Space-Time Codes for Mode-Multiplexed Optical Fiber Transmission Systems

Elie Awwad*(1), Ghaya Rekaya-Ben Othman(1), Yves Jaouën(1) and Yann Frignac(2)

(1) Institut Mines-Télécom, TELECOM ParisTech, CNRS UMR5141,
46 rue Barrault, 75634 Paris, France

(2) Institut Mines-Télécom, TELECOM SudParis, CNRS SAMOVAR UMR5157,
9 rue Charles Fourier, 91011 Evry, France

**elie.awwad@telecom-paristech.fr*

Abstract: We propose space-time codes to mitigate mode dependent loss in spatial division multiplexed optical transmission systems using multi-mode fibers. The codes can replace or complement mode scrambling used to reduce loss disparities between the modes.

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1. Introduction

Space is currently the last available degree of freedom that can be used in an optical fiber transmission system to increase its capacity since time, frequency, phase and polarization state were all exhausted [1]. Spatial division multiplexing (SDM) can be achieved by deploying more single mode fibers (SMF) or by replacing existing SMFs with multi-mode fibers (MMF) or multi-core fibers (MCF). Recently, researchers are thoroughly evaluating the multiplexing gains as well as the setup costs, complexity and energy consumption of MMF/MCF-based systems. Despite the spectral efficiency gain, additional constraints are to be met in order to recover the transmitted signals at the receiver. Indeed, the modes in a MMF propagate at different velocities leading to differential mode dispersion (DMD). The modes also suffer from non-unitary crosstalk and losses while propagating in inline optical components like amplifiers and multiplexers or through imperfect connectors and splices [2].

While DMD remains a unitary impairment that can be treated through time-domain or frequency-domain equalization techniques [3], the accumulated crosstalk and losses lead to loss disparities between modes, known as mode dependent loss (MDL) which is a capacity limiting impairment [2]. In order to reduce MDL, stringent requirements must be applied to splice losses and optical components. In [2,4], discrete mode permutation or scrambling is proposed to reduce the accumulated MDL. However, this technique remains conceptual and real mode converters still suffer from imperfections [4]. In our work, we propose Space-Time (ST) codes to mitigate MDL arising from misaligned fibers at splices and connectors in mode-multiplexed optical links and hence increase their maximum transmission distance. ST coding was proven to be efficient in mitigating polarization dependent loss (PDL) in polarization-multiplexed optical systems [5] and therefore is a good candidate for MDL-impaired systems. We show, through numerical simulations, that ST codes are able to absorb the MDL induced penalties. They can be used as a standalone solution or can be coupled to the insertion of mode scramblers in the optical link.

2. SDM Channel Model

We consider a MMF based SDM system with M propagating modes where discrete linear modal crosstalk is introduced at connectors and splices. Polarization crosstalk is not addressed in our model. Neglecting any fiber non-linearity, the resulting multiple-input multiple-output (MIMO) channel model is given by [2]:

$$\mathbf{Y}_{M\times T} = \mathbf{H}_{M\times M}\mathbf{X}_{M\times T} + \mathbf{N}_{M\times T} = \sqrt{\alpha}\prod_{k=1}^{K}(\mathbf{T}_k\mathbf{C}_k\mathbf{P}_k)\,\mathbf{X}_{M\times T} + \mathbf{N}_{M\times T} \tag{1}$$

where $\mathbf{X}_{M\times T}$ (resp. $\mathbf{Y}_{M\times T}$) are the emitted (resp. the received) symbols on the M modes and during T time slots, $\mathbf{H}_{M\times M}$ is the $M\times M$ channel matrix that consists of a concatenation of K fiber sections. The fibers are modeled with a diagonal $M\times M$ matrix $\mathbf{T}_k$ with random phase entries $\exp(i\phi_m)$ and $\phi_m \in [0:2\pi]$. DMD is not considered since it does not affect the capacity of the system and can be equalized using time domain filters or OFDM format with a suitable cyclic prefix [3]. $\mathbf{C}_k$ represents random non-unitary modal coupling due to fiber misalignments at splices or connectors. These $M\times M$ matrices are computed using an overlap integral of the electrical field distributions of the propagating modes over the fiber cross section as in [2]. We assume independent Gaussian distributed fiber misalignments in directions x and y with zero mean and standard deviation $\sigma_{x,y}$. $\mathbf{P}_k$ corresponds to the scramblers and are replaced by a random $M\times M$ permutation matrix when k is a multiple of the scrambling period K_{scr}, or an identity matrix otherwise. α is a normalization factor used to compensate the common link loss. Finally, $\mathbf{N}_{M\times T}$ is the noise assumed to be additive white Gaussian with variance $2N_0$ per complex dimension.

