Synchronization and Digital Receivers

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Synchronization algorithms
(Single carrier systems, Gaussian channels)

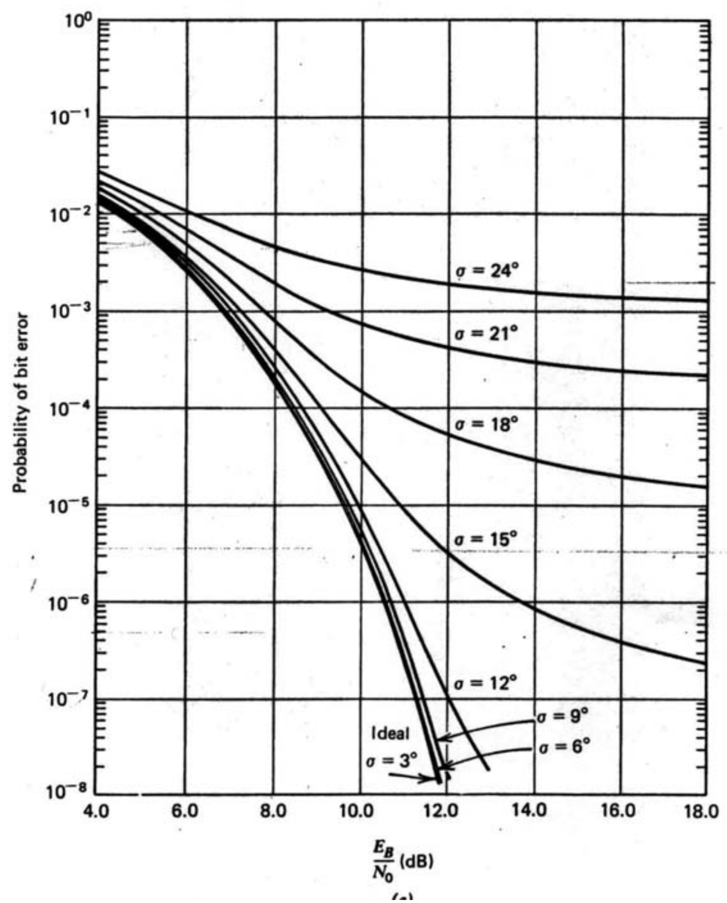
Contents

Impact of synchronization errors (1)

➢ Carrier phase error:

BPSK, « NRZ »filter

➢ Maximum phase jitter is determined by the implementation loss in the link budget.

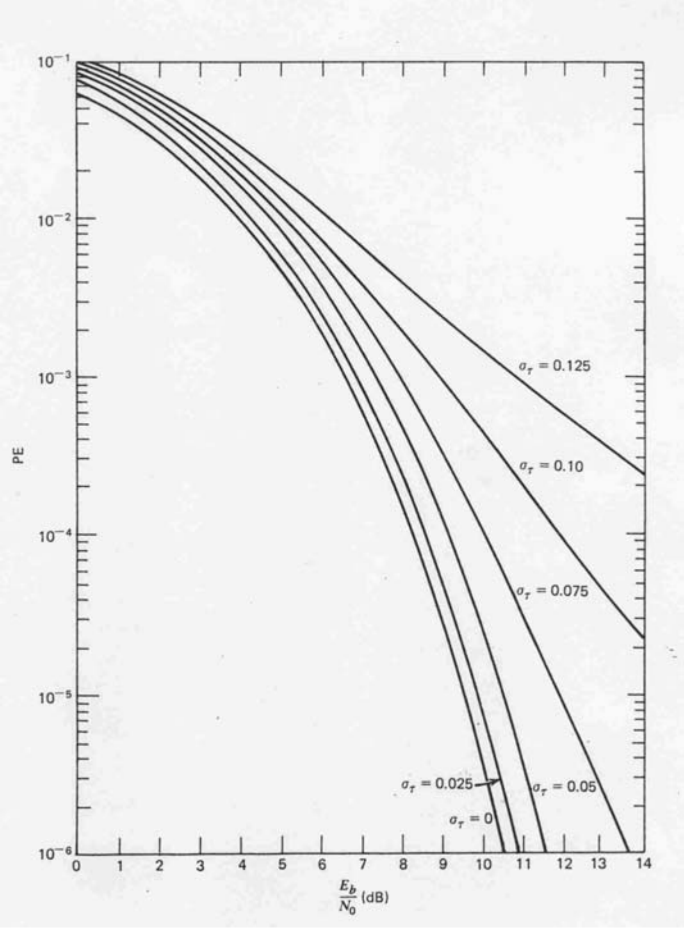


Impact of synchronization errors (2)

➢ Timing error

BPSK, « NRZ »filter

➢ Maximum timing jitter is determined by the implementation loss in the link budget.



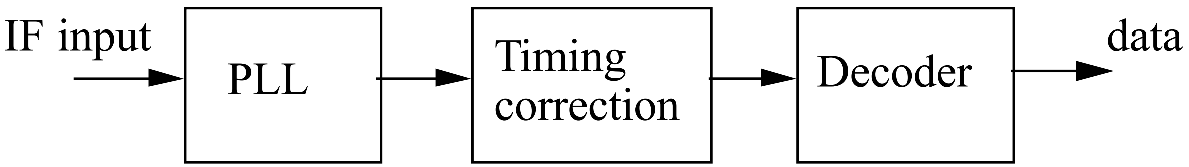
Demodulation

- Functions to be implemented
  - Baseband conversion
  - I,Q generation
  - Carrier recovery
  - Timing recovery
  - Matched filtering
  - Demodulation/decoding

Analog demodulators

• Typical analog demodulator architecture

PLL : baseband conversion + carrier frequency/phase correction



PLL : baseband conversion + carrier frequency/phase correction

Timing correction : FF/FB structure **AFTER PLL**

Digital demodulators

A digital demodulator is **NOT** the sampled version of the equivalent analog demodulator.

⇒ Specific algorithms suited to digital implementation have been developped.

Main differences between digital and analog demodulators:

- Down conversion is INDEPENDENT from phase recovery
- Timing recovery is performed BEFORE phase recovery

Receiver input signal

$$y(t) = \mathrm{Re}\left(x(t)e^{2j\pi f_0 t}\right) + \mathrm{Re}\left(n(t)e^{2j\pi f_0 t}\right)$$

$$x(t) = e^{j\varphi(t)} \sum_k d_k h(t - kT - \tau)$$

$$\varphi(t) = 2\pi\Delta f t + \varphi_0$$

f_0 : carrier frequency, Δf :carrier frequency uncertainty

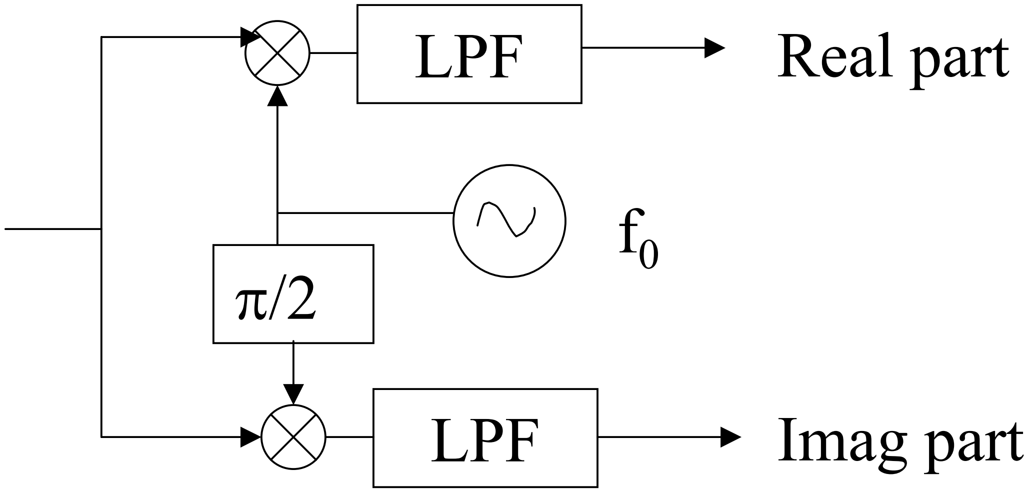
ϕ_0 : phase offset, τ : timing offset

d_k : emitted symbols

h(t): emission filter (wideband channel assumed)

