Estimation Theory for Wirelss Communication, 24-28 Oct 2005, PARIS.

Channel Estimation for Cyclic Prefixed Block Transmissions

M. Ghogho University of Leeds, UK

Aims

- To describe a few channel estimation techniques for cyclic-prefixed (CP) block transmissions, including OFDM and single-carrier (SC-)CP systems
- To address the issue of optimum training design and power allocation
- To introduce a new bandwidth efficient pilot assisted transmission technique

Outline

- Introduction
- Channel estimation for OFDM
  - OFDM signal model and preliminaries
  - Pilot-based channel estimation for OFDM
  - Blind channel estimation for OFDM
- Channel estimation for general CP systems
  - Affine precoding and MMSE channel estimation
  - Full rank orthogonal precoding
  - Rank-deficient orthogonal precoding
- Summary

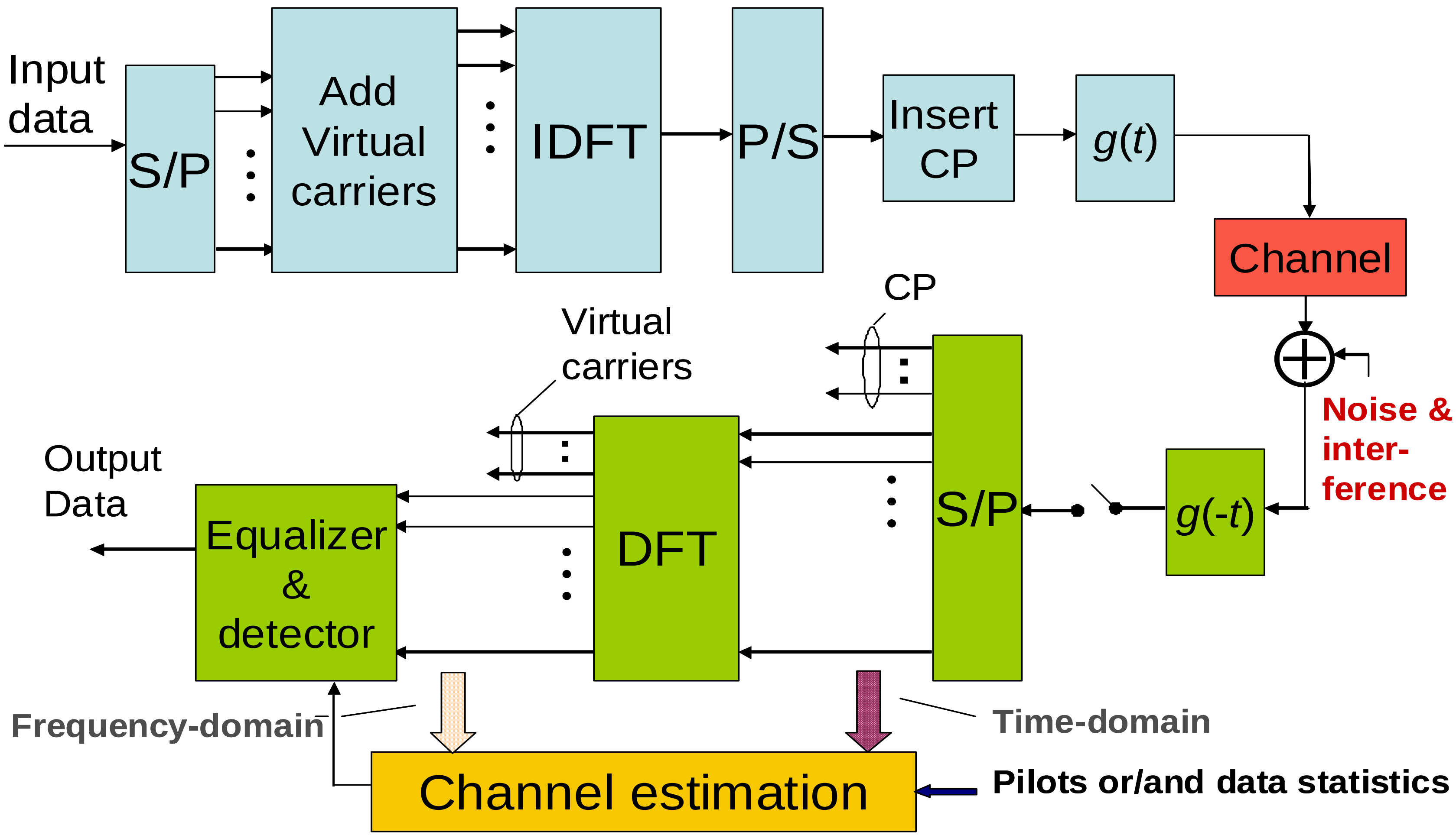
Introduction

- Why block transmissions?
  - existence of zero-forcing equalizer
  - block-by-block processing
- Why cyclic prefix?
  - FFT-based channel equalization
- Why channel estimation
  - required for coherent communication systems

Part 1: Channel Estimation for OFDM

OFDM signal model and preliminaries

- Block diagram



OFDM signal model and preliminaries (2)

- Frequency-domain (F-D) methods: either pilot-based or (semi-)blind
- Time-domain (T-D), generally (semi-)blind.

Assumptions:

- Channel impulse response (CIR) constant during each OFDM symbol

$$h(t) = \sum_{\ell=0}^{L} h_\ell \delta(t - \tau_\ell)$$

- $\tau_\ell = \ell T_s$, $T_s = T/N$ and T: duration of 1 OFDM block w/0 CP.
- $\boldsymbol{h} := [h_0 \cdots h_L]^T \sim \mathcal{CN}(0, \mathbf{R}_h)$, $\mathbf{R}_h = \mathrm{diag}\{\sigma^2_{h_\ell}, \ell = 0 \cdots L\}$
- length of CP $= L$. Additive noise is Gaussian and white with variance σ^2_v.

OFDM signal model and preliminaries (3)

- Notations

- N: DFT size • N_a: # active carriers • N_p: # pilot carriers
- $\mathcal{A}$ ($\mathcal{P}$): set of active (pilot) carriers; $\mathcal{P} \subseteq \mathcal{A} \subseteq \{0, \cdots N-1\}$
- $\mathbf{F} = (1/\sqrt{N})\{\exp(-j2\pi nk/N)\}_{n,k=0}^{N-1}$ • $\mathbf{W} = (\sqrt{N})\mathbf{F}(:, 0 : L)$
- $\mathbf{T}_a$: active carriers selection matrix ($N \times N_a$)
- $\mathbf{T}_p$: pilot carriers selection matrix ($N \times N_p$)
- $\mathbf{T}_d$: data carriers selection matrix ($N \times N_d$) with $N_d = N_a - N_p$
- $\mathbf{W}_a = \mathbf{T}_a^T\mathbf{W}$ • $\mathbf{W}_{\mathcal{P}} = \mathbf{T}_p^T\mathbf{W}$ • $\mathbf{W}_{\mathcal{D}} = \mathbf{T}_d^T\mathbf{W}$
- σ_p^2 (resp σ_s^2) total power of pilot (resp. data) carriers; ;$\sigma_t^2 := \sigma_p^2 + \sigma_s^2$.
- $\mathbf{D}_{\boldsymbol{z}} = \mathrm{diag}\{\boldsymbol{z}\}$.

OFDM signal model and preliminaries (4)

- VC insertion: $\mathbf{T}_{vc}$: N_a columns of a $N \times N$ identity matrix
- CP insertion: $\mathbf{T}_{cp} = \begin{bmatrix} \mathbf{0}_{L\times(N-L)}, & \mathbf{I}_L \\ \multicolumn{2}{c}{\mathbf{I}_N} \end{bmatrix}$
- Transmitted block: $\boldsymbol{u}_{\rm cp}(i) = \mathbf{T}_{cp}\mathbf{F}^{\mathcal{H}}\mathbf{T}_{sc}\boldsymbol{s}(i)$
- Input-output relationship $(N \geq N_a, P = L + N)$

$$x_{\rm cp}(n) = \sum_{l=0}^{L} h(l)u_{\rm cp}(n-l) + v_{\rm cp}(n)$$

OFDM signal model and preliminaries (5)

- Received blocks

$$\boldsymbol{x}_{\rm cp}(i) = [\mathbf{H}_1 \boldsymbol{u}_{\rm cp}(i) + \mathbf{H}_2 \boldsymbol{u}_{\rm cp}(i-1)] + \boldsymbol{v}(i)$$

- Discard CP to avoid IBI: $\mathbf{R}_{cp} := [\mathbf{0}_{N\times(P-N)}, \mathbf{I}_N] \rightarrow \mathbf{R}_{cp}\mathbf{H}_2 = \mathbf{0}$.
- Channel matrix: $\mathbf{H}_1$ Toeplitz $\Rightarrow$ $\mathbf{H}_c = \mathbf{R}_{cp}\mathbf{H}_1\mathbf{T}_{cp}$ circulant; so

$$\mathbf{F}\mathbf{H}_c\mathbf{F}^{\mathcal{H}} = \mathrm{diag}(H_0 \cdots H_{N-1}) =: \mathbf{D}_H$$

where $H_k = \sum_{\ell=0}^{L} h_\ell e^{-j2\pi\ell k/N}$

- Received blocks after CP removal

$$\boldsymbol{x}(i) \;=\; \mathbf{R}_{cp}\boldsymbol{x}_{\rm cp}(i) = \mathbf{F}^{\mathcal{H}}\mathbf{D}_H\mathbf{T}_{sc}\boldsymbol{s}(i) + \boldsymbol{v}(i)$$

and after FFT

$$\tilde{\boldsymbol{x}}(i) = \mathbf{D}_H\mathbf{T}_{sc}\boldsymbol{s}(i) + \tilde{\boldsymbol{v}}(i)$$

OFDM signal model and preliminaries (6)

- F-D received signal at the data carriers (dropping block index):

$$\tilde{x}_n = H_n s_n + \tilde{v}_n \qquad n \in \mathcal{D}$$

s_n: data symbol on nth carrier and $H_n = \sum_{\ell=0}^{L} h_\ell e^{-2j\pi\ell n/N}$.

