OFDM Systems

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  - Synchronization algorithms

Recall on multipath mobile channels (1)

- <u>Coherence bandwidth : $(\Delta f)_c$</u>
  - –Two carriers separated by $(\Delta f)_c$ are affected by « more or less » the same attenuation.

$$T_m = \frac{1}{(\Delta f)_c}$$

W : occupied bandwidth

W<< $(\Delta f)_c$ => non frequency selective channels

W>> $(\Delta f)_c$ => frequency selective channels

<u>Nota :</u> $(\Delta f)_c$ is not related to the relative mobility emitter/receiver (ex: cables)

Recall on multipath mobile channels (2)

- <u>Coherence time $(\Delta t)_c$</u>

Two signal samples separated by less than $(\Delta t)_c$ are affected by « more or less « the same attenuation.

$$B_d = \frac{1}{(\Delta t)_c}$$

B_d: doppler bandwidth

Principles of OFDM systems (1)

- <u>Frequency selective channels</u>

⇒ Use of multiple carriers

The « elementary channel » (one carrier) is now non frequency selective.

- <u>Spectral efficiency</u>

⇒ Use of overlapping orthogonal carriers

- <u>Diversity</u>

⇒ Use of ECC

COFDM

Principles of OFDM systems (2)

- <u>Expression of OFDM signal (complex envelop)</u>

Carrier #i :

$$x_i(t) = \sum_k d_{ik} h(t - kT) \exp\left(2j\pi f_i t\right)$$

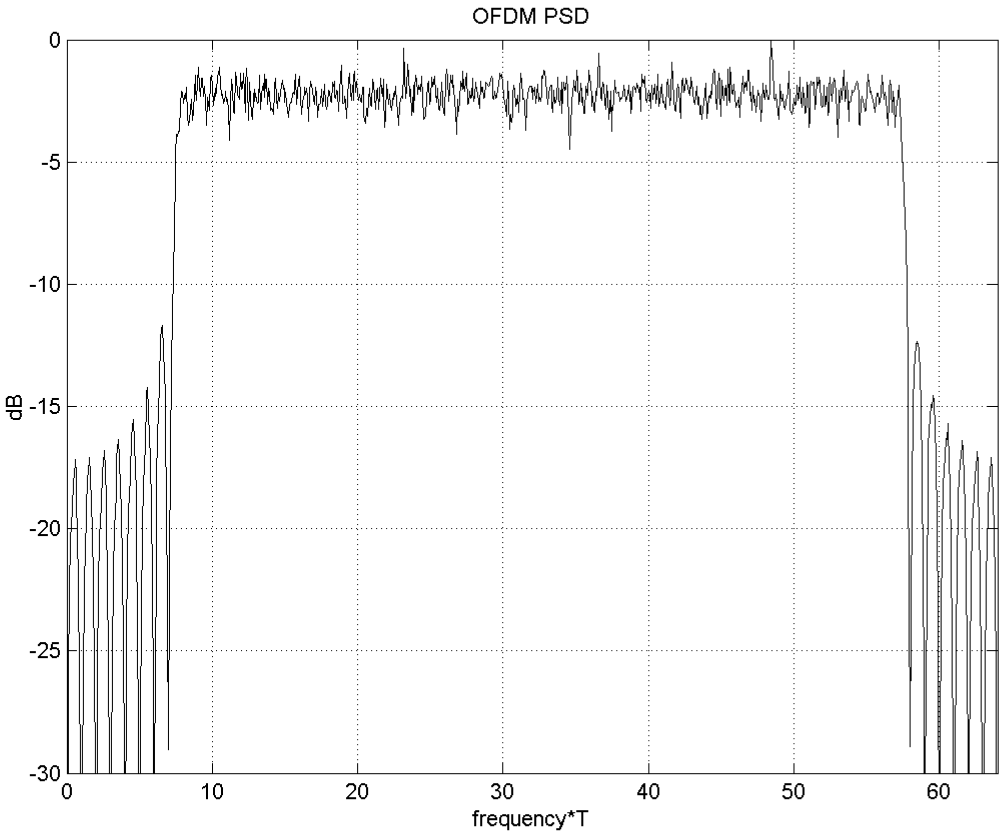
h(t): rectangle of width T (NRZ)

$f_i = i/T$

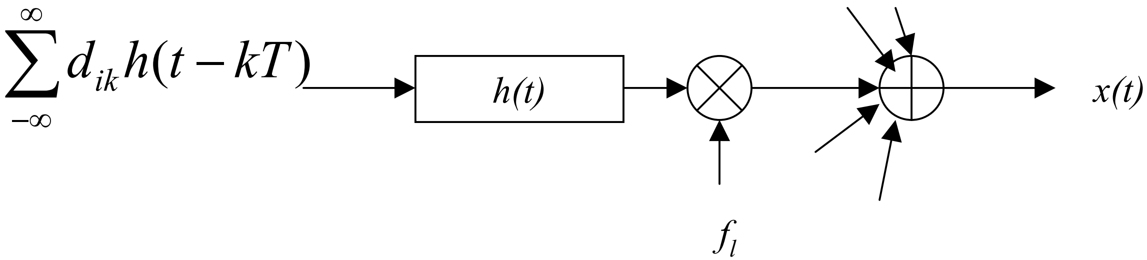
Frequency multiplex

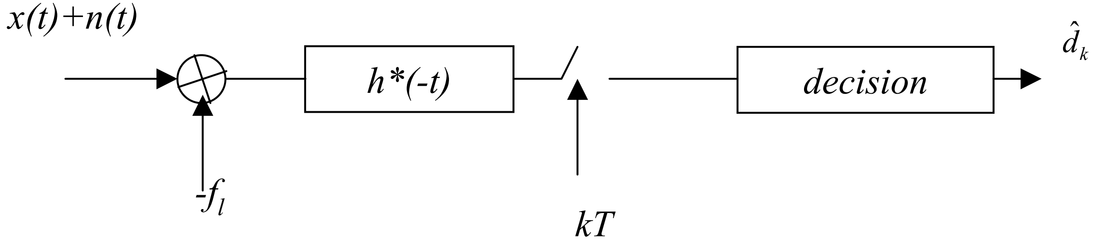
$$x(t) = \sum_{i=0}^{N-1} \sum_k d_{ik} h(t - kT) \exp\left(2j\pi f_i t\right)$$

Principles of OFDM systems (3)