Baseband signal generation (1)

➢ Analog implementation



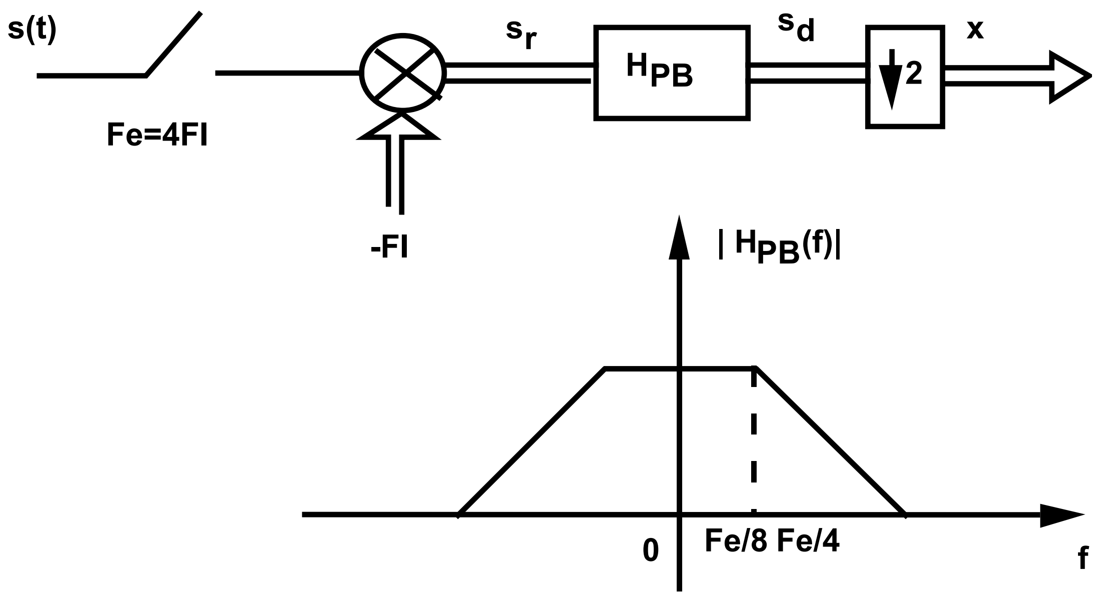
This process can be digitally implemented

(DAF : digital anti aliasing filter)

Baseband signal generation (2)

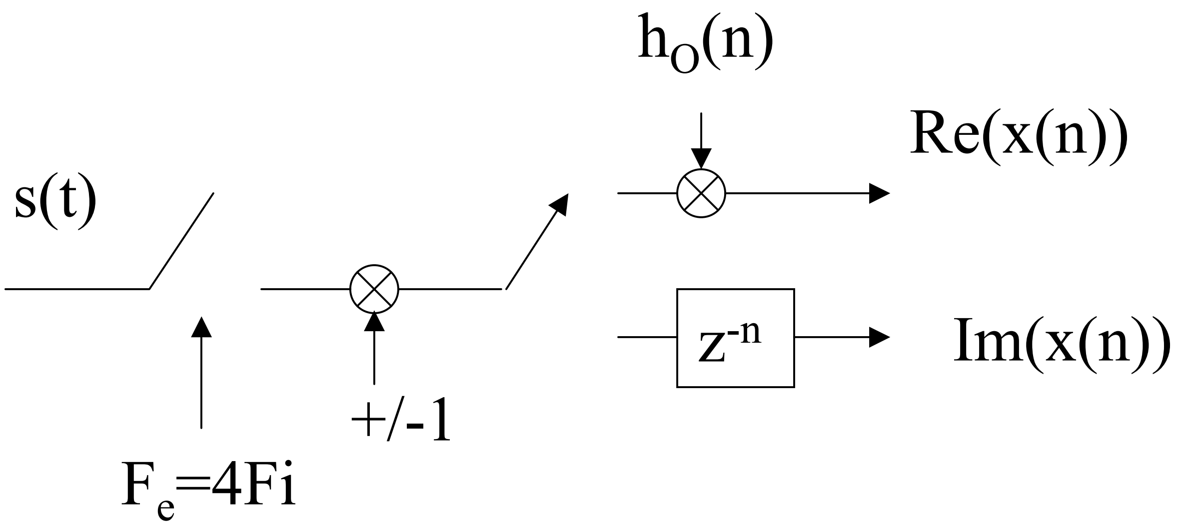
➢ Digital implementation (1)

s(t) is the real received passband signal (allocated bandwidth : FI, centred at f_0=FI)



Baseband signal generation (3)

- Digital implementation (2)



Likelihood functions (1)

$$r(t) = x(t) + n(t)$$

$$x(t) = e^{j\varphi(t)} \sum_k d_k h(t - kT - \tau)$$

$$\varphi(t) = 2\pi \Delta f t + \varphi_0$$

$\rho(T_0)$: signal observed during a period of duration T_0

$\Phi = \{\varphi_0, \Delta f, \tau, \{d_k\}\}$ Vector of unknown parameters

$\hat{\Phi} = \{\hat{\varphi}_0, \Delta\hat{f}, \hat{\tau}, \{\hat{d}_k\}\}$ Vector of parameters estimates

Likelihood functions (1)

$$\Lambda(\tilde{\Phi}) = \Pr(\rho(T_0) / \tilde{\Phi})$$

In Gaussian channel:

$$\Lambda(\tilde{\Phi}) = \exp\left(-\frac{1}{N_0} \int_{T_0} \left| r(t) - s(t,\tilde{\Phi}) \right|^2 dt \right)$$

$$s(t,\tilde{\Phi}) = Ae^{2j\pi\Delta\tilde{f}t + j\tilde{\varphi}_0} \sum_k \tilde{d}_k h(t - kT - \tilde{\tau})$$

$s(t,\tilde{\Phi})$: signal replica

Likelihood functions (2)

Sub-optimal likelihood functions :

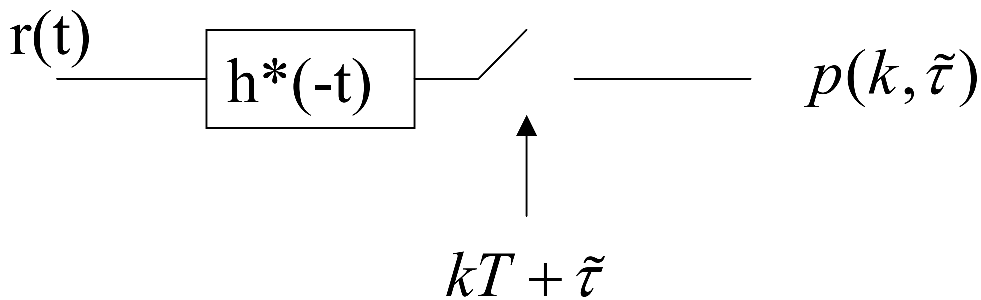
- DD : Decision Directed
- NDA : Non-data aided (depends on modulation)

These sub-optimal likelihood functions are derived for timing, phase and frequency.