- F-D signal at the pilot carriers, $\mathcal{P} = \{i_1, \cdots, i_{N_p}\} \subseteq \mathcal{A} = \mathcal{D} \cup \mathcal{P}$,

$$\tilde{x}_{i_m} = H_{i_m} c_m + \tilde{v}_{i_m}, \qquad m = 1, \cdots, N_p$$

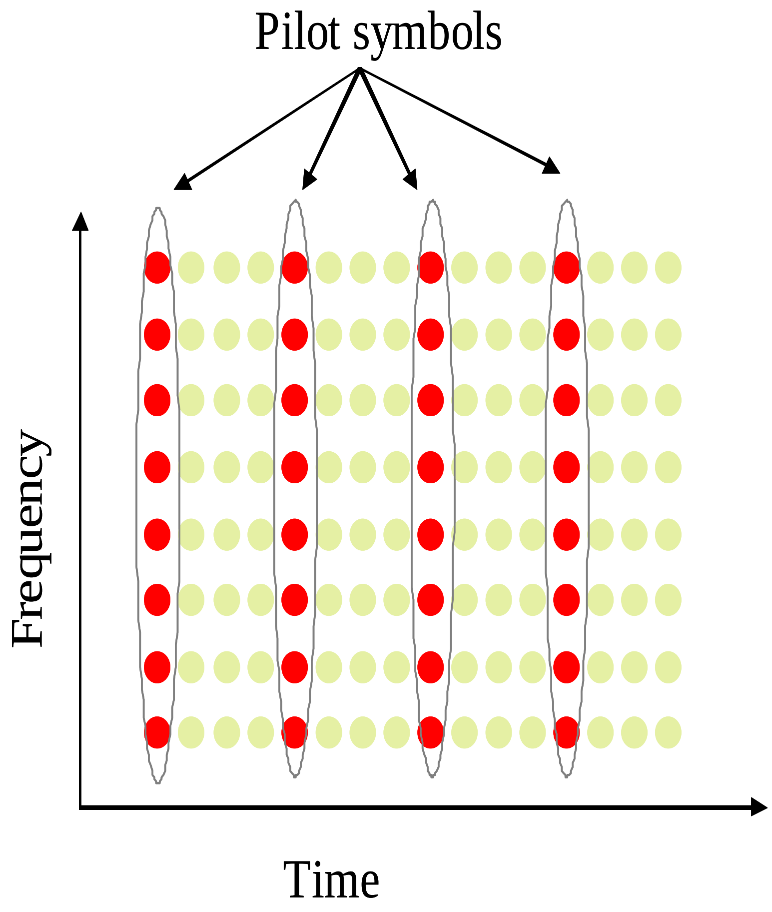
In vector form:

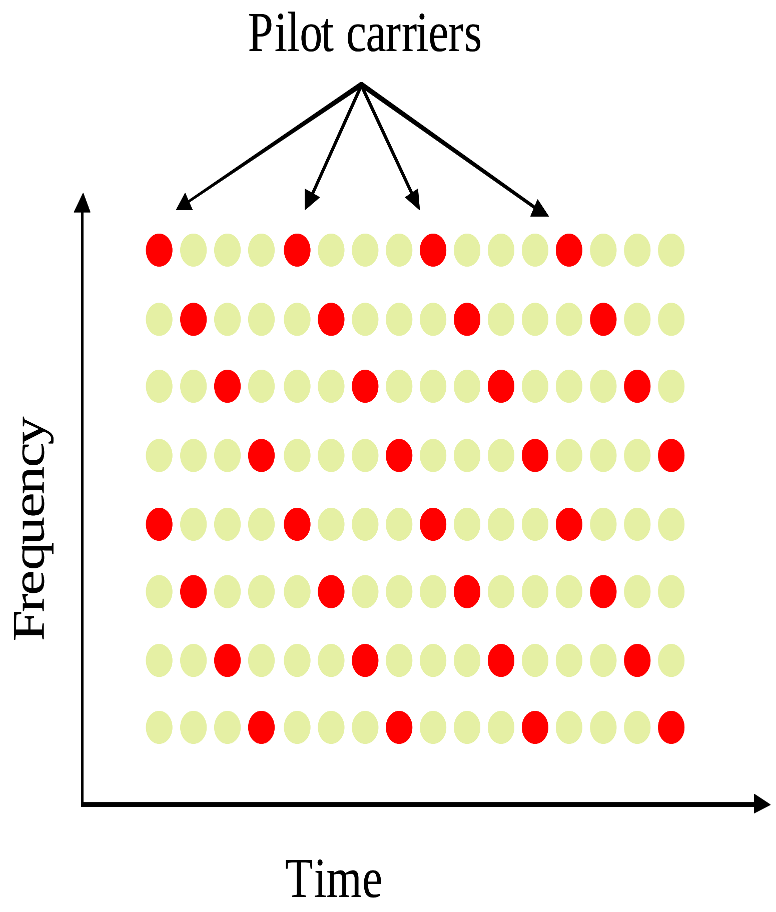
$$\tilde{\boldsymbol{x}}_{\mathcal{P}} = \mathbf{D}_{\boldsymbol{c}} \mathbf{W}_{\mathcal{P}} \boldsymbol{h} + \tilde{\boldsymbol{v}}_{\mathcal{P}}$$

$\boldsymbol{c} = [c_1, \cdots, c_{N_p}]^T$: known pilot symbols.

Pilot-Based Channel Estimation for OFDM

- Pilot placement





- Time-invariant or slowly varying channels

- Time-varying channels

Pilot-Based Channel Estimation for OFDM (2)

- Minimum Mean Square Error (MMSE) Method

❑ MMSE CIR estimator

$$\hat{\boldsymbol{h}} = \left(\sigma^2 \mathbf{R}_h^{-1} + \mathbf{W}_{\mathcal{P}}^{\mathcal{H}} \mathbf{D}_{\boldsymbol{\rho}} \mathbf{W}_{\mathcal{P}}\right)^{-1} \mathbf{W}_{\mathcal{P}}^{\mathcal{H}} \mathbf{D}_{\boldsymbol{c}}^{\mathcal{H}} \tilde{\boldsymbol{x}}_{\mathcal{P}}$$

where $\mathbf{D}_{\boldsymbol{\rho}} = \text{diag}\{|c_m|^2,\ m = 1 \cdots N_p\}$.

☞ The least square (LS) estimator is obtained by setting $\mathbf{R}_h^{-1} = 0$.

❑ Identifiability condition (since $c_m \neq 0$):

$$\text{rank}\left(\mathbf{D}_{\boldsymbol{c}} \mathbf{W}_{\mathcal{P}}\right) = L + 1 \quad \Longleftrightarrow \quad N_p \geq L + 1$$

❑ MMSE estimate of H_n

$$\hat{H}_n = \boldsymbol{w}_n^{\mathcal{H}} \hat{\boldsymbol{h}}$$

where $\boldsymbol{w}_n^{\mathcal{H}} := \mathbf{W}(n, :)$

Pilot-Based Channel Estimation for OFDM (3)

- Performance of MMSE estimates

❑ MSEs of $\hat{\boldsymbol{h}}$ and the $\hat{H}_n$'s:

$$\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}} := E\left\{(\hat{\boldsymbol{h}}-\boldsymbol{h})(\hat{\boldsymbol{h}}-\boldsymbol{h})^{\mathcal{H}}\right\} = \left(\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{W}_{\mathcal{P}}^{\mathcal{H}}\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}\right)^{-1}$$

$$\gamma_n := E\left\{|\hat{H}_n - H_n|^2\right\} = \boldsymbol{w}_n^{\mathcal{H}}\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\boldsymbol{w}_n$$

$$\bar{\gamma}_{\text{mmse}} := \sum_{n\in\mathcal{D}}\gamma_n = \text{Tr}\left\{\mathbf{W}_{\mathcal{D}}\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\mathbf{W}_{\mathcal{D}}^{\mathcal{H}}\right\}$$

☞ MSEs of LS estimates obtained by setting $\mathbf{R}_h^{-1} = 0$

Pilot-Based Channel Estimation for OFDM (4)

- Optimum pilot design for MMSE channel estimation

  ❑ Equalization carried out in F-D; so criterion based on γ_n. Minimizing the total (or average) mse:

$$\begin{aligned}\{\boldsymbol{\rho}^o, \mathcal{P}^o\} &= \arg\min_{\boldsymbol{\rho},\mathcal{P}} \bar{\gamma}_{\text{mmse}} \\ &= \arg\min_{\boldsymbol{\rho},\mathcal{P}} \text{Tr}\left\{\mathbf{W}_{\mathcal{D}}\left(\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{W}_{\mathcal{P}}^{\mathcal{H}}\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}}\right)^{-1}\mathbf{W}_{\mathcal{D}}^{\mathcal{H}}\right\}\end{aligned}$$

under the constraints

$$\mathcal{P} \subseteq \mathcal{A}; \qquad \sum_{n=1}^{N_p} \rho_n = \sigma_p^2 \qquad (\mathbf{C1})$$

Pilot-Based Channel Estimation for OFDM (5)

- Optimum pilot design for MMSE channel estimation: no VC

❑ For any $(L \times L)$ positive-definite matrix, $\mathbf{B} = \{b_{k,\ell}\}_{k,\ell=0}^{L}$, we have

$$\mathrm{Tr}\left\{\mathbf{B}^{-1}\right\} \geq \sum_{\ell=0}^{L} \frac{1}{b_{\ell,\ell}}$$

with equality iff $\mathbf{B}$ is diagonal.

❑ Since $\mathbf{R}_h$ is diagonal, $\bar{\gamma}_{\mathrm{mmse}}$ is minimized if

$$\mathbf{W}_{\mathcal{D}}^{\mathcal{H}}\mathbf{W}_{\mathcal{D}} = N_d\mathbf{I} \;\; \text{and} \;\; \mathbf{W}_{\mathcal{P}}^{\mathcal{H}}\mathbf{D}_{\boldsymbol{\rho}}\mathbf{W}_{\mathcal{P}} = \sigma_p^2\mathbf{I}$$

which is possible in the no-VC case

Pilot-Based Channel Estimation for OFDM (6)

- Optimum pilot design for MMSE channel estimation: no VC (cont.)

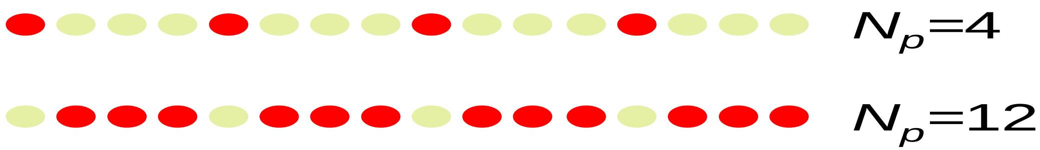
  - An optimum design is

$$\boldsymbol{\rho}^o = \frac{\sigma_p^2}{N_p}\mathbf{1}^T$$

$$\mathcal{P}^o = \begin{cases} \mathcal{P}_1^o := \{t + iQ,\ i = 0, \cdots, N_p - 1\} & \text{if } Q := \frac{N}{N_p} \text{ integer} \\ \mathcal{P}_2^o := \{0, \cdots, N-1\} - \mathcal{P}_1^o & \text{if } Q := \frac{N}{N-N_p} \text{ integer} \end{cases}$$

where t is arbitrary integer from $[0, Q-1)$.

