Sequence Selection for Nonlinear Interference Mitigation: Current Approaches & Open Challenges

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***Abstract*—In optical fiber transmission systems, nonlinear interference (NLI) significantly limits the achievable data rates for a reliable communication. In this talk, we review recent approaches to sequence selection as a method for minimizing NLI. By carefully selecting at the transmitter side symbol sequences that generate minimal NLI, it is possible to enhance system performance. We underline the various metrics used to assess the NLI generated by a symbol sequence in a transmission link, highlighting their theoretical bases and practical applications. Despite these recent advancements, numerous challenges remain unsolved, such as the complexity of predicting NLI accurately and on-the-fly at the transmitter, the computational burden of sequence selection, and the quest for novel schemes drawing upon the lessons learned from sequence selection. This talk discusses these open challenges and suggests potential directions for future research to address them.**

***Index Terms*—Optical fiber communication, Probabilistic shaping, Nonlinear fiber channel.**

I. Introduction

Under the assumption of low-to-negligible bandwidth-narrowing filtering and accumulated polarization dependent loss, linear impairments in terrestrial metropolitan to long-haul optical fiber communication systems can be compensated for on the receiver side almost perfectly, allowing us to approach the capacity of the linear additive white Gaussian noise (AWGN) channel by increasing the signal-to-noise ratio (SNR). However, due to non-negligible nonlinear Kerr effects in silica fibers and to the energy confinement of the propagating light in the fiber core, unmanaged nonlinear interference (NLI) constrain the achievable information rate even when increasing the SNR. Above a certain power threshold, significant NLI is generated, leading to a degradation in the transmission performance.

The capacity of the optical fiber channel in the nonlinear regime remains an open problem, with currently unknown tight bounds [1], [2]. While the upper limit is constrained by the linear capacity of the channel, the lower limit was defined by numerous bounds provided in [3]–[5]. These bounds are typically established by computing an achievable information rate (AIR) while considering a fixed simple input distribution such as independent and identically distributed (i.i.d.) samples with a Gaussian distribution or discretized 2D Gaussian constellations creating concentric ring constellations [5]–[9]. The nonlinear propagation channel is simplified based on a perturbation analysis in which only nonlinear contributions up to the first order in the nonlinear coefficient of the fiber are considered [6]. The optimization of decoding metrics considering these distributions leads to the estimation of AIR and consequently, to the understanding of the capacity bounds. While the exact capacity is not yet known, understanding these bounds is crucial for designing and optimizing modulation schemes for optical fiber transmission systems.

To effectively minimize NLI in optical fiber transmission systems, it is crucial to understand its nature and how it builds up in a fiber link. A first partitioning of NLI can be made based on its statistical nature: deterministic or stochastic. Deterministic interactions are primarily between various data-carrying signal components, while stochastic interactions involve both data-carrying signals and noise (and noise-noise interactions that have a negligible impact in our studied scenarios) [5]. It is worth mentioning that inter-channel interactions can also be considered stochastic when the data on adjacent channels is unknown to the channel under consideration. The reduction of NLI has been demonstrated through two main approaches: digital equalization at the transmitter side and/or receiver side as recalled in the recent work [10] and its cited references, or coding and modulation schemes tailored to the nonlinear fiber channel, an approach known as nonlinear shaping implemented using geometric and probabilistic constellation shaping over multiple dimensions [11], [12].

Recently, several works explored metrics that directly assess NLI power or predicts the potential of a sequence of data symbols in generating NLI, which is an important tool in the design of nonlinear tolerant transmission schemes [13]–[19]. In this talk, we trace the development of sequence selection strategies essentially based on a trial-and-error approach consisting in the evaluation of a given metric for different sequences and the selection of the sequence that generates the lower amount of NLI. Among these metrics, we focus on one of our recent works in which we introduced a dispersion-aware metric, denoted D-EDI, or energy dispersion index of dispersed sequences [19]. The energy dispersion index (EDI) evaluates the variations of energy of a sequence over a given temporal window and was first introduced in [15], [16]. In our proposal, instead of evaluating EDI only over the original data sequences, we average EDI values computed over sequences altered by digital filters emulating the accumulated chromatic dispersion over the considered transmission link.