Likelihood functions (3)

Timing :

$$L_{NDA}(\tilde{\tau}) = \sum_k \left| p(k,\tilde{\tau}) \right|^2$$



Timing recovery is performed prior to phase recovery

Likelihood functions (4)

Carrier phase:

- DD likelihood function

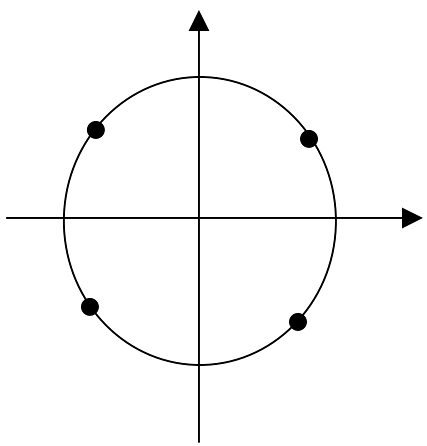
$$L_{DD}(\tilde{\varphi}) = \sum_k \hat{a}_k \operatorname{Re}\left(p(k,\hat{\tau})e^{-j\tilde{\varphi}} \right) + \sum_k \hat{b}_k \operatorname{Im}\left(p(k,\hat{\tau})e^{-j\tilde{\varphi}} \right)$$

- NDA lokelihood function for general rotationnaly symetric signal constellation (2π/N symetry)

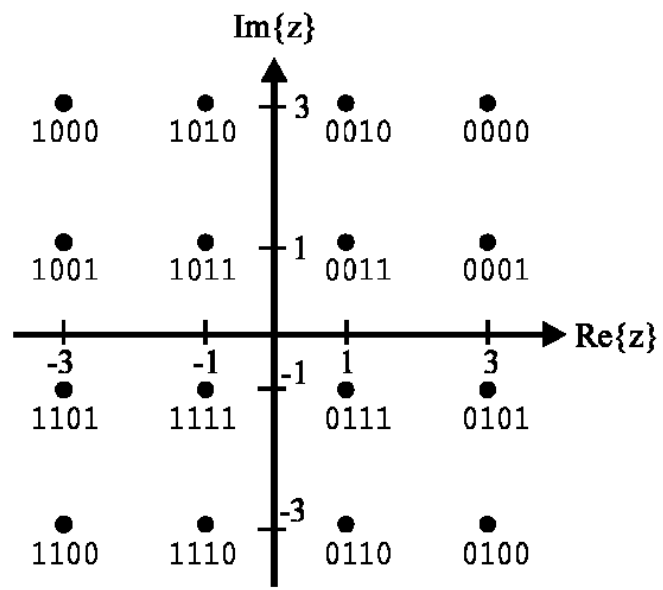
$$L_{NDA}(\tilde{\varphi}) = \operatorname{Re}\left(E\left(d_k^{*N} \right) \sum_k p^N(k,\hat{\tau})e^{-jN\tilde{\varphi}} \right)$$

Likelihood functions (5)

Examples of general rotationnaly symetric signal constellation



QPSK N=4



16QAM N=4

Likelihood functions (6)

- <u>Carrier frequency recovery</u>

QAM

$$L\left(\{a_k\},\Delta f,\varphi\right)=\sum_k \left|d_k\right|^2+2\sum_k \mathrm{Re}\left\{p(k,\hat{\tau})d_k^* e^{-j\left(2\pi\Delta f kT+\varphi\right)}\right\}$$

PSK

$$L\left(\{a_k\},\Delta f,\varphi\right)=\sum_k \mathrm{Re}\left\{p(k,\hat{\tau})d_k^* e^{-j\left(2\pi\Delta f kT+\varphi\right)}\right\}$$

Carrier phase recovery : DDMLFB (1)

Derivation of detector expression from Likelihood function

$$\frac{d}{d\tilde{\varphi}} L_{DD}(\tilde{\varphi}) = 0 \text{ for } \tilde{\varphi} = \hat{\varphi}$$

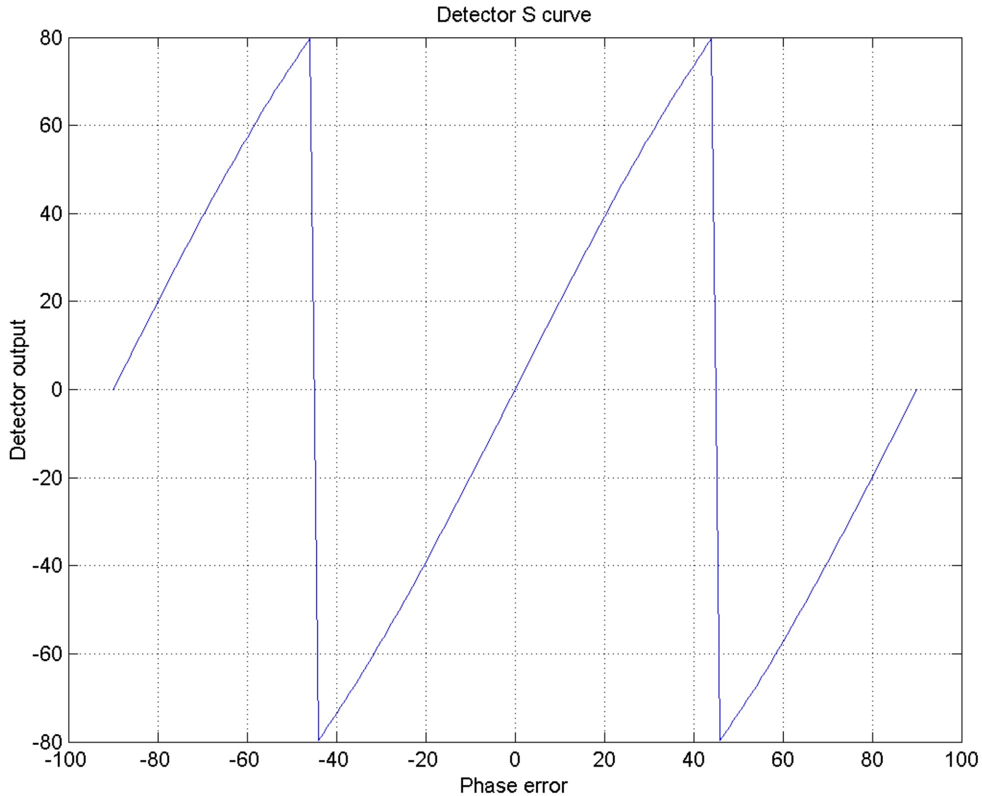
$$\Rightarrow \sum_k \mathrm{Im}\left(d_k^* p(k, \hat{\tau}) e^{-j\tilde{\varphi}} \right) = 0 \text{ for } \tilde{\varphi} = \hat{\varphi}$$

$$\Rightarrow \; u(k) = \mathrm{Im}\left(d_k^* p(k, \hat{\tau}) e^{-j\tilde{\varphi}} \right) \text{ is a phase detector}$$

Carrier phase recovery : DDMLFB (2)

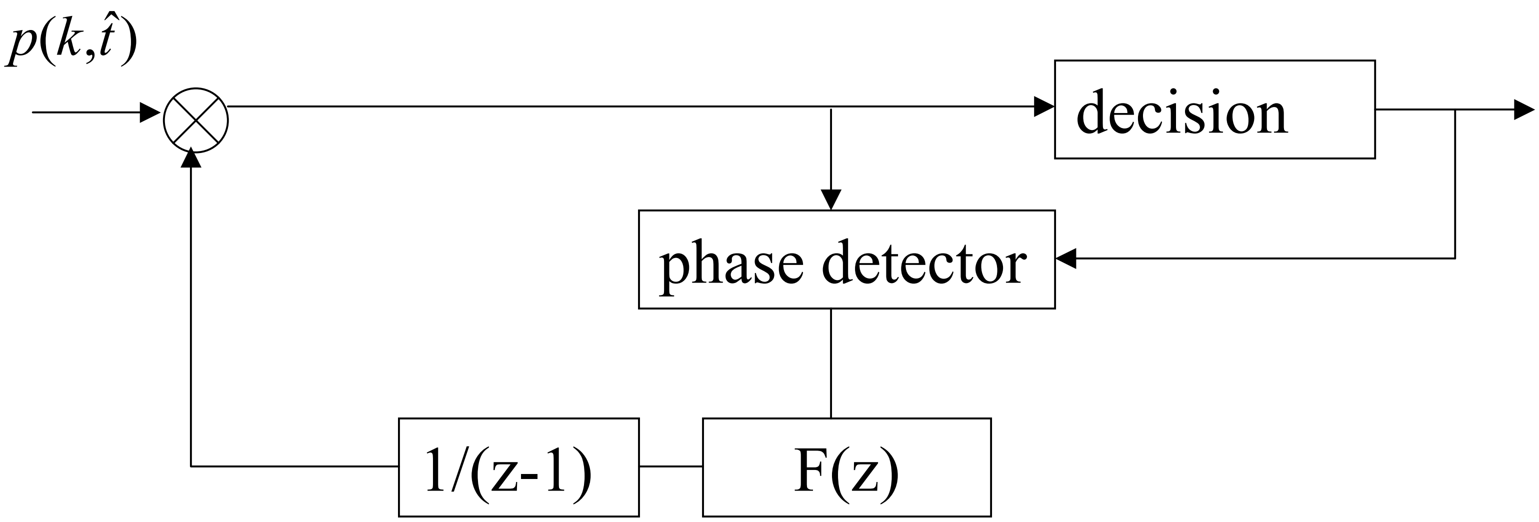
S curve (example for QPSK)