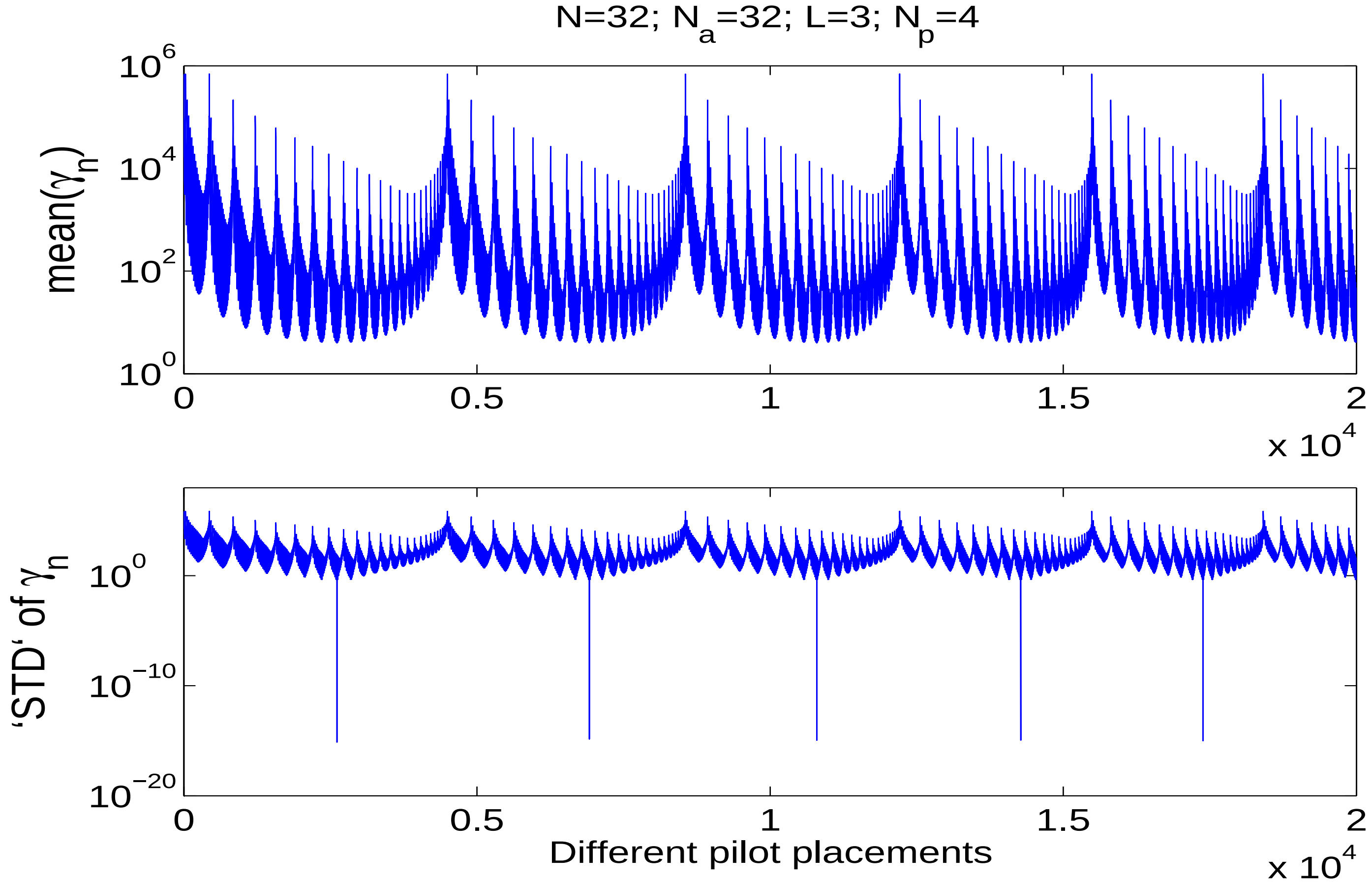
Example: $(N = 16)$



Pilot-Based Channel Estimation for OFDM (7)

- Optimum pilot design for MMSE channel estimation: no VC (cont.)

Illustration of the effect of pilot placement on estimation performance ($\boldsymbol{\rho} = \sigma_p^2 / N_p \mathbf{1}^T$)



Pilot-Based Channel Estimation for OFDM (8)

- Optimum pilot design for MMSE channel estimation: no VC (cont.)

❑ The minimum of $\bar{\gamma}_{\text{mmse}}$ is (using $N_d = N - N_p$ since no VC)

$$\bar{\gamma}^o_{\text{mmse}} = (N - N_p)\gamma^o = (N - N_p)\sum_{\ell=0}^{L} \frac{\sigma_v^2\ \sigma_{h_\ell}^2}{\sigma_v^2 + \sigma_p^2 \sigma_{h_\ell}^2}$$

☞ The MSE, $\bar{\gamma}_{\text{LS}}$, of LS estimate obtained using $\sigma_{h_\ell}^2 = \infty$.

☞ Pilot design minimizing $\bar{\gamma}_{\text{mmse}}$ also minimizes the γ_n's individually, and with optimal design, all carriers experience the same channel estimation MSE, i.e. $\gamma_n^o = \gamma^o$.

☞ Minimizations of the MSE in the F-D and T-D are equivalent:

$$\arg\min_{\boldsymbol{\rho},\mathcal{P}} \bar{\gamma}_{\text{mmse}} = \arg\min_{\boldsymbol{\rho},\mathcal{P}} \text{Tr}\left\{\boldsymbol{\Sigma}_{\hat{\boldsymbol{h}}}\right\}$$

Pilot-Based Channel Estimation for OFDM (9)

- Optimum pilot design for MMSE channel estimation: no VC (cont.)

☞ For fixed σ_p^2 and with $\boldsymbol{\rho}^o$ and $\mathcal{P}^o$, γ^o is independent of N_p. But this is not exactly true if there is a mismatch between the assumed and the actual channel models, e.g. fractional path delays!

☞ Optimum pilot placement and power distribution design not unique, in general. However if $N_p = L + 1$, only equipowered and equispaced pilot carriers achieve minimum MSE.

☞ In the case of colored noise with unknown spectral density, use pilot carrier hopping, e.g. t in the above optimum design should vary across the blocks.

Pilot-Based Channel Estimation for OFDM (10)

- Under the above optimal placement and power distribution of the pilots, what are the optimal value of N_p, the optimal power allocation and the optimal data power distribution? We use a capacity-bound criterion

- Channel unknown at transmitter ⇒ ideal training-based capacity maximized when $\sigma_s^2(n) := E\left\{|s_n|^2\right\} = \sigma_s^2/N_d$:

$$C_{\text{ideal}} = \frac{N_d}{N+L} E\left\{\log\left(1 + \beta_{\text{ideal}}|g|^2\right)\right\} \qquad \text{(bits/symbol)}$$

where $g \sim \mathcal{CN}(0,1)$ and β_{ideal} is the ideal SNR ($\sigma_H^2 = \sum_\ell \sigma_{h_\ell}^2$)

$$\beta_{\text{ideal}} := \frac{\sigma_H^2 \sigma_s^2}{N_d \sigma_v^2}$$

Pilot-Based Channel Estimation for OFDM (11)

- Incorporating estimation error into signal model

❑ Treating estimation error as extra noise:

$$\tilde{x}_n = H_n s_n + \tilde{v}_n = \hat{H}_n s_n + \underbrace{e_n s_n}_{\text{extra noise}} + \tilde{v}_n$$

where $e_n = \hat{H}_n - H_n$ and $E\left\{|e_n s_n + \tilde{v}_n|^2\right\} = \gamma_n \sigma_s^2(n) + \sigma_v^2$.

❑ Orthogonality principle: $E\left\{\hat{H}_n e_n\right\} = 0$. Thus

$$E\left\{|\hat{H}_n|^2\right\} = \sigma_H^2 - \gamma_n < \sigma_H^2$$

☞ Equivalent to a *known channel* $\hat{H}_n$ system subjected to an additive noise $\tilde{v}'_n = e_n s_n + \tilde{v}_n$ which is *neither Gaussian nor independent* (though uncorrelated) of the data.

Pilot-Based Channel Estimation for OFDM (12)

- Effect of estimation on capacity

  - Since noise $\tilde{v}'_n$ is *uncorrelated* from data, the capacity is lower bounded by that of a system subjected to Gaussian noise with same power as $\tilde{v}'_n$.

$$C > \underline{C} = \frac{1}{N+L} \sum_{n \in \mathcal{D}} E\left\{\log\left(1 + \beta(n)|g|^2\right)\right\}$$

where $\beta(n)$ effective SNR at nth carrier

$$\beta(n) := \frac{E\left\{|\hat{H}_n|^2\right\} E\left\{|s_n|^2\right\}}{E\left\{|\tilde{v}'_n|^2\right\}} = \frac{(\sigma_H^2 - \gamma_n)\sigma_s^2(n)}{\gamma_n \sigma_s^2(n) + \sigma_v^2}$$

Pilot-Based Channel Estimation for OFDM (13)

- Optimum data power distribution, no VC

  - In this case, $N_d = N - N_p$ and with optimal design, $\gamma_n = \gamma^o, \forall n$. Hence, $\underline{C}$ maximized when $\sigma_s^2(n) = \sigma_s^2 / N_d$:

  - Maximum lower bound:

$$\underline{C} = \frac{N - N_p}{N + L} E \left\{ \log \left(1 + \beta |g|^2 \right) \right\}$$

where

$$\beta := \frac{(\sigma_H^2 - \gamma^o)\sigma_s^2}{\gamma^o \sigma_s^2 + (N - N_p)\sigma_v^2}$$

Pilot-Based Channel Estimation for OFDM (14)

- Optimal number of pilots: no VC

❑ Treating N_p as a continuous variable ν, it can be shown that

$$\frac{\partial \underline{C}}{\partial \nu} = \frac{1}{N+L} E\left\{ -\log\left(1+\beta|g|^2\right) + (N-\nu)\frac{\partial \beta}{\partial \nu}\frac{|g|^2}{1+\beta|g|^2} \right\} < 0$$

$\Rightarrow$ μ should be as small as possible, i.e.

$$N_p^o = L + 1$$

☞ $N_p = L+1$ also minimizes complexity at the receiver and maximizes bandwidth efficiency. However, $N_p = L+1$ might not be optimal in the case of channel modeling mismatch.

Pilot-Based Channel Estimation for OFDM (15)

- Optimum power allocation: no VC

❑ Let $\alpha = \sigma_s^2/\sigma_t^2$. Using $N_p = L+1$, $\mathcal{P}^o$, $\boldsymbol{\rho}^o$ we maximize $\underline{C}$

$$\alpha^o := \arg\max_{\alpha} \underline{C} = \arg\max_{\alpha} \beta$$

$\Rightarrow$ For the general case, solution can be found by polynomial rooting. Let β^o denote maximum value of β.

❑ Let $\xi = \sigma_H^2 \sigma_t^2/(N-N_p)\sigma_v^2$, i.e. data SNR when $\sigma_s^2 = \sigma_t^2$.

