, conventional coding scheme is nowadays close to the fundamental limit.

In Fig. 4 (resp. Fig. 5), we plot the *SNR gap* versus the mean PDL for the different coding schemes when the Maxwellian model (resp. the phenomenological model) is considered. The SNR gap associated with a coding scheme is defined as the ratio between the SNR needed to yield a required BER with this coding scheme and the SNR needed to have the same value of out-

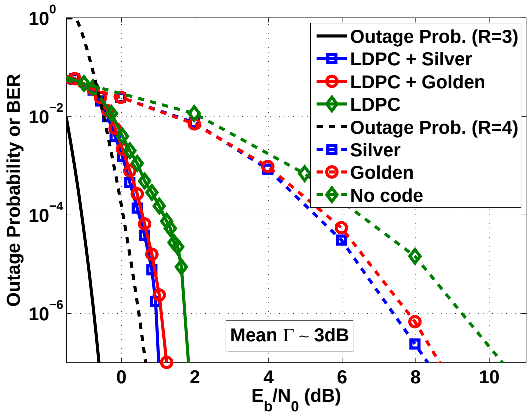

Fig. 2. Comparison for Maxwellian model

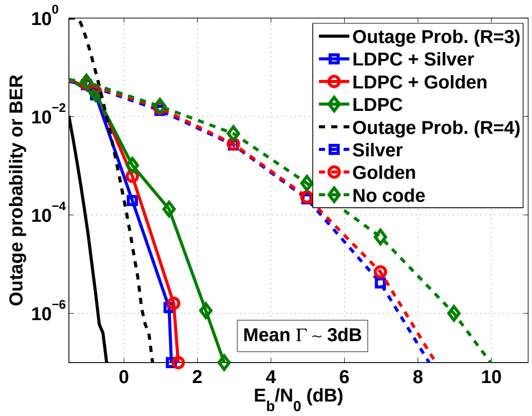

Fig. 3. Comparison for phenomenological model

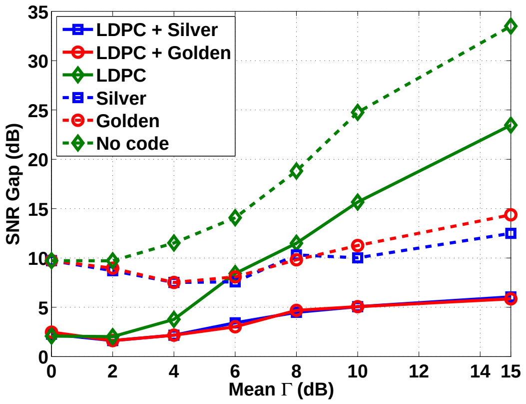

Fig. 4. SNR Gap for Maxwellian model

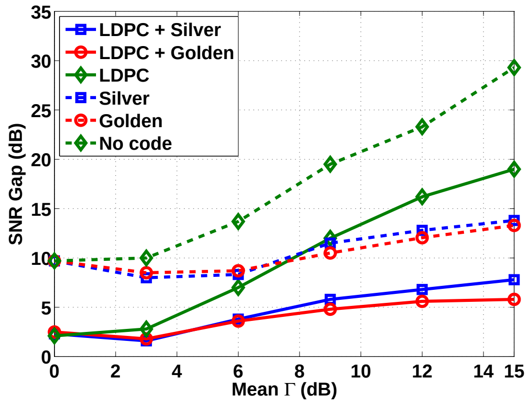

Fig. 5. SNR Gap for phenomenological model

age probability as the required BER. We have fixed the required BER and outage probability to 10^{-7}. We remark that the LDPC code is greatly affected by high mean PDL. In contrast, the PT codes are significantly less sensitive to the PDL phenomenon. More importantly, the concatenation of PT and LDPC really manages to keep the SNR gap quite weak even when the mean PDL is very large. Furthermore, the phenomenological model appears to be slightly more severe. Thus, whatever the value of the mean PDL, it is still a relevant and challenging task to fill up the SNR gap since significant improvement (at least a few dB) can be provided. Consequently, designing good LDPC and PT for mitigating PDL phenomenon remains of great interest.

Conclusion

In presence of PDL, various BERs for different code combinations (with/without PT coding, with/without LDPC) have been numerically evaluated and compared to the fundamental limits given by the outage probability. We have seen that i) the gains provided by the LDPC and the PT are cumulative, and ii) the performance of the LDPC alone are not satisfactory. Finally, we have shown that designing better new FEC codes and/or PT codes will at best offer around 1.5dB of SNR gain when a 3dB mean PDL is considered.

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