=> Phase ambiguity (solved by using differential encoding/decoding)



Carrier phase recovery : DDMLFB (3)

DPLL



Carrier phase recovery : DDMLFB (4)

Other possible detectors

$$p(k,\hat{\tau}) = w(k)$$

$$u_1(k) = \mathrm{Im}\left(w^*(k).\mathrm{sgn}\left\{ w(k) - \hat{d}_k \right\} \right)$$

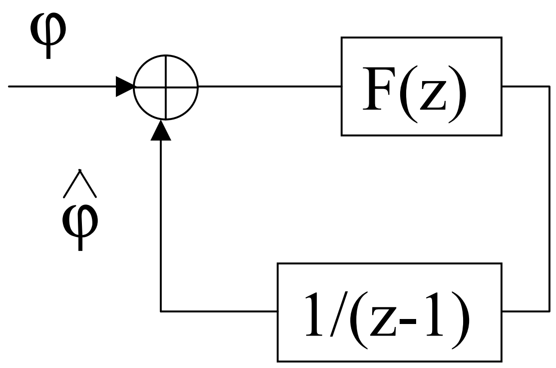
$$u_2(k) = \mathrm{Im}\left(\hat{d}_k^* \right)\left[w(k) - \hat{d}_k \right]$$

$$u_3(k) = \mathrm{Im}\left(d_k^*.c\,\mathrm{sgn}\left\{ w(k) - \hat{d}_k \right\} \right)$$

$$u_4(k) = \mathrm{Im}\left(\hat{d}_k^* \right)\mathrm{sgn}\left[w(k) - \hat{d}_k \right]$$

Carrier phase recovery : DDMLFB (5)

Phase equivalent scheme



$$H(z) = \frac{\hat{\varphi}(z)}{\varphi(z)}$$

$$2B_L T = \frac{1}{2j\pi} \oint_{\gamma} H(z) H^*(z^{-1}) \frac{dz}{z}$$

$$\sigma^2 \propto \frac{B_L T}{E_s / N_0}$$

Carrier phase recovery : NDAMLFB (1)

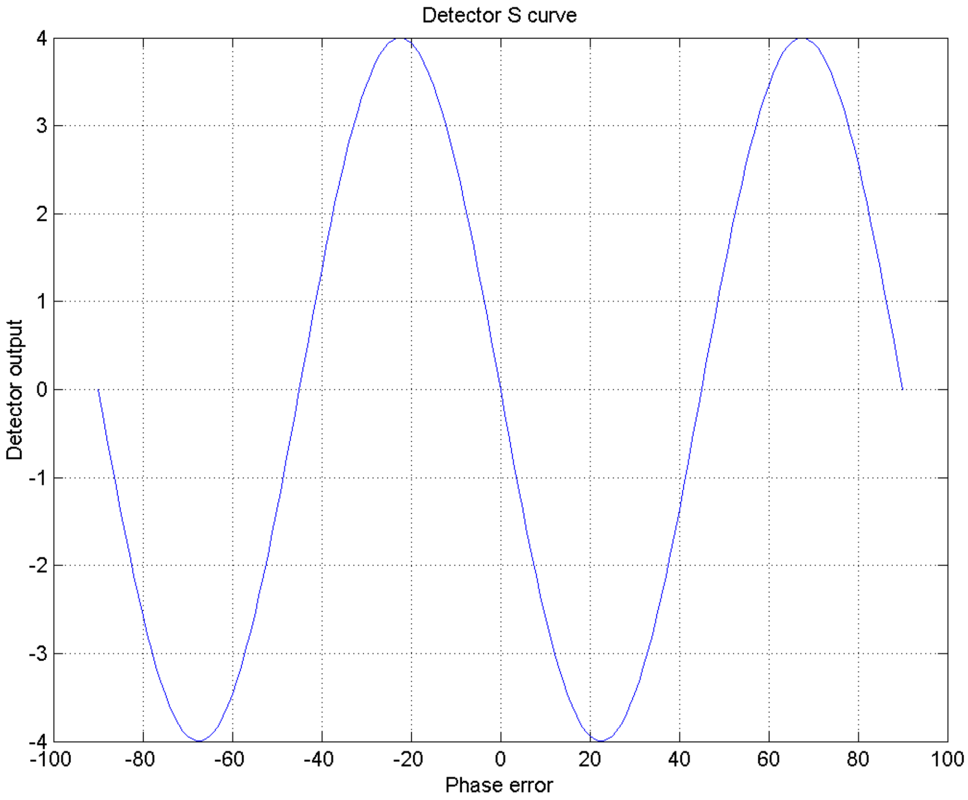
Example for QPSK

$$\frac{d}{d\tilde{\varphi}} L_{NDA}(\tilde{\varphi}) = 0 \text{ for } \tilde{\varphi} = \hat{\varphi}$$

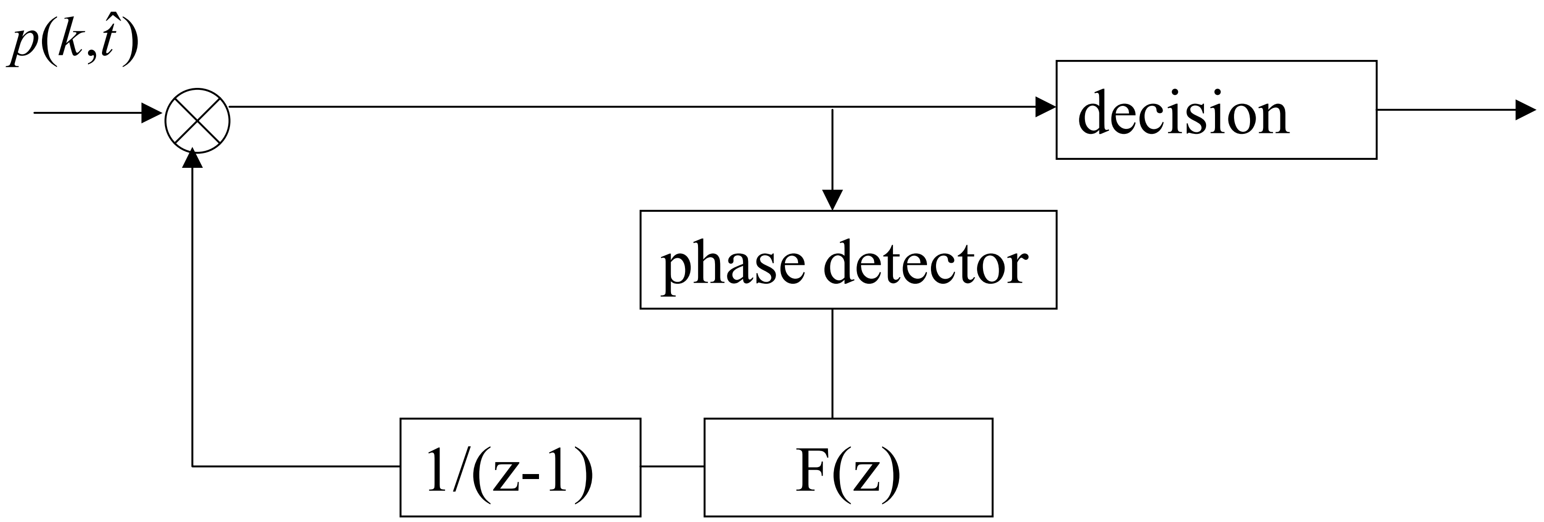
$$\Rightarrow \sum_k \mathrm{Im}\left(\left\{ p(k,\hat{\tau}) e^{-j\tilde{\varphi}} \right\}^4 \right) = 0 \text{ for } \tilde{\varphi} = \hat{\varphi}$$

$$\Rightarrow \; u(k) = \mathrm{Im}\left(\left\{ p(k,\hat{\tau}) e^{-j\tilde{\varphi}} \right\}^4 \right) \text{ is a phase detector}$$