Nonlinear gains were demonstrated through the selection of sequences using several metrics, namely the computation of the NLI power of transmitted sequences using a noiseless fiber propagation model [14], the sign-independent energy dispersion index (EDI) [16], and sign-dependent metrics such as perturbation-model-based low-pass-filtered symbol-amplitude sequence (LSAS) metric [17], [18] and more-recently D-EDI [19] and perturbation-based sequence selection [20]. The paper is structured as follows: in Section II, we briefly review the concept of sequence selection and recall the basic principles of the main metrics used for selection. In Section III, we discuss the limitations of NLI-mitigation schemes based on sequence selection and conclude by providing some insights for designing novel nonlinear shaping schemes drawn from recent investigations.

II. Review of Metrics for Sequence Selection

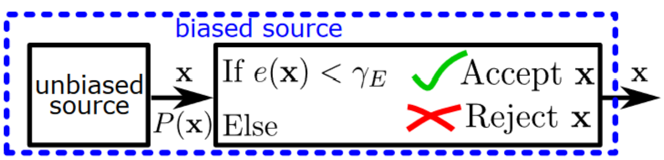


Fig. 1. The biased source obtained by a selection procedure from an unbiased source. Figure taken from [13].

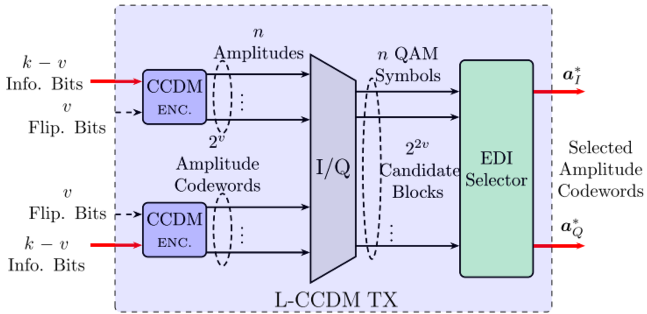
The basic principle of sequence selection is shown in Fig. 1. Sequence selection uses a certain metric to select ‘good’ sequences for transmission and reject ‘bad’ sequences. A random sequence of symbols $\mathbf{X}$ is initially drawn from an unbiased source with probability distribution $P(\mathbf{X})$. Each sequence is then evaluated against a chosen measure $e(\mathbf{X})$. If it meets the criteria (e.g. comparison to a threshold $e(\mathbf{X}) < \gamma_E$), it is accepted as the actual transmission source; otherwise, it is discarded. This method creates biased sources from unbiased ones and generates symbol sequences as the channel inputs. The unbiased source can be chosen arbitrarily. For a signal source with N complex-valued symbols, it can be a continuous constellation (such as a Gaussian constellation), or a discrete constellation (such as quadrature and amplitude modulated (QAM) symbols). The symbols can be independent or correlated. In recent research works, the symbols are generated from a probabilistic-amplitude-shaped (PAS) transmitter obtained by a distribution matcher (DM) with finite block length. In the following, we briefly review the main metrics listed in the introduction and specify, for each one, its benefits and limitations.

Initial attempts of performing coarse sequence selection over PAS-symbol streams can be found in [21]–[23]. These methods primarily involve changing the set of sequences generated by the DM based on certain criterion. For instance, in [21], the kurtosis of symbol energies in the sequence is minimized, or the trellis tree of the 1D energies [22] or 4D energies [23] of enumerative-sphere-shaped (ESS) symbols are trimmed to generate sequences with smaller energy fluctuations. The main limitation of these schemes is the large rate loss arising from the coarse selection process. This rate loss exceeds the achieved nonlinear shaping gains obtained by NLI reduction yielding positive net gains limited to single-span transmissions.

A. Nonlinear interference (NLI) computation from SSFM

Moving towards more refined sequence selection strategies for transmission of PAS constellations, one thoroughly-investigated approach involves generating multiple candidate output sequences by varying the input source bit sequence of the DM, rather than altering the DM itself. This concept of sequence selection (SS) was first introduced in [13] and was further studied in [24]. This signaling approach provided valuable insights into the computation of a lower bound of the optical fiber channel capacity or equivalently an upper bound on the performance that current generation methods of sequences of PAS symbols can achieve. The metric used in [13], [14], [24] is the NLI variance computed for each candidate sequence through noiseless split-step Fourier method (SSFM) simulations. The candidate sequences are randomly chosen from a set of N-symbol long sequences where each symbol comes from an M-QAM PAS constellation. The total number of tested sequences N_S is usually much smaller than M^N when N is large due to the high complexity of an exhaustive search. Using NLI variance as a metric offers a reliable gauge of the potential improvements and inherent limitations of sequence selection. However, it requires performing several SSFM simulations in parallel to select a single sequence among the N_S candidates for the transmission of each N-symbol long sequence, which represents a hindrance to adopting it for implementation in real-time transmitters. As a matter of fact, any selection scheme based on trial-and-error, even if it adopts lower-complexity metrics, will add complexity (and latency) in the DSP chain at the transmitter side that might not be acceptable.