❑ SNR losses due to channel estimation, estimation errors and both:

$$\frac{\xi}{\beta_{\text{ideal}}}\left(=\frac{1}{\alpha}\right), \qquad \frac{\beta_{\text{ideal}}}{\beta}, \qquad \frac{\xi}{\beta}$$

Pilot-Based Channel Estimation for OFDM (16)

- Optimum power allocation: no VC (cont.) High SNR regime:
  - ❑ Approximations:

$$\sigma_H^2 - \gamma^o \approx \sigma_H^2, \quad \gamma^o \approx \frac{\sigma_v^2\ (L+1)}{\sigma_p^2}$$

$$\beta = (N - N_p)\xi \frac{\alpha(1-\alpha)}{(L+1)\alpha + (N-N_p)(1-\alpha)}$$

  - ❑ Take $N_p = L + 1$. For fixed pair (N, L), optimal value of α and β:

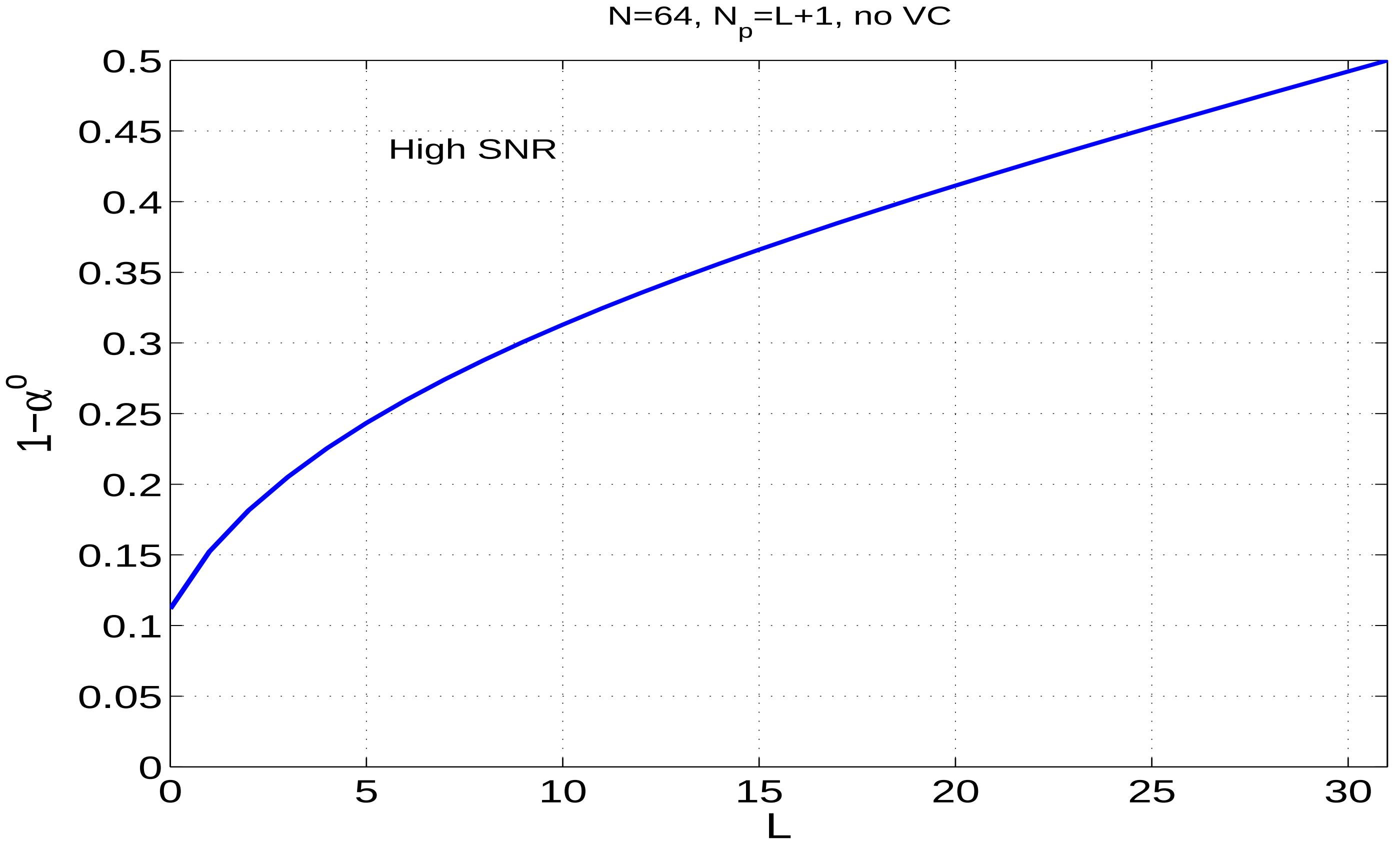
$$\alpha_\infty := \alpha^o|_{\text{high snr}} = \frac{1}{1 + \sqrt{\frac{L+1}{N-L-1}}}; \quad \beta_\infty := \beta^o|_{\text{high snr}} = \xi\ \alpha_\infty^2$$

  - ❑ For typical $N > 2(L+1)$, $\alpha \geq 0.5$ and maximum SNR losses (at high SNR) are resp 3dB, 3dB and 6dB. ☞ SNR loss decreases with N/L and $\to 0$ when $N >> L$.

Pilot-Based Channel Estimation for OFDM (17)

- Optimum power allocation: no VC (cont.)

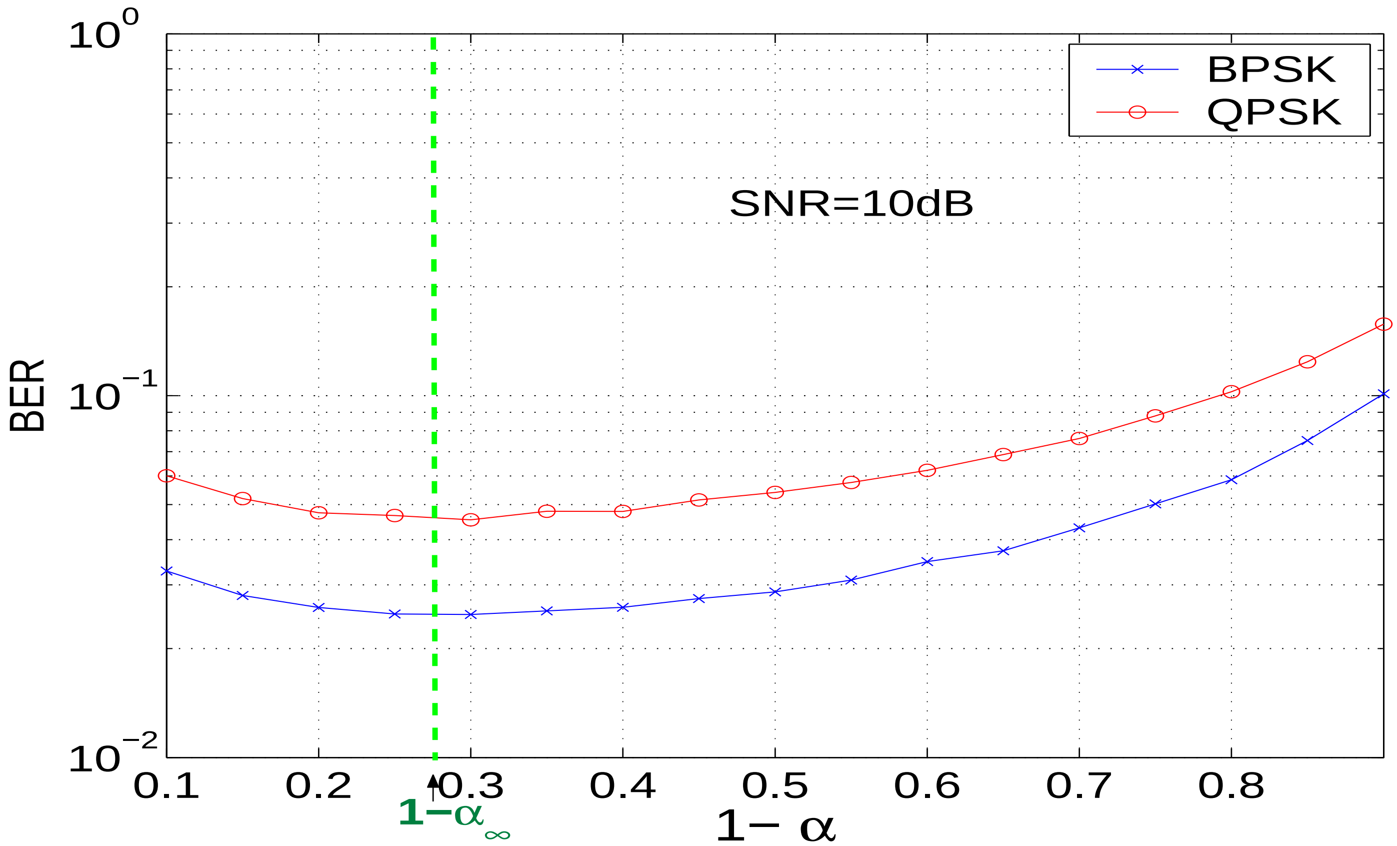
Optimum pilot power allocation at high SNR



Pilot-Based Channel Estimation for OFDM (18)

- Optimum power allocation: no VC (cont.)

BER performance: Rayleigh channel with exponential delay profile; $N = 64$ and $N_p = L + 1 = 8$.



Pilot-Based Channel Estimation for OFDM (19)

- Optimum power allocation: no VC (cont.)

❑ Rayleigh channels with equipowered taps, i.e. $\sigma_{h_\ell} = \sigma_h$:

$$\alpha_{\text{iid}} = \frac{1}{1 + \sqrt{1 - 1/\xi}} \qquad \text{where} \quad \xi := \frac{N - N_p}{N - N_p - L - 1}\left(1 + \frac{L+1}{(N - N_p)\xi}\right)$$