Carrier phase recovery : NDAMLFB (2)



Carrier phase recovery : NDAMLFB (3)



Carrier phase recovery : NDAMLFF (1)

- Suited for burst transmission
- Two types of structures : block window, sliding window
- Example for QPSK

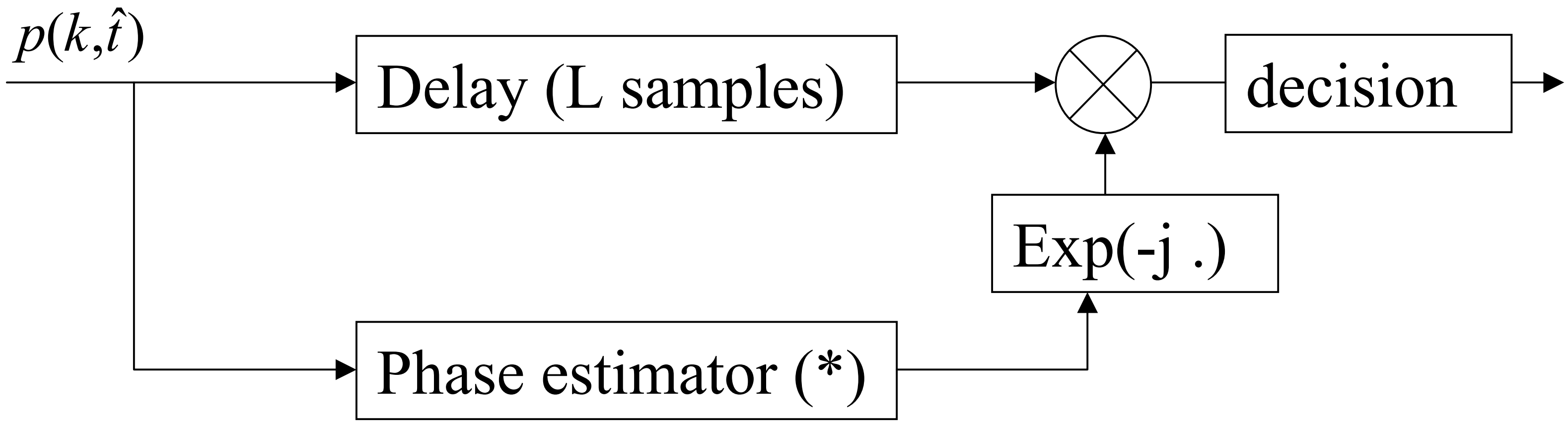
$$\sum_k \mathrm{Im}\left(\left\{ p(k,\hat{\tau}) e^{-j\tilde{\varphi}} \right\}^4 \right) = 0 \text{ for } \tilde{\varphi} = \hat{\varphi}$$

$$\Rightarrow \boxed{\hat{\varphi} = \frac{1}{4} Arg\left(\sum_k p^4(k,\hat{\tau}) \right) + k\frac{\pi}{2}}$$

$\Rightarrow$ Phase ambiguity ($k\pi/2$)

Carrier phase recovery : NDAMLFF (2)

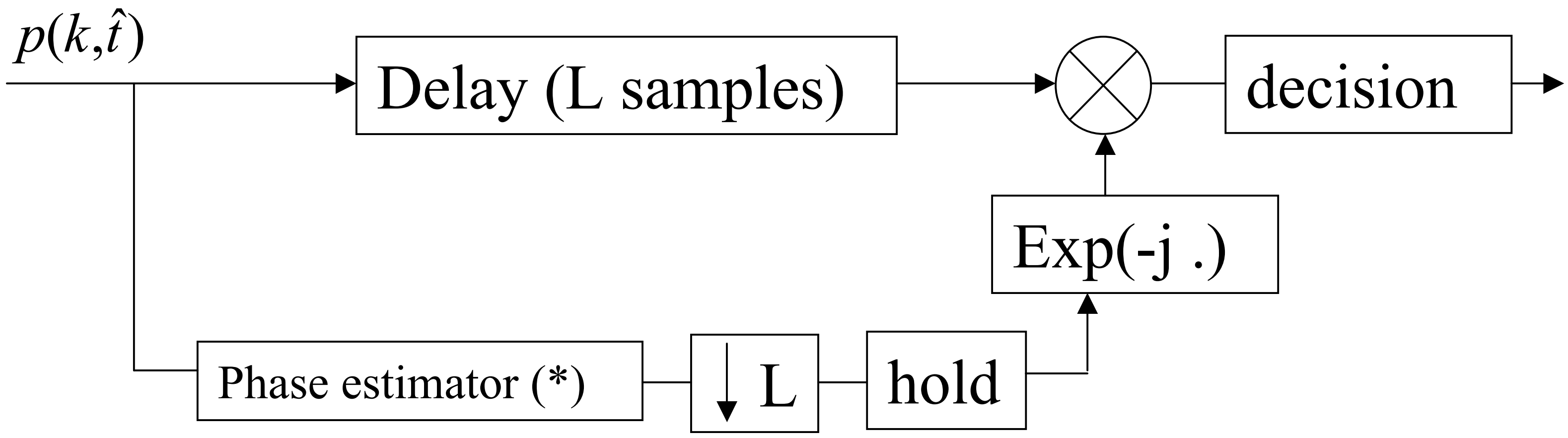
« Sliding window » estimator



(*): averaging over 2L+1 samples

Carrier phase recovery : NDAMLFF (3)

- « Block » estimator



FF structures vs FB structures

- Advantage
  - No acquisition time

- Drawbacks
  - Smaller $B_L T$ => higher jitter, higher cycle slip probability
  - Sensitivity to frequency deviation

Timing recovery (1)

$$L_{NDA}(\tilde{\tau}) = \sum_k \left| p(k,\tilde{\tau}) \right|^2 = \sum_k \mathrm{Re}^2\left(p(k,\tilde{\tau})\right) + \sum_k \mathrm{Im}^2\left(p(k,\tilde{\tau})\right)$$

$$\frac{d}{d\tau} L_{NDA}(\tilde{\tau}) = 2\sum_k \mathrm{Re}(p(k,\tilde{\tau}))\frac{d}{d\tilde{\tau}}\mathrm{Re}(p(k,\tilde{\tau})) + 2\sum_k \mathrm{Im}(p(k,\tilde{\tau}))\frac{d}{d\tilde{\tau}}\mathrm{Im}(p(k,\tilde{\tau}))$$

⇒Derivative vs timing is approximated by a difference

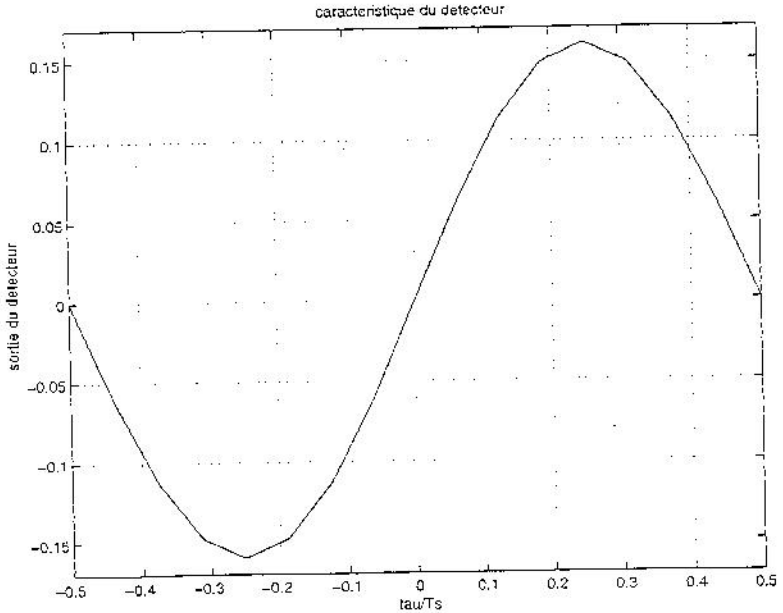
$$\mathrm{Re}(p(k,\tilde{\tau})) \propto \mathrm{Re}(p(k+\lambda,\tilde{\tau})) - \mathrm{Re}(p(k-\lambda,\tilde{\tau}))$$
$$\mathrm{Im}(p(k,\tilde{\tau})) \propto \mathrm{Im}(p(k+\lambda,\tilde{\tau})) - \mathrm{Im}(p(k-\lambda,\tilde{\tau}))$$

Timing recovery (2)

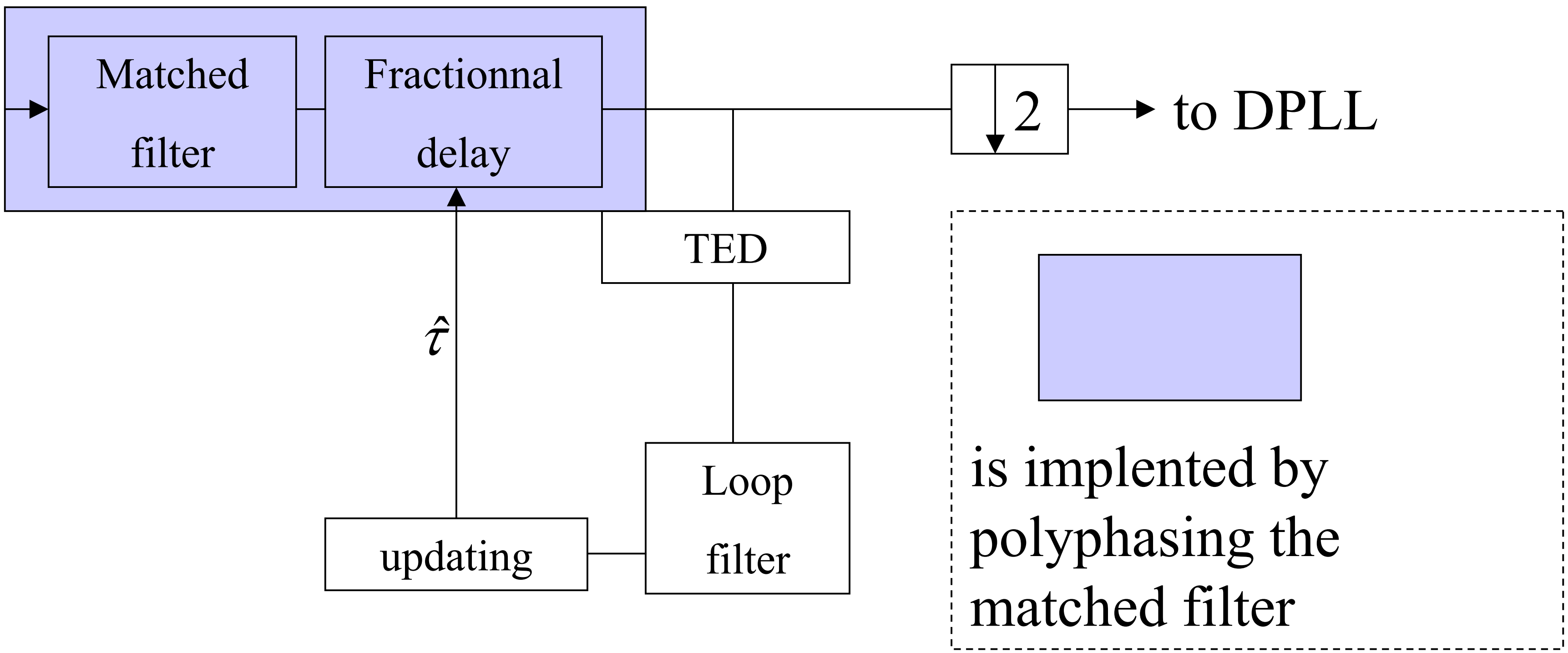
Gardner:

λ=1/2 => detector output is independent from carrier phase error.

$$GA(k) = \mathrm{Re}(p(k+1/2,\tilde{\tau}))\left\{\mathrm{Re}(p(k,\tilde{\tau})) - \mathrm{Re}(p(k+1,\tilde{\tau}))\right\}$$
$$+\mathrm{Im}(p(k+1/2,\tilde{\tau}))\left\{\mathrm{Im}(p(k,\tilde{\tau})) - \mathrm{Im}(p(k+1,\tilde{\tau}))\right\}$$

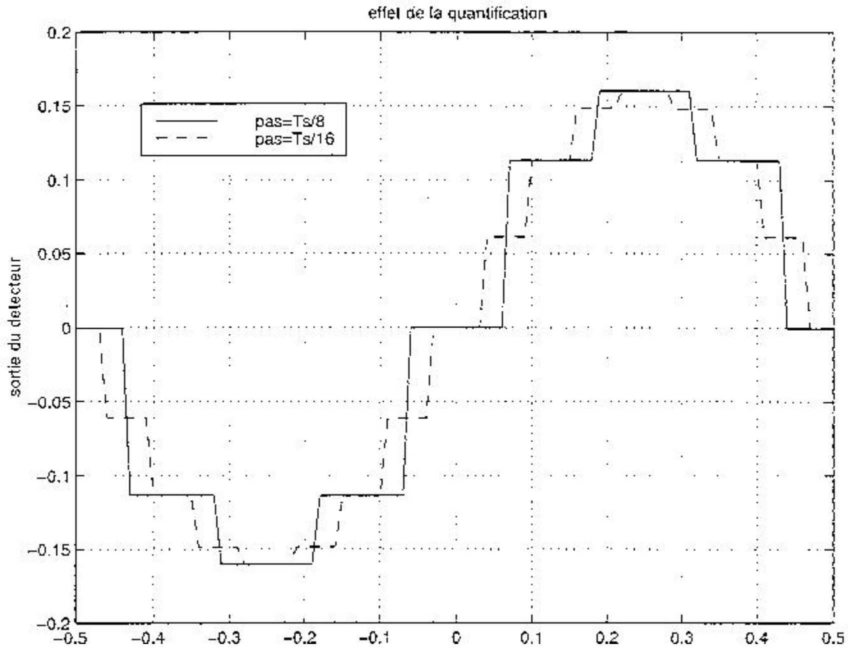


Timing recovery (3)



Timing recovery (4)

➢ S curve (Gardner, quantized)



Timing recovery (5)

➢ Timing estimator (Oerder and Meyr)

$$\frac{\hat{\tau}}{T} = -\frac{1}{2\pi} Arg\left(\sum_{k=0}^{L-1} \sum_{n=0}^{N-1} \left| p(k,n) \right|^2 e^{2j\frac{\pi n}{N}} \right)$$

$$p(k,n) \triangleq p(kT + nT/N)$$

where N is the number of samples per second

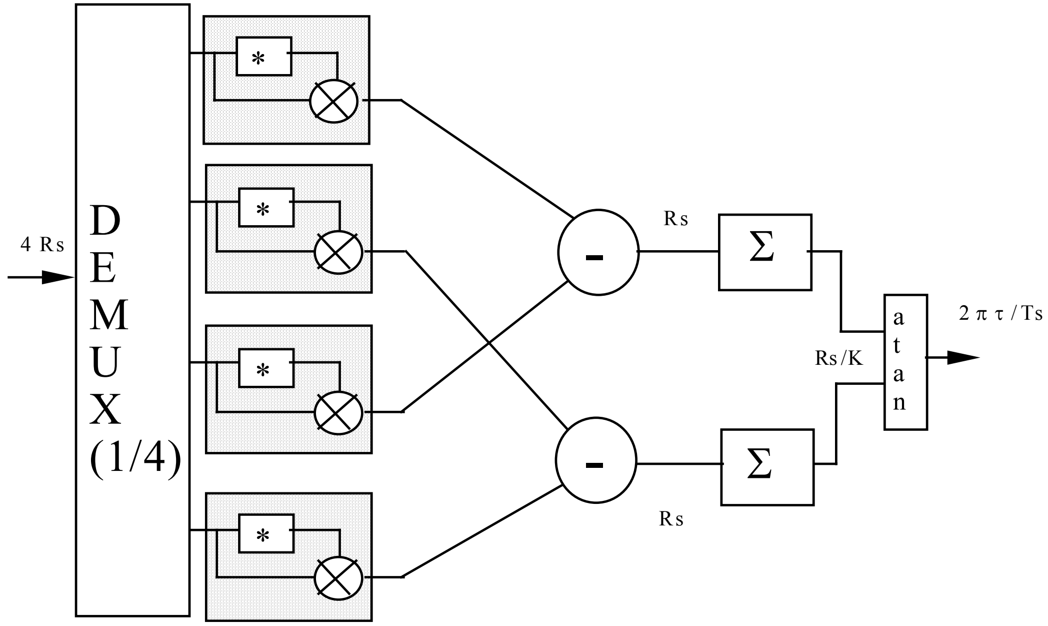
Example : N=4

$$\frac{\hat{\tau}}{T} = -\frac{1}{2\pi} Arg\left(\sum_{k=0}^{L-1} \sum_{n=0}^{3} \left| p(k,n) \right|^2 j^n \right)$$

Timing recovery (6)

Implementation of Oerder and Meyr

$$\frac{\hat{\tau}}{T} = -\frac{1}{2\pi} Arg\left(\sum_{k=0}^{L-1} \left\{ |p(k,0)|^2 - |p(k,2)|^2 \right\} + j \sum_{k=0}^{L-1} \left\{ |p(k,1)|^2 - |p(k,3)|^2 \right\} \right)$$

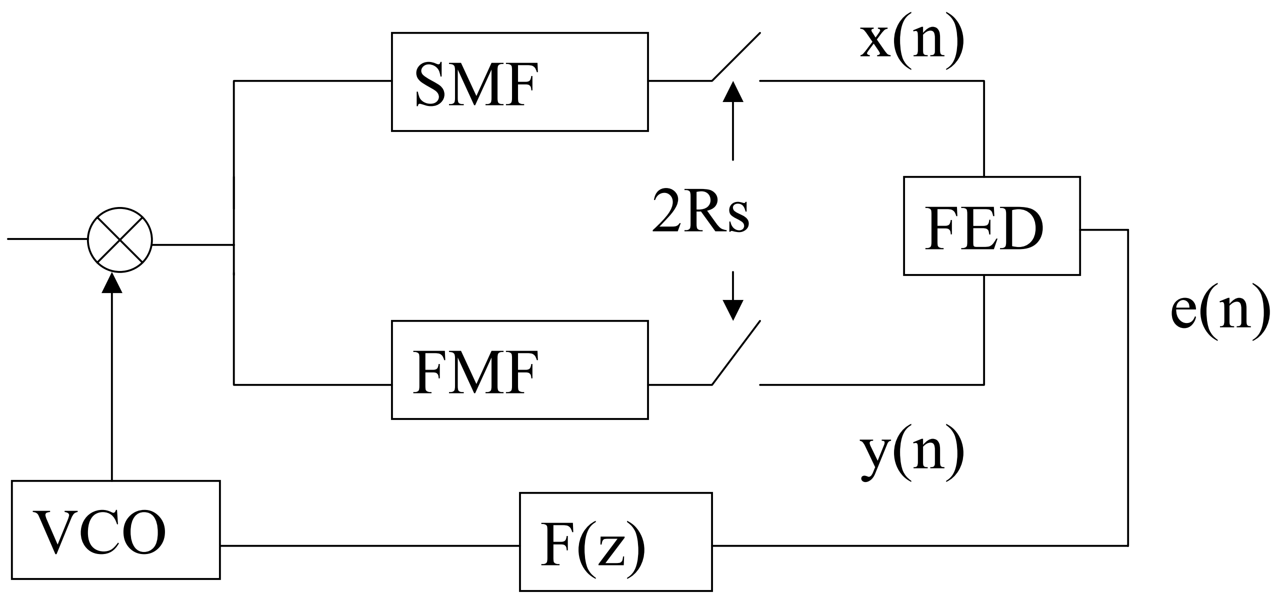


Frequency recovery : general

- Feedback structures
  - « Frequency » detectors
  - « Time » detectors
- Feedforward structures
  - Type 1
  - Type 2

Frequency recovery:FB structures (1)

➢ « Frequency » detector (1)



Frequency recovery:FB structures (2)

- « Frequency » detector (2)

SMF : signal matched filter : g(t)
FMF : frequency matched filter : $-2j\pi t g(-t)$

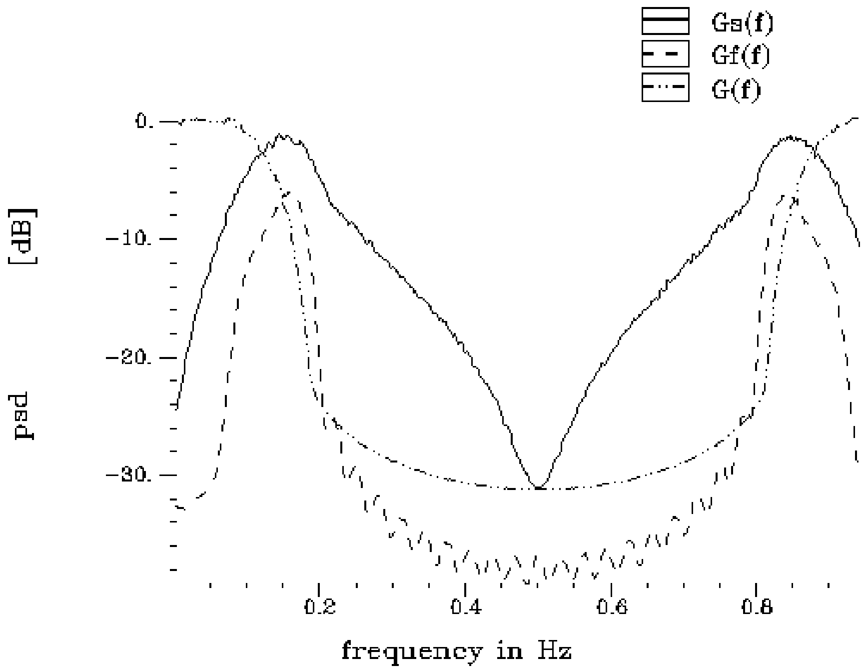
$e(n)=Im(x(n)y^*(n))$

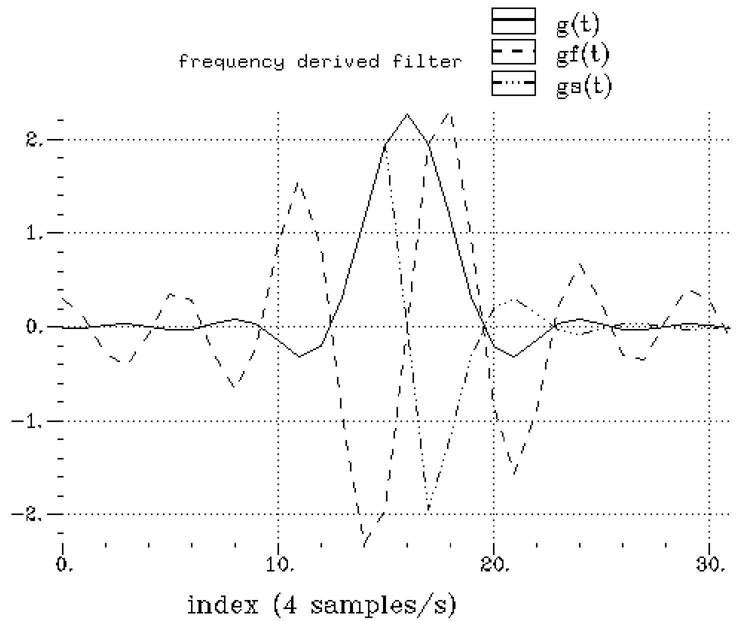
A simpler filter (SFMF) derived from FMF can be used
$(g(t)=-j\, sgn(t)\, g(-t)$