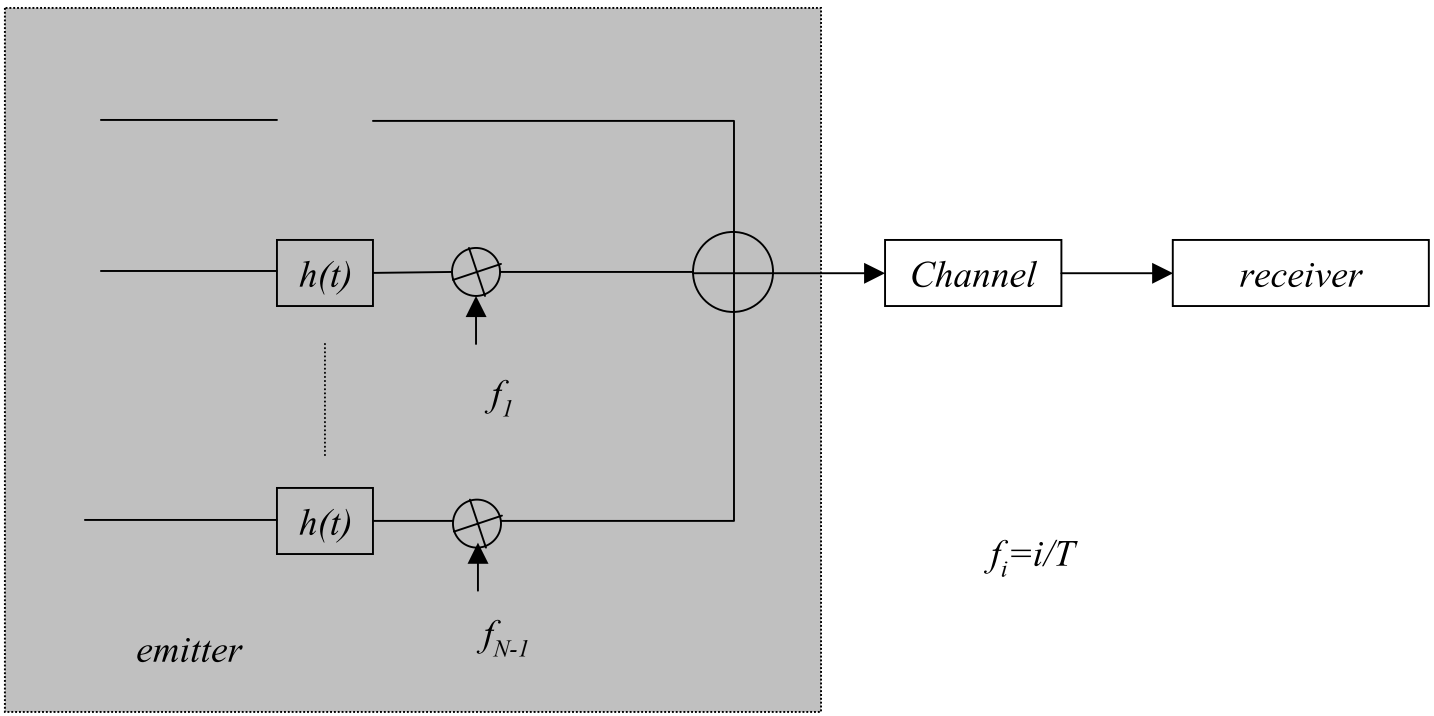
➢ Modulator / demodulator for carrier # l (ideal case)





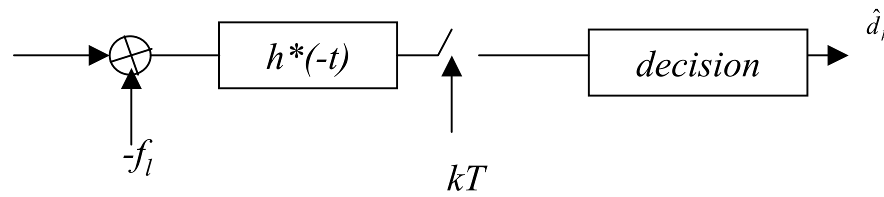
OFDM systems and filter banks (1)

- OFDM modulator/demodulator can be seen as a synthesis/analysis filter bank (no guard time, no coding)

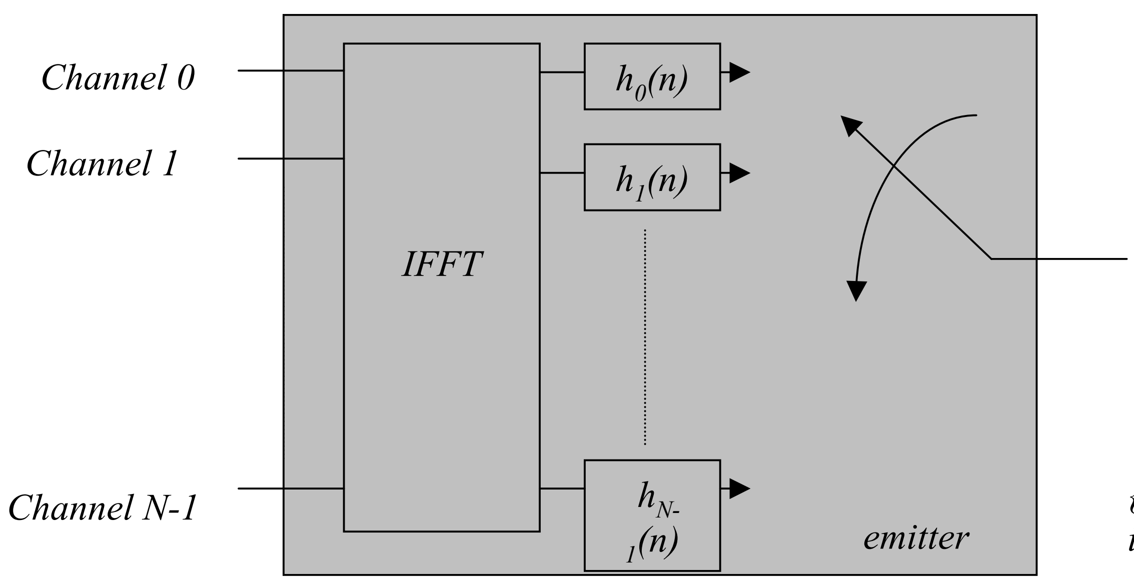


OFDM systems and filter banks (2)