➪ Max effective data SNR

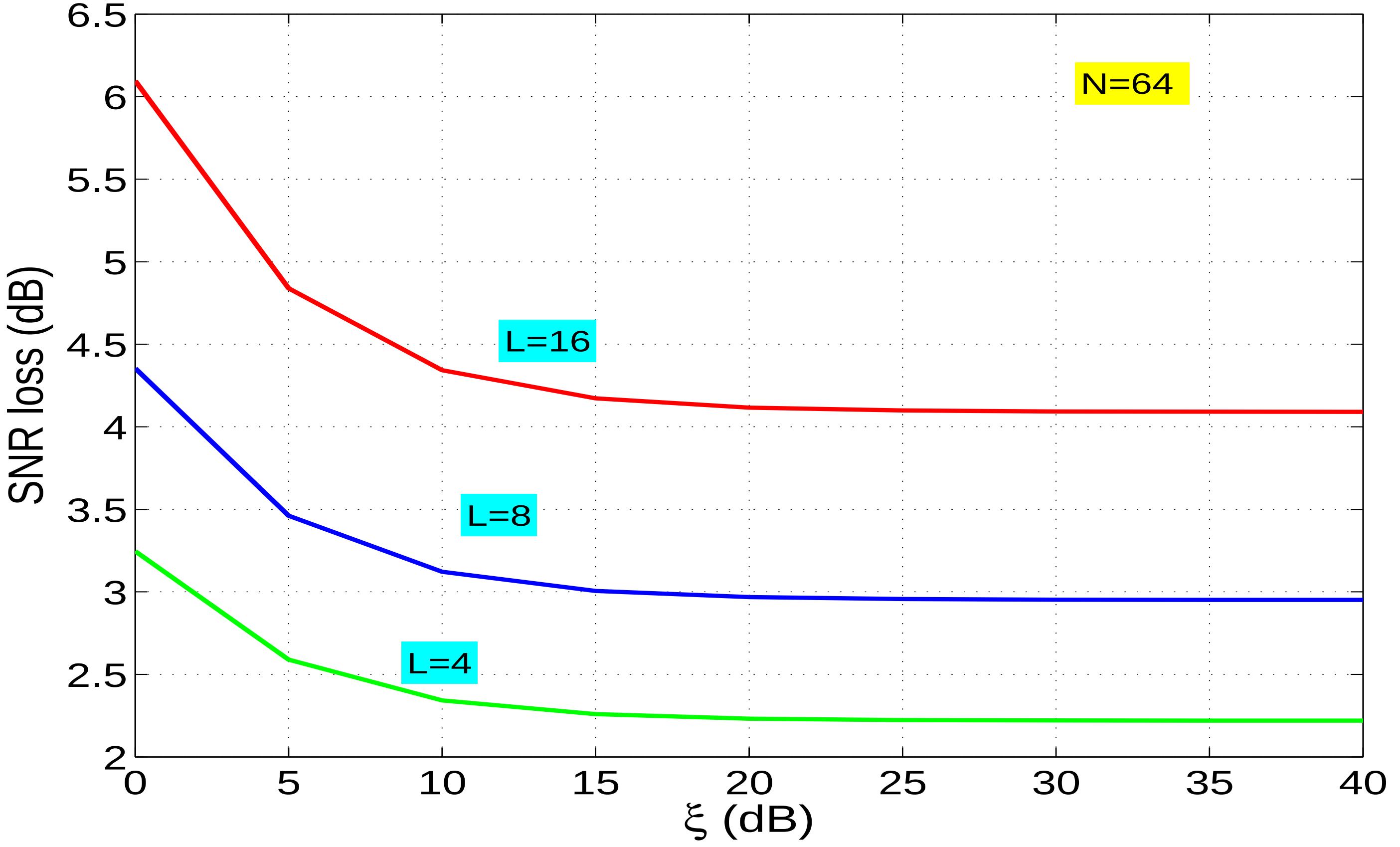
$$\beta_{\text{iid}} = \frac{\xi}{1 + \frac{L+1}{(N-N_p)\xi}} \quad \alpha_{\text{iid}}^2$$

➪ Data SNR loss due estimation depends on both N/L and ξ.

Pilot-Based Channel Estimation for OFDM (20)

- Optimum power allocation: no VC (cont.)

SNR loss vs ξ, $N_p = L + 1$.



Pilot-Based Channel Estimation for OFDM (21)

- Optimum pilot design for LS channel estimation: VC present
  - Optimization wrt both $\mathcal{P}$ and $\boldsymbol{\rho}$ untractable in general.
  - Complexity reduced if LS is used and $N_p = L + 1$ (i.e. $\mathbf{W}_{\mathcal{P}}$ square).
  - If total MSE, $\bar{\gamma}$, is used as criterion:

$$\{\boldsymbol{\rho}^o, \mathcal{P}^o\} = \arg\min_{\boldsymbol{\rho},\mathcal{P}} \bar{\gamma}_{\text{LS}} = \arg\min_{\boldsymbol{\rho},\mathcal{P}} \sum_{n=1}^{N_p} \frac{\psi_{n,n}}{\rho_n}$$

under $(\mathbf{C1})$ where $\boldsymbol{\Psi} := {\mathbf{W}_{\mathcal{P}}^{-1}}^{\mathcal{H}} \mathbf{W}_{\mathcal{D}}^{\mathcal{H}} \mathbf{W}_{\mathcal{D}} \mathbf{W}_{\mathcal{P}}^{-1}$.

  - Minimizing wrt to $\boldsymbol{\rho}$ under $\sum \rho_n = \sigma_p^2$ gives

$$\rho_n^o = \sigma_p^2 \frac{\sqrt{\psi_{n,n}}}{\sum_{i=1}^{N_p} \sqrt{\psi_{i,i}}}, \ \forall n = 1, \cdots, N_p$$

Pilot-Based Channel Estimation for OFDM (22)

- Optimum pilot design: VC present (cont.)

  - Optimization reduced to:

$$\mathcal{P}^o = \arg\min_{\mathcal{P}\subset\mathcal{A}} \left(\sum_{n=1}^{N_p} \sqrt{\psi_{n,n}} \right)^2$$

    - ☞ Minimum total MSE of LS estimates:

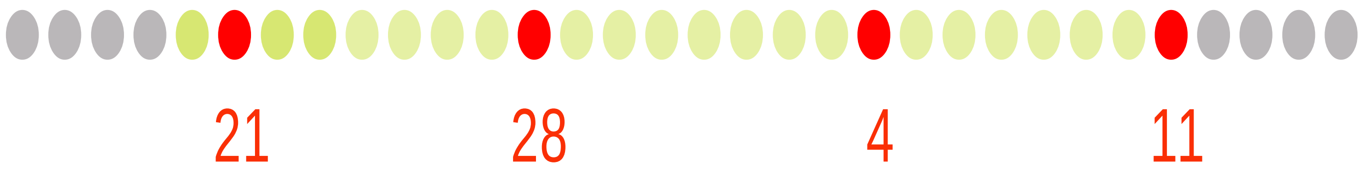
$$\frac{\sigma_v^2}{\sigma_p^2} \left(\sum_{n=1}^{N_p} \sqrt{\psi_{n,n}} \right)^2$$

  - Exhaustive search over all N_p-point subsets of $\mathcal{A}$.

Pilot-Based Channel Estimation for OFDM (23)

- Optimum pilot design: VC present (cont.)

❑ Example: $N = 32,\ N_a = 24,\ N_p = L + 1 = 4$:



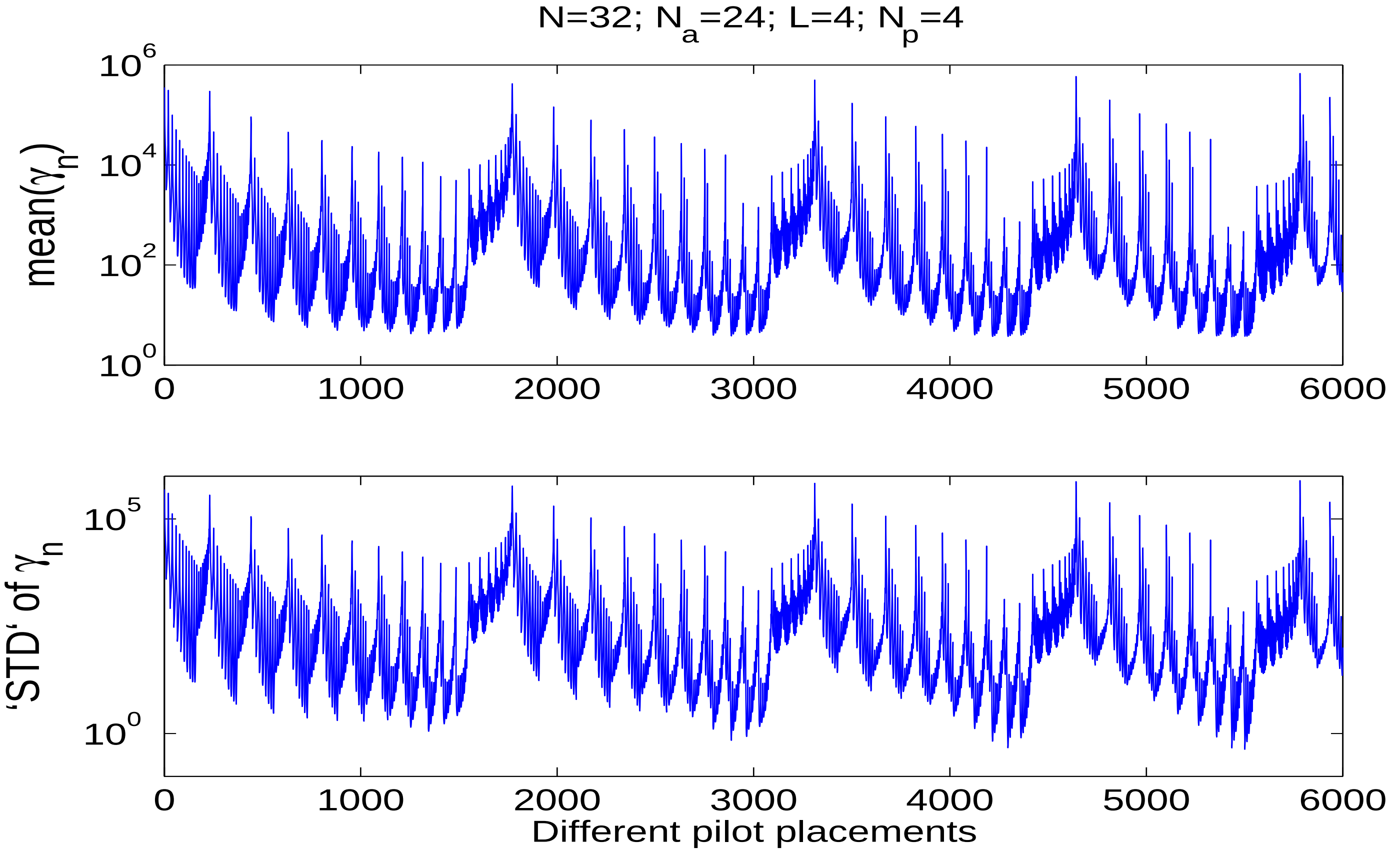
☞ Equispacing pilots in the active carrier region with one pilot placed near each edge of the VCs seems to be optimal.

☞ Pilot power ρ_n decreases when pilot close to VCs.

❑ Numerical examples show that setting $\boldsymbol{\rho}$ to be constant and optimizing wrt $\mathcal{P}$ lead to *almost* the same design

Pilot-Based Channel Estimation for OFDM (24)

- Optimum pilot design: VC present (cont.)



☞ $\mathcal{P}^o$ *almost* also minimizes 'STD' of γ_n. Perfect 'Fairness' in terms of estimation accuracy at different data carriers is impossible in general.

Pilot-Based Channel Estimation for OFDM (25)

- Optimum pilot design: VC present (cont.)

❑ The general problem is that of maximizing

$$\underline{C} = \frac{N}{N+L} \sum_{n \in \mathcal{D}} E\left\{ \log\left(1 + \frac{(\sigma_H^2 - \gamma_n)\sigma_s^2(n)}{\gamma_n \sigma_s^2(n) + \sigma_v^2}\right) |g|^2 \right\}$$

wrt $\mathcal{P}$, $\boldsymbol{\rho}$ σ_p^2 and the $\sigma_s^2(n)$'s for a constant σ_t^2; (orthogonality is valid only for MMSE estimator!)