Furthermore, to compute the NLI variance in the case of polarization-multiplexed transmissions, the signs assigned to each amplitude over the four modulated dimensions of the optical field (I and Q components of two orthogonal polarization tributaries) need to be known, while previous coarse selection methods based on the symbol energies do not use the sign information. Hence, in [14], NLI variance was coined as ‘sign-dependent’ metric while the kurtosis of symbol energies and the energy dispersion index were categorized as ‘sign-independent’ metrics. In particular, the analysis in [14] clearly showed that there is considerable room for improvement of nonlinear shaping gains in long-haul optical fiber transmissions, by emphasizing the importance of performing sequence selection using sign-dependent metrics. The authors demonstrated that while sign-independent metrics do not yield significant rate improvements over long-haul transmissions, a sequence selection based on the computation



(a) L-CCDM in the PAS transmitter.

Fig. 2. List CCDM in the probabilistic amplitude shaping transmitter. Figure taken from [16].

of NLI variance achieved throughput enhancements even when an optimized carrier phase recovery (CPR) is applied at the receiver side. This last condition is crucial when assessing any nonlinear shaping gain from tailored modulation schemes as CPR already mitigates the part of NLI that manifests itself as a phase rotation of the constellation in the I-Q plane of each polarization tributary. A first experimental demonstration of this sequence selection strategy was reported in [25] showing some nonlinear shaping gain for long-haul transmissions in the presence of a CPR at the receiver side.

B. Sign-independent metrics

Among sign-independent metrics that perform the selection based on the symbol energies while discarding their signs in 1D (or equivalently their phases in the 2D I-Q plane), we mention the Energy Dispersion Index (EDI) used in the list-CCDM transmitter described in [16] – where CCDM stands for constant composition distribution matching –, and the Low-pass-Filtered Symbol-Amplitude Sequence (LSAS) metric for the LSAS-ESS transmitter presented in [18].

List-CCDM [15], [16] is an evolution of the conventional CCDM where v flip bits are allocated to change the DM output as shown in Fig. 2, followed by the selection of a subset of sequences based on the minimization of the energy dispersion index (EDI), which is, in simple terms, a moving-window statistic that measures the windowed energy fluctuation in a sequence of symbols $\mathbf{X}$. Based on this windowed-energy process, EDI is defined as the ratio of the variance of the windowed energy to its mean. The exact formula of EDI and the choice of the window size are defined in [16].

Another metric for the selection of shaped sequences that minimizes NLI is the LSAS method which stands for Low-pass-Filtered Symbol-Amplitude Sequence. The method assesses the nonlinear tolerance of a finite-length PAS sequence in a given system configuration (such as link length and baud rate) by modeling the nonlinear distortion term from signal-signal interactions based on the first-order perturbation analysis of a multi-span polarization-multiplexed WDM system and by retaining only the symbol-energy dependent terms as detailed in the appendix of [18]. The proposed metric, denoted as λ_{LSAS} and defined by equation (16) in [18], accounts for the filtering effects imposed by the fiber channel on the signal amplitudes in a sequence and provides a comprehensive measure of sequence quality with respect to nonlinear distortion. LSAS sequence selection operates similarly to List-CCDM, but it adopts ESS as its DM, resulting in superior performance compared to the EDI metric.

While both EDI and LSAS sequence selection methods have substantially improved the nonlinear tolerance of ESS or CCDM transmissions, significant gains were observed either over short-distance single-span links or in specific unconventional transmission scenarios such as single-polarization high-order QAM modulations. The benefits tend to disappear over long-haul transmissions, where the accumulation of dispersion and other impairments over extended distances limits the effectiveness of these sequence selection strategies. Finally, the nonlinear phase noise (NPN) metric was proposed in [26], which is based on the frequency-resolved logarithmic perturbation model. Compared with LSAS, it can further explain the interaction between CPR and finite-block-length DM.