In a conventional MMF-based SDM transmitter, spatial multiplexing (SM) is performed by sending independent data streams on each mode at each time slot, hence $T=1$ and the codeword $\mathbf{X}_{M\times 1}$ is a vector of independent q-QAM symbols. To implement a $M\times T$ space-time code at the transmitter, we create linear combinations of the q-QAM symbols before multiplexing them on the M modes during T time slots. A good code would provide a better performance in terms of bit error rates (BER) than a simple multiplexing scheme without reducing the transmission rate. ST codes were designed and applied to wireless Rayleigh fading MIMO schemes where they provide diversity and coding gains [6]. 2×2 codes such as the Golden and Silver codes, were also implemented in a PDL-impaired polarization-multiplexed optical transmission system where they provided important coding gains [5].

To illustrate the benefits of using ST coding to mitigate MDL, we consider an SDM system with $M=3$. At the transmitter, in the SM scheme, we send a 4-QAM symbol $s_{m=1,2,3}$ of unit energy $E_S=1$ over each mode, providing a spectral efficiency of six bits per time slot (or channel use c.u.). In the coded case, we apply a well-known full-rate 3×3 code of the linear threaded algebraic space-time (TAST) code family [7]. The codeword is given by:

$$\mathbf{X}_{3\times 3}=\frac{1}{\sqrt{3}}\begin{pmatrix} s_{1,1}+\theta s_{1,2}+\theta^2 s_{1,3} & \phi^{2/3}(s_{3,1}+j\theta s_{3,2}+j^2\theta^2 s_{3,3}) & \phi^{1/3}(s_{2,1}+j^2\theta s_{2,2}+j\theta^2 s_{2,3}) \\ \phi^{1/3}(s_{2,1}+\theta s_{2,2}+\theta^2 s_{2,3}) & s_{1,1}+j\theta s_{1,2}+j^2\theta^2 s_{1,3} & \phi^{2/3}(s_{3,1}+j^2\theta s_{3,2}+j\theta^2 s_{3,3}) \\ \phi^{2/3}(s_{3,1}+\theta s_{3,2}+\theta^2 s_{3,3}) & \phi^{1/3}(s_{2,1}+j\theta s_{2,2}+j^2\theta^2 s_{2,3}) & s_{1,1}+j^2\theta s_{1,2}+j\theta^2 s_{1,3} \end{pmatrix} \tag{2}$$

where $\phi=\exp(i\pi/12), j=\exp(i2\pi/3)$, $\theta=\exp(i\pi/9)$ and $s_{k=1:3,l=1:3}$ are 4-QAM symbols. ϕ and θ are chosen to maximize the coding and diversity gains over a wireless fading channel [7]. Full-rate means that the code introduces no spectral efficiency penalty since we have a total of nine 4-QAM symbols sent on $T=3$ time slots in each codeword. A rate of six bits per c.u. is maintained.