❑ Maximization is untractable. A suboptimum solution is to use $\mathcal{P}$, $\boldsymbol{\rho}$ which minimize $\bar{\gamma}_{LS}$ and use the individual γ_n to maximize $\bar{C}$ wrt the $\sigma_s^2(n)$'s. ☞ Numerical examples show that no significant gain is obtained by accounting for the slight differences between the $gamma_n$'s.

Blind Channel Estimation for OFDM

- Two main classes of methods
  - methods exploiting the redundancy introduced by CP or/and virtual carriers: require large number of OFDM symbols.
  - methods exploiting the finite-alphabet (FA) property of the symbols: performance deteriorates with size of constellation.

When the channel varies rapidly across the blocks, only the FA-based methods may be suitable.

Blind Channel Estimation for OFDM (2)

- FA-based blind channel estimation

  - Assume $E\left\{s_n^M\right\} = \mu_M \neq 0$ and $E\left\{s_n^J\right\} = 0$ for $J < M$, e.g. $M = 2$ for BPSK and $M = 4$ for QPSK and QAM.

  - Received ith block after CP removal and DFT (assume $N_a = N$):

$$\tilde{x}_n(i) = H_n s_n(i) + \tilde{w}_n(i), \qquad n = 0, \cdots, N-1$$

  - Then

$$\tilde{y}_n(i) := [\tilde{x}_n(i)]^M = H_n^M s_n^M(i) + \xi_n(i)$$

where $E\left\{\xi_n(i)\right\} = 0.$ and

$$H_n^M = [1, \; e^{-j2\pi n/N}, \; \cdots, e^{-j2\pi n M(L)/N}](\boldsymbol{h} *_M \boldsymbol{h}) =: \Omega(n,:)\boldsymbol{h}_M$$

  - In vector form

$$[H_0^M, \cdots, H_{N-1}^M]^T =: \boldsymbol{H}_M = \boldsymbol{\Omega}\boldsymbol{h}_M$$

Blind Channel Estimation for OFDM (3)

- FA-based blind channel estimation (cont)

❑ Blind estimate of $\boldsymbol{H}_M$ and $\boldsymbol{h}_M$ using K blocks:

$$\begin{aligned} [\hat{\boldsymbol{H}}_M]_n &:= \widehat{H_n^M} = \frac{1}{\mu_M}\frac{1}{K}\sum_{i=1}^{K}\tilde{\boldsymbol{y}}(i) \\ \hat{\boldsymbol{h}}_M &= \boldsymbol{\Omega}^{\dagger}\hat{\boldsymbol{H}}_M = (1/N)\boldsymbol{\Omega}^{\mathcal{H}}\hat{\boldsymbol{H}}_M \end{aligned}$$

❑ Necessary condition: $N \geq ML + 1$. For PSK, identifiability guaranteed even with one OFDM symbol.

❑ Blind estimate of $\boldsymbol{h}$:

$$\hat{\boldsymbol{h}} = \arg\min_{\boldsymbol{h}} \|\hat{\boldsymbol{h}}_M - \boldsymbol{h} *_M \boldsymbol{h}\|$$

Blind Channel Estimation for OFDM (4)

- FA-based blind channel estimation (cont)
- Minimum Distance Algorithm

❑ Estimate $\hat{H}_n$ using

$$\hat{H}_n = \lambda_n \left[\widehat{H_n^M} \right]^{1/M}$$

where $\lambda_n \in \{e^{j(2\pi/M)m}\}_{m=0}^{M-1}$ is the scalar ambiguity.

❑ Using exhaustive search over all M^N possible vectors $\boldsymbol{\lambda}$, and for each $\boldsymbol{\lambda}$, estimate time-domain vector $\hat{\boldsymbol{h}}$ and compute

$$||\hat{\boldsymbol{h}}_M - \hat{\boldsymbol{h}} *_M \hat{\boldsymbol{h}}||$$

❑ Final estimate of $\boldsymbol{h}$ is the minimizer of the above criterion.
☞ Reduced complexity because of discrete search. Other simpler algorithms exist.

Part 2: Channel Estimation for General CP Systems

Affine Precoding and MMSE Channel Estimation

- Assume
  - frequency-selective channel, constant over $K(\geq 1)$ blocks
- Received signal after CP removal

$$\begin{aligned} \boldsymbol{x}_i &= \mathbf{H}\boldsymbol{u}_i + \boldsymbol{v}_i \qquad i = 1 \cdots K \\ \boldsymbol{u}_i &= \boldsymbol{\Theta}_i \boldsymbol{s}_i + \boldsymbol{b}_i \end{aligned}$$

- $\boldsymbol{\Theta}_i$ $(N \times N)$ precoding matrix • $\boldsymbol{s}_i$: ith transmitted data block
- $\boldsymbol{b}_i$: ith pilot sequence • $\mathbf{H} = \text{circ}([h_0 ... h_L 0 ... 0])$
- $\boldsymbol{v}_i$: AWGN, variance σ_v^2 • $\boldsymbol{s}_i$: independent of $\boldsymbol{v}_i$.

☞ Affine precoding includes TDM and superimposed training.

Affine Precoding and MMSE Channel Estimation (2)

Assume

- **(A1)** The non-zero elements of the $\boldsymbol{s}_i$'s are unknown, i.i.d zero-mean random variables drawn from a finite alphabet $\mathcal{M}$.
- Design criteria assume a fixed total pilot power in the frame

$$\sigma_b^2 = \frac{1}{K}\sum_{i=0}^{K-1} \sigma_b^2(i) \ ,$$

but the training power can vary from block to block.

Affine Precoding and MMSE Channel Estimation (3)

- Collecting K blocks:

$$\boldsymbol{x}_i = \mathbf{H}\boldsymbol{\Theta}_i \boldsymbol{s}_i + \mathbf{B}_i \boldsymbol{h} + \boldsymbol{v}_i, \quad i = 0, ..., K-1$$

  - $\mathbf{B}_i$: leading $(N \times L)$ of $\text{circ}(\boldsymbol{b}_i)$ • $\boldsymbol{h} = [h_0 \cdots h_L]^T$.

- MMSE channel estimate:

$$\hat{\boldsymbol{h}} = \frac{1}{\sigma_v^2}\left(\mathbf{R}_h^{-1} + \frac{1}{\sigma_v^2}\mathbf{B}^{\mathcal{H}}\mathbf{B}\right)^{-1}\mathbf{B}^{\mathcal{H}}\boldsymbol{x}.$$

  - $\boldsymbol{x} = [\boldsymbol{x}_1^T \cdots \boldsymbol{x}_K^T]^T$ • $\mathbf{B} = [\mathbf{B}_1^T \cdots \mathbf{B}_K^T]^T$

Affine Precoding and MMSE Channel Estimation(4)

- Identifiability condition:

$$\text{rank}(\mathbf{B}) = L + 1 \qquad \text{(C2)}$$

- Frequency-domain counterpart:
  - let $\tilde{\boldsymbol{b}}_i :=$ DFT of $\boldsymbol{b}_i$ and

$$\rho_n := \sum_{i=1}^{K} |\tilde{b}_i(n)|^2, \quad n = 0, ..., N-1$$

  - Let N_p: number of nonzero entries of $\boldsymbol{\rho} := [\rho_0 \cdots \rho_{N-1}]$

➩ $\text{rank}(\mathbf{B}) = \min(N_p, L+1)$

$$(C2) \qquad \Longleftrightarrow \qquad N_p \geq L + 1$$

i.e. combined training power across the blocks is non-zero at at least $L+1$ frequencies.

Affine Precoding and MMSE Channel Estimation(5)

- Orthogonal precoding

❑ Condition for decoupled channel estimation and data detection:

$$\tilde{b}_i^*(n)\ [\mathbf{F}^{\mathcal{H}}\boldsymbol{\Theta}_i \boldsymbol{s}_i]_n = 0, \quad \forall n, i$$

$$\Updownarrow$$

$$\mathbf{T}_i\mathbf{F}^{\mathcal{H}}\boldsymbol{\Theta}_i \boldsymbol{s}_i = 0, \quad \forall i \qquad \text{(C3)}$$

where $\mathbf{T}_i = \text{diag}\{t_i(n), n = 0, \cdots, N-1\}$ with

$$t_i(n) = \begin{cases} 1 & \text{if } n \in \mathcal{P}_i \\ 0 & \text{otherwise} \end{cases}$$

Affine Precoding and MMSE Channel Estimation(6)

- Optimal training for orthogonal precoding

Result 1 Assume that $Q = N/N_p$ is an integer. Under (C3) and the constraint of fixed training power σ_b^2, the MSE of $\hat{\boldsymbol{h}}$ in orthogonal precoders is minimized when

$$\rho_n = \begin{cases} \frac{\sigma_b^2 N}{N_p} \sum_{\ell=0}^{N_p-1} \delta(n - \ell Q - m) & \text{if } Q := \frac{N}{N_p} \text{ integer} \\ \frac{\sigma_b^2 N}{N_p} \sum_{\ell=0}^{N_p-1} [1 - \delta(n - \ell Q - m)] & \text{if } Q := \frac{N}{N-N_p} \text{ integer} \end{cases} \quad \text{(C4)}$$

- m : arbitrary integer from $[0, ..., Q-1]$

Affine Precoding and MMSE Channel Estimation(7)

- Result 1 implies that the pilot frequencies should be equispaced and that their *average* powers across the K blocks should be identical. Therefore, channel estimation performance is the same regardless of the distribution of the training power across the blocks.
- the minimum MSE of $\hat{\boldsymbol{h}}$ is independent of N_p, the number of pilot frequencies.
- Time-division multiplexing (TDM) is not an orthogonal precoding scheme. Condition (C3) implies that training should be superimposed onto the data in the time domain (but orthogonal in the frequency domain).
- The $K > 1$ scenario gives more flexibility for designing precoders. It is also useful if frequency hopping is desired.

Full-Rank Orthogonal Precoding

- Let $\mathcal{P}_i$: set of pilot frequencies during ith block
- **Result 2** Assume that $\mathbf{\Theta}_i$, $i = 1, ..., K-1$, are full rank, assumption (A1) holds and maximum possible data-rate is required. Then, the orthogonality condition (C3) is satisfied if and only if the nth entry of $\mathbf{\Lambda}_i \boldsymbol{s}_i$ is identically zero for $n \in \mathcal{P}_i$, where $\mathbf{\Lambda}_i$ is any permutation matrix, and the precoding matrix has the following form

$$\mathbf{\Theta}_i = \mathbf{F}^{\mathcal{H}} \left[\mathbf{T}_i \mathbf{W}_i \mathbf{T}_i + (\mathbf{I} - \mathbf{T}_i) \mathbf{A_i} \right] \mathbf{\Lambda}_i$$

where $\mathbf{W}_i$ and $\mathbf{A_i}$ are any $(N \times N)$ matrices such that $(\mathbf{T}_i \mathbf{W}_i \mathbf{T}_i + (\mathbf{I} - \mathbf{T}_i)\mathbf{A_i})$ is full-rank.

