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Carrier Frequency-Offset for OFDM and Related Multicarrier Systems

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Aims and General Outline

Aims:

- To present data-aided and (semi-)blind CFO estimation algorithms for OFDM
- To give a unified framework for several existing algorithms

General outline

- Motivation and context
- Null-subcarrier-based CFO estimation
- Blind CFO estimation exploiting data properties

Motivation and Context

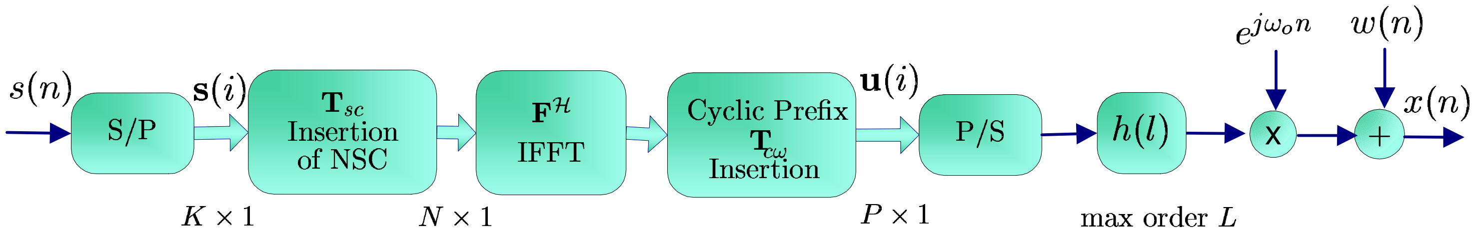
- High data rates (up to 54 Mbps) with Coded-OFDM
  - ⋆ IEEE802.11a, HIPERLAN/2, MMAC; DAB, DVB
- OFDM turns frequency-selective to flat fading channels
  - ⋆ Timing-Offset (TO) as a pure-delay channel
- Low-complexity equalization and easy decoding
  - ⋆ convolutional coded OFDM (across subcarriers)
- Challenges
  - Non-constant modulus $\Rightarrow$ large peak-to-average power ratio
  - Sensitivity to Carrier Frequency-Offset (CFO)
    - ☞ Inter-Carrier Interference (ICI)
    - ✍ At $E_s/N_0 = 19$dB: CFO/subcarrier spacing $= 1.26\%$
      $\Longrightarrow$ SNR degradation 10 dB

Part 1: Null-Subcarrier-based CFO Estimation

Outline

- Signal model
- Deterministic ML estimator
- Identifiability issues
- CRB and optimal placement of null subcarriers
- Performance analysis
- Repetitive Slot-Based CFO Estimation
- Comparisons
- Summary

Signal Model



- NSC insertion: $\mathbf{T}_{sc}$: K cols of a $N \times N$ permutation matrix
- CP insertion: $\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{L\times(N-L)}, & \mathbf{I}_L \\ & \mathbf{I}_N \end{bmatrix}$
- Transmitted block: $\boldsymbol{u}_{\mathrm{cp}}(i) = \mathbf{T}_{cp}\mathbf{F}_N^{\mathcal{H}}\mathbf{T}_{sc}\boldsymbol{s}(i)$
- Input-output relationship $(N \geq K, P = L + N)$

$$x_{\mathrm{cp}}(n) = e^{j\omega_o n}\sum_{l=0}^{L} h(l)u_{\mathrm{cp}}(n-l) + w_{\mathrm{cp}}(n)$$

Goal: Estimate CFO ω_o based only on knowledge of $\mathbf{T}_{sc}$ without channel state information

Signal Model (2)

- Received blocks

$$\boldsymbol{x}_{\text{cp}}(i) = e^{j\omega_o iP}\mathbf{D}_P(\omega_o)\left[\mathbf{H}_1\boldsymbol{u}(i) + \mathbf{H}_2\boldsymbol{u}(i-1)\right] + \boldsymbol{w}(i)$$

where $\mathbf{D}_P(\omega_o) = \text{diag}(e^{jk\omega_o}, k = 0, ..., P-1)$

- Discard CP to avoid IBI: using $\mathbf{R}_{cp} := [\mathbf{0}_{N\times(P-N)}, \mathbf{I}_N]$:

$$\mathbf{R}_{cp}\mathbf{H}_2 = \mathbf{0}, \quad \mathbf{R}_{cp}\mathbf{D}_P(\omega_o) = \mathbf{D}_N(\omega_o)\mathbf{R}_{cp}, \qquad \mathbf{R}_{cp}\mathbf{D}(\omega_o)\mathbf{H}_2 = \mathbf{0}$$

- Channel matrix: $\mathbf{H}_1$ Toeplitz $\Rightarrow$ $\mathbf{H}_c = \mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}$ circulant; so

$$\mathbf{F}_N\mathbf{H}_c\mathbf{F}_N^{\mathcal{H}} == \text{diag}(H_0 \cdots H_{N-1}) =: \mathbf{D}_H$$

where $H_k = \sum_{\ell=0}^{L} h_\ell \exp(-j2\pi\ell k/N)$

Signal Model (3)

- Received blocks after CP removal

$$\boldsymbol{x}(i) \; = \; \mathbf{R}_{cp}\boldsymbol{x}_{\mathrm{cp}}(i) = e^{j\omega_o iP}\mathbf{D}_N(\omega_o)\mathbf{F}_N^{\mathcal{H}}\mathbf{D}_H\mathbf{T}_{sc}\boldsymbol{s}(i) + \boldsymbol{w}(i)$$

- Perform FFT:

$$\begin{aligned}\tilde{\boldsymbol{x}}(i) &= \mathbf{F}_N\boldsymbol{x}(i)\\ &= e^{j\omega_o iP}\underbrace{\left[\mathbf{F}_N\mathbf{D}_N(\omega_o)\mathbf{F}_N^{\mathcal{H}}\right]}_{\text{diagonal?}}\mathbf{D}_H\mathbf{T}_{sc}\boldsymbol{s}(i) + \tilde{\boldsymbol{w}}(i)\\ &= \mathbf{D}_H\mathbf{T}_{sc}\boldsymbol{s}(i) + \tilde{\boldsymbol{w}}(i) \qquad \text{iff } \omega_o = 0\end{aligned}$$

- $\hookrightarrow$ CFO causes ICI; degrades BER

Signal Model (4)

- After discarding CP, but before FFT (dropping block index)

$$x(k) = \sum_{n \in \mathcal{A}} H_n s_n e^{j2\pi k(n+\nu_o)/N} + w(k) \quad k = 0, ..., N-1$$

• $\nu_o = N\frac{\omega_o}{2\pi}$ is unknown CFO ; $-N/2 < \nu_o \leq N/2$ s_n unknown data symbols

• $\mathcal{A} \subset \mathcal{N} = \{-N/2+1, ..., N/2\}$: active sub-carriers
$\mathcal{Z} = \mathcal{N} - \mathcal{A}$: set of NSC's

-

$$\begin{aligned} a(k) &= \sum_{n \in \mathcal{A}} H_n s_n e^{j2\pi kn/N} \\ x(k) &= a(k) \exp(j2\pi k \xi_o / N) + w(k) \end{aligned}$$

• Estimate CFO in additive + multiplicative noise

Deterministic ML Estimator

- Treat $\alpha_n := H_n s_n$ as non-random unknowns
- Receiver knows NSC set

$$\boldsymbol{x} = \mathbf{D}(\nu_o)\, \Phi_{\mathcal{A}}\, \boldsymbol{\alpha} \;+\; \boldsymbol{w}$$

$$\begin{aligned}
\mathbf{D}(\nu_o) &= \mathrm{diag}\{1, e^{j2\pi\nu_o/N}, ..., e^{j2\pi(N-1)\nu_o/N}\} \\
\Phi_{\mathcal{A}} &= \mathbf{F}_N^{\mathcal{H}} \mathbf{T}_{sc} \\
\boldsymbol{\alpha} &= [\alpha_{n_1} \;\; \ldots \;\; \alpha_{n_{N_a}}]^T; \qquad n_\ell \in \mathcal{A},\; \ell = 1, ..., N_a
\end{aligned}$$

Deterministic ML Estimator (2)

- Gaussian Problem. Concentrate LLF wrt α_n's:

$$\begin{aligned}
\hat{\nu}_o &= \arg\max_{\nu} \sum_{\tau} r(\tau)\psi^*_{\mathcal{A}}(\tau)e^{-j2\pi\tau\nu/N} \\
r(\tau) &= \sum_{k=0}^{N-1-\tau} y^*(k)y(k+\tau) = r^*(-\tau) \\
\psi_{\mathcal{A}}(\tau) &= \frac{1}{N_a}\sum_{n\in\mathcal{A}} e^{j2\pi n\tau/N}
\end{aligned}$$

- Peak-pick windowed correlogram; window dictated by $\mathcal{A}$.

- $N_a = N \Rightarrow \psi_{\mathcal{A}}(\tau) = \delta(\tau) \Rightarrow$ CFO is *not* identifiable
  $\hookrightarrow$ Need NSC's

Deterministic ML Estimator (3)

- Interpretation of DML

  - MLE maximizes $J_A(\nu)$ or minimizes $J_z(\nu)$

$$\hat{\nu}_o = \arg\max J_a(\nu) = \arg\min J_z(\nu)$$

where

$$J_a(\nu) = \sum_{n \in \mathcal{A}} |X(\nu + n)|^2 \qquad J_z(\nu) = \sum_{n \in \mathcal{Z}} |X(\nu + n)|^2$$

with $X(f)$=DTFT of $\boldsymbol{x}$

  - Peak-pick (null-pick) sum of shifted periodograms
  - $\hat{\nu}$: frequency shift that minimizes total energy at NSC's

Identifiability Issues

- Identifiability study assumes noiseless case

❑ Identifiability is guaranteed iff

$$|\mathbf{D}(\nu_o)\mathbf{\Phi}_{\mathcal{A}}\alpha - \mathbf{D}(\nu)\mathbf{\Phi}_{\mathcal{A}}\alpha\|_2 \neq 0 \;\; \forall \; \nu \neq \nu_o$$

❑ Equivalently $J(\nu) < J(\nu_o)$ where

$$J(\nu) = \boldsymbol{\alpha}^{\mathcal{H}} \; \mathbf{G}_{\mathcal{A}}(\nu - \nu_o) \; \boldsymbol{\alpha}$$

with

$$\mathbf{G}_{\mathcal{A}}(\epsilon) = \mathbf{T}_{sc}^{\mathcal{H}}\mathbf{F}\mathbf{D}^{\mathcal{H}}(\epsilon)\mathbf{F}^{\mathcal{H}}\mathbf{T}_{sc}$$

➩ $J(\nu_o) = |\boldsymbol{\alpha}|^2$.

➩ Channel zeros $\alpha_n = 0$: it suffices to have $N_a \geq L + 1$

Identifiability Issues (2)

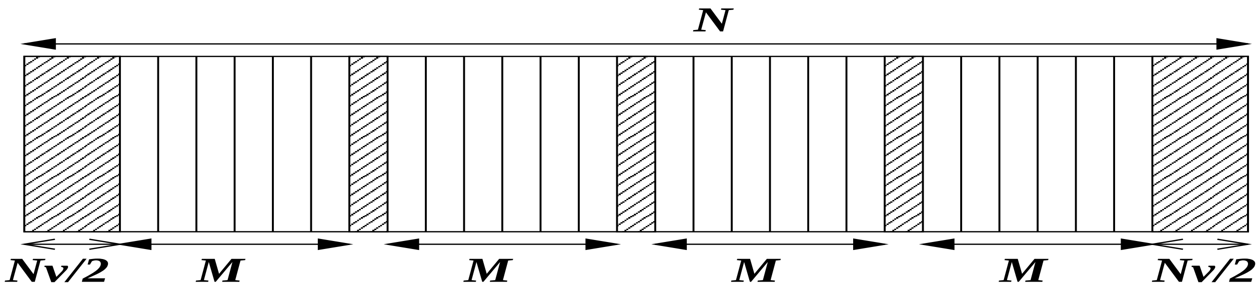
- Ambiguity due to number and location of NSC's
  - Global maxima of $J(\nu)$ at $\nu = \nu_o + m$; unique global at m=0?
  - For $\nu = \nu_o + m$, $\mathbf{G}_{\mathcal{A}}$ is diagonal of ones and zeros
  - $J(m + \nu_o) = \sum_{n_\ell \in \mathcal{A}} |\alpha_{n_\ell} g_{n_\ell}(m)|^2$
  - If for some $m \neq 0$, $g_{n_i}(m) \neq 0$ whenever $\alpha_{n_i} \neq 0$:
    $\hookrightarrow$ Identifiability lost
  - Identifiability is restored in $(-M/2, M/2]$ by choosing $\mathcal{A}$ st. $\forall m \in [1, M/2]$, $g_{n_i}(m) = 0$ for at least $L + 1$ values of i, $n_i \in \mathcal{A}$. (because channel has a maximum of L zeros)

Identifiability Issues (3)

- Let $P(m) := \{n_p : n_p \neq n_k + m, n_p, n_k \in \mathcal{A}\}$. Need $P(m) \geq L + 1$, for $0 < |m| \leq M/2$
- For consecutive NSC, $P(m) = \min(m, N_z, N_a)$. With $m = 1 \rightarrow L = 0 \rightarrow$ VSC-based estimator is viable only for AWGN channel.
- If $M \geq 2$, need $\min(N_a, N_z) > L$.
- For equi-spaced NSC's, CFO is uniquely identifiable in $(-N/2N_z, N/2N_z)$, if $L < N_z < N - L$.
- For equi-spaced active sub-carriers, CFO is uniquely identifiable in $(-N/2N_a, N/2N_a)$, if $L < N_a < N - L$.
- For NSC with distinct spacing, CFO is uniquely identifiable in $[-N/2, N/2)$ iff $L + 1 < N_z < N - L$.

Identifiability Issues (4)

- *If the number of consecutive NSC $N_v > L$, the number of equispaced NSC $N_n > L$ and the spacing between the equispaced NSC is $M > L$, then the CFO is uniquely identifiable in the entire acquisition range $(-N/2, N/2]$ regardless of the channel zeros.*



- Tradeoffs between acquisition range, performance, maximum tolerable delay spread.
- Identifiability conditions are relaxed if multiple blocks used and null-subcarrier hopping is performed.

CRB and Optimal Placement of Null Subcarriers

- Conditional CRB (CCRB)

❑ CCRB treats $\alpha_n = H_n s_n$ as non-random unknowns

$$CCRB_{\mathcal{A}}(\nu_o) = \frac{\sigma^2}{8\pi^2 N}\left[\boldsymbol{\alpha}^{\mathcal{H}}\Phi_{\mathcal{A}}^{\mathcal{H}}\mathbf{Q}\left(\mathbf{I} - \frac{N_a}{N}\Psi_{\mathcal{A}}\right)\mathbf{Q}\Phi_{\mathcal{A}}\boldsymbol{\alpha}\right]^{-1}$$

$$\mathbf{Q} = N^{-3/2}\text{diag}\{0, ..., N-1\}$$

$$\Phi_{\mathcal{A}} = \mathbf{F}^{\mathcal{H}}\mathbf{T}_{sc}, \quad \Psi_{\mathcal{A}} = \Phi_{\mathcal{A}}\Phi_{\mathcal{A}}^{\mathcal{H}}$$

❑ If no NSC i.e. $N_a = N \longrightarrow CCRB(\nu_o) = \infty$.

❑ CCRB is channel-dependent.

CRB and Optimal Placement of Null Subcarriers (2)

- Modified CRB (MCRB)
  - Rayleigh fading $\mathbf{R}_h = E\{\tilde{\mathbf{h}}\tilde{\mathbf{h}}^{\mathcal{H}}\}$.
  - $\alpha_n := H_n s_n$; $\mathbf{S} = \text{diag}\{s_n, n \in \mathcal{A}\}$; $\quad \mathbf{R}_\alpha = \mathbf{S}\mathbf{R}_h\mathbf{S}^H$
  - Channel-independent CRB:

$$MCRB_{\mathcal{A}}(\nu_o) = \frac{1/(8\pi^2 N)}{\text{Tr}\left\{\mathbf{R}^{-1}\mathbf{Q}\mathbf{R}\mathbf{Q} - \mathbf{Q}^2\right\}}$$

where

$$\mathbf{R} = \Phi_{\mathcal{A}}\mathbf{R}_\alpha\Phi_{\mathcal{A}}^{\mathcal{H}} + \sigma^2\mathbf{I}$$

  - Blind case: reasonable to assume $\mathbf{R}_\alpha$ diagonal

CRB and Optimal Placement of Null Subcarriers (3)

- → MCRB is a function of $\mathcal{A}$: # and placement of NSC's:

$$MCRB_{\mathcal{A}}(\nu_o) = \frac{1/(8\pi^2 N\eta)}{\frac{N}{N_a}\mathrm{Tr}\left\{\mathbf{Q}^2\right\} - \mathrm{Tr}\left\{\Psi_{\mathcal{A}}\mathbf{Q}\Psi_{\mathcal{A}}\mathbf{Q}\right\}}$$

  - $\eta = N_a\gamma^2/(N_a + N\gamma)$ is channel-independent
  - $\gamma = E|H_n|^2/\sigma^2$ is the average SNR

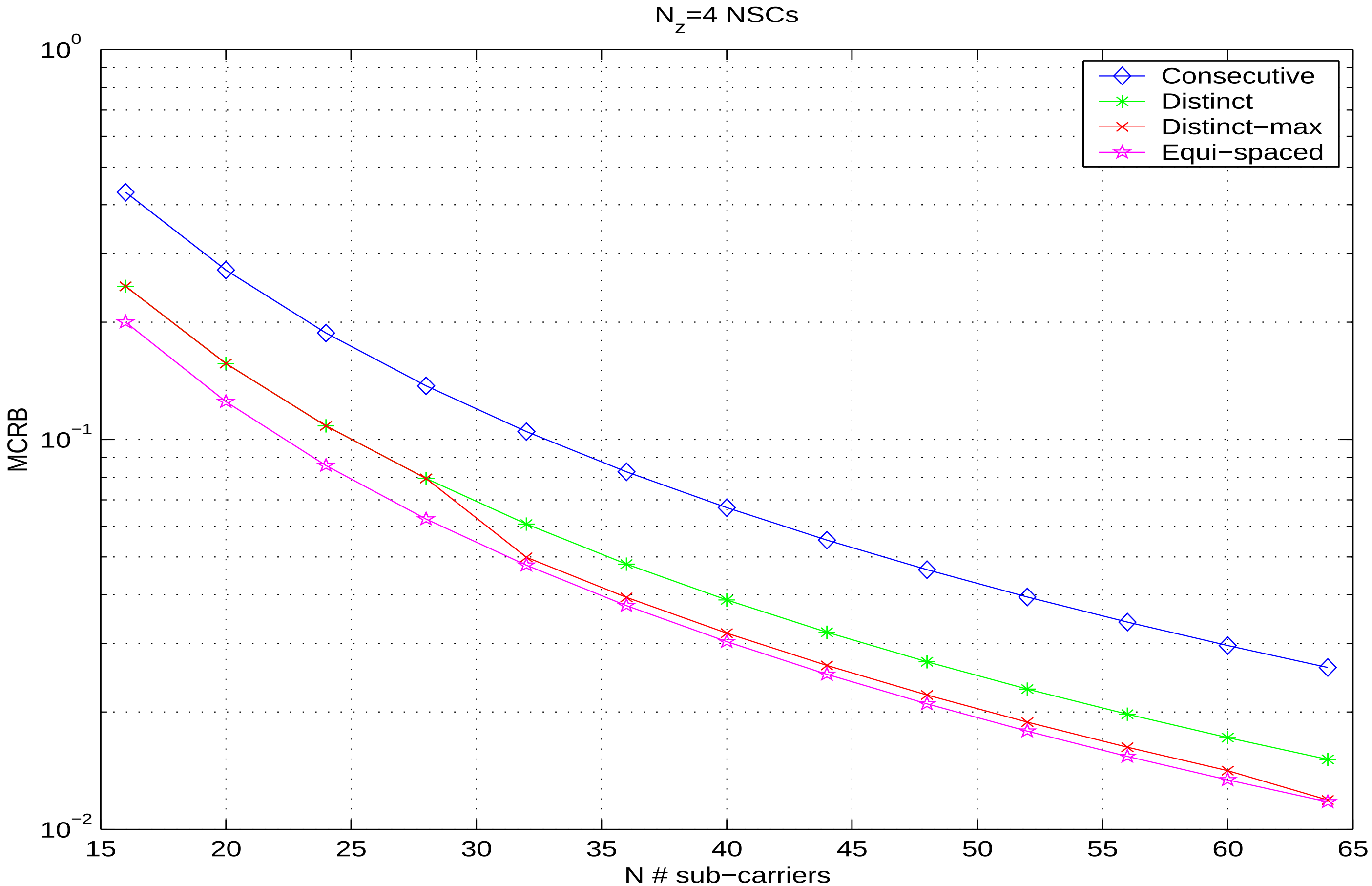
- *The optimal (in the sense of minimum MCRB) placement of a fixed number of active sub-carriers, N_a, is given by*

$$\mathcal{A}^* = \arg\min_{\mathcal{A}} \sum_{k,\ell=0}^{N-1} k\ell|\psi_{\mathcal{A}}(k,\ell)|^2$$

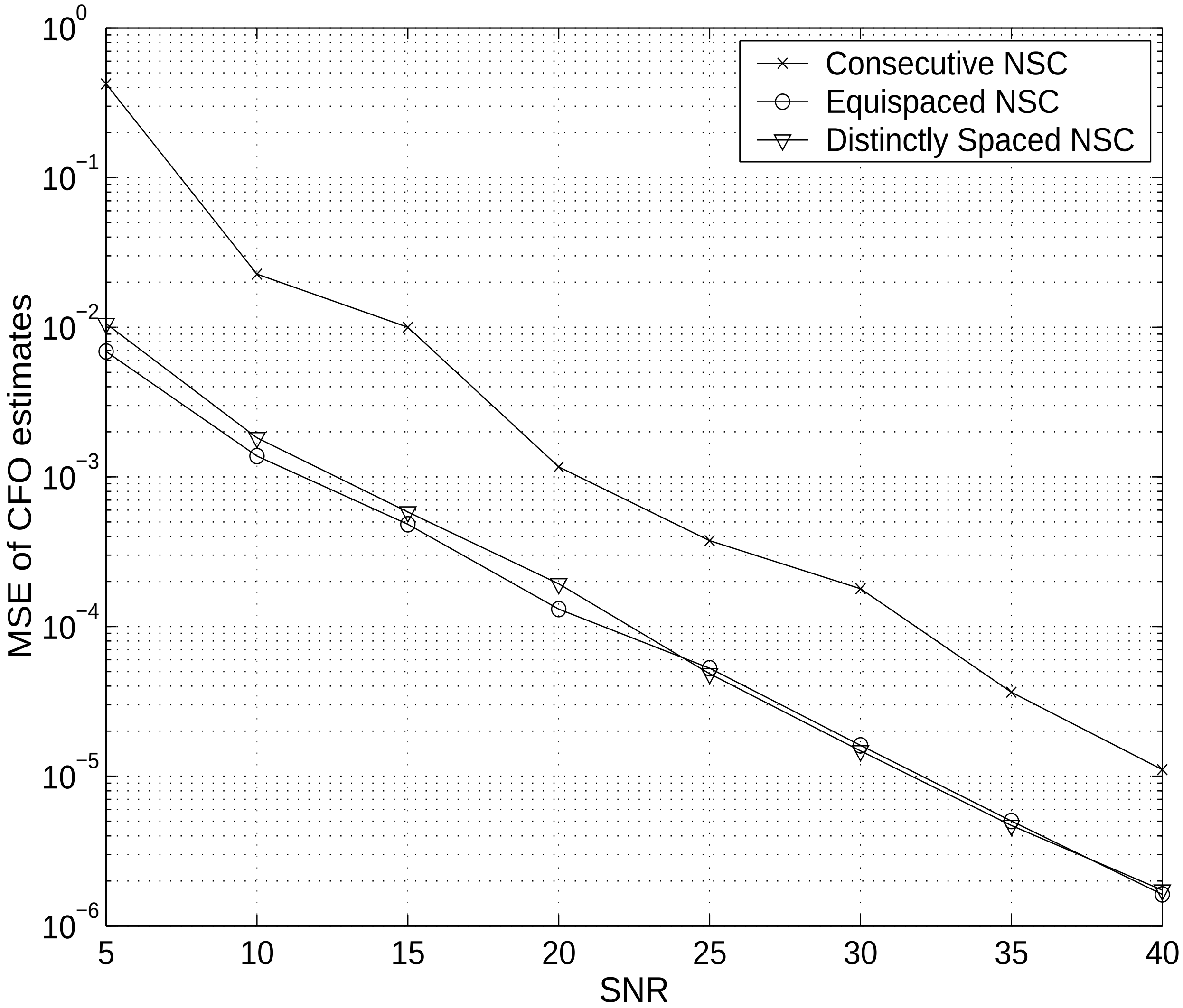
For $N_a \leq N/2$: equispace active sub-carriers
For $N_a \geq N/2$: equispace null sub-carriers
Average performance improves with # NSC's $N_z = N - N_a$

CRB and Optimal Placement of Null Subcarriers (4)

MCRB for different NSC placements; $N_z = 4$; one block



Performance Analysis



1 OFDM block

N = 64

$N_a = 54$

$N_z = 10$

$\nu_o \in [-2,2)$

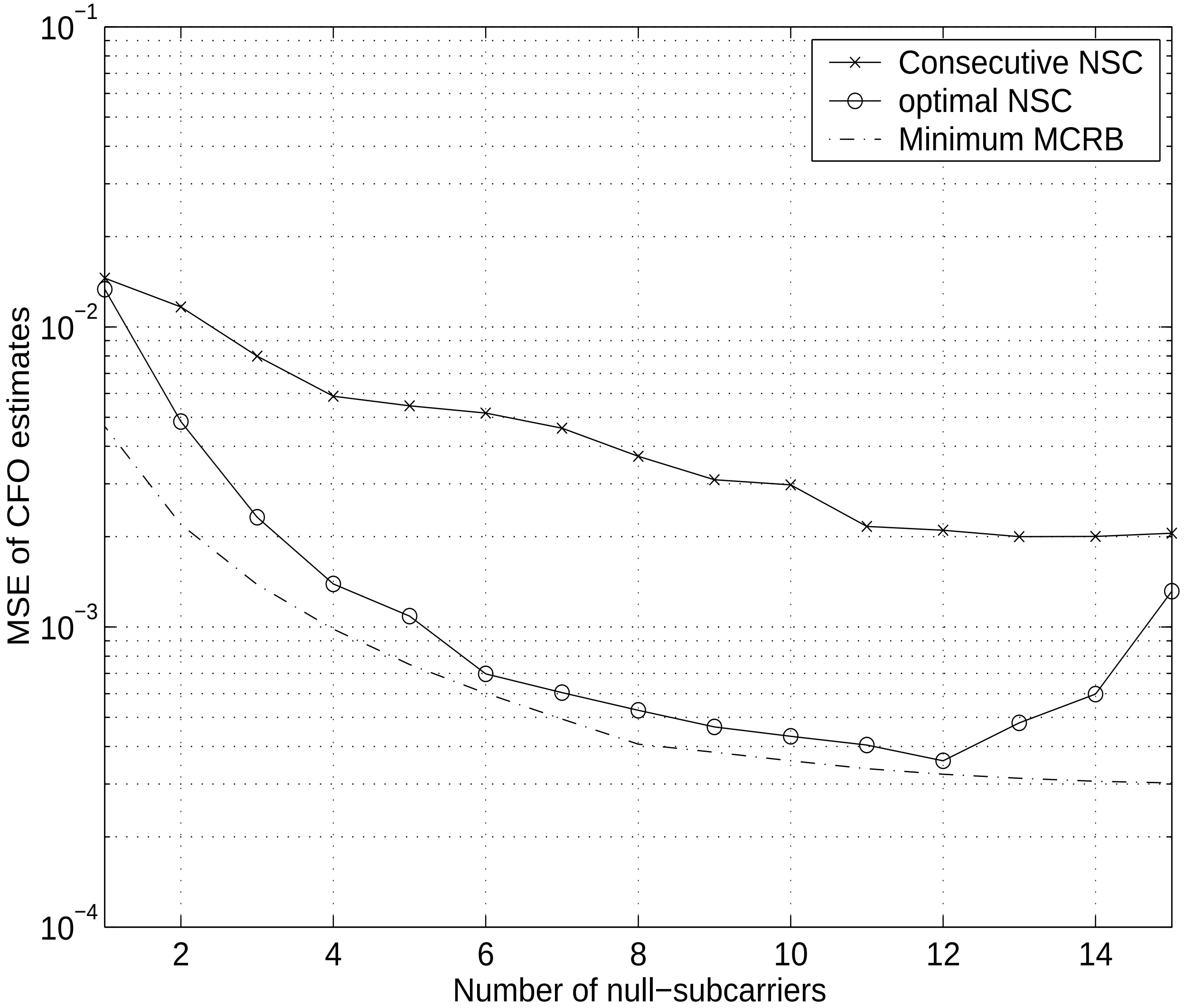
L = 8

$E\left\{|h_l|^2\right\} = e^{0.2l}$

Distinct:

1,2,4,7,...,56

Performance Analysis (2)



1 OFDM block

N = 16

L = 4

$E\left\{|h_l|^2\right\} = e^{0.2l}$

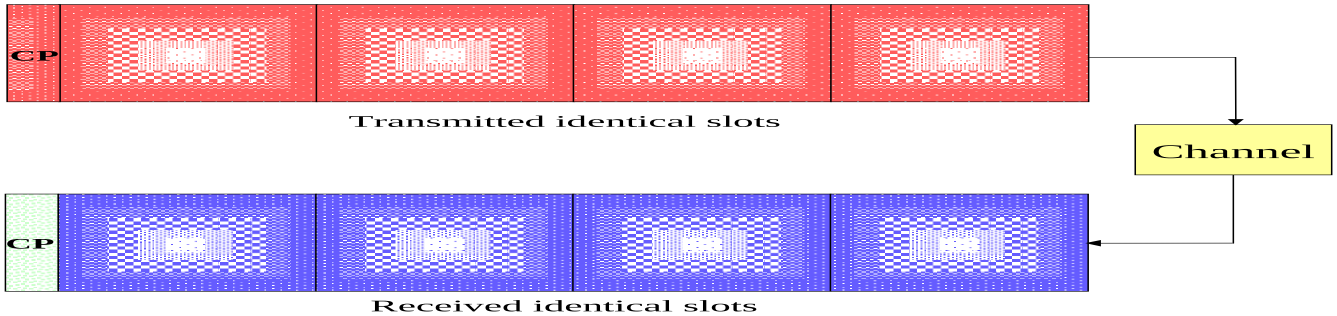
$\nu_o \in [-2,2)$

SNR = 15 dB

QPSK

Repetitive Slot-Based CFO Estimation

Motivation: CFO acquisition not requiring channel estimation



☞ J identical slots obtained by nulling all carriers not multiples of J

- $\boldsymbol{u} := \mathbf{F}^H \boldsymbol{s}$ made of J identical slots $(N = JQ) \rightarrow u(k) = u(k + \ell Q)$, $k = 0 \ldots Q-1;\ \ell = 0 \ldots J-1$

$$\hookrightarrow \quad x(k+\ell Q) = z(k)\, e^{j2\pi\nu\ell/J} + w(k+\ell Q)$$

$$z(k) = e^{j2\pi\nu k/N} H_c(k,:)\boldsymbol{u}$$

Repetitive Slot-Based CFO Estimation (2)

- We ignore the dependence between $\boldsymbol{z}$ and ν. Nonlinear Least Squares Estimator (NLLS):

$$\{\hat{\nu}_{REP}, \hat{\boldsymbol{z}}\} = \min_{\nu, \boldsymbol{z}} \sum_{\ell=0}^{J-1} \sum_{k=0}^{Q-1} \left| x(k + \ell Q) - z(k) e^{j2\pi\nu\ell/J} \right|^2$$

$$\hookrightarrow \qquad \hat{\nu}_{REP} = \arg\max_{\nu} \sum_{k=0}^{Q-1} \xi_\nu(k)$$

$$\xi_\nu(k) = \frac{1}{J} \left| \sum_{\ell=0}^{J-1} e^{-j2\pi\ell\nu/J} x(k + \ell Q) \right|^2$$

✍ Acquisition range increases with J: $-\frac{J}{2} \leq \hat{\nu}_{REP} < \frac{J}{2}$

Repetitive Slot-Based CFO Estimation (3)

- NLS estimator can be rewritten as

$$\hat{\nu}_{REP} = \arg\max_{\nu} \sum_{m=1}^{J-1} \mathrm{Re}\left[r(mQ) e^{-j2\pi m\nu/J} \right]$$

$$r(\tau) = \sum_{k=0}^{M-\tau-1} x^*(k) x(k+\tau)$$

  - if $J = 2$, $\rightarrow$ closed-form solution (Schmidl/Moose algorithms)

$$\hat{\nu}_{REP} = \frac{1}{\pi} \arg\{r(N/2)\}$$

  - if $J > 2$, $\rightarrow$ no closed-form solution...

Repetitive Slot-Based CFO Estimation (4)

- Relationship between DML and NLS estimators

  - Repetition of identical slots: VSC absent

    - $\mathcal{K} = \{mJ,\ m = 0, ..., M/J - 1\}$ and

$$\psi_{\mathcal{K}}(\tau) = \frac{K}{M}\delta(\tau - mQ) \qquad m = 0, \pm 1, \pm 2, ...$$

    - The repetitive slot-based and NSC-based are identical:

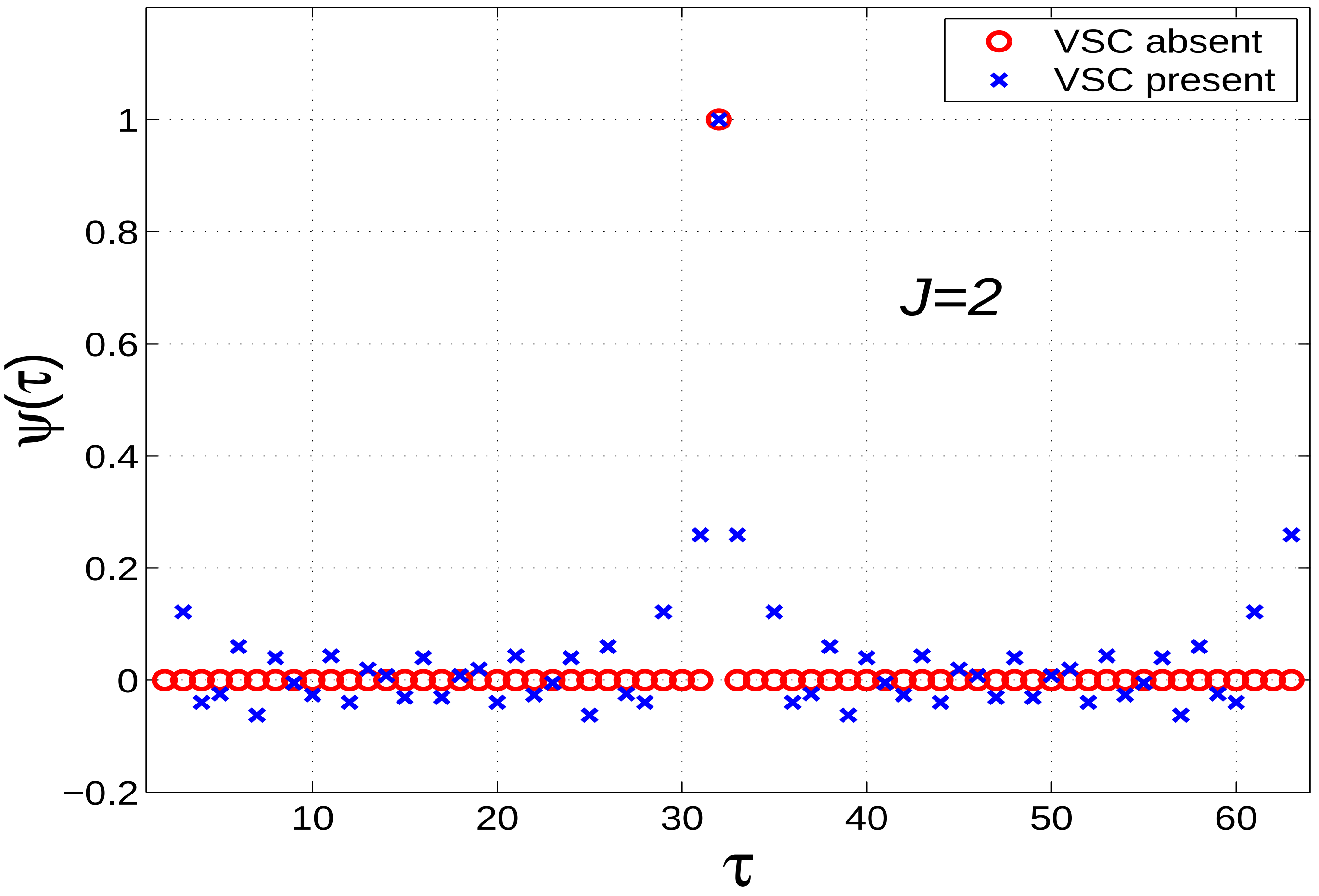
$$\hat{\nu}_{REP} \equiv \hat{\nu}_{NSC}$$

if no VSC (consecutive NSC dictated by system design)

Repetitive Slot-Based CFO Estimation (5)

- Relationship between DML and NLS estimators (cont.)

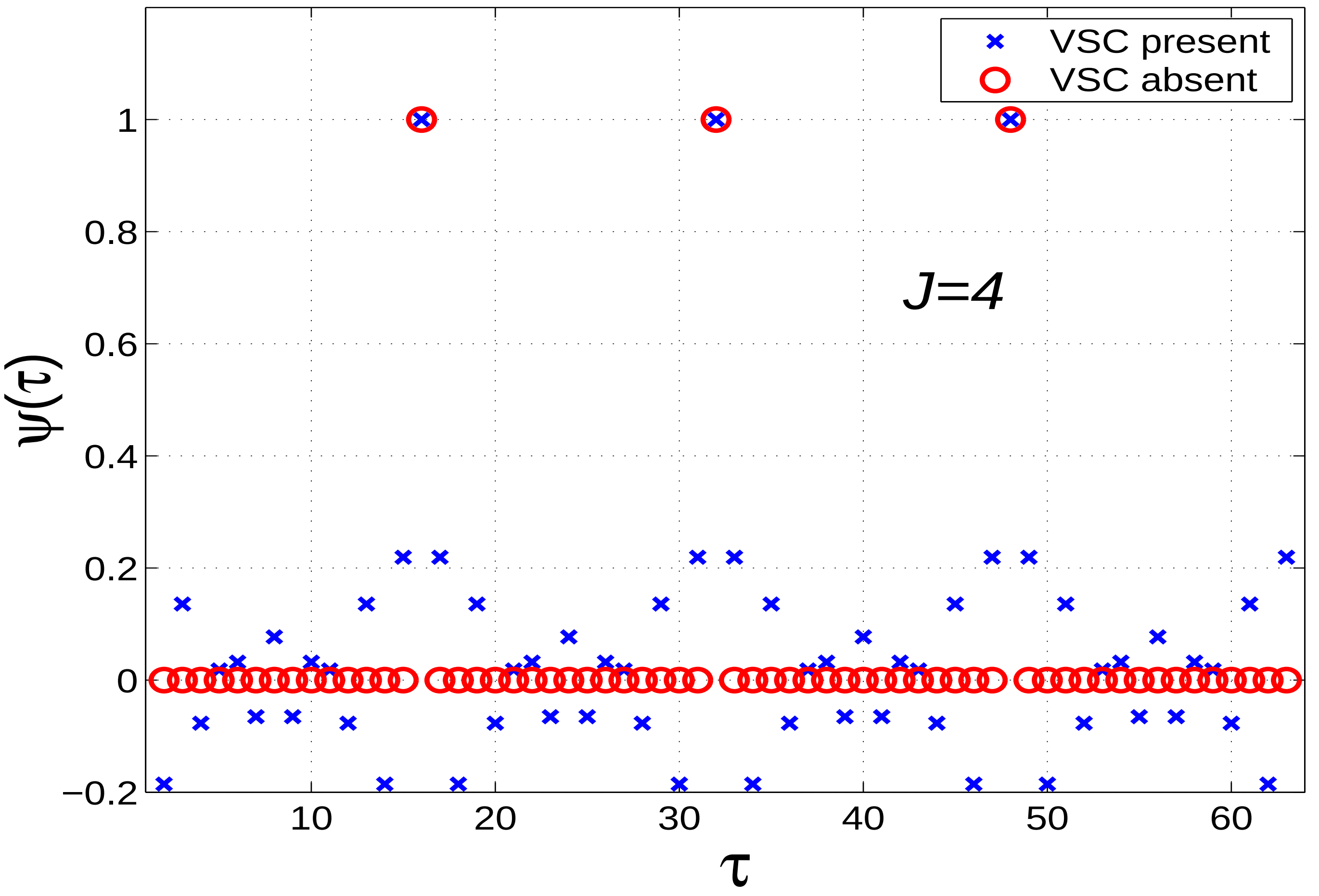
Plot of $\psi(\tau)$; $N = 64$;15 VSCs



Repetitive Slot-Based CFO Estimation (6)

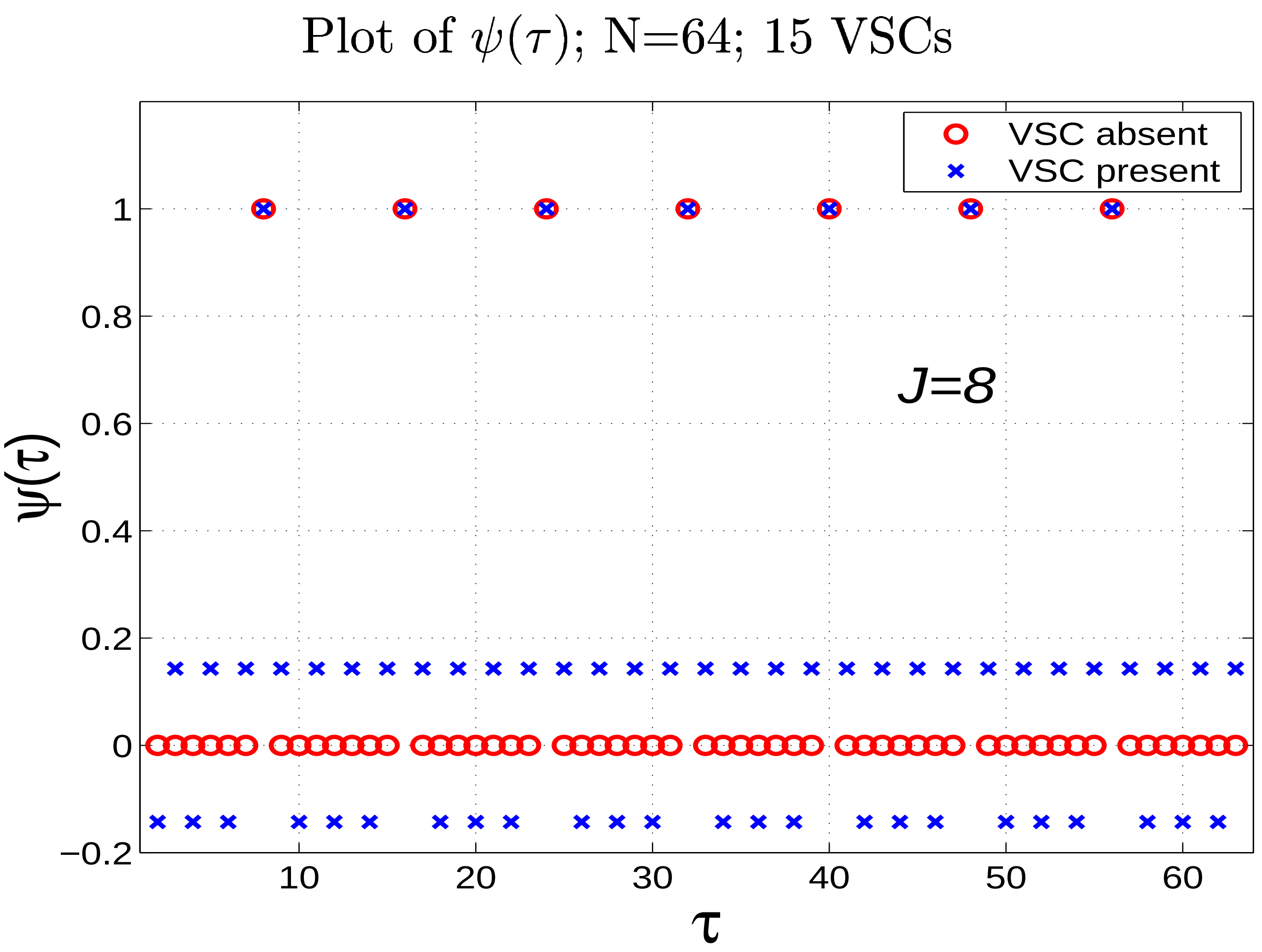
- Relationship between DML and NLS estimators: J=4

Plot of $\psi(\tau)$; N=64; 15 VSCs



Repetitive Slot-Based CFO Estimation (7)

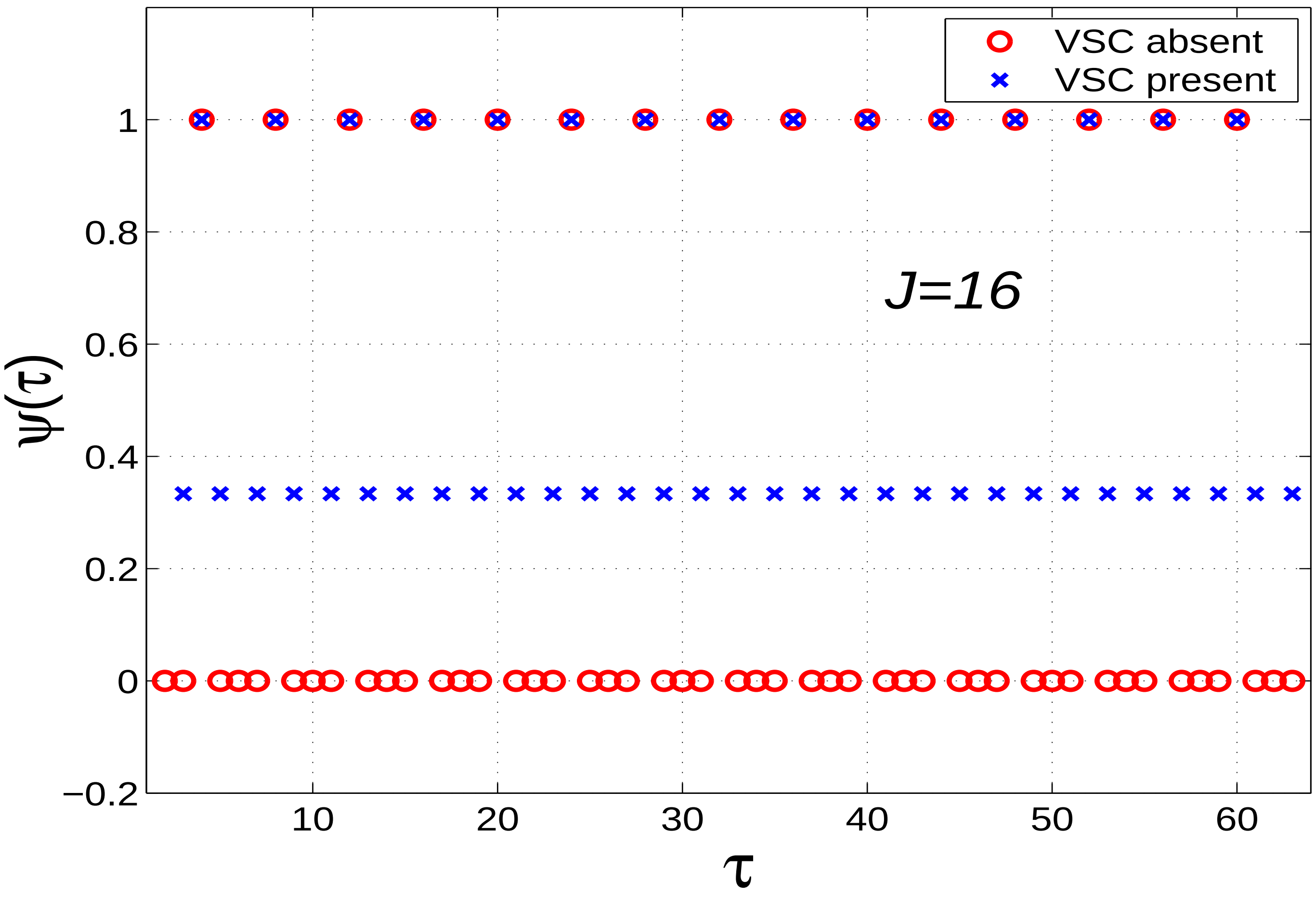
- Relationship between DML and NLS estimators: J=8



Repetitive Slot-Based CFO Estimation (8)

- Relationship between DML and NLS estimators: J=16

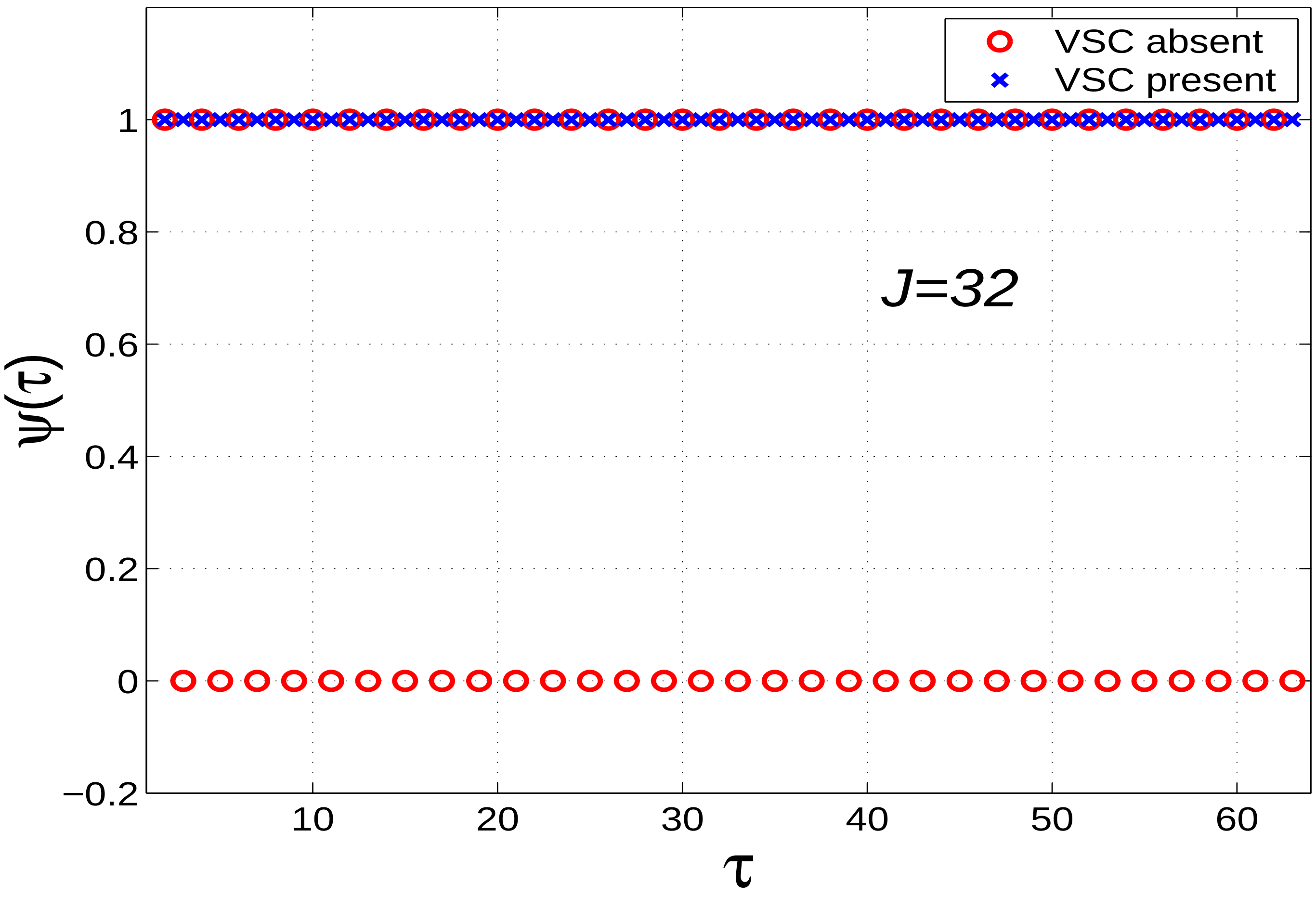
Plot of $\psi(\tau)$; N=64; 15 VSCs



Repetitive Slot-Based CFO Estimation (9)

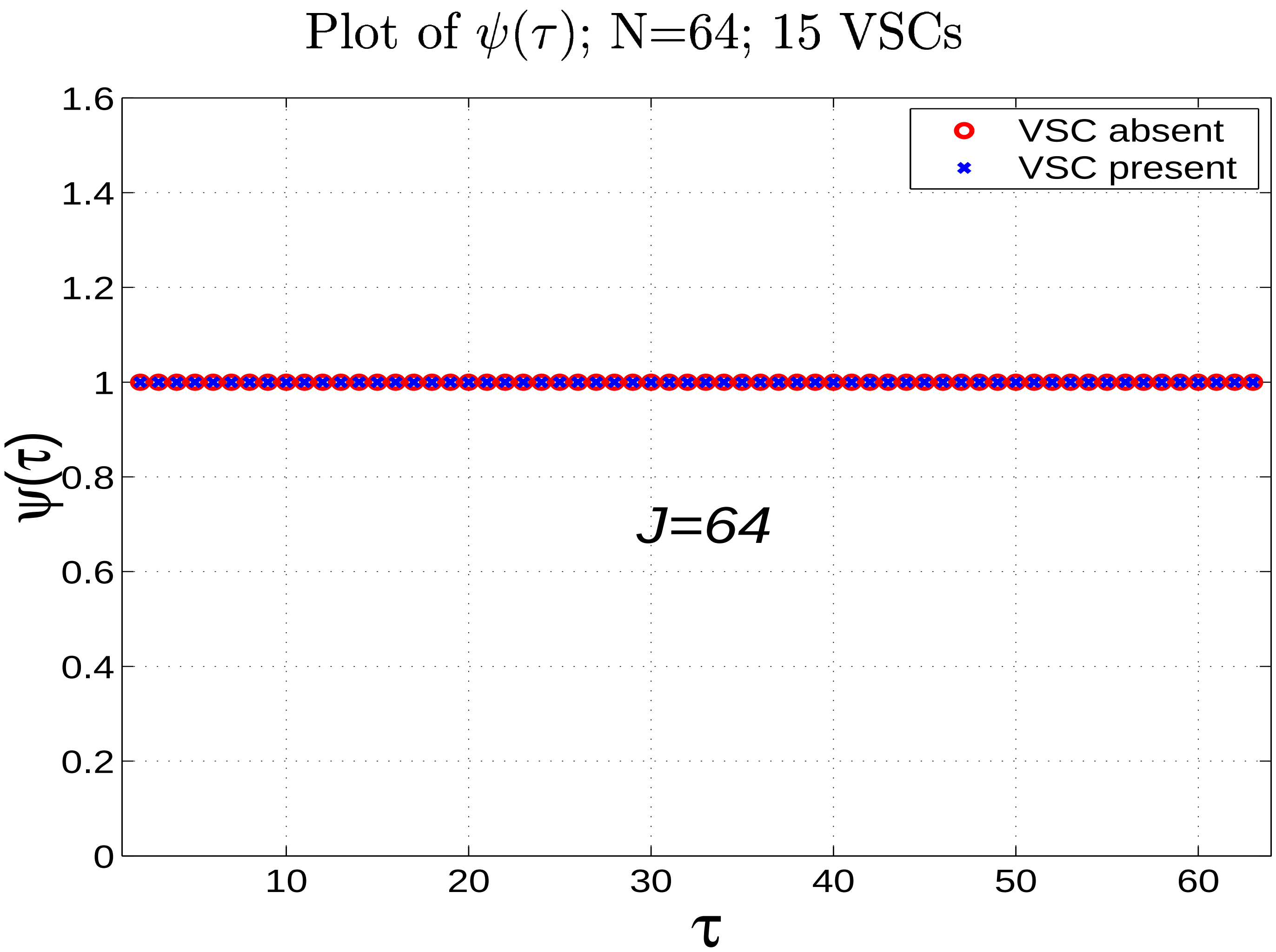
- Relationship between DML and NLS estimators: J=32

Plot of $\psi(\tau)$; N=64; 15 VSCs



Repetitive Slot-Based CFO Estimation (10)

- Relationship between DML and NLS estimators: J=64



Repetitive Slot-Based CFO Estimation (11)

- Relationship between DML and NLS estimators (cont.)

  - Repetition of identical slots: VSC present

    - Most of the correlation coefficients contribute to the ML estimator
    - $\hat{\nu}_{REP}$ consists of using only the $(J-1)$ highest correlation coefficients, and is therefore an approximate ML estimator.
    - DML is computationally more demanding than NLS.
    - If $J=2$, NLS is obtained in closed-form. If $J>2$, no closed-form expression. Approximations given by the following algorithms.

Repetitive Slot-Based CFO Estimation (12)

- The 'BLUE' estimator: optimal combining of the correlations' phases.
  - To avoid phase wrapping, the algorithm is based on

$$\varphi(m) = [\arg\{r(mQ)\} - \arg\{r((m-1)Q)\}]_{2\pi}$$

  - Deriving the average (over Rayleigh channel) statistics of the $\varphi(m)$'s, the BLUE estimator is

$$\breve{\nu}_{REP} = \frac{J}{2\pi} \sum_{m=1}^{p} w(m)\varphi(m)$$

p: design parameter (optimum value=$J/2$) and

$$w(m) = 3\frac{(J-m)(J-m+1) - p(J-p)}{p(4p^2 - 6pJ + 3J^2 - 1)}$$

☞ The amplitude of the correlations not exploited in BLUE...

Repetitive Slot-Based CFO Estimation (13)

- Approximate NLLS (ANNLS) estimator

  - Rewrite the NLS criterion

$$\sum_{m=1}^{J-1} |r(mQ)| \cos(\phi_m - 2\pi m\nu/J)$$

ϕ_m: unwrapped phase of $r(mQ)$

  - Small error approx. $\sin(\phi_m - j2\pi m\nu/J) \approx (\phi_m - j2\pi m\nu/J) \rightarrow$ ANLS estimator:

$$\tilde{\nu}_{REP} = \frac{J}{2\pi} \frac{\sum_{m=1}^{J-1} m|r(mQ)|\phi_m}{\sum_{m=1}^{J-1} m^2|r(mQ)|}$$

Repetitive Slot-Based CFO Estimation (14)

- Optimum number of identical slots (cont.)

❑ The repetitive-slot structure-based Conditional CRB:

$$\mathrm{CCRB}(\nu) = \frac{3}{2\pi^2 N(1 - 1/J^2)\ SNR}\ \frac{1}{\gamma_H}$$

where we assumed no VSC and $|s_m| = 1, \forall m$ and where

$$\gamma_H = \sum_{m=0}^{N/J-1} \frac{|H_{nJ}|^2}{\sigma_H^2};$$

frequency diversity decreases with J

❑ Averaged CCRB:

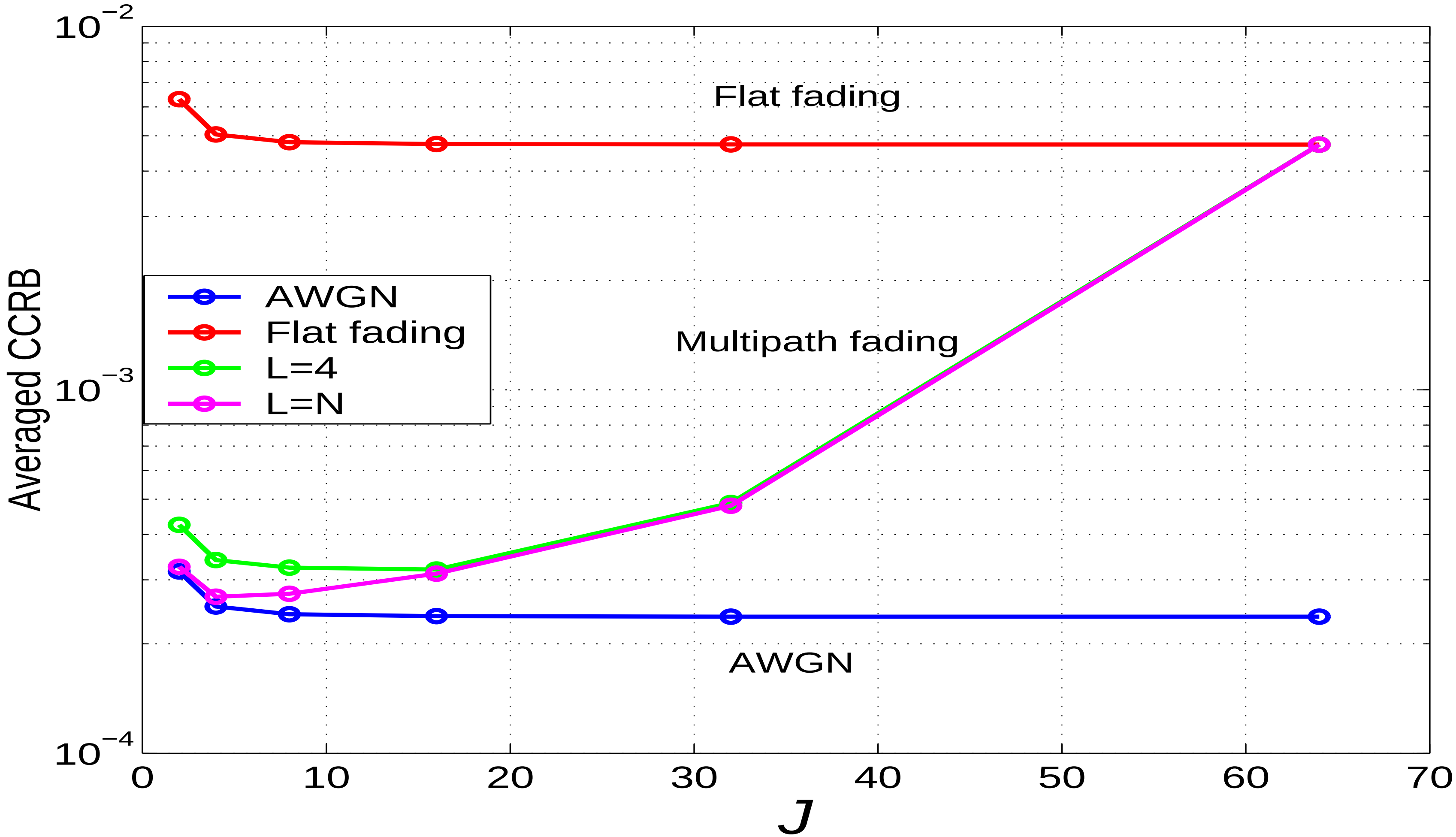
$$\mathrm{ACCRB}(\nu) = \frac{3}{2\pi^2 N(1 - 1/J^2)\ SNR} E\left\{\frac{1}{\gamma_H}\right\}$$

→ no closed-form expression

→ Monte-Carlo simulations

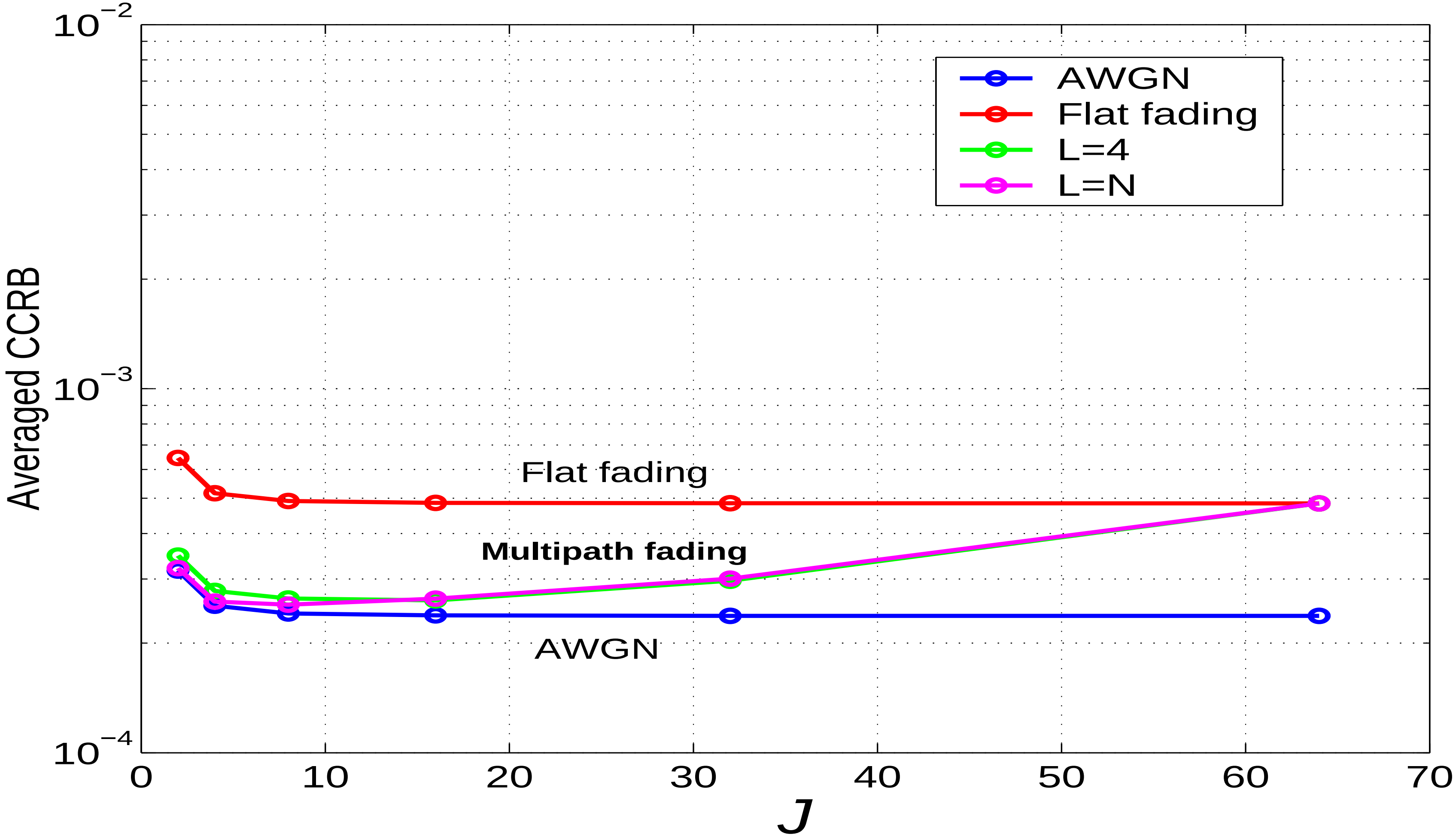
Repetitive Slot-Based CFO Estimation (15)

- Optimum number of identical slots (cont.) Rayleigh channel



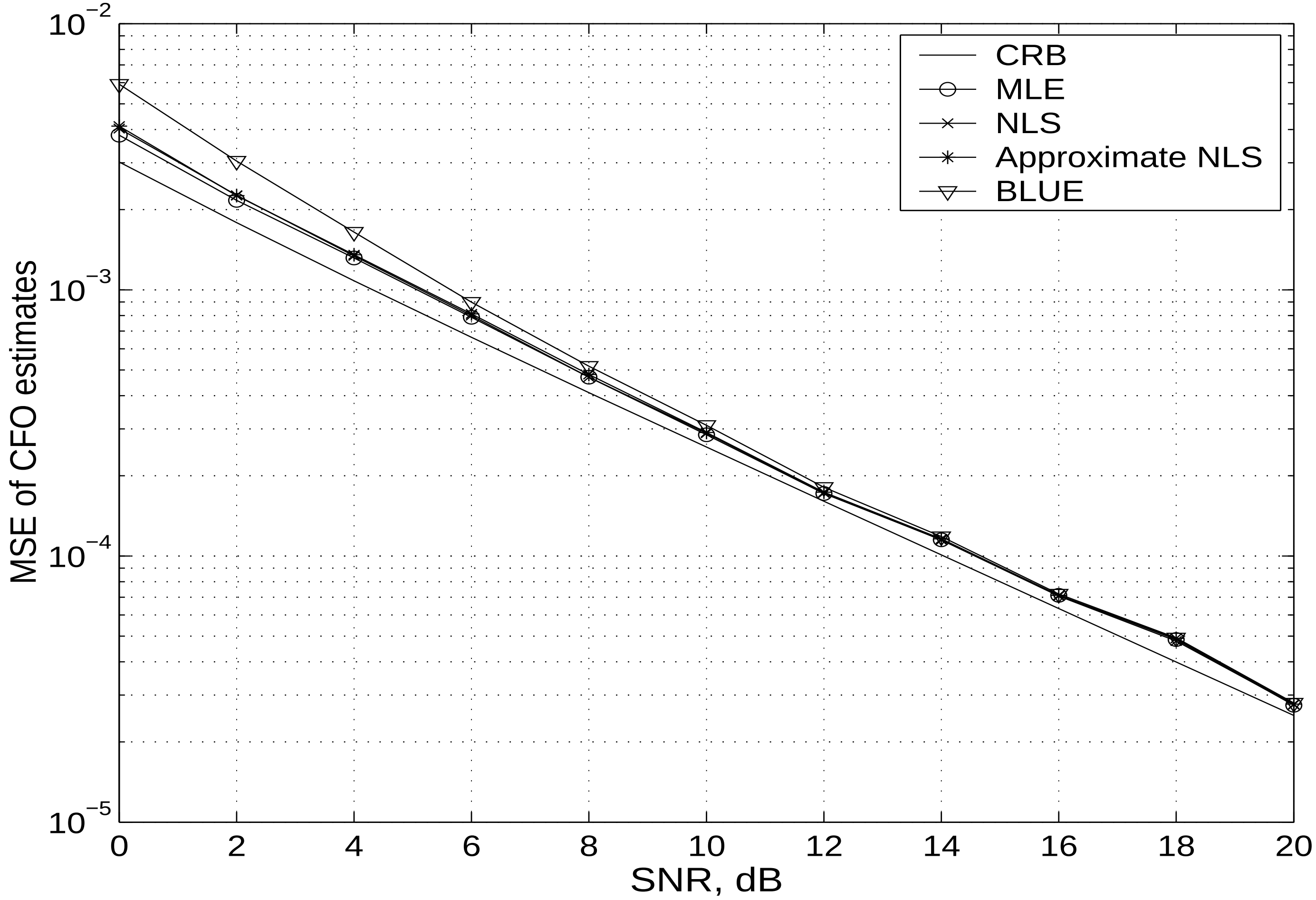
Repetitive Slot-Based CFO Estimation (16)

- Optimum number of identical slots (cont.) Ricean channel $\kappa = 4$



Comparisons

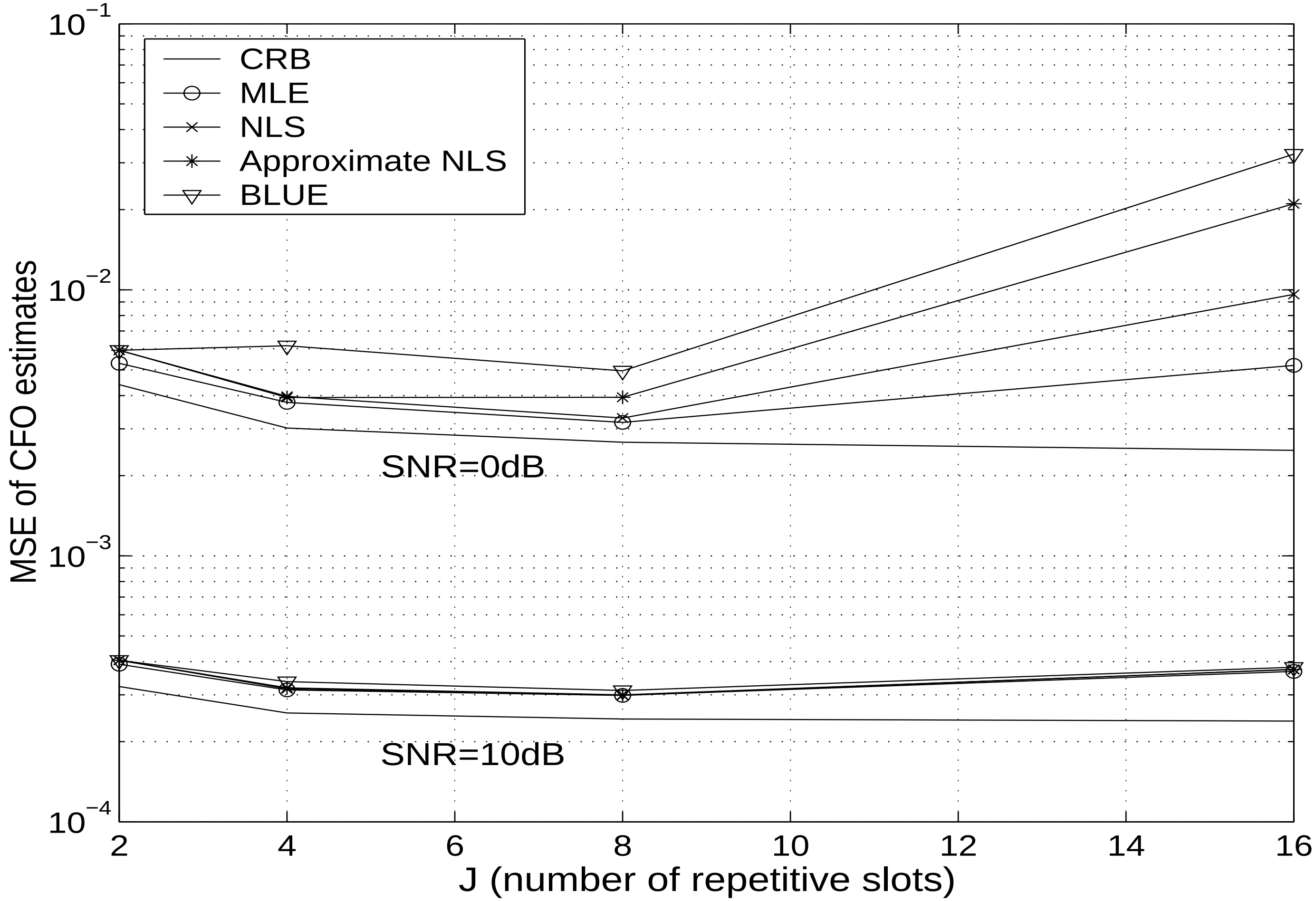
- MSE vs. SNR, J=4



$N = 64,\ N_a = 49,\ L = 15$, CFO $\in [-2, 2]$, $E\left\{|h_\ell|^2\right\} = e^{-0.2\ell}$, QPSK

Comparisons (2)

- MSE vs. # Repeated slots, J



Summary

- A computationally efficient algorithm
- Analytical performance analysis and CRB
- Relationship between the repetitive slot-based and the NSC-based MLE
  - Equivalent in the absence of VSC's
  - NSC is better if VSC's present

Part 2: Blind CFO estimation

Outline

- Constant-modulus algorithm
- Finite-alphabet algorithm
- Comparative study

Constant-Modulus Algorithm

- Assuming $|s_n| = 1, \ \forall n$, wlog

$$\hookrightarrow \quad H_n s_n = |H_n| e^{j\theta_n}; \qquad \theta_n = \angle H_n s_n$$

$$\hookrightarrow \quad x(k) = e^{j2\pi k\nu_o/N} \sum_{n\in\mathcal{A}} |H_n| e^{j\theta_n} e^{j2\pi kn/N} + w(k), \quad k = 0, ..., N-1$$

- The $|H_n|$'s are parameterized by only $(L+1)$ coefficients, the h_ℓ's
- The $H_n s_n$'s are parameterized by only $(N_a + L + 1)$ coefficients instead of $2N_a$, $\qquad (N_a = card(\mathcal{A}))$
- $w(k)$ is assumed AWGN

Constant-Modulus Algorithm (2)

- Deterministic Max-Likelihood
  - Treat $\{|H_n|\}, \{\theta_n\}$ as non-random unknowns
  - DML criterion

$$J(\nu, |\mathbf{H}|, \boldsymbol{\theta}) = \sum_{k=0}^{N-1} \left| x(k) - e^{j2\pi k\nu/N} \sum_{n\in\mathcal{A}} |H_n| e^{j\theta_n} e^{j2\pi kn/N} \right|^2$$

- can be rewritten as

$$J(\nu, |\mathbf{H}|, \boldsymbol{\theta}) = \sum_{k=0}^{N-1} |x(k)|^2 + \sum_{n\in\mathcal{A}} |H_n|^2 - 2N\mathrm{Re}\left[\sum_{n\in\mathcal{A}} |H_n| X(n+\nu) e^{-j\theta_n}\right]$$

- $X(f)$: DTFT of $\{x(k)\}$ at frequency f/N

$$X(f) = \sum_{k=0}^{N-1} x(k) e^{-j2\pi kf/N}$$

Constant-Modulus Algorithm (3)

- Deterministic Max-Likelihood, cont.

❑ Setting $\partial J/\partial\theta_n = 0$,

$$\widehat{\theta}_n = \arg\{X(n+\nu)\}$$

- If $|H_n| = 0$, θ_n becomes non-identifiable
- $N_a > L$ ensures that $H_n \neq 0$, $\forall n \in \mathcal{A}$

❑ DML of $\{H_n\}$ and ν_o obtained by minimizing

$$\begin{aligned} J(\nu, |\mathbf{H}|) &= J_{VSC}(\nu) + J_A(\nu, |\mathbf{H}|) \\ J_{VSC}(\nu) &= \sum_{n\in\mathcal{Z}} |X(n+\nu)|^2 && \text{due to VSC} \\ J_A(\nu, |\mathbf{H}|) &= \sum_{n\in\mathcal{A}} (|X(n+\nu)| - |H_n|)^2 && \text{due to CM} \end{aligned}$$

Constant-Modulus Algorithm (4)

- Non-Dispersive Channel

  - $H_n = h_0, \ \forall n \in \mathcal{A}$. Criterion becomes

$$\begin{aligned} J(\nu, |\mathbf{H}|) &= \sum_{n\in\mathcal{Z}} |X(n+\nu)|^2 + \sum_{n\in\mathcal{A}} \left(|X(n+\nu)| - |h_0|\right)^2 \\ &= \sum_{n=0}^{N-1} |X(n+\nu)|^2 + N_a|h_0|^2 - 2|h_0| \sum_{n\in\mathcal{A}} |X(n+\nu)| \end{aligned}$$

  - DML of CFO:

$$\hat{\nu}_o = \arg\max_{\nu} \sum_{n\in\mathcal{A}} |X(n+\nu)|$$

☞ VSC-based estimator is equivalently obtained by maximizing the L_2-norm

$$\arg\min_{\nu} J_{VSC}(\nu) = \arg\max_{\nu} \sum_{n\in\mathcal{A}} |X(n+\nu)|^2$$

Constant-Modulus Algorithm (5)

- Dispersive Channel
  - $J_{VSC}(\nu)$ is not a function of $|\mathbf{H}|$
  - $J_A(\nu, |\mathbf{H}|)$ should be minimized wrt $|\mathbf{H}|$ under the constraint:

$$|H_n|^2 = \sum_{l,p=0}^{L} h_l h_p^* e^{-j2\pi(l-p)n/N}$$

  - we modify $J_A(\nu, |\mathbf{H}|)$ into

$$J'_A(\nu, |\mathbf{H}|) = \sum_{n \in \mathcal{A}} \left(|X(n+\nu)|^2 - |H_n|^2 \right)^2$$

Constant-Modulus Algorithm (6)

- Dispersive Channel, cont.

  - $|H_n|^2$ can be re-parameterized as

$$|H_n|^2 = \boldsymbol{c}_n^T \boldsymbol{\lambda}, \qquad n \in \mathcal{A}$$

$$\begin{aligned}
\boldsymbol{c}_n &= [1, \sqrt{2}\cos(2\pi n/N), \cdots, \sqrt{2}\cos(2\pi nL/N), \\
&\quad \sqrt{2}\sin(2\pi n/N), \cdots, \sqrt{2}\sin(2\pi nL/N)]^T \\
\boldsymbol{\lambda} &= [g_0, \sqrt{2}\mathrm{Re}[g_1], \cdots, \sqrt{2}\mathrm{Re}[g_L], \sqrt{2}\mathrm{Im}[g_1], \cdots, \sqrt{2}\mathrm{Im}[g_L]]^T \\
g_i &= \sum_{l=0}^{L-i} h_l^* h_{l+i}
\end{aligned}$$

Constant-Modulus Algorithm (7)

- Dispersive Channel, cont.

❑ $\boldsymbol{\lambda}$ estimate:

$$\hat{\boldsymbol{\lambda}} = \arg\min_{\boldsymbol{\lambda}} J'_A(\nu, |\mathbf{H}|) = \mathbf{C}_2^{\dagger} \sum_{n\in\mathcal{A}} |X(n+\nu)|^2 \boldsymbol{c}_n \ ,$$

$$\mathbf{C}_2 := \sum_{m\in\mathcal{A}} \boldsymbol{c}_m \boldsymbol{c}_m^T \ .$$

❑ CFO estimate: obtained by minimizing $J(\nu) = J_{VSC}(\nu) + J_{CM}(\nu)$

$$J_{VSC}(\nu) = \sum_{n\in\mathcal{Z}} |X(n+\nu)|^2; \quad J_{CM}(\nu) = \sum_{n\in\mathcal{A}} \left(|X(n+\nu)| - \sqrt{Y(n;\nu)}\right)^2$$

$$Y(n;\nu) = \boldsymbol{c}_n^T \mathbf{C}_2^{\dagger} \sum_{n\in\mathcal{A}} |X(n+\nu)|^2 \boldsymbol{c}_n$$

Constant-Modulus Algorithm (8)

- Dispersive Channel, cont.
  - The proposed VSC&CM estimate:

$$\hat{\nu}_o = \arg\min_{\nu} \sum_{n\in\mathcal{A}} \left(Y(n;\nu) - 2|X(n+\nu)|\sqrt{Y(n;\nu}\right)$$

$$Y(n;\nu) = \boldsymbol{c}_n^T \mathbf{C}_2^\dagger \sum_{n\in\mathcal{A}} |X(n+\nu)|^2 \boldsymbol{c}_n$$

$$\mathbf{C}_2 := \sum_{m\in\mathcal{A}} \boldsymbol{c}_m \boldsymbol{c}_m^T \qquad (\mathrm{pre-computatble})$$

$$X(f) = \frac{1}{N} \sum_{k=0}^{N-1} x(k) e^{-j2\pi kf/N}$$

Constant-Modulus Algorithm (9)

- Extension to Multiple Blocks: Time-Invariant Channel
  - Signal model for M blocks: (CFO and fading assumed constant across the set of blocks)

$$x_m(k) = e^{j2\pi k\nu_o/N} \sum_{n\in\mathcal{A}} H_n s_{m,n} e^{j2\pi kn/N} + w_m(k), \qquad m = 1, .., M$$

  - VSC&CM CFO estimate:

$$\hat{\nu}_o = \arg\min_{\nu} \sum_{n\in\mathcal{A}} \left[Z(n;\nu) - 2\left(\frac{1}{M}\sum_{m=1}^{M} |X_m(n+\nu)|\right)\sqrt{Z(n;\nu)} \right]$$

$$Z(n;\nu) = \boldsymbol{c}_n^T \mathbf{C}_2^\dagger \sum_{n\in\mathcal{A}} \left(\frac{1}{M}\sum_{m=1}^{M} |X_m(n+\nu)|^2\right) \boldsymbol{c}_n$$

Constant-Modulus Algorithm (10)

- Extension to Multiple Blocks: Time-varying Channel

  - Signal model for M blocks:

$$x_m(k) = e^{j2\pi k\nu_o/N} \sum_{n\in\mathcal{A}} H_{m,n} s_{m,n} e^{j2\pi kn/N} + w_m(k)$$

  - VSC&CM CFO estimate:

$$\hat{\nu}_o = \arg\min_{\nu} \sum_{m=1}^{M} J_m(\nu)$$

$$J_m(\nu) = \sum_{n\in\mathcal{A}} \left(Y_m(n;\nu) - 2|X_m(n+\nu)|\sqrt{Y_m(n;\nu}\right)$$

Finite-Alphabet Algorithm

❑ PSK constellations of size M satisfy:

$$s_n^M = 1$$

→ In the noiseless case

$$[X(n+\nu_o)]^M = H_n^M = \left[\sum_{l=0}^{L} h_l e^{-j2\pi ln/N}\right]^M = \sum_{l=0}^{ML} v_l e^{-j2\pi ln/N} = \boldsymbol{\gamma}_n^H \mathbf{v}$$

- $\boldsymbol{\gamma}_n = [1, e^{j2\pi n/N}, ..., e^{j2\pi MLn/N}]^T$; $\mathbf{v}$: $(ML+1) \times 1$

Finite-Alphabet Algorithm (2)

- Proposed criterion:

$$
\begin{aligned}
J(\nu) &= wJ_{VSC}(\nu) + (1-w)\bar{J}_{FA}(\nu, \mathbf{v}) \\
\bar{J}_{FA}(\nu, \mathbf{v}) &= \sum_{n \in \mathcal{A}} \left| [X(n+\nu)]^M - \boldsymbol{\gamma}_n^H \mathbf{v} \right|^2
\end{aligned}
$$

- If $ML + 1 < N_a$, $\boldsymbol{u}$ can be estimated as:

$$
\hat{\mathbf{v}} = \boldsymbol{\Gamma}^{\dagger} \sum_{n \in \mathcal{A}} [X(n+\nu)]^M \boldsymbol{\gamma}_n \ ,
$$

$$
\boldsymbol{\Gamma} := \sum_{n \in \mathcal{A}} \boldsymbol{\gamma}_n \boldsymbol{\gamma}_n^H \ .
$$

Finite-Alphabet Algorithm (3)

❑ The finite alphabet-based criterion becomes

$$J_{FA}(\nu) = \sum_{n \in \mathcal{A}} \left| [X(n+\nu]^M - Z(n;\nu) \right|^2$$

- $Z(n;\nu) = \boldsymbol{\gamma}_n^H \boldsymbol{\Gamma}^\dagger \sum_{n \in \mathcal{A}} [X(n+\nu)]^M \boldsymbol{\gamma}_n$

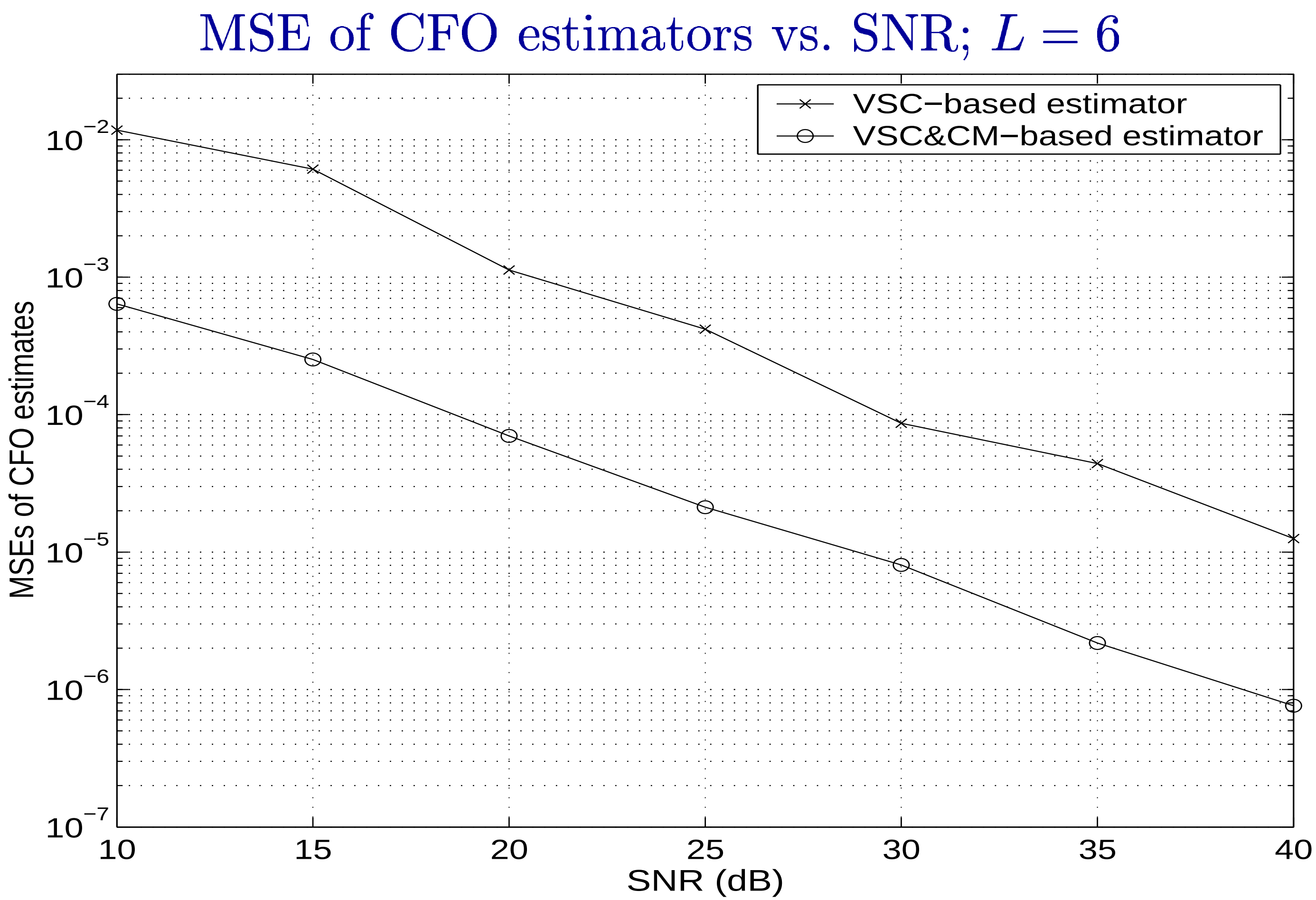
↪ Proposed VSC&FA-based estimator:

$$\hat{\nu}_o = \arg\min_{\nu} \; [\; w \; J_{VSC}(\nu) \; + \; (1-w) \; J_{FA}(\nu) \;]$$

w: weight parameter to be adjusted. If no VSC, $w = 0$.

Comparative Study

- VSC vs CM: performance vs SNR.

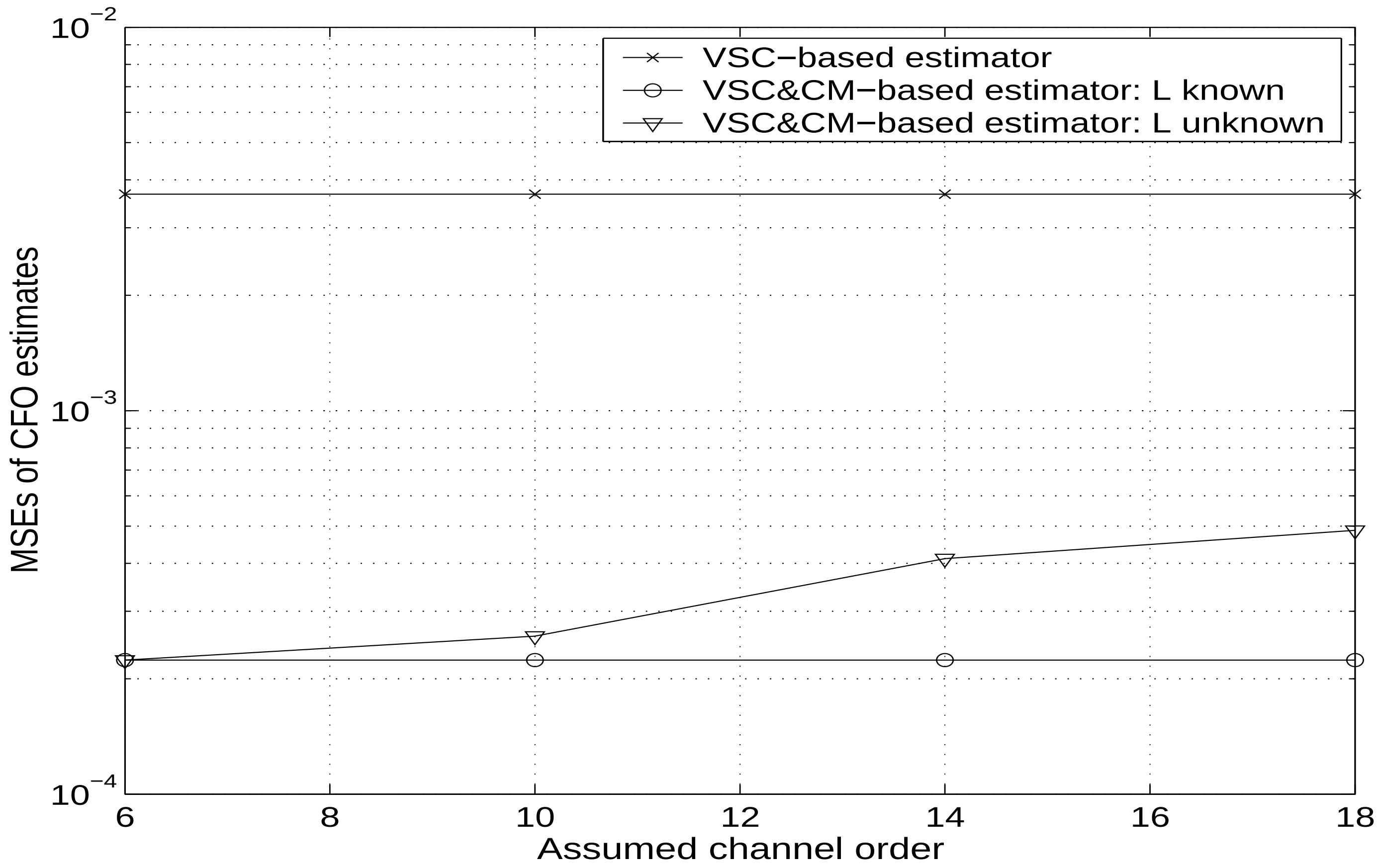


$N = 64$, $N_a = 49$, CFO in $[-2, 2]$ and $E\left\{|h_\ell|^2\right\} = e^{-0.2\ell}$; 8PSK.

Comparative Study (2)

- VSC vs CM: unknown channel order.

MSE of CFO estimators vs. assumed L; actual $L = 6$

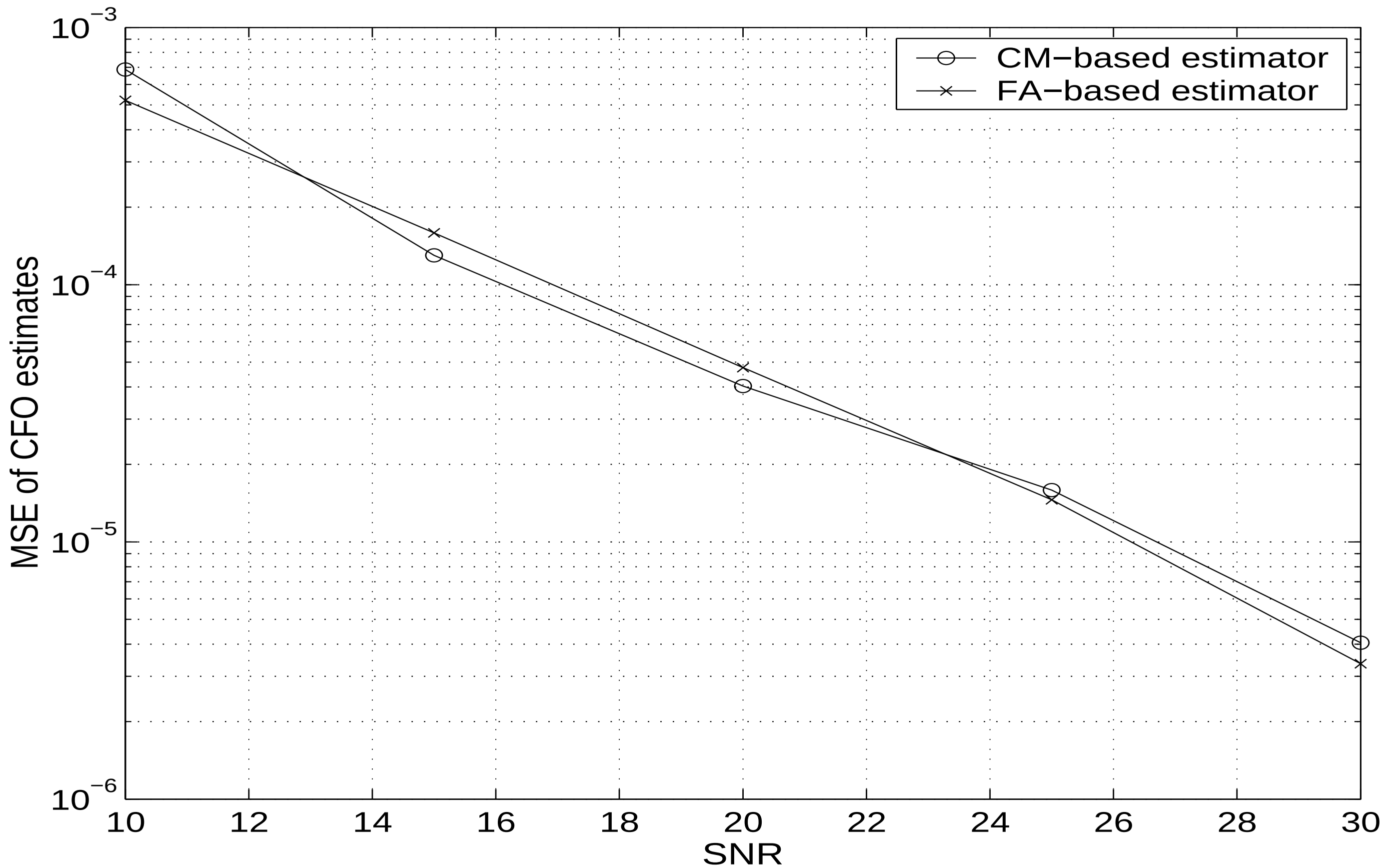


$N = 64,\ N_a = 49$, CFO in $[-2, 2]$ and $E\left\{|h_\ell|^2\right\} = e^{-0.2\ell}$; 8PSK.

Comparative Study (3)

- CM versus FA: BPSK case

MSE of CFO estimators vs. SNR; actual $L = 6$



$N = N_a = 64$, CFO in $[-2, 2]$ and $E\left\{|h_\ell|^2\right\} = e^{-0.2\ell}$

Summary

- CMA greatly outperforms VSC-based estimators
- CMA works even when the system is fully loaded
- CMA outperforms FA for M-PSK with $M > 2$
- Performance of CM close to data-aided algorithms
- Complexity is however greater than VSC and data-aided algorithms.

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