Acquisition range : $+/-(1+\alpha)R_s$

No prior timing correction required

Frequency recovery:FB structures (3)





Frequency recovery:FB structures (3)

- « Time » detectors

Any estimator can be used as a time detector.

Frequency offset range is +/- R_s/M

Timing has to be corrected prior to frequency detection

1 sample/symbol is sufficient.

Frequency recovery:FF structures (1)

- Bellini

$$\Delta\hat{f}T = \left(\sum_{-N}^{N} i\alpha_i\right) \Big/ \left(8\pi T \sum_{-N}^{N} i^2\right)$$

=> Cycle slip

α_i: unwrapped phase

- RCFE (reduced complexity frequency estimator)

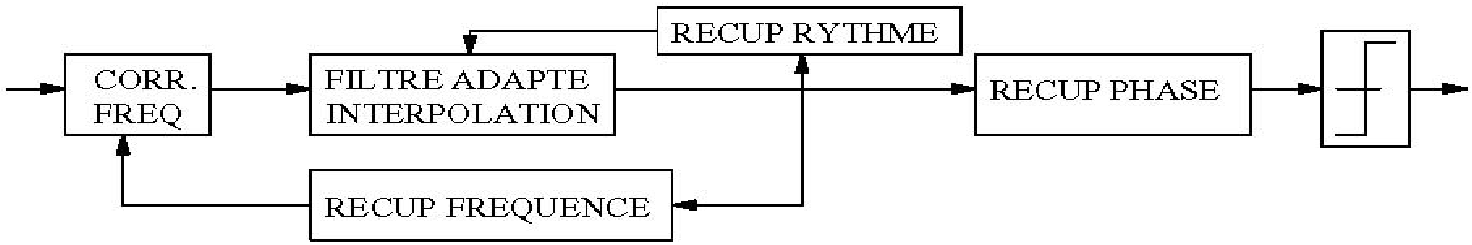
$$2\pi\Delta\hat{f}T = \frac{1}{MD} Arg\left\{\sum_{k} d_k \left(r_k r_{k-D}^{*}\right)^{M}\right\}$$

$$r_k = p(k, \hat{\tau})$$

Large D leads to better performances but to lower frequency range.

Digital demodulators (1)

Typical FeedForward Architecture



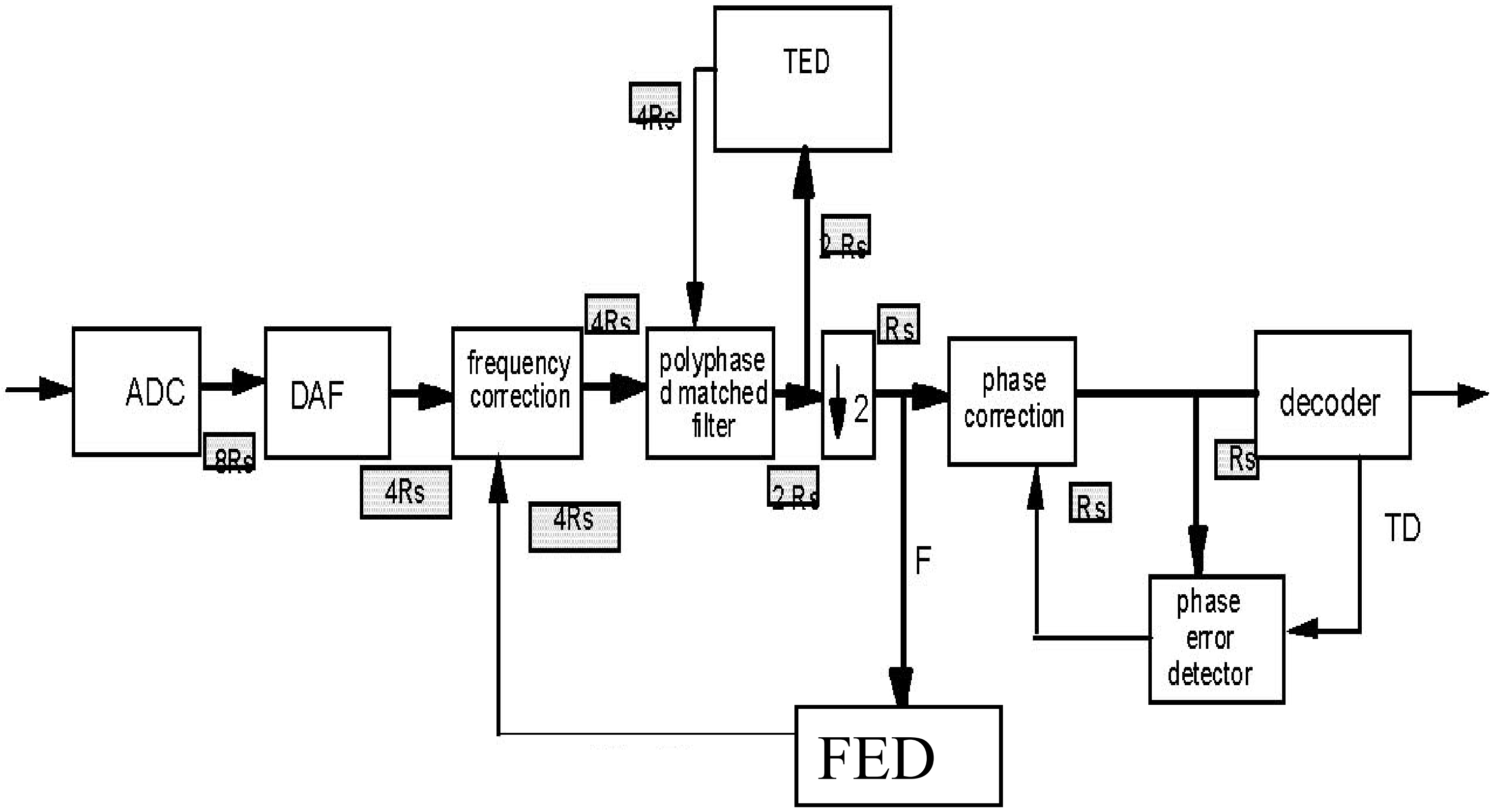
Typical Feedback Architecture

Digital demodulators (2)

Choice of algorithms depends on specifications such as:

- Acquisition time (=> FF/FB structures)
- Maximum frequency deviation (=> frequency circuitry needed)
- Eb/No (=> use of TD if low)
- .....

Digital demodulators (3)



Example: Receiver for TCM (in cooperation with CNES)

Advanced topics

➢Evolutions of input specifications (for satellite communications)

- Low Eb/No (use of efficient coding schemes such as Turbo-Codes and LDPC)
- Bursty transmission
- Large frequency deviation (low-cost terminals, non GEO sat.)

➢Critical function : **phase recovery** (classical algorithms fail)

➢There is a need to develop new synchronisation schemes

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