Full-Rank Orthogonal Precoding (2)

- $\mathbf{W}_i = \mathbf{A_i} = \mathbf{I} \rightarrow \mathbf{\Theta_i} = \mathbf{F}^{\mathcal{H}} \equiv$ OFDM with reserved pilot tones.
- Uncoded OFDM has poor performance because only diversity order one is possible through Rayleigh fading channels. This problem is overcome by employing either Galois field channel coding or LP-OFDM - LCP-OFDM.
- Here, we focus on SC-CP systems. Although such systems do not have full multipath diversity, their performance at realistic SNR values approaches that of maximum diversity systems. Further, maximum diversity at high SNR can be achieved if the constellations are first rotated prior to SC-CP modulation.
- Conventional SC-CP where $\mathbf{\Theta}_i = \mathbf{I}$ is not an orthogonal precoding scheme.

Full-Rank Orthogonal Precoding (3)

- Full-rank orthogonal single carrier (FROSC) precoding

  - Let $\mathbf{T}_{\mathcal{D}_i}$ and $\mathbf{T}_{\mathcal{P}_i}$ be the data and pilot selection matrices, and $\bar{\mathbf{A}}_i$= non-zero $((N - N_{p_i}) \times N)$ submatrix of $(\mathbf{I} - \mathbf{T}_i)\mathbf{A_i}$
  - FROSC is obtained by choosing $\mathbf{\Theta}$ to be the same as $\mathbf{I}$ except for the N_{p_i} pilot rows. This is achieved by

$$\mathbf{W}_i = \mathbf{I}, \quad \text{and} \quad \bar{\mathbf{A}} = (\mathbf{T}_{\mathcal{D}_i}^T \mathbf{F}^{\mathcal{H}} \mathbf{T}_{\mathcal{D}_i})^{-1} \mathbf{T}_{\mathcal{D}_i}^{\mathcal{H}} (\mathbf{I} - \mathbf{F}^{\mathcal{H}} \mathbf{T}_{\mathcal{P}_i} \mathbf{T}_{\mathcal{P}_i}^T)$$

  - Bandwidth efficiency of FROSC:

$$\zeta_{FROSC}(i) = \frac{N - N_{p_i}}{N + L}$$

Full-Rank Orthogonal Precoding (4)

- FROSC precoding (cont.)

❑ The Θ_i's are the same as $\mathbf{I}$ except for P_i rows are obtained using $\mathbf{A_i}$. An example of the structure of $\boldsymbol{\Theta}_i$ when $N = 8$ and $\mathcal{P}_i = \{0, 4\}$ is

$$\boldsymbol{\Theta}_i = \begin{pmatrix} \times & \times & \times & \times & \times & \times & \times & \times \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \times & \times & \times & \times & \times & \times & \times & \times \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad \boldsymbol{s}_i = \begin{pmatrix} 0 \\ \times \\ \times \\ \times \\ 0 \\ \times \\ \times \\ \times \end{pmatrix}$$

Full-Rank Orthogonal Precoding (5)

- FROSC precoding (cont.)
  - Effectively, the precoding is redundant (or tall):

$$\mathbf{\Theta}_i \boldsymbol{s}_i = \bar{\mathbf{\Theta}}_i \bar{\boldsymbol{s}}_i \quad \text{with } \bar{\mathbf{\Theta}}_i := \mathbf{\Theta}_i \mathbf{T}_{\mathcal{D}_i}^T \text{ and } \bar{\boldsymbol{s}}_i = \mathbf{T}_{\mathcal{D}_i} \boldsymbol{s}_i$$

Previous example:

$$\bar{\mathbf{\Theta}}_i = \begin{pmatrix} \times & \times & \times & \times & \times & \times \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \times & \times & \times & \times & \times & \times \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} ; \quad \bar{\boldsymbol{s}}_i = \begin{pmatrix} \times \\ \times \\ \times \\ \times \\ \times \\ \times \end{pmatrix}$$

Full-Rank Orthogonal Precoding (6)

- FROSC precoding: symbol detection

❑ Linear equalization: $\mathbf{H}$ is circulant $\Rightarrow$ equalization in the F-D

$$\widehat{\boldsymbol{s}}_i = \lfloor \bar{\boldsymbol{\Theta}}_i^{\dagger} \mathbf{F}^{\mathcal{H}} (\mathbf{I} - \mathbf{T}_i) \mathbf{G} \mathbf{F} \boldsymbol{x}_i) \rfloor_{\mathcal{M}}$$

where $\mathbf{G} = \text{diag}\{g(k),\ k = 0, \cdots, N-1\}$ is the MMSE equalizer:

$$g(k) = \hat{H}_n^* / (|\hat{H}_n|^2 + \sigma_v^2)$$

❑ Ignoring the $n \in \mathcal{P}_i$ rows of $\bar{\boldsymbol{\Theta}}_i$, a simpler detection scheme is

$$\widehat{\boldsymbol{s}}_i = \lfloor \mathbf{T}_{\mathcal{D}_i} \mathbf{F}^{\mathcal{H}} (\mathbf{I} - \mathbf{T}_i) \mathbf{G} \mathbf{F} \boldsymbol{x}_i) \rfloor_{\mathcal{M}}$$

Rank-Deficient Orthogonal Precoding

- Rank-deficient orthogonal single carrier (DROSC) precoding

  - Full data-rate under (C3) requires ($\mathrm{rank}(\boldsymbol{\Theta}_i) = N - P_i$)

$$[\mathbf{F}\boldsymbol{\Theta}_i]_n = 0, \quad n \in \mathcal{P}_i$$

  - $\boldsymbol{s}_i$ cannot be recovered linearly. However, using the finite-alphabet property detection is still possible.

  - DROSC is obtained by designing $\boldsymbol{\Theta}_i$ as

$$\boldsymbol{\Theta}_i^o = \min_{\boldsymbol{\Theta}_i;\ \mathbf{F}_{\mathcal{P}_i}\boldsymbol{\Theta}_i = \mathbf{0}} \sum_{i=0}^{K-1} \|\boldsymbol{\Theta}_i - \mathbf{I}\|_2$$

$$\Downarrow$$

$$\boldsymbol{\Theta}_i^o = \mathbf{F}^{\mathcal{H}}(\mathbf{I} - \mathbf{T}_i)\mathbf{F}$$

Rank-Deficient Orthogonal Precoding (2)

- **Result 3** Assume $N/(L+1) = Q$ and $M = (L+1)/K$ are integers. A bandwidth efficient orthogonal precoding scheme is obtained as follows
  - for $i = 0, ..., K-1$ chose $\mathcal{P}_i = \{nKQ + iQ, n = 0, ..., M-1\}$
  - set $\mathbf{\Theta}_i = \mathbf{F}^{\mathcal{H}} (\mathbf{I} - \mathbf{T}_i) \mathbf{F}$
  - add a training sequence according to condition (C3).
- Bandwidth efficiency of DROSC:

$$\zeta_{DROSC} = \frac{N}{N+L}$$

Rank-Deficient Orthogonal Precoding (3)

- Symbol detection

  - Received signal $\boldsymbol{x}_i = \mathbf{H}\left[(\mathbf{I}-\mathbf{J})\boldsymbol{s}_i + \boldsymbol{b}_i\right] + \boldsymbol{v}_i$ with $\mathbf{J} = \mathbf{F}^{\mathcal{H}}\mathbf{T}_i\mathbf{F}$

  - Remove training related term

$$
\begin{aligned}
\boldsymbol{z}_i &:= (\mathbf{I}-\mathbf{J})\ \boldsymbol{x}_i \\
&= (\mathbf{I}-\mathbf{J})\ \mathbf{H}\boldsymbol{x}_i + (\mathbf{I}-\mathbf{J})\ \boldsymbol{v}_i, \\
&= \mathbf{H}\ (\mathbf{I}-\mathbf{J})\boldsymbol{x}_i + \tilde{\boldsymbol{v}}_i \\
&= \mathbf{H}\ (\mathbf{I}-\mathbf{J})[(\mathbf{I}-\mathbf{J})\boldsymbol{s}_i + \boldsymbol{b}_i] + \tilde{\boldsymbol{v}}_i \\
&= \mathbf{H}\ (\mathbf{I}-\mathbf{J})\boldsymbol{s}_i + \tilde{\boldsymbol{v}}_i \qquad \text{since } (\mathbf{I}-\mathbf{J})^2 = \mathbf{I}-\mathbf{J}
\end{aligned}
$$

  - MMSE equalizer: $\mathbf{G} = \mathrm{diag}\{|[\hat{H}_n|^2 + \tilde{\sigma}^2]^{-1}\hat{H}_n, n = 0, \cdots N-1\}$

$$
\boldsymbol{u}_i = \mathbf{F}^{\mathcal{H}}\mathbf{G}\mathbf{F}\boldsymbol{z}_i
$$

Rank-Deficient Orthogonal Precoding (4)

- Symbol detection, cont.
  - Even if channel estimation is perfect and no noise, $\boldsymbol{u}_i \neq \boldsymbol{s}_i$:

$$\boldsymbol{u}_i = (\mathbf{I} - \mathbf{J})\boldsymbol{s}_i + \boldsymbol{\epsilon}_i \qquad (\boldsymbol{\epsilon}_{\mathrm{i}} : \text{ due to noise \& estimation errors})$$

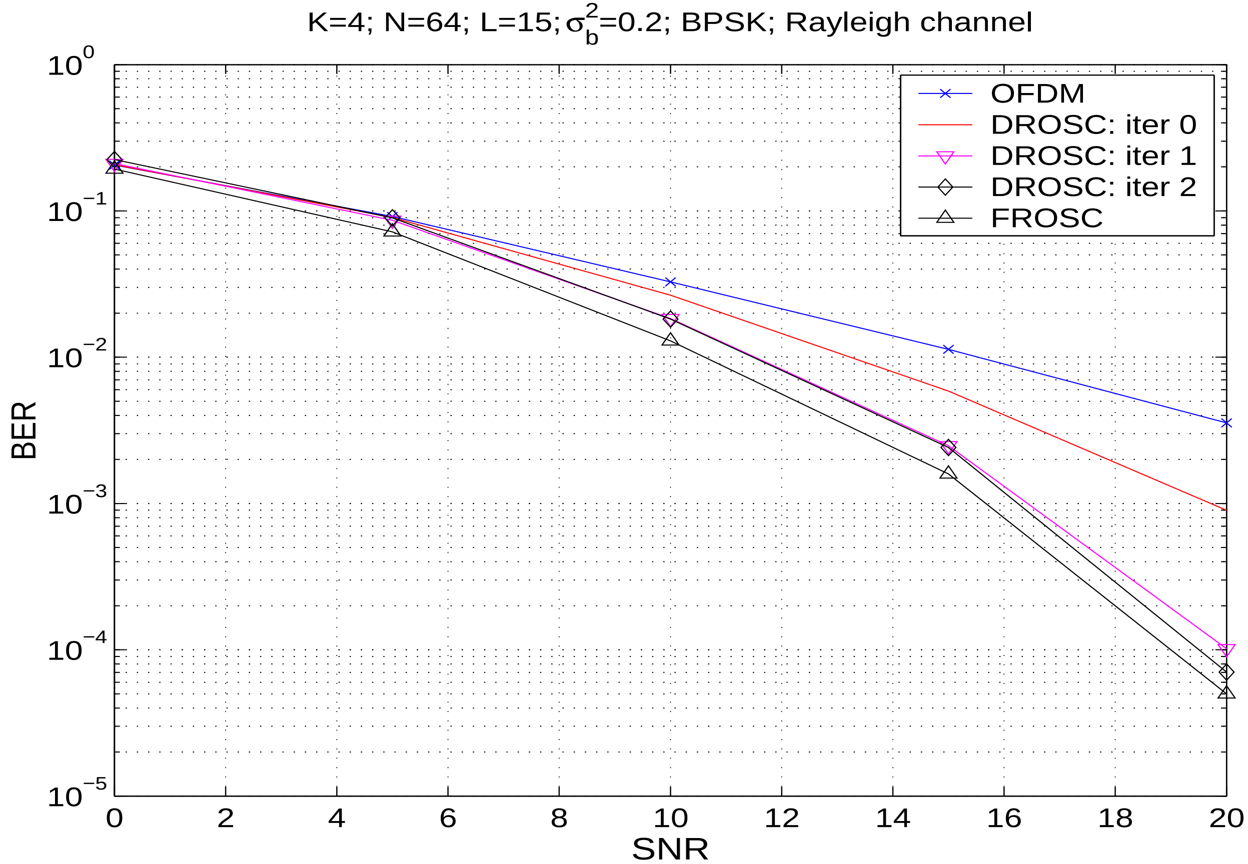
  - $\mathbf{I} - \mathbf{J}$: rank-deficient $\Rightarrow$ $\boldsymbol{s}_i$ cannot be recovered linearly
  - Using finite alphabet property:
    - Symbol vector detection $\leftarrow$ prohibitive
    - Iterative symbol-by-symbol detection: (1-2 iterations suffice)