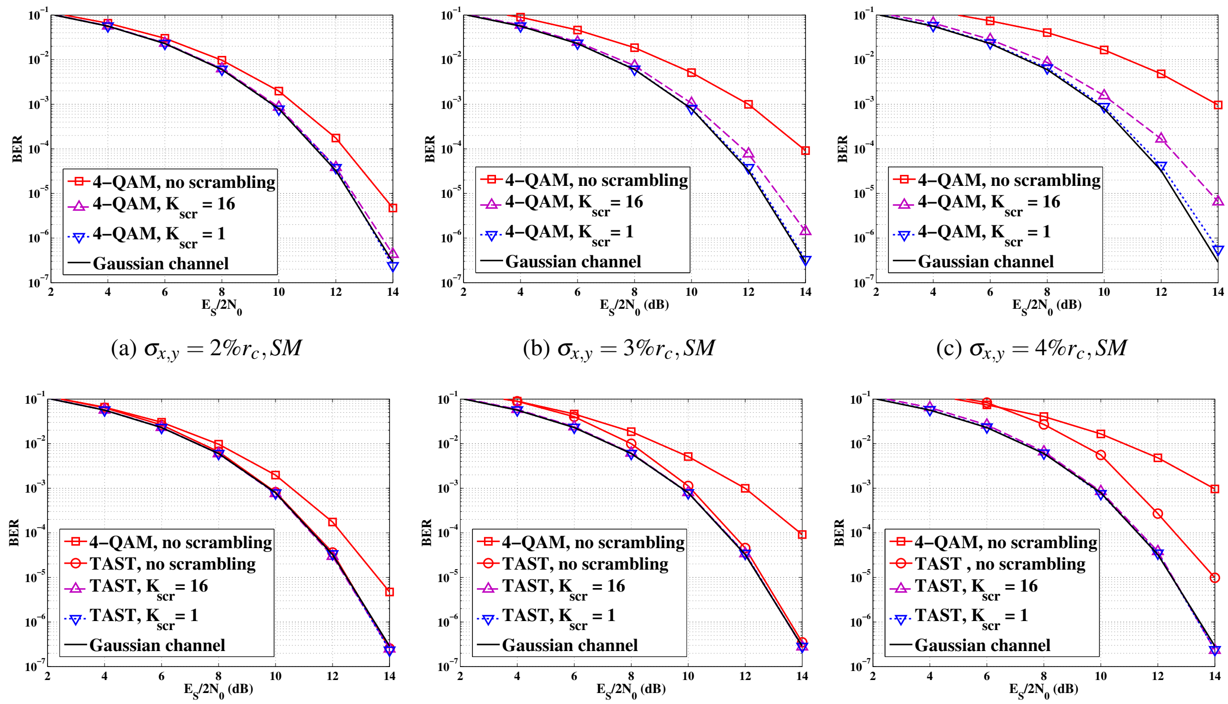
3. Simulation Results

In this section, we simulate a 3 mode-multiplexed SDM system as defined in (1) using the following parameters: graded-index fibers with a parabolic index profile, a core radius $r_c=6\mu m$ and a numerical aperture $NA=0.205$ at a wavelength $\lambda=1550nm$. The random couplings $\mathbf{C}_k$ are computed using the field distributions approximated by Laguerre-Gauss modes as in [2]. $K=300$ misaligned fiber slices with a random Gaussian misalignment of zero mean and standard deviation $\sigma_{x,y}=\{2\%,3\%,4\%\}\,r_c$ in the x and y directions. We measured the performance in terms of BER curves versus the signal-to-noise ratio $E_S/2N_0$, of both, SM and ST-coded schemes with and without the scrambling option. Scrambling periods were set to $K_{scr}=\{1,16\}$. At the receiver, the data symbols, in both schemes, are retrieved using an optimal maximum-likelihood (ML) decoder. The obtained results are shown in Fig. 1.

From the square marked red curves in Fig. 1a, 1b and 1c, we notice that the SNR penalty (i.e. the gap at a certain BER to a perfect Gaussian channel) induced by fiber misalignment increases from 0.8dB for a misalignment standard deviation (std) of $2\%r_c$ to 4dB for a misalignment std of $4\%r_c$, at a BER $=10^{-3}$. For a misalignment std lower than $2\%r_c$, the channel is almost unitary and no penalty is induced whereas for an std larger than $4\%r_c$, the penalties become extremely severe. Indeed, at a higher misalignment, the resulting accumulated MDL is also higher and the performance degradation increases. In fact, as the authors in [2] have pointed out, the system performance will depend on the overall link MDL which is a function of the single element MDL and the total number of MDL elements.

When mode scramblers are inserted each 16 fiber sections, the SNR penalty at BER $=10^{-3}$ is fully absorbed at $2\%r_c$, a small penalty of 0.3dB remains at $3\%r_c$ and 0.7dB at $4\%r_c$. When scrambling at each section, the penalty is fully absorbed in all cases. Scrambling reduces MDL by averaging the losses over all the modes [4]. On the other hand, when the TAST code is used alone (circle marked red curves in Fig. 1d, 1e and 1f), the achieved performance is close to the one obtained with a scrambling period of 16. We absorb all the penalty at $2\%r_c$, and are left with a penalty of 0.5dB at $3\%r_c$ and 1dB at $4\%r_c$ which is equivalent to an important coding gain of 3dB. Combining the code with the scramblers produces no penalty in all cases and not only at a BER of 10^{-3} but also for any achieved BER.

It is worthy to note that the choice of the scrambling periods K_{scr} is purely conceptual and optimistic comparing to the number of mode scramblers that may be installed in a real mode-multiplexed optical link. The techniques proposed in [4] to achieve mode scrambling have imperfections such as important insertion losses. Moreover, in a real optical link, mode scramblers would be installed mainly at the optical amplification stage which limits the number of scramblers in the link. On the other hand, the above results show that ST coding is an interesting candidate for MDL mitigation and has the advantage of being a software based solution implemented at the transmitter. Hence, it can be adopted as an alternative to mode scrambling or as a complementary solution to limit the number of installed scramblers in the optical link.



(a) $\sigma_{x,y} = 2\% r_c, SM$ (b) $\sigma_{x,y} = 3\% r_c, SM$ (c) $\sigma_{x,y} = 4\% r_c, SM$

(d) $\sigma_{x,y} = 2\% r_c, ST\,coding$ (e) $\sigma_{x,y} = 3\% r_c, ST\,coding$ (f) $\sigma_{x,y} = 4\% r_c, ST\,coding$

Fig. 1: BER curves of the 3×3 SDM system with a spectral efficiency = 6 bits/c.u. and using ML decoding.

4. Conclusion

In this paper, we have shown that ST coding is a promising technique for MDL-impaired SDM systems. It can be used as a standalone solution or to complement other optical-components based solutions such as mode scrambling. The efficiency of these codes in mitigating MDL and relaxing the requirements for ideal optical components paves the way for further investigation of ST coding schemes in mode-multiplexed optical links having different MDL sources such as few mode amplifiers with mode dependent gain as well as SDM systems with a higher number of modes M.

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