- Receiver for carrier n°l



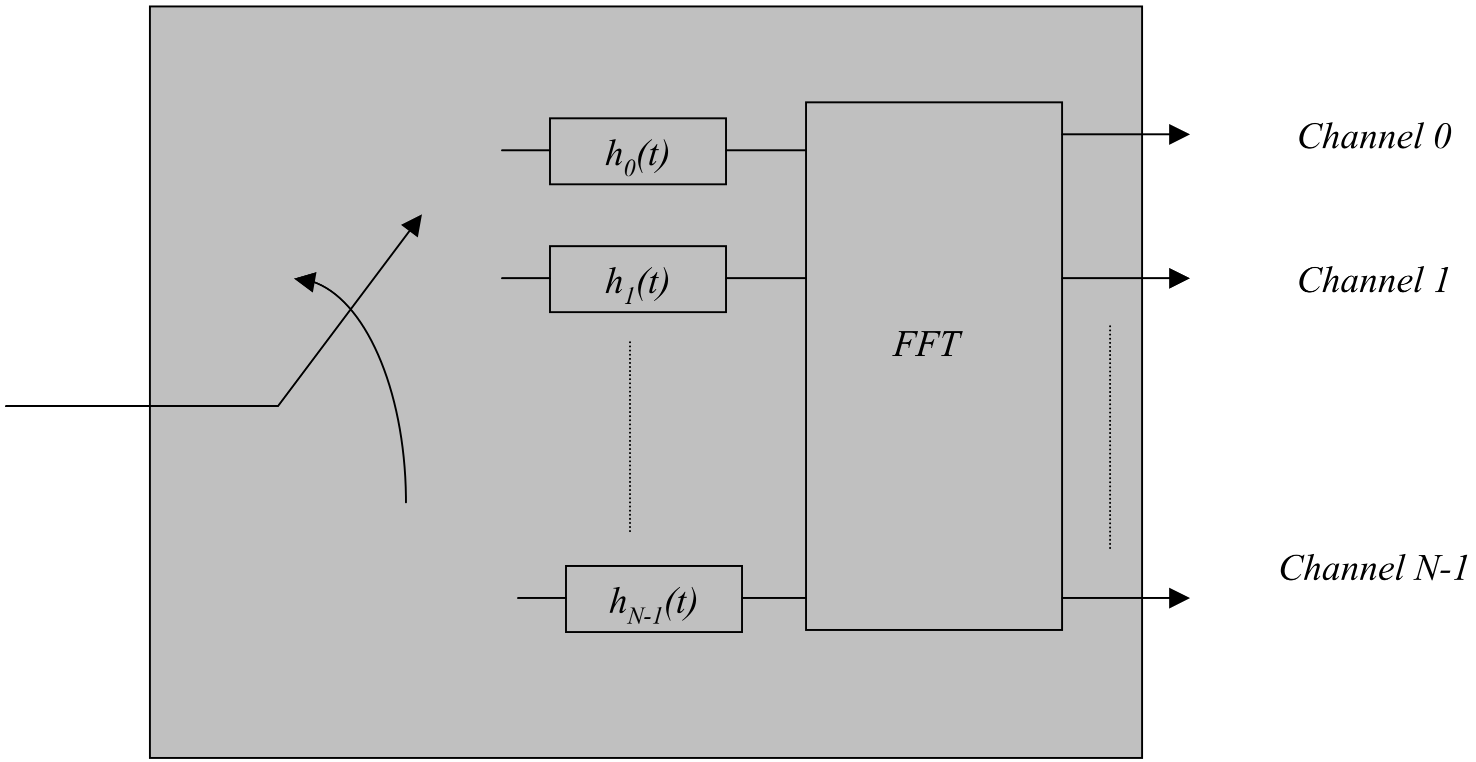
- Efficiently implemented via FFT^{-1} (emitter) and FFT (receiver)



$\{h_i(n)\}$: polyphase implementation of h(n

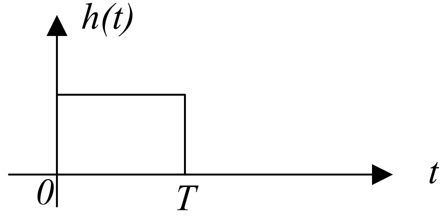
OFDM systems and filter banks (3)

- OFDM receiver



OFDM systems and filter banks (4)

- Application : classical OFDM



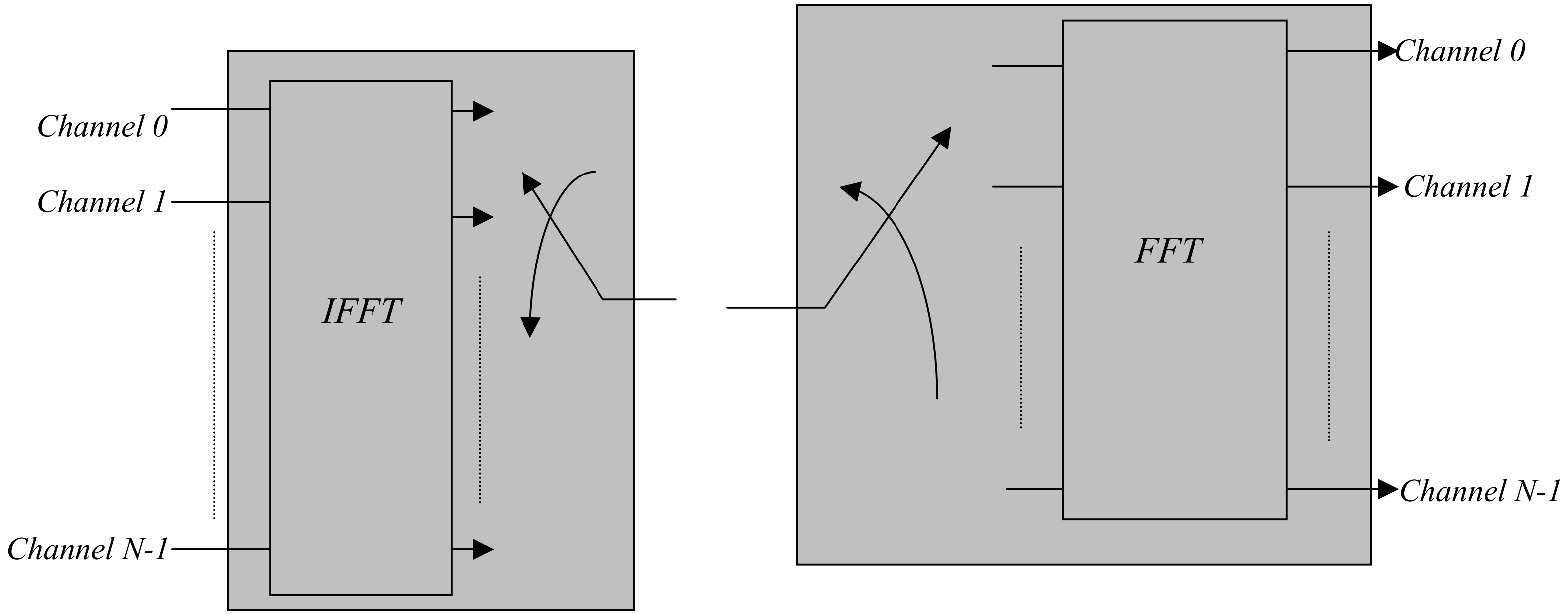
$F_e = N/T$

$h(n) = 1$ for $n = 0, \ldots, N-1$

$h_i(n) = 1$ for $n = 0$

$h_i(n) = 0$ elsewhere

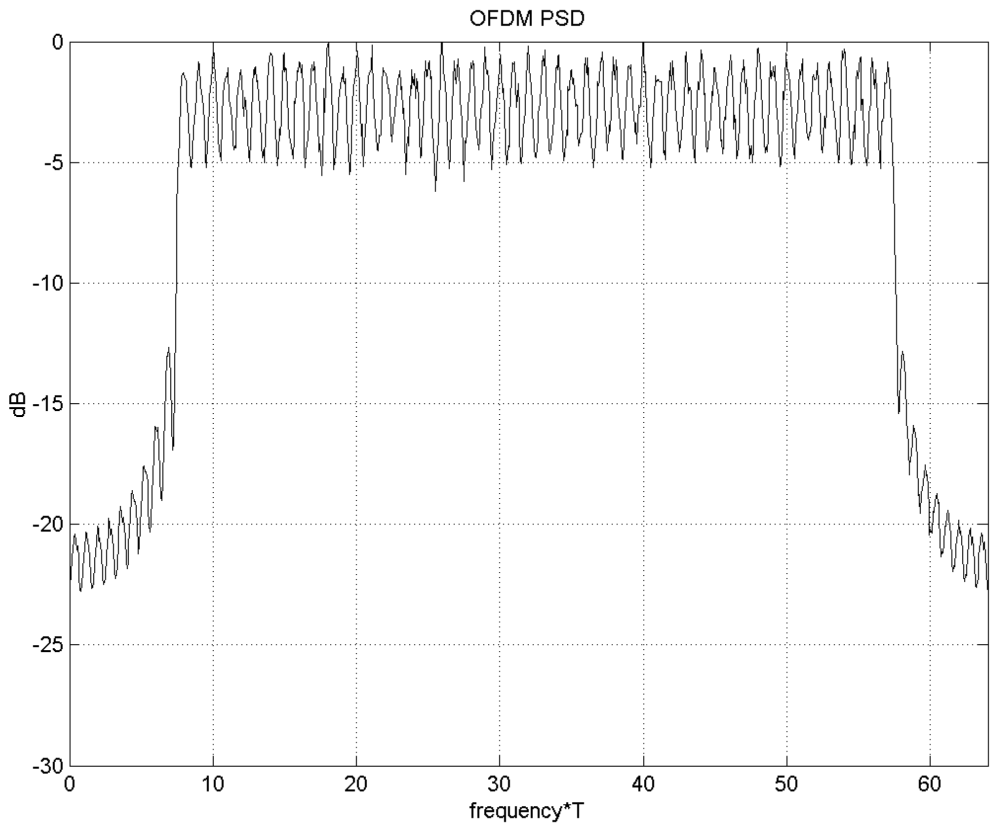
- Implementation with polyphase+FFT filter banks



OFDM system with guard interval (1)

- Guard interval is used to removed residual intersymbol interference (ISI)
- Guard interval is inserted by copying the [kT, kT+ΔT[part of original OFDM symbol => no discontinuity in the signal!
- Resulting OFDM symbol period is T+ΔT (ΔT : guard interval)

OFDM system with guard interval (2)



OFDM system with guard interval (3)

- The FFT output is (symbol # i, carrier #j):

$X_{i,j}=H_j s_{i,j}$ (without noise)

=> flat fading channel at sub-carrier level

- Cyclic prefix is used in order to:
  – Avoid equalization
  – Increase robustness against sampling time error

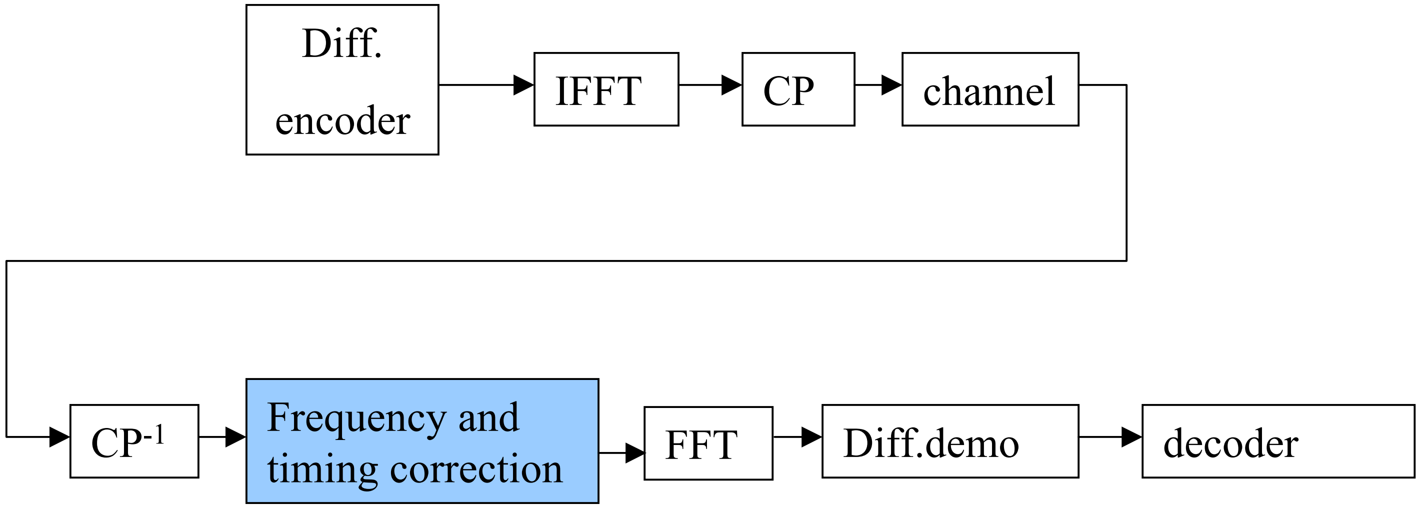
Advantages/drawbacks of OFDM systems

- Advantages:
  - Emitter and receiver are efficiently implemented with FFT/IFFT
  - No equalization is required
  - Spectral efficiency
  - Diversity

- Drawbacks
  - Sensitivity to synchronization errors
  - Sensitivity to non linearities (Amplifiers)
  - Mainly used in broadcasting applications

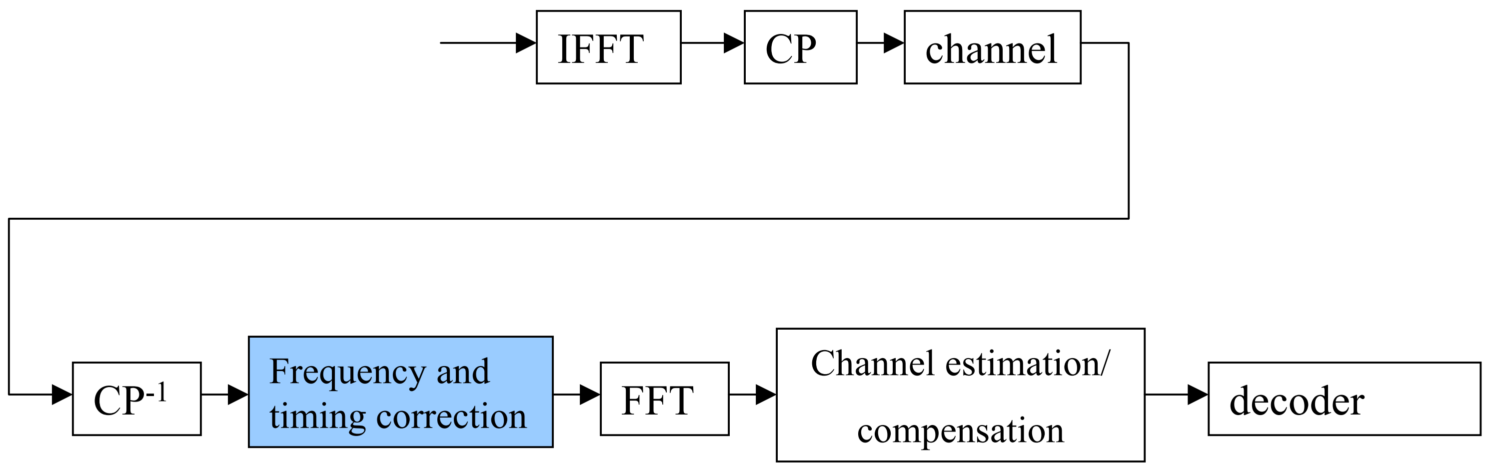
Receiver Architecture (1)

➢ Differential demodulation (ex: DAB)



In non-coherent communication, differential encoding/decoding avoids the use of channel estimation.

➢ Coherent demodulation (ex: DVB-T)



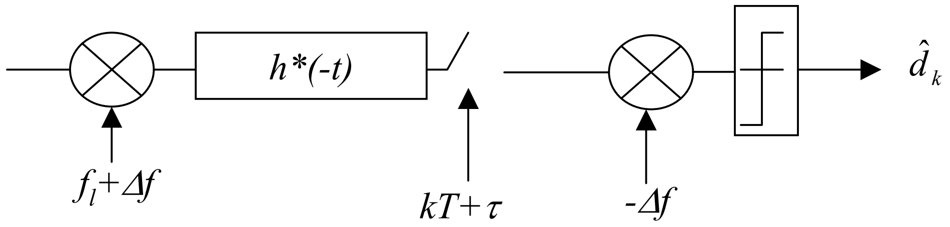
➢Specificity of OFDM system w.r.t synchronization issue

•OFDM systems are much more sensitive to synchronization errors than single carrier systems.

•Synchronization algorithms suited to single carrier systems are inefficient for OFDM.

Impact of a synchronization error (1)

- <u>System model (Gaussian channel)</u>
  - –Carrier : n° l
  - –Frequency offset : Δf
  - –Timing error : τ



Impact of a synchronization error (2)

➢ Timing error τ

– $\tau < \Delta - L$: phase rotation (compensated by channel estimation/correction=

– $\tau > \Delta - L$: n^{th} symbol, carrier n° i

$$Y_{i,n} = e^{2j\pi(n/N)\tau} \frac{N-\tau}{N} X_{i,n} H_{i,n} + n_{i,n} + n_{\tau}(i,n)$$

➡ SNR loss

➡ ICI/ISI

Impact of a synchronization error (3)

➢ Frequency error : Δf

$Y_{m,l}$=p(Δf)exp[2jπ(m+1/2)ΔfT]d_{ml}+ICI

with

$$ICI = \sum_{n\neq l} \exp\left(2j\pi(k-l)(m+1/2)\right)\sin_c\left(\pi\left(n-l+\Delta f T\right)\right),\;\; p(\Delta f) = \sin_c\left(\pi\Delta f T\right)$$

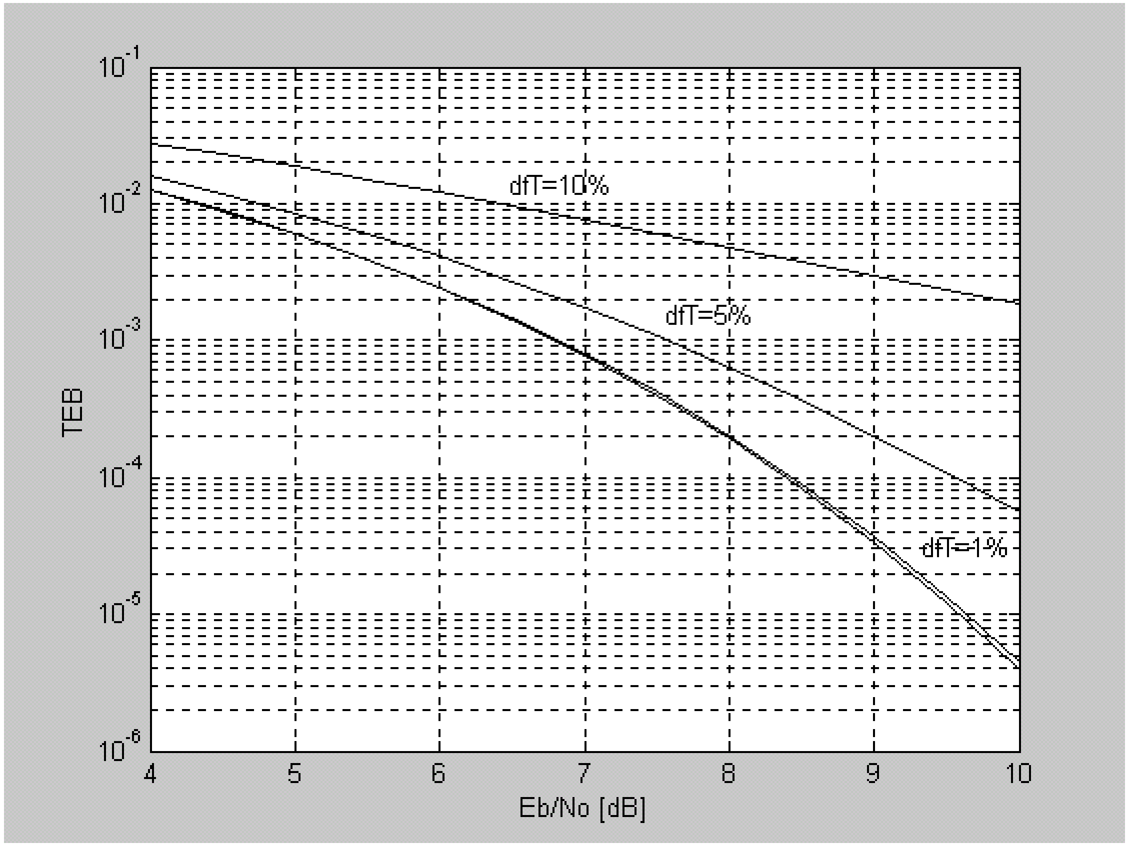
For $|\tau| < G$ (G: guard time)

$$\left|I_{n,i,k}\right| = \left|\frac{\sin\left[\pi\left\{\left(n-l\right)+\Delta f \times T\right\}\right]}{\pi\left[\left(n-l\right)+\Delta f \times T\right]}\right|$$

$$TEB = \frac{1}{4} erfc\left(\sqrt{\frac{E_b}{N_0}}\left|I_{n,n,k}\right| \times \left(1+2\frac{E_b}{N_0}\sum_{i\neq n}\left|I_{n,i,k}\right|^2\right)^{-\frac{1}{2}}\right)$$

Impact of a synchronization error (4)

BER degradation due to a frequency error (gaussian channel)



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Impact of a synchronization error (5)

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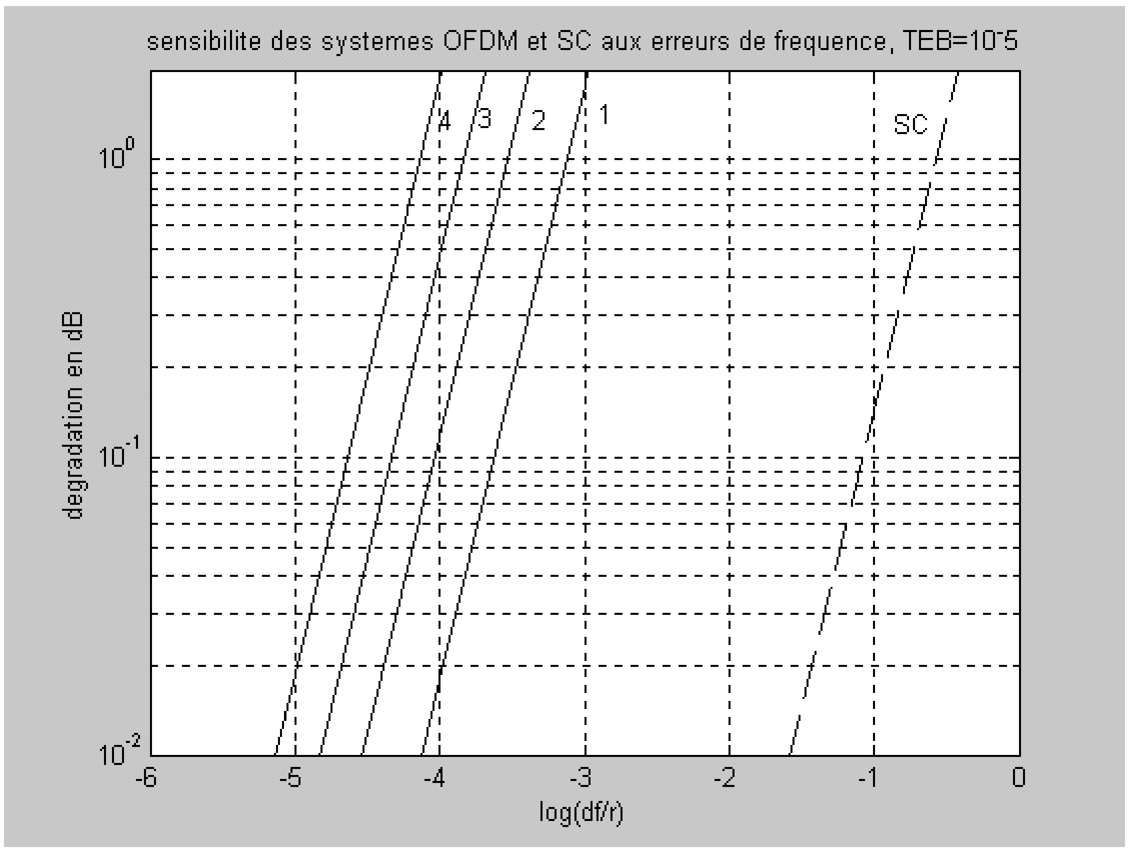
BER degradation due to a frequency error (gaussian channel) : single and MC case



1: single carrier

2: OFDM, N=100

3: OFDM, N=256

3: OFDM, N=512

4: OFDM, N=1024

Impact of phase noise

$$D \approx \begin{cases} \dfrac{10}{\ln 10}\dfrac{11}{60}\left(4\pi N\dfrac{\beta}{R}\right)\dfrac{E_s}{N_O} & \text{(OFDM)} \\ \dfrac{10}{\ln 10}\dfrac{11}{60}\left(4\pi\dfrac{\beta}{R}\right)\dfrac{E_s}{N_O} & \text{(SC)} \end{cases}$$

β : 3 dB BW (SSB) in Wiener model

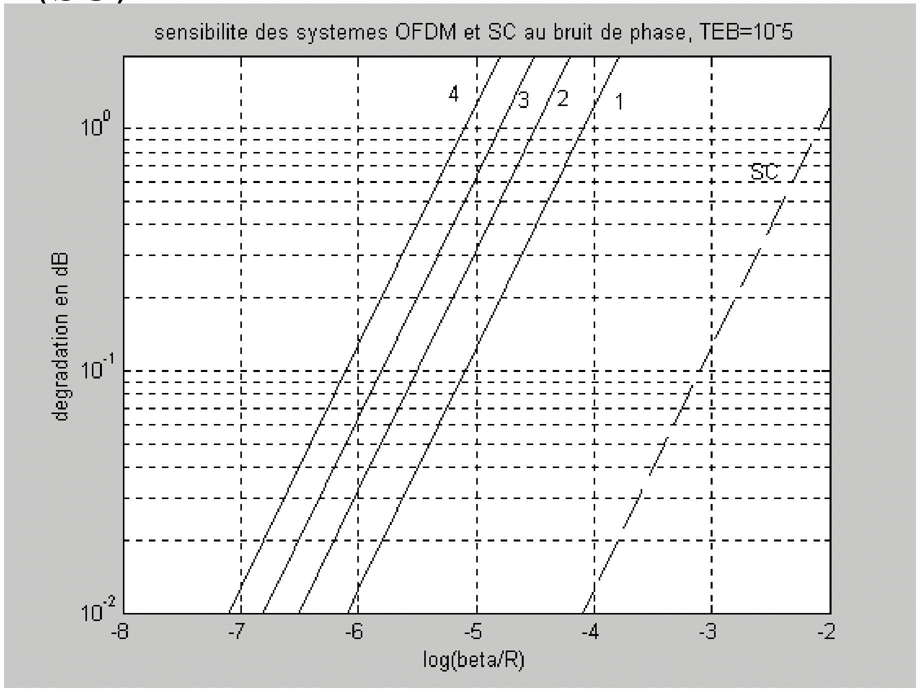
1: single carrier

2: OFDM, N=100

3: OFDM, N=256

3: OFDM, N=512

4: OFDM, N=1024



Timing/frequency estimators (1)

- Estimators using pilot symbols
  - Moose
  - Schmidl et Cox

- Estimators not using pilot symbols
  - Van de Beek

- These estimators are suited to frequency selective channels
  - Guard time is necessary for other reason
  - Each elementary channel (FFT output) is modelled by a different complex multiplicative coefficient.

Moose estimator (1)

- <u>Principle :</u> Emission of 2 identical OFDM symbols
- Timing has to be corrected first
- Hypothesis : the channel impulse response is constant over some OFDM symbols

Moose estimator (2)

First OFDM received symbol : $[r_0\ r_1 \ldots r_{N-1}]$

Second OFDM received symbol : $[r_N\ r_{N+1} \ldots r_{2N-1}]$

CIR constant over 1 OFDM symbol => $r_{n+N} = r_n \exp(2j\pi\Delta f N T_e) = r_n \exp(2j\pi\varepsilon)$

with ε=1/T (inter carrier spacing)

FFT output (first symbol) : $y(k) = \sum_{n=0}^{N-1} r_n \exp\left(2j\pi \frac{nk}{N}\right)$

FFT output (second symbol): $y(k+N) = \sum_{n=0}^{N-1} r_{n+N} \exp\left(2j\pi \frac{nk}{N}\right)$

$y(k+N) = y(k)\exp(2j\pi\varepsilon)$ $k \in \{0,1,\ldots,N-1\}$ =>The signal and ICI are affected exactly in the same way by the frequency offset.

Moose estimator (3)

MLE estimator:

$$\hat{\varepsilon} = \frac{1}{2\pi} \text{Arg}\left\{ \sum_{\text{k=0}}^{\text{N-1}} \text{y(k+N)y}^*(k) \right\}$$

$$|\varepsilon| < 1 \;\Rightarrow\; |\Delta f| < \frac{1}{T} \;\Rightarrow -\frac{1}{2T} < \Delta f < \frac{1}{2T}$$

Frequency unbiguity has to be removed.

Schmidl et Cox estimator (1)

- ❑ Estimation of both timing and frequency errors
- ❑ Principle:

➢ 2 dedicated pilot symbols

- First symbol : null odd carriers
- Second symbol : 2 interleaved PN sequences (odd/even carriers)

➢ Estimation

- First symbol is used for timing and frequency estimation (2/T ambiguity)
- Second symbol is used to remove ambiguity on frequency estimation

Schmidl et Cox estimator (2)

First symbol : null odd carriers

$$y_n = \sum_{k=0}^{N-1} x_k \exp\left\{2j\pi \frac{nk}{N}\right\}$$
$$= \sum_{k=0}^{N/2-1} x_{2k} \exp\left\{2j\pi \frac{nk}{N/2}\right\}$$

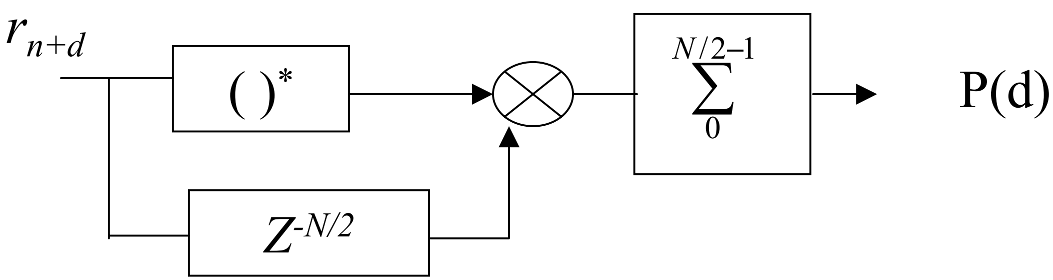
$y_{n+N/2}=y_n$ => OFDM symbols with 2 identical halves

Schmidl et Cox estimator (3)

Received OFDM symbol: $r_n, 0 \le n \le N-1$

Timing metric:

$$M(d) = \frac{|P(d)|^2}{(R(d))^2} \qquad R(d) = \sum_{m=0}^{N/2-1} |r_{d+m+N/2}|^2$$

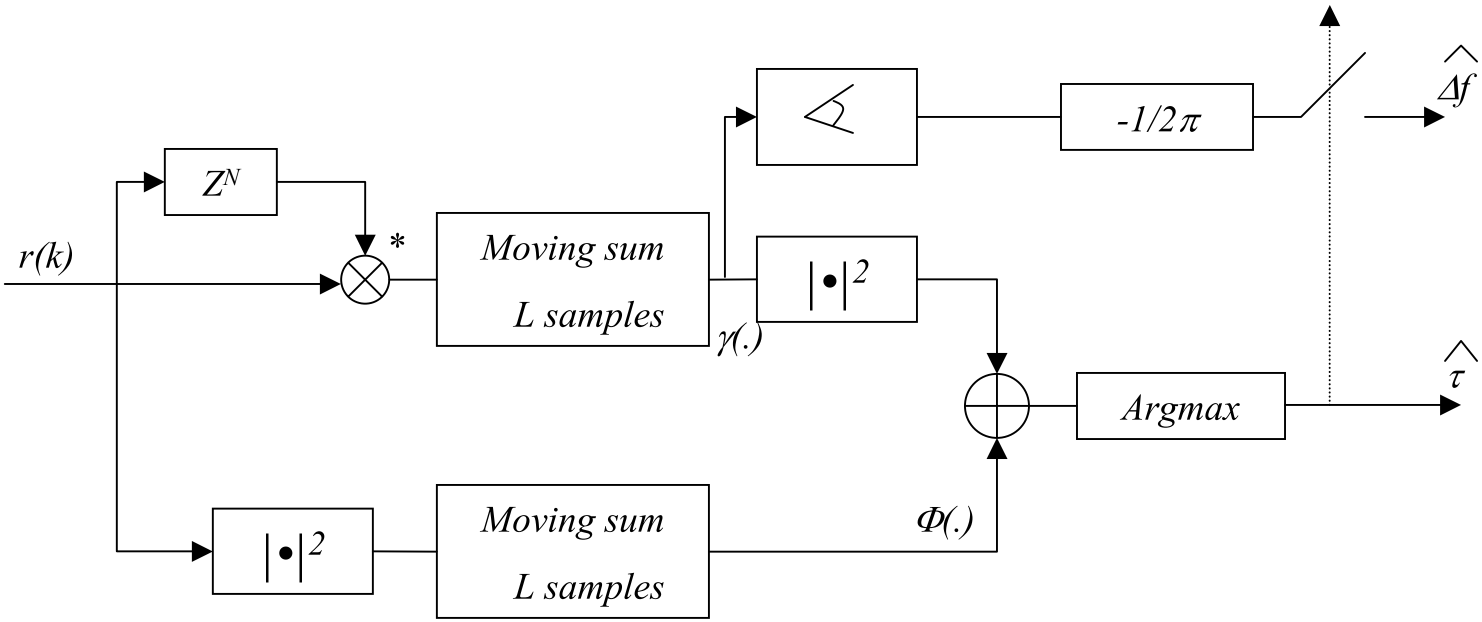


Timing estimate: $\hat{d} = \arg_d \{\max(M(d))\}$

Frequency estimate: $\hat{\varepsilon} = \frac{1}{\pi} angle\{P(\hat{d})\}$

$$|\varepsilon/2| < 1 \Rightarrow |\Delta f| < \frac{2}{T} \Rightarrow -\frac{1}{T} < \Delta f < \frac{1}{T}$$

Van de Beek estimator

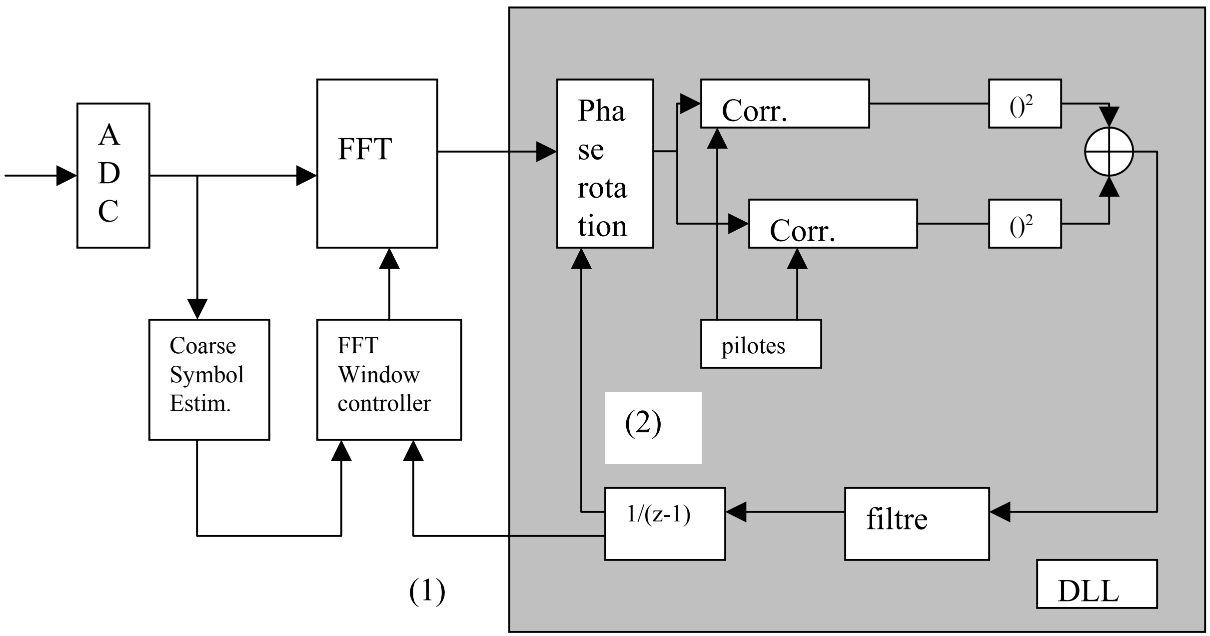


$$\hat{\tau}_{ML} = \arg\max\left\{\left|\gamma(\theta)\right| - \Phi(\theta)\right\}$$

$$\Delta\hat{f}_{ML} = -\frac{1}{2\pi} \angle \gamma\left(\hat{\theta}_{ML}\right)$$

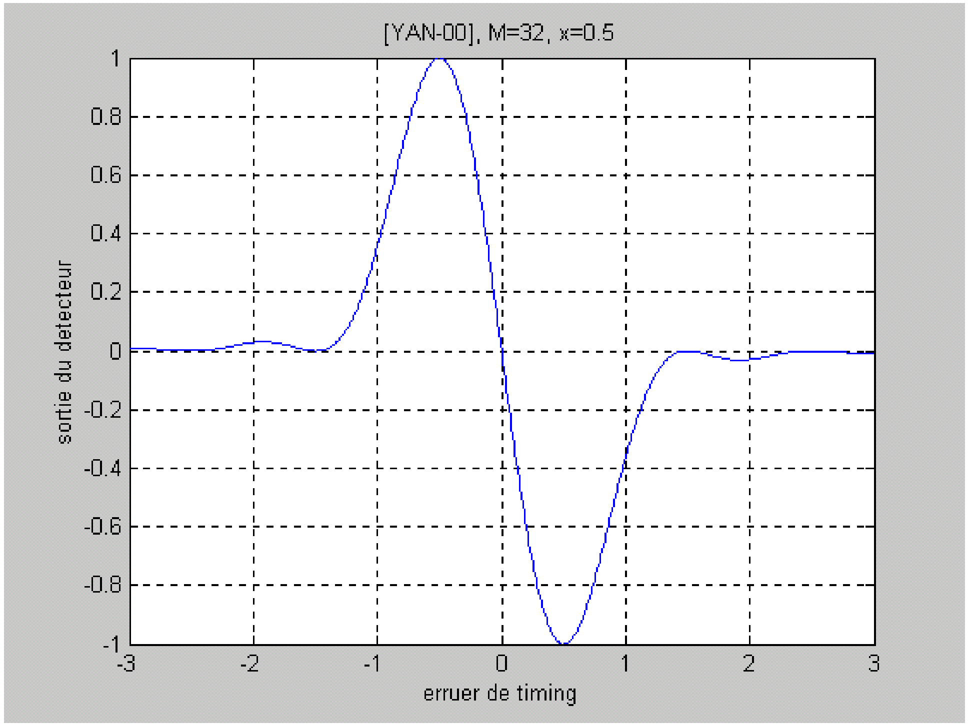
Yang estimator (timing) (1)

Idea : exploit the fact that a timing error introduces a phase error at the FFT output which depends on the carrier number.



Yang estimator (timing) (2)

$$S(\varepsilon,\xi)=\left(\frac{\sin(\pi(\varepsilon+\xi))}{M\sin[\pi(\varepsilon+\xi)/M]}\right)^2-\left(\frac{\sin(\pi(\varepsilon-\xi))}{M\sin[\pi(\varepsilon-\xi)/M]}\right)^2$$



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