$$\begin{aligned} \hat{\boldsymbol{s}}_i^{(0)} &= \lfloor \boldsymbol{u}_i \rfloor \\ \hat{\boldsymbol{s}}_i^{(m)} &= \lfloor \boldsymbol{u}_i + \mathbf{J}\hat{\boldsymbol{s}}_i^{(m-1)} \rfloor \end{aligned}$$

Rank-Deficient Orthogonal Precoding (5)

- Simulation Results

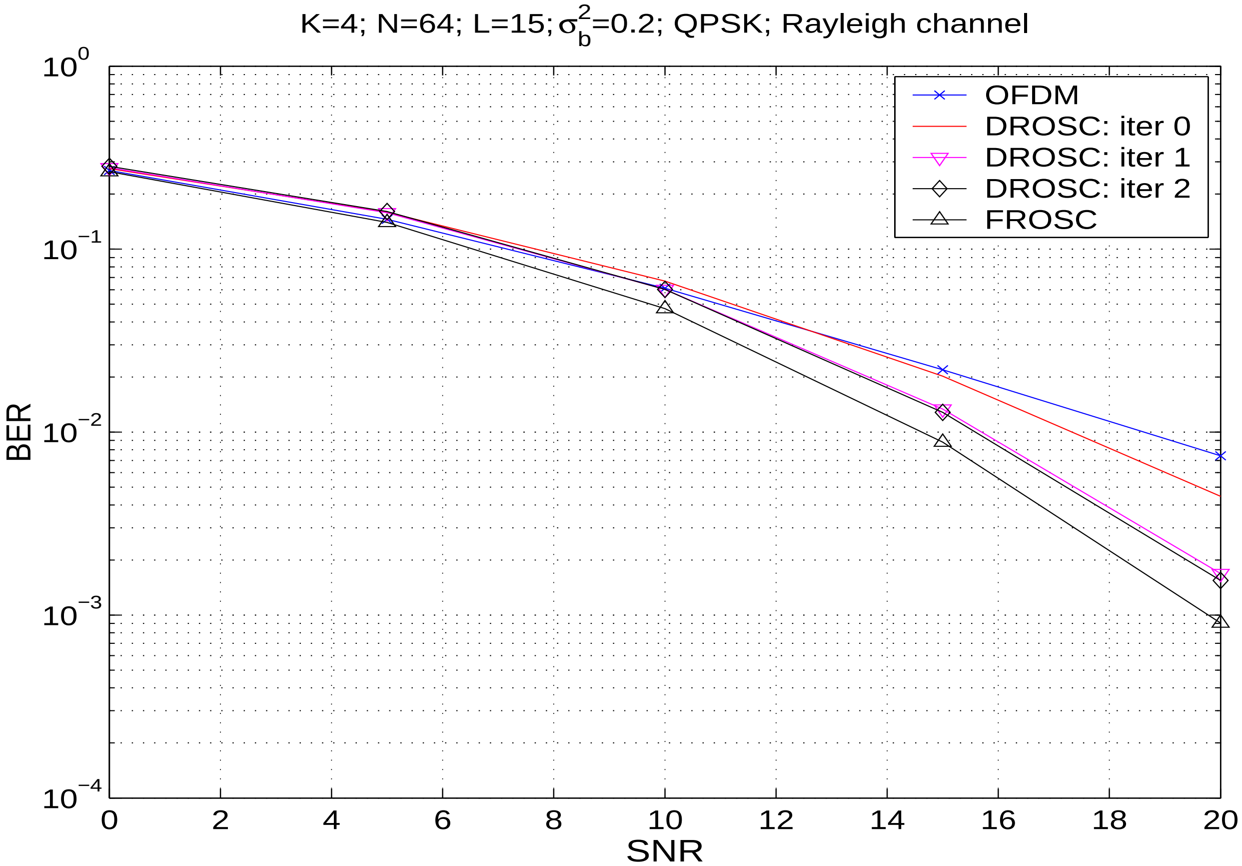
❑ BER vs SNR; $K = 4,\ N = 64,\ L = 15,\ \sigma_b^2 = 0.2$, BPSK.



Rank-Deficient Orthogonal Precoding (6)

- Simulation Results, cont.

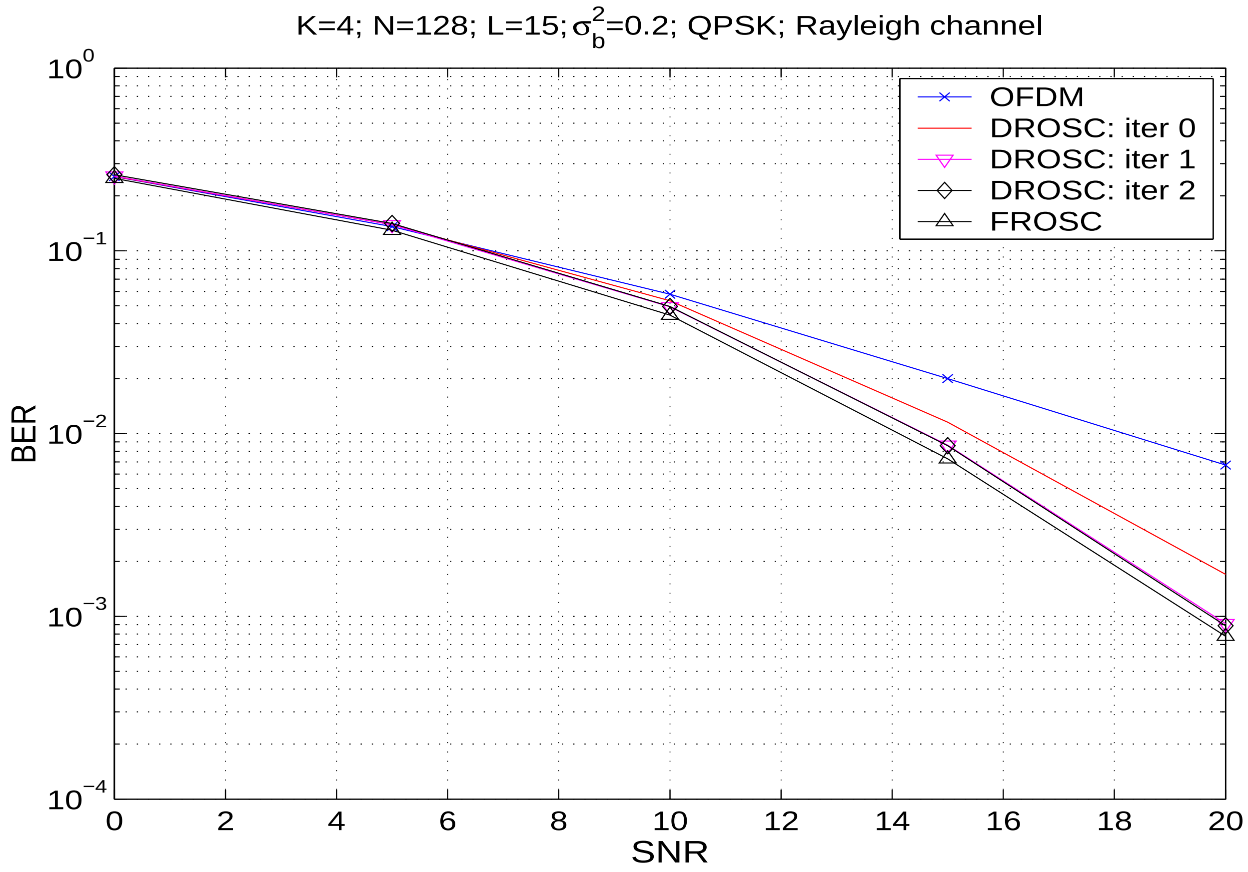
  - BER vs SNR; $K = 4,\ N = 64,\ L = 15,\ \sigma_b^2 = 0.2$, QPSK.



Rank-Deficient Orthogonal Precoding (7)

- Simulation Results, cont.

  - BER vs SNR; $K = 4,\ N = 128,\ L = 15,\ \sigma_b^2 = 0.2$, BPSK.



Summary

$\surd$ Pilot carrier design dramatically affects system performance

$\surd$ Blind techniques for OFDM may be more promising than for serial single-carrier systems

$\surd$ Affine precoding gives a general framework for block transmission schemes

$\surd$ OFDM or single-carrier CP systems? the saga continues...

References

[1] R. Negi and J. Cioffi, "Pilot tone selection for channel estimation in a mobile OFDM system," *IEEE Trans. Consumer Electron.*, vol. 44, Aug. 1998.

[2] R. Heath and G.B. Giannakis, "Exploiting input cyclostationarity for blind channel identification in OFDM systems," *IEEE Trans. Signal Processing*, vol. 47, March 1999.

[3] M. Medard, "The effect upon channel capacity in wireless communication of perfect and imperfect knowledge of the channel," *IEEE Trans. Information Theory*, vol. 46, May 2000.

[4] S. Zhou and G. B. Giannakis, "Finite-alphabet based channel estimation for OFDM and related multicarrier systems," *IEEE Trans. Communications*, vol. 49, Aug. 2001.

[5] C. Li and S. Roy, "Subspace based blind channel estimation for OFDM by exploiting virtual carