Space-Time Coding and Optimal Scrambling for Mode Multiplexed Optical Fiber Systems

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***Abstract*—Approaching the capacity limits of single-mode fiber based optical transmission systems, new fibers supporting the propagation of up to six orthogonal spatial modes, called few-mode fibers, stand as promising candidates for future high-capacity systems. Extensive research is being carried out to further increase the number of modes to multiplex more data. This technique is known as spatial division multiplexing (SDM). However, the co-existence of modes in the same space leads to inevitable modal crosstalk that may induce a loss of their orthogonality as well as power disparities. This phenomenon is called mode dependent loss (MDL) and mainly arises from optical components such as few-mode amplifiers. Although optical solutions were suggested to reduce MDL by inserting mode scramblers or using fibers with strong modal coupling, MDL was unfortunately not completely removed. In this work, we propose a DSP solution based on Space-Time coding, originally designed for multi-antenna channels, to mitigate MDL in SDM systems. We show that a combination of ST coding at the transmitter and an optimal distribution of mode scramblers in the optical link can completely absorb the penalties induced by important levels of MDL in 6-mode SDM systems. Later on, we address the complexity and scalability of the ST-coding solution and propose a sub-optimal decoding scheme that keeps the MDL-induced penalty low while considerably reducing the decoding complexity.**

I. Introduction

Spatial division multiplexing (SDM) holds out the prospects of increasing the capacities of optical fiber transmission links [1], especially long-haul and regional links that form the backbone of Internet and many emerging applications of information technology such as telepresence, Internet of things and human-centric communications promised by future telecommunication standards. In current optical transmission systems, single-mode fibers (SMFs) with a small core, in which only one mode named the fundamental mode can propagate, are deployed. In [2], it was shown that the capacity of SMF-based optical links has a fundamental limit due to the non-linear effects in the fiber that arise with high injected powers, thus defining a non-linear Shannon capacity. Moreover, the authors in [2], [3] point out that the achieved data rates in transmission experiments are getting closer to this limit. Hence, SMF, despite the use of all its degrees of freedom, namely wavelength, time and the amplitude, phase and polarization state of the propagating optical field for multiplexing data, is not able to cope with the ever-growing demands for higher capacities. The remaining degree of freedom in the fiber is space that can be explored through the insertion of multiple cores in the same cladding (multi-core fibers or MCFs) or through the enlargement of the fiber core to allow the propagation of several spatial modes (multi-mode fibers or MMFs). The capacity can be thus multiplied by the number of orthogonal spatial channels.

However, a tight packing of multiple pathways in a single waveguide such as modes sharing the same core in an MMF or closely spaced cores in an MCF will inevitably lead to crosstalk. Furthermore, in MMFs, the different spatial modes propagate at distinct velocities leading to differential modal delays and causing temporal inter-symbol interference (ISI). The fiber design (core radius, refractive index profiles of core and cladding) is crucial in order to guarantee the lowest possible differential dispersion and the lowest modal crosstalk which facilitates the separation of modes at the receiver using multiple-input-multiple-output (MIMO) signal processing. With recent advances in fiber design, fibers supporting three and six orthogonal spatial modes with low differential modal dispersion, called few-mode fibers (FMFs), were manufactured [4]. Few-mode optical components also emerged to address simultaneously all the modes such as few-mode optical amplifiers [5] periodically inserted for potential long-haul applications ($\geq 1000\text{km}$). Yet, scaling beyond 6 modes while maintaining low modal delays is currently very challenging.

The availability of low-dispersion fibers facilitates the management of ISI at the receiver using multi-tap time-domain MIMO filters for single-carrier formats, or single-tap frequency-domain MIMO filters for multi-carrier formats such as orthogonal frequency division multiplexing (OFDM) along with a cyclic prefix to absorb interference. The latter technique was found to achieve the lowest DSP complexity for long-haul systems [6]. Yet, SDM systems are also impaired by a more deleterious effect which is mode dependent loss (MDL) arising from inline components: optical amplifiers, couplers, multiplexers, as well as from non-unitary crosstalk in the fiber and at fiber splices and connectors [5], [7]. Due to imperfections in optical components, the modes experience differential losses or gains when propagating through the optical link which leads to signal-to-noise ratio disparities and a loss of orthogonality between modes. MDL is a capacity-limiting effect that cannot be rescinded at the receiver and is thus able to reduce the multiplexing benefit of SDM systems.

Optical solutions or enhanced equalizers were previously suggested to reduce, yet not completely remove, the accumulated MDL in the link, through the use of strong coupling fibers or mode scramblers [7]–[9]. Moreover, the opticalcomponent based solutions set stringent requirements in order to obtain the desired MDL reduction. Inspired by our work on mitigating polarization dependent loss (PDL) in polarization-multiplexed optical transmission systems [10], we propose Space-Time (ST) coding, a digital signal processing technique for MIMO systems originally designed for multi-antenna wireless channels, to mitigate MDL in mode-multiplexed optical transmission systems. We show that the combination of ST coding solutions with in-line mode scramblers or fibers with inherently strong coupling can completely absorb the SNR penalties induced by MDL in 6×6 mode-multiplexed systems where MDL levels up to 10 dB were observed. Furthermore, we notice that fewer mode scramblers than previously suggested in [7], [9], are actually needed when ST codes are used. Later on, we end with a brief complexity analysis of the proposed ST coding solutions and test the performance of a reduced-complexity sub-optimal ST decoding using a zero-forcing with decision feedback equalizer (ZF-DFE) that surprisingly appears to be close to the optimal solution.

Accordingly, the paper is organized as follows: we start by describing the channel model of a mode-multiplexed OFDM optical transmission system impaired by MDL arising from few-mode amplifiers (FMAs) in section II then we present the ST code that will be applied for MDL mitigation in section III. The performance of the coded scheme using an optimal maximum-likelihood (ML) decoder is analyzed in section IV. Later, an optimal distribution of MDL-reducing mode scramblers in ST-coded systems is suggested in section V. Finally, a low-complexity sub-optimal decoding is suggested and the coding gains are compared to the ones obtained with an optimal ML decoding in section VI.

II. Mode-multiplexed OFDM optical system

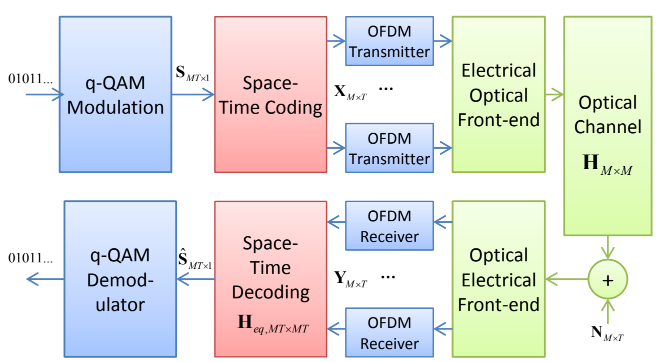


Fig. 1: Mode-multiplexed Space-Time coded OFDM optical transmission system.

We consider a mode-multiplexed optical transmission system with six single-polarization propagating modes. The number of propagating spatial modes in a fiber depends on a cut-off frequency defined for each fiber design. The modes are usually defined within the linearly-polarized LP mode classification [11, Chap.4]. The fundamental mode is defined as LP_{01} where the subscripts describe the transverse spatial geometry of the mode. When increasing the core radius of the fiber, the cut-off frequency of the LP_{11} mode is reached. This mode exhibits a degeneracy into two orthogonal modes: LP_{11a} and LP_{11b} resulting in a total of three spatial modes. By increasing the core radius more, the cut-off frequency of three other modes is reached simultaneously: the LP_{02}, LP_{21a} and LP_{21b} modes, ending up in a total of six spatial modes.

An OFDM signal along with a suitable cyclic prefix is used to modulate the modes that will propagate through a long-haul optical link containing FMFs with modal crosstalk and FMAs with modal gain offsets. The strength of crosstalk in the fiber depends on the propagation constant values of the modes (closer values result in more crosstalk) as well as on fiber imperfections [12], [13]. Moreover, fiber core misalignments at splices and bending losses can induce non-unitary crosstalk (and hence MDL) [7]. However, we intentionally choose to keep the fiber-generated crosstalk unitary in order to focus on modal gain disparities at FMAs that are hard to cancel due to technological limits and induce larger MDL values [5], [14]. The transmission scheme can be seen in Fig. 1. Neglecting any fiber non-linearity and focusing on the linear impairments of the system to evaluate its performance, the resulting MIMO channel for each OFDM subcarrier is given by:

$$\begin{aligned}\mathbf{Y}_{M\times T} &= \mathbf{H}_{M\times M}\mathbf{X}_{M\times T} + \mathbf{N}_{M\times T} \\ &= \sqrt{\alpha}\prod_{l=1}^{L}\left(\mathbf{P}_l\mathbf{G}_l\mathbf{F}_l\right)\mathbf{X}_{M\times T} + \mathbf{N}_{M\times T}\end{aligned} \tag{1}$$

where $\mathbf{X}_{M\times T}$ (resp. $\mathbf{Y}_{M\times T}$) are the emitted (resp. the received) complex symbols on the $M = 6$ modes and during T time slots. $\mathbf{H}_{M\times M}$ is the channel matrix consisting of L independent fiber spans $\mathbf{F}_l$ given by:

$$\mathbf{F}_{span,M\times M} = \prod_{k=1}^{K}\left(\mathbf{T}_k\mathbf{R}_k\right) \tag{2}$$

Each fiber span consists of K independent sections modeled as a product of a diagonal matrix $\mathbf{T}_k$ with random phase entries uniformly drawn in $[0 : 2\pi]$ representing modal phase shifts, and a real orthogonal rotation matrix $\mathbf{R}_k$ representing the distributed modal crosstalk. The mode coupling angles of the real rotation matrices $\mathbf{R}_k$ are fixed by the crosstalk levels generated at "fictional" displaced cores of two fiber sections, computed by overlap integrals of two different propagating modes over the fiber cross section as in [7]. The displacements Δx and Δy for each section are drawn from a uniform distribution over $[-\sigma r_c : \sigma r_c]$ where σ is a percentage of the core radius r_c determining the coupling strength. Note that in [7], the authors use overlap integrals to model real core displacements generating non-unitary matrices whereas here, we are using the overlap of different modes at fictional misalignments to emulate unitary coupling.

A fiber span is followed by an FMA modeled as a diagonal matrix $\mathbf{G}_l$, as well as a mode scrambler $\mathbf{P}_l$. The gains in $\mathbf{G}_l$ are assigned as follows: the LP_{01} mode has a unit gain and the gain level of the $LP_{\mu\nu}$ mode is given by $\exp(\Delta G_{01-\mu\nu}\ln 10/20)$, $\Delta G_{01-\mu\nu}$ being the gain offset in dB.

$\mathbf{P}_l$ are random permutation matrices obtained by randomly permuting the rows of an identity matrix $\mathbf{I}_M$, representing perfect mode mixers as in [7]. α is a normalization factor compensating the common link loss. It can be seen that modal dispersion is not considered since it does not affect the capacity of the system. Hence, the investigated channel matrix $\mathbf{H}$ is frequency independent, observed at the level of a single OFDM subcarrier. Finally, $\mathbf{N}_{M\times T}$ is an additive noise assumed to be zero-mean white Gaussian of variance $2N_0$ per complex dimension per mode added at the receiver.

Before simulating the performance of this transmission system, we look into the statistics of the accumulated MDL in our channel model. Previous works have shown that strong coupling in the fiber [8] and insertion of mode scramblers [7], [9] at the amplification stages enhanced the system performance by averaging the losses observed by the multiplexed signals, thus reducing gain disparities. To evaluate MDL under different coupling scenarios, we consider an SDM system with $L = 8$ spans where FMFs have a parabolic index profile with a core radius $r_c = 8.7\mu$m and a numerical aperture of 0.205 at $\lambda = 1550$nm, supporting hence 6 spatial modes. The field distributions of the modes are approximated by Laguerre-Gauss modes as in [7]. Each span consists of $K = 200$ sections. The amplifiers present the following gain offsets $\Delta G_{01-11} = -1.3$dB, $\Delta G_{01-21} = -2$dB and $\Delta G_{01-02} = -0.2$dB corresponding to a promising FMA technology presented in [5]. For $K = 200$ sections, three coupling strengths are investigated by drawing core displacements from a uniform distribution with σ tuned to 0.6%, 3% and 5% of r_c to emulate weak, medium and strong mode coupling respectively. 10^6 channel realizations were numerically simulated and the accumulated MDL levels computed for each scenario, MDL being defined as the ratio in dB between the squares of the highest and the lowest singular values of $\mathbf{H}$. The obtained MDL probability distribution functions (PDF) are shown in Fig. 2 for the different coupling levels, with and without mode scrambling.

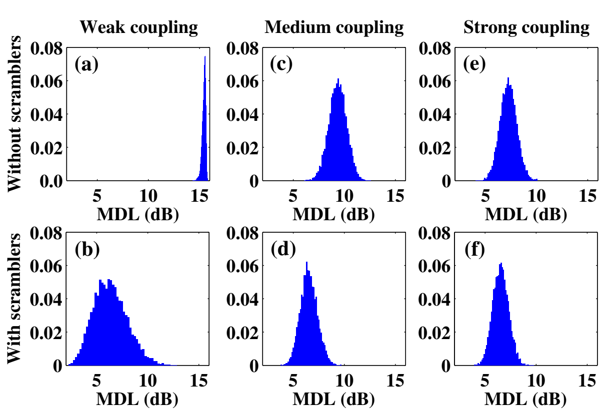


Fig. 2: PDF of the accumulated MDL in different coupling and scrambling scenarios ($L = 8$ spans, 2dB of MDL per span).

First, we notice that coupling and scrambling significantly reduce the average MDL as well as its variance. In weak coupling and without scramblers, the 2dB MDL of each amplifier sums up resulting in an accumulated MDL of $8 \times 2\text{dB} = 16\text{dB}$. At medium coupling, the modes are only partially correlated and the average MDL decreases to 10dB. Strong coupling and mode scrambling reduces the average MDL to 6dB which is very close to $\sqrt{8} \times 2\text{dB} = 5.7\text{dB}$, the expected accumulated MDL value when full, random coupling occurs between identical MDL sources [15]. However, MDL is not completely eliminated. The impact of MDL on the capacity C of the MIMO channel in (1) is illustrated in Fig. 3 where the cumulative distribution functions (CDF) of the capacity, at $\text{SNR}_{dB} = 10\log_{10}(E_S/2N_0) = 10\text{dB}$ per mode, are given. C is defined as [16, Chap.15]:

$$C = \sum_{i=1}^{M=6} \log_2 (1 + \text{SNR}\lambda_i) \tag{3}$$

where $\{\lambda_i\}$ are the squares of the singular values of $\mathbf{H}$ and E_S the average symbol energy. The capacity of a 6×6 MDL-free additive white Gaussian noise (AWGN) channel is also shown as a reference.

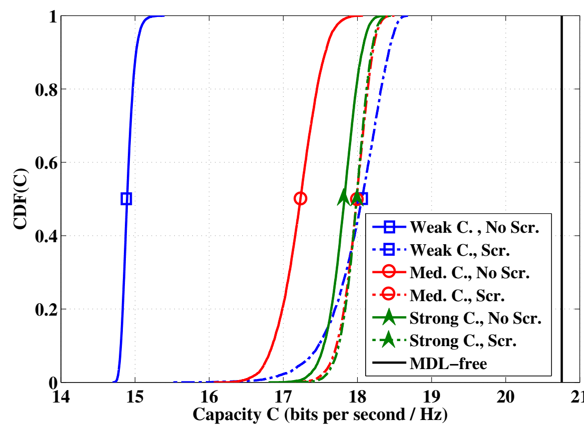


Fig. 3: CDF of the capacity of the 6×6 SDM system in the different coupling and scrambling scenarios at $\text{SNR} = 10\text{dB}$.

III. ST CODES FOR MDL MITIGATION

In [10], we proved that ST codes efficiently mitigates PDL in single-mode polarization-multiplexed optical transmission systems, and showed that the offered performance enhancement obeyed different criteria than those in wireless systems. MDL, a gain disparity between modes, can be seen as a generalization of PDL, a gain disparity between polarization states. Hence, ST coding is an interesting DSP solution for MDL mitigation. Instead of using the modes simply for multiplexing which consists in sending a vector of independent q-QAM symbols $\mathbf{S}_{M\times 1}$ on M modes at a single time slot, we can benefit from the two degrees of freedom: space and time of the MIMO scheme to insert multiple copies of a data symbol over different modes at different time slots. Then, at the receiver, we can exploit these copies to get a better estimate of the data since it would have experienced various channel states, and the channel disparities would be further reduced. This technique is known as Space-Time (ST) coding and was originally designed to combat fading in wireless MIMO communications [17].

A linear space-time code is implemented at the transmitter by creating linear combinations of the q-QAM symbols before sending them on M modes at T time slots. The linearity property leads to an easier decoding of the data symbols at the receiver using lattice decoders [18], [19]. Many ST code families were designed for various wireless MIMO schemes. We will focus on a specific category of codes: Space time block codes (STBC) in which a codeword is represented by a matrix $\mathbf{X}_{M\times T}$ obtained by multiplying a symbol vector $\mathbf{S}_{MT\times 1}$ with a generator matrix $\mathbf{M}_G$ and rearranging the obtained vector into an $M \times T$ matrix. This operation takes place at the transmitter side in the "Space-Time Coding" block in Fig. 1. Among STBCs, we will choose the ones that fulfill the following requirements: first, the codeword matrices place each data symbol on a different mode at each time slot while maintaining a minimum decoding delay; second, the code must be full-rate meaning that MT q-QAM information symbols are sent in each codeword and thus does not reduce the spectral efficiency of the SDM system; finally, a uniform average energy must be transmitted per mode. A code family answering these requirements is the one of linear threaded algebraic space-time (TAST) codes [18], well-known for its generality (codes exist for any M) and performance. For instance, the 6×6 TAST codeword matrix is given by:

$$\mathbf{x}_{\mathcal{T}} = \frac{1}{\sqrt{6}} \begin{bmatrix} f_1(\mathbf{s}_1) & \phi^{\frac{5}{6}} f_2(\mathbf{s}_6) & \phi^{\frac{4}{6}} f_3(\mathbf{s}_5) & \phi^{\frac{3}{6}} f_4(\mathbf{s}_4) & \phi^{\frac{2}{6}} f_5(\mathbf{s}_3) & \phi^{\frac{1}{6}} f_6(\mathbf{s}_2) \\ \phi^{\frac{1}{6}} f_1(\mathbf{s}_2) & f_2(\mathbf{s}_1) & \phi^{\frac{5}{6}} f_3(\mathbf{s}_6) & \phi^{\frac{4}{6}} f_4(\mathbf{s}_5) & \phi^{\frac{3}{6}} f_5(\mathbf{s}_4) & \phi^{\frac{2}{6}} f_6(\mathbf{s}_3) \\ \phi^{\frac{2}{6}} f_1(\mathbf{s}_3) & \phi^{\frac{1}{6}} f_2(\mathbf{s}_2) & f_3(\mathbf{s}_1) & \phi^{\frac{5}{6}} f_4(\mathbf{s}_6) & \phi^{\frac{4}{6}} f_5(\mathbf{s}_5) & \phi^{\frac{3}{6}} f_6(\mathbf{s}_4) \\ \phi^{\frac{3}{6}} f_1(\mathbf{s}_4) & \phi^{\frac{2}{6}} f_2(\mathbf{s}_3) & \phi^{\frac{1}{6}} f_3(\mathbf{s}_2) & f_4(\mathbf{s}_1) & \phi^{\frac{5}{6}} f_5(\mathbf{s}_6) & \phi^{\frac{4}{6}} f_6(\mathbf{s}_5) \\ \phi^{\frac{4}{6}} f_1(\mathbf{s}_5) & \phi^{\frac{3}{6}} f_2(\mathbf{s}_4) & \phi^{\frac{2}{6}} f_3(\mathbf{s}_3) & \phi^{\frac{1}{6}} f_4(\mathbf{s}_2) & f_5(\mathbf{s}_1) & \phi^{\frac{5}{6}} f_6(\mathbf{s}_6) \\ \phi^{\frac{5}{6}} f_1(\mathbf{s}_6) & \phi^{\frac{4}{6}} f_2(\mathbf{s}_5) & \phi^{\frac{3}{6}} f_3(\mathbf{s}_4) & \phi^{\frac{2}{6}} f_4(\mathbf{s}_3) & \phi^{\frac{1}{6}} f_5(\mathbf{s}_2) & f_6(\mathbf{s}_1) \end{bmatrix}$$

where $\phi = \exp(i\pi/12)$, $\mathbf{s}_{1:6}$ a vector of 6 q-QAM symbols, $f_n(\mathbf{x}) = \Sigma_{k=1:6} x_k (j^{n-1}\theta)^{k-1}$ with $j = \exp(i2\pi/6)$, $\theta = \exp(i\pi/18)$. ϕ and θ are chosen to maximize the coding and diversity gains over a wireless Rayleigh fading 6×6 multi-antenna channel [18]. 36 symbols are sent on 6 time slots in each codeword, achieving a full rate of 6 symbols/time slot. Its generator matrix, which is unitary (i.e. $\mathbf{M}_G \mathbf{M}_G^{\dagger} = \mathbf{I}$), can be found in [18]. The unitary coding matrix does not increase the energy of the new transmitted symbols after encoding the q-QAM information symbols.

At the receiver side, the original data symbols are estimated using a maximum likelihood (ML) decoder. Assuming that the channel matrix $\mathbf{H}$ is known (or perfectly estimated at the receiver) and constant during T time slots, and that all emitted codewords $\mathbf{X}$ are equiprobable, the optimal detection scheme of the channel in (1) should satisfy the ML criterion that consists in estimating the codeword $\mathbf{X}$ with $\hat{\mathbf{X}}_{ML}$ that minimizes the following Euclidean distance:

$$\hat{\mathbf{X}}_{ML} = \underset{\mathbf{X}_{M\times T} \in C}{\operatorname{argmin}} \|\mathbf{Y} - \mathbf{H}\mathbf{X}\|^2 \tag{4}$$

where C is the set of all possible transmitted codewords. The ML criterion can be further developed to explicitly show the original q-QAM symbols when an STBC is used. To this end, we use a column-wise vectorized form of (1) containing the generator matrix of the ST code and define an equivalent channel $\mathbf{H}_{eq}$ [18]:

$$\begin{aligned} \text{vec}_{\mathbb{C}}(\mathbf{Y}) &= \begin{bmatrix} \mathbf{H} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{H} \end{bmatrix} \text{vec}_{\mathbb{C}}(\mathbf{X}) + \text{vec}_{\mathbb{C}}(\mathbf{N}) \\ \mathbf{Y}'_{MT\times 1} &= \mathbf{H}'_{MT\times MT} \mathbf{M}_G \mathbf{S}_{MT\times 1} + \mathbf{N}'_{MT\times 1} \\ &= \mathbf{H}_{eq}\mathbf{S} + \mathbf{N}' \end{aligned} \tag{5}$$

where $\mathbf{M}_G$ is the generator matrix of the coding scheme. In the case of simple spatial multiplexing, $T = 1$ and $\mathbf{M}_G$ is replaced by the identity matrix. Given that $\mathbf{H}$ is a full-rank matrix and $\mathbf{M}_G$ is unitary, the ML decoding rule can be reinterpreted as:

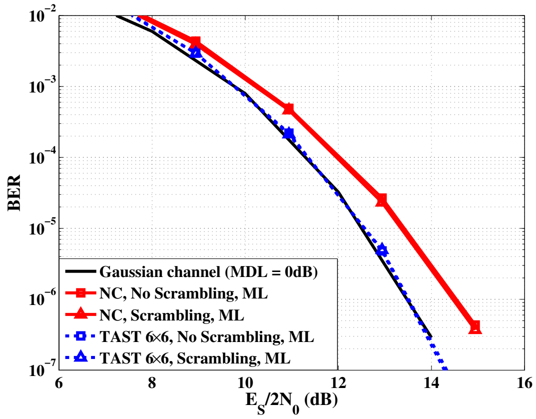
$$\hat{\mathbf{S}}_{ML} = \underset{\mathbf{S}_{MT\times 1} \in C'}{\operatorname{argmin}} \|\mathbf{Y}' - \mathbf{H}_{eq}\mathbf{S}\|^2 \tag{6}$$

where C' is the set of all possible transmitted q-QAM symbols. Hence, after a complex-to-real transformation of (6), C' can be seen as a subset of the lattice $\mathbb{Z}^{2MT}$ and the ML criterion can be implemented through reduced-search lattice decoders such as the sphere decoder [19] for both uncoded and ST-coded systems.

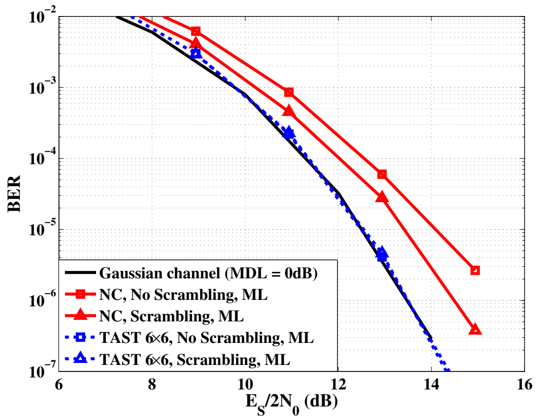
IV. ST-CODED SDM SYSTEM PERFORMANCE

After presenting the SDM transmission system and the chosen ST code, we simulate the benefits of using ST coding to mitigate MDL in the 6-mode SDM system in (1) with the link parameters defined in section II. At the transmitter, in the uncoded (or no coding: NC) scheme, a vector of 4-QAM symbols $S_{m=1:6}$ of unit energy $E_S = 1$ is sent over the modes, providing a spectral efficiency of 12 bits per time slot. For the coded case, a 6×6 TAST code is used. The performance in terms of average bit-error-rate (BER) curves versus SNR_{dB} of both NC and ST-coded schemes is measured through Monte-Carlo simulations. A minimum of 100 bit errors are registered per simulation point. Three coupling scenarios with and without scrambling at the FMAs are presented in Fig. 4. At the receiver, in all scenarios, the data symbols are retrieved using a sphere decoder.

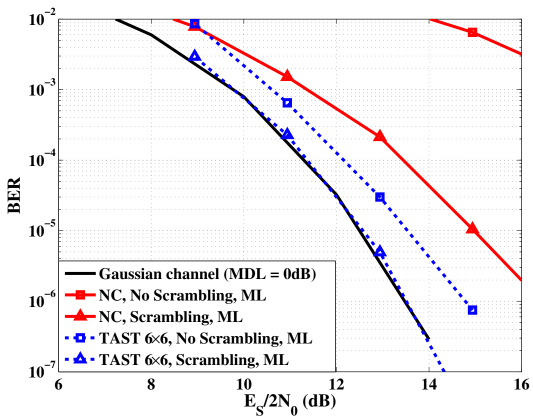
From the square marked curves corresponding to NC without mode scramblers, we notice that the SNR penalty at $\text{BER} = 10^{-3}$ induced by MDL (i.e. the gap at a given BER to a perfect MDL-free Gaussian channel) decreases from more than 6dB for weak coupling, to 1.2dB for medium coupling and to 0.4dB for strong coupling. Adding mode scramblers at FMAs (triangle marked curves) reduces these penalties to 1.7dB for weak coupling and 0.4dB for medium coupling, while it has no effect in strong coupling regime because the modes are already fully coupled in the fiber and MDL cannot be further reduced by scramblers, as seen from the MDL distributions in Fig. 2. The reduction of accumulated MDL with strong fiber coupling and mode scramblers was already observed in previous works [8], [9].



(a) 6×6 system, strong coupling.



(b) 6×6 system, medium coupling.



(c) 6×6 system, weak coupling.

Fig. 4: Average BER versus SNR of 6×6 SDM systems (8 spans of fibers with various coupling strengths and FMAs with a maximum modal gain offset of 2dB).

On the other hand, when the 6×6 TAST code is used alone (square-marked dashed curves), it outperforms the scrambling solution for weak coupling, reducing the SNR penalty at BER $= 10^{-3}$ to 0.8dB. Furthermore, the code absorbs all MDL-induced penalty in the medium and strong coupling scenarios. Finally, combining ST coding with mode scrambling results in no penalty in all schemes, and not only at BER $= 10^{-3}$ but for any given BER. These results prove the efficiency of ST coding solutions in mitigating MDL by averaging the losses experienced by the mode-multiplexed data symbols, making it an interesting DSP solution for SDM systems. ST coding can be adopted as an alternative to mode scrambling or as a complementary solution depending on the coupling strength in the installed FMFs.

V. Optimally scrambled ST-coded MDM systems

We have seen that full absorption of high MDL levels was possible using ST codes along with mode scramblers at each amplification stage. On the other hand, we also observed that modal coupling and scrambling are able to reduce the accumulated MDL. However, this reduction is lower bounded by an average MDL that grows as the square root of the number of MDL sources (FMAs in our case) in the link as explained in [15]. This minimum average value can be reached using solely fibers with inherently strong coupling as seen from the MDL statistics in Fig. 2 or with regular mode scrambling when there is not enough coupling in the fiber. In weak coupling, mode scramblers are needed at each amplification stage. However, for intermediate coupling strengths, a lower number of scramblers might be installed.

The placement and number of scramblers required for an optimal reduction of MDL are important parameters for the design of long-haul SDM systems because real mode scramblers are not as perfect as modeled in the numerical studies (perfect permutation matrices). Practical implementations of mode scramblers include few meters of strong-coupling fibers or mode converters with free-space optical components having non-negligible crosstalk and insertion losses. In order to find the optimal scrambling map for the studied SDM system, we define the ratio $r = N_{scr}/L$ where N_{scr} denotes the number of scramblers per L fiber spans, with $0 \leq r \leq 1$ allowing at most for a single mode scrambler per span. Considering the parameters of the 6-mode system in section II, we draw the evolution of the average accumulated MDL with the number of spans L for different coupling strengths (obtained by varying the maximum core displacements σ) and in the absence of mode scramblers. The resulting curves are drawn in Fig. 5 along with the lower bound $2\sqrt{L}$ where 2dB stands for the MDL per FMA. Then, for each σ, we vary the ratio of mode scramblers r in the link and determine the minimum ratio r_{opt} that leads to an average MDL within 1dB of its lower bound $2\sqrt{L}$dB. For a small number of spans, the optimal MDL reduction cannot be reached because the modes are not yet fully coupled as we can see from Fig. 2 in the weak coupling scenario with mode scramblers at each FMA ($r = 1$). In this case, r_{opt} is determined for a longer SDM system in which the modes are fully coupled. Fig. 6 shows the optimal values r_{opt} obtained for different coupling strengths as well as a linear interpolation for intermediate values. We see that for $\sigma \leq 1\%$, a mode scrambler is needed at each span and beyond 5%, mode scramblers are no longer needed.

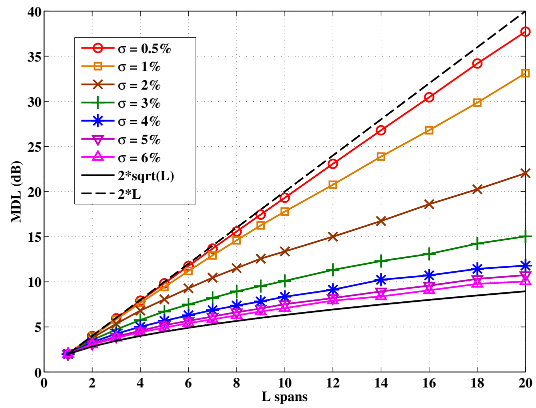


Fig. 5: Evolution of the average accumulated MDL with the number of spans L for various coupling strengths in absence of mode scramblers (2dB MDL per span).

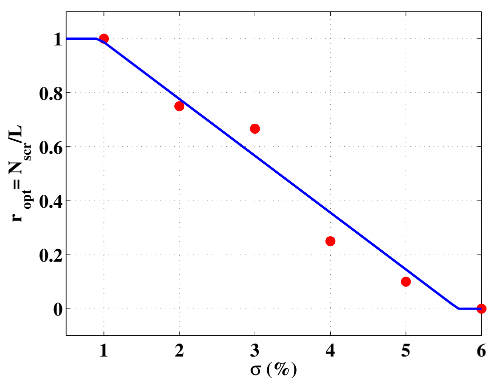


Fig. 6: Ratio of mode scramblers required to optimally reduce MDL for various coupling strengths determined by σ.

VI. Low-complexity decoding with ZF-DFE

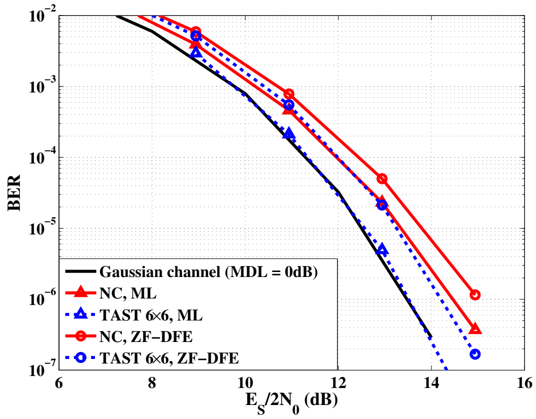
ST coding can be generalized to larger $M \times M$ SDM systems. However, the downside of this solution is its increased decoding complexity that grows as $\mathcal{O}((MT)^6)$ using a sphere decoder with an appropriate initial radius [19]. This can turn ST codes into a prohibitive MIMO solution for larger SDM systems. Therefore, we suggest a variant that trades a portion of the optimal coding gains for a reduction in complexity and a better scalability. In [10], we showed that the performance enhancement offered by ST coding in PDL-impaired systems obeyed different criteria than those in wireless systems. An analytic expression of the minimum distance between the possible emitted symbols after propagating in a PDL-impaired channel is given in [10]. ST-coded schemes exhibited a larger minimum distance compared to uncoded schemes which translates into a better performance. Hence, only a coding gain is brought by the codes while no diversity gain is needed which is not the case for Rayleigh fading MIMO channels. We conjecture that the same behavior is observed in MDL-impaired systems and propose sub-optimal decoding of ST schemes to check whether it can achieve any coding gain.

For that, we replace the sphere decoder with a sub-optimal zero-forcing with decision feedback equalizer (ZF-DFE). In wireless MIMO channels, ZF-DFE demonstrates a performance gain over the classic ZF decoder that performs a simple channel inversion, notably through its successive interference cancellation while retrieving the data symbols [20]. However, both decoders perform far worse than an ML decoder because they fail to attain the full diversity of the wireless channel. ZF-DFE decoding consists in performing a QR decomposition of $\mathbf{H}_{eq}$ that rewrites the channel as a product of a unitary matrix $\mathbf{Q}$ and an upper triangular matrix $\mathbf{R}$. An equivalent channel is obtained by applying the inverse of $\mathbf{Q}$ on (5). Then, each symbol in the vector $\hat{\mathbf{S}}_{ZF-DFE}$ is estimated by solving the linear system $\tilde{\mathbf{Y}} = \mathbf{RS}$. $\mathbf{R}$ being upper triangular, this can be done in an iterative fashion starting from the last symbol and performing a threshold decision on each estimated symbol before feeding it to the previous equation [20]. The complexity of this algorithm is fixed by the QR decomposition of the channel matrix and the resolution of the linear system. A rough estimation of this complexity in flops defined as the number of required complex scalar multiplications in order to decode a transmitted symbol $\mathbf{S}$ gives $\mathcal{O}((MT)^3)$, nearly the same complexity as for a ZF equalization (simple channel inversion) for large MIMO systems and negligible compared to the complexity of a sphere decoder.

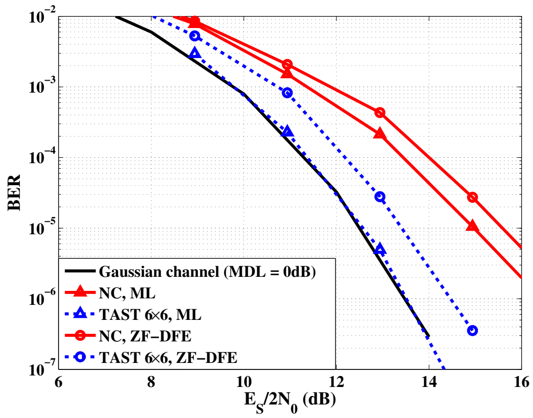
Monte-Carlo simulations were carried to assess the performance of ZF-DFE on the previously defined 6×6 SDM system with $L = 8$ spans. The average BER curves of both, NC and TAST-coded schemes for three fiber coupling strengths are given in Fig 7. In all scenarios, mode scramblers are used at FMAs. Strong or medium coupling along with mode scrambling induce the same MDL distributions, hence the same BER performance given in Fig. 7a. Obviously, ZF-DFE performs worse than ML decoding for the NC scheme (circle marked curves) because of noise enhancement. On the other hand, when ST coding is used along with ZF-DFE decoding (dashed curves), a performance gain is obtained in all cases. At BER $= 10^{-3}$, the ZF-DFE decoded ST scheme has the same penalty of 0.4dB as the optimal ML-decoded NC scheme for medium and strong coupling while it outperforms the NC scheme for weak coupling, providing a coding gain of 1.2dB. Surprisingly, this observed performance of ST codes is completely different from the one obtained on a wireless channel where ZF-DFE decoding of the codewords would not bring any gains due to the different nature of the channel. In all investigated scenarios, the low-complexity decoded ST scheme for a 6×6 SDM system is at worst at 1dB from its corresponding ML-decoded scheme that matches the performance over an MDL-free Gaussian channel. We have also tested a ZF decoding of the ST scheme that had a poorer performance than ZF-DFE.

VII. Conclusion

In this paper, we have shown, through numerical simulations, that ST coding is a promising solution for MDL mitigation in SDM systems. It can be used as a standalone solution or to complement other optical components-based solutions such as mode scrambling depending on the coupling



(a) Strong or medium coupling + mode scramblers.



(b) Weak coupling + mode scramblers.

Fig. 7: Average BER versus SNR of ZF-DFE decoded 6×6 ST schemes with mode scrambling ($L = 8$ spans of fibers, FMAs with $\mathrm{MDL} = 2\mathrm{dB}$).

strength in the installed fibers. The ML performance of the applied codes was investigated, showing a complete mitigation of MDL levels up to 10dB in a 6-mode system, relaxing thus the gain offset requirements in optical components such as FMAs and improving the reach of the transmission system. The requirements in terms of coupling strengths and mode scrambling in the optical link were also investigated in order to find the configurations that minimize MDL to its lowest value, letting the ST code manage the residual MDL. Furthermore, we showed that a performance enhancement can also be brought to MDL-impaired systems using a low-complexity sub-optimal ZF-DFE decoding of the ST schemes, trading thus a portion of the coding gains for a complexity reduction in order to ensure that the DSP remains tractable. These observations pave the way for further investigation of the capabilities of ST coding such as the maximum loss disparities that can be mitigated, allowing to further reduce the number of required scramblers (r_{opt}), as well as the study of MDM optical links having different MDL sources, a higher number of modes M or polarization-multiplexed MDM channels where PDL can be added as another performance-limiting effect.

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