C. Sign-dependent metrics

In [19], we introduced a sign-dependent metric named D-EDI and we showed through numerical simulations that it varies in opposite direction with respect to the effective SNR (measured from the constellations at the end of the DSP chain at the receiver side) for high-rate multi-span transmission scenarios while EDI failed to predict the effective SNR. To further capture the energy variations within a finite-length sequence propagating over a fiber link, D-EDI is a dispersion-aware EDI defined as the average of EDI values computed at multiple points along a linear dispersive and lossless fiber. This is shown in the lower part of Fig. 3 in which D-EDI is denoted as Ψ_D and the average is computed over EDI values of sequences that propagated over $i \times L_D$ kilometers of an ideal dispersive fiber where $i : 0 \to m_D$ and m_D is the maximum number of dispersion operations. Compared to the SSFM method [14], D-EDI is less complex. As a brief complexity discussion, we can compare the number of FFT (Fast Fourier Transform) operations used in each span. For our D-EDI, we only need to apply one FFT and one IFFT per span, whereas the SSFM needs to add dispersion and nonlinear effects progressively through several steps per span, requiring one FFT and one IFFT in each step.

D-EDI was then utilized as a metric for sequence selection of probabilistic amplitude shaped symbols with ESS serving as the DM. The scheme was denoted D-SS and the proposed transmitter is shown in the top half of Fig. 3. The fundamental concept behind D-SS draws inspiration from the list-CCDM [16]. Our approach extends the concept for the use of any DM and the concatenation of the outputs of several DMs to generate longer candidate sequences. Input bits are fed into 1D DMs, each DM having a block length l (i.e. generating l amplitudes). The input bit stream is composed of two parts: $k - \nu$ information bits and ν prefix flipping bits. Altering these ν prefix flipping bits can cause pronounced changes

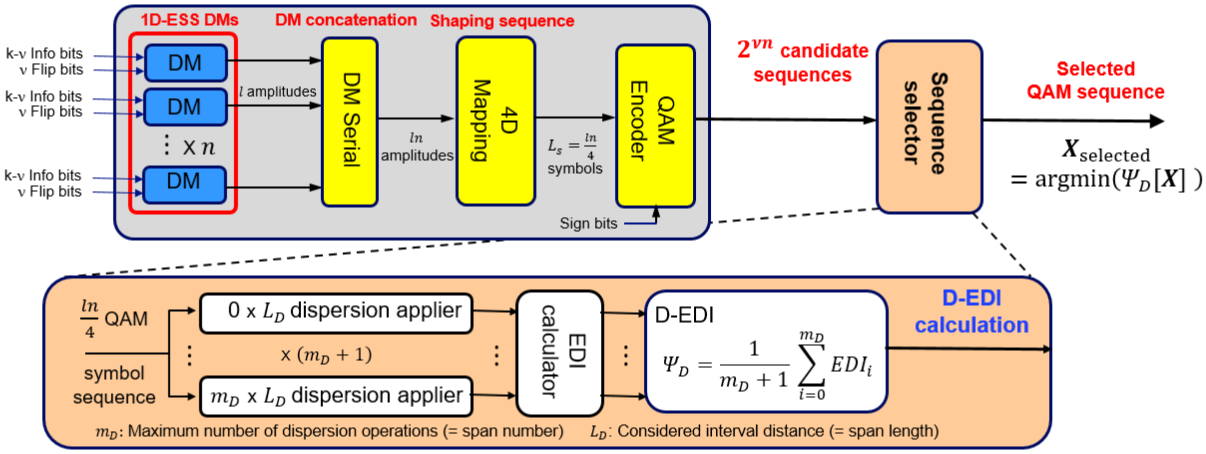


Fig. 3. Block diagram of FEC independent D-SS in the PCS transmitter: EDI of dispersed sequences (D-EDI) is determined by averaging the calculated EDI at multiple locations along an ideal dispersive fiber. The dispersion is applied at 1 sample per symbol. EDI is a special case of D-EDI where $m_D = 0$. The sign bits are fixed through a multi-block FEC-independent sequence selection process as shown in [14].

in the indices of the sequences during the ESS encoding process. Such changes are reflected as considerable alterations in the amplitudes across the block. The outputs from n 1D-DMs are then sequentially concatenated, from head-to-tail, to create extended sequences, which can then undergo 4D mapping, QAM encoding with assigned sign bits, and finally a selection process. For n cascaded DMs, where each DM includes ν flipping bits, we can generate $2^{\nu n}$ different candidate sequences. The sign bits can be fixed through a multi-block FEC-independent sequence selection process as proposed in [14]. We thoroughly investigated and optimized the performance of the D-SS scheme in [19]. The D-SS scheme demonstrated superior performance compared to ESS without sequence selection across various block lengths and complexity levels, in both single-channel and WDM transmission scenarios with optimized CPR. Remarkably, it performed on par with the ideal SSFM-based sequence selection [14] and it consistently delivered throughput enhancements across various block lengths and various selected sequence lengths.

Recently, in [20], the same authors of [18] introduced a new sign-dependent sequence-selection metric for PAS schemes that consists in a perturbation-based alternative to SSFM. The new metric is an evolution of the LSAS and consists in the l^2-norm of the difference between the transmitted symbol sequence and the dispersion-compensated received sequence impacted by NLI through symbol-energy dependent perturbation terms (as for LSAS) and additionally symbol-sign dependent perturbation terms. To predict the performance of their scheme, the authors used the perturbative model in [27] to estimate the achieved SNR gains. Table I summarizes the main features of the presented sequence-selection metrics. Lastly, in [28], the authors propose a trellis-shaping technique to implement sequence selection for NLI mitigation in inter-datacenter single-span links. They use the generalized mutual information of a sequence (GMI, i.e. the achievable information rate) as the selection metric after propagating the sequence in a fiber model (SSFM) or a perturbation-based model.

III. Discussion and Challenges

After presenting the main sequence-selection methods for NLI minimization, we address the major challenges that they should overcome to get sequence selection closer towards a practical implementation. In particular, we discuss the choice of the selection metric and the generation of the sequences.

The first challenge to be addressed is the choice of the selection metric. Some key findings presented in [14] already brought interesting answers. To gauge the potential benefits of a given nonlinear shaping, the metrics discussed in the previous sections can be evaluated over a stream of selected sequences transmitted over the studied link as detailed in [13]. Crucially, this estimation process involves averaging the impact of adjacent sequences, thereby accounting for the inter-block NLI. Through this process, one can assess not only the self-NLI impacting a given sequence but also how the sequence might influence or be influenced by the neighboring sequences. Applying this averaging process on the assessment of the NLI variance, the authors in [13] compared three shaping strategies: first, both amplitude and sign are shaped according to the average NLI metric; second, only the amplitude is shaped and the chosen metric is a sign-independent one, such as EDI, LSAS, NPN or kurtosis; third, an intermediate case where the signs of the symbols are known but fixed and the average NLI metric is used to shape the amplitudes. This third approach recognizes sign information but does not allow its optimization as part of sequence selection. By comparing the three strategies, they made several interesting conclusions:

- First, an effective sequence-selection strategy necessitates the use of sign-dependent metrics.
- Selection with unshaped but known signs nearly matches the performance of sequence selection with shaped signs.

TABLE I

MAIN FEATURES OF THE SEQUENCE-SELECTION METRICS FOR PAS TRANSMISSIONS AND TRANSMITTERS UTILIZING THESE METRICS

Metric	Sign dependence	Channel awareness	Generation of candidate sequences
NLI variance from SSFM [14]	Yes	CD[a] map & Kerr effect, single-channel SSFM[b]	Bit scrambling or symbol interleaving [14]; Ideal M-B[c] , CCDM or ESS transmitter
EDI [15]	No	Channel memory	Multiple DMs[d] with flipping bits [16]; CCDM transmitter
LSAS [18]	No	Partial perturbation analysis	Multiple DMs[d] with flipping bits [18]; CCDM or ESS transmitter
D-EDI [19]	Yes	CD map	Multiple DMs[d] with flipping bits [19]; ESS transmitter
Perturbation-based [20]	Yes	Improved perturbation analysis	Symbol interleaving [20]; Ideal M-B[c] transmitter

[a]CD: Chromatic dispersion, [b] SSFM: Split-step Fourier method,[c] M-B: Maxwell-Boltzmann, [d] DM: Distribution matcher.

- For any given sign sequence, there are ‘good' and ’bad' amplitude sequences that can be identified or discarded respectively to improve performance.

These findings show the complexity of simultaneously selecting signs and amplitudes. However, they also tell us that it is enough to maintain an i.i.d. sign distribution, while selectively choosing the amplitudes using a sign-dependent metric, to achieve performance gains at reduced complexity. Using D-EDI as a selection metric in [19] instead of computing NLI variance from SSFM-based simulations, we came to the same conclusions. Moreover, we have shown that selection using a moderate-complexity version of D-EDI (by applyig a lower number of digital dispersion operations to compute the metric) retained a robust performance and outperformed conventional PAS schemes across different block lengths.

The second challenge deals with finding a low-complexity sequence generation method including a bit-to-symbol mapping rule. All sequence-selection methods presented above relied on the generation of a selection pool to find sequences with minimized nonlinear distortions followed by a trial-and-error scheme in which the sequences are compared using a given a performance metric. This process is not convenient for real-time implementation in optical transmitters due to complexity and latency issues. To allow for a wider range of performance fluctuations between the sequences within the selection pool, and in consequence a higher probability of reaching the upper limit of sequence-selection performance, several generation methods were explored. In [16], [19], prefix bits in each input bit-block to a DM were allocated to create variations in the output amplitude sequence. In [14], bit-scrambling at the input of the DM was proposed through XOR operations as well as symbol-interleaving through the insertion of pilot symbols. From [14], [19], [20], [28], we saw that the major part of the nonlinear gain is achieved with only 8 to 32 candidate sequences. Moreover, in [19], we found that a selection based on single-channel D-EDI achieves the same gain as a selection based on single-channel SSFM simulation when sequences are generated with the suggested D-SS transmitter shown in Fig. 3. This implies that we have reached the single-channel performance limit of sequence selection by the proposed transmitter (flipping bits to generate sequences). To achieve further gains, we need to find better methods for the generation of sequences that exhibit possibly larger performance variations, thus improving the upper limit of sequence-selection performance.

Finally, as PAS QAM modulation became an industry-wide accepted format [11] thanks to its rate maximization and finely-tuneable information rate despite its higher implementation complexity compared to uniform QAM, the generation of low-NLI-producing information sequences should cope with the implemented PAS transmitters including the interactions with the FEC encoder [14] to minimize the overall complexity and to reduce latency when matching bits to FEC codewords.

This summary of results and challenges paves the way for further optimizations of shaping schemes for long-distance transmissions and confirm the necessity of using sign-dependent and channel-aware metrics for effectively evaluating nonlinear fiber distortions for any transmitted sequence. Among other works, in [10], [12], [29], performance comparisons of several multidimensional-coded, PAS-QAM or uniform-QAM schemes were made in diverse transmission scenarios: various distances, different number of subcarriers, etc. The measured results emphasized on the facts that no constellation is universal and that the balance between the linear shaping gain (over an ideal AWGN channel) and the nonlinear shaping gain (over a fiber channel) should be found for each specific transmission scenario including the configuration of the transmission link, the transmission format (modulation scheme, number of subcarriers, etc.) and the implemented nonlinearity-compensation techniques. Nevertheless, all the findings in the published literature tell us that there is still room for improvement. We conclude this keynote by suggesting some directions for future research:

- Choosing sequences by taking into account the dispersion map, and in particular the shape of the waveform at the locations in the link where the optical power of the propagated signal is high, always gives the best performance results. As the transmission rates continue to increase, the channel memory can spread over hundreds or thousands of symbols for both multi-carrier and single-carrier transmissions. Hence, modifications and optimizations of sequence generation for high-rate long-distance transmissions while maintaining a reduced complexity is still an open question.

- Another improvement of the sequence generation scheme can address the inter-channel nonlinear effects and the signal-noise nonlinear interactions. For instance, in a digital multi-carrier transmission, the sequence selection may be jointly made across the different subcarriers.
- Ultimately, one may ask if sequence selection will ever be implemented or if we should rather use it as an inspiration to build new resilient schemes coded over time, frequency and polarization. [14] advocates for the latter. Conducting an analysis of selected sequences from SSFM simulations or through lower-complexity channel models based on perturbation analysis [27], [30] and improved performance-prediction models for ultra-wide band systems [31], [32], could help in understanding their properties and in designing simpler sequence generation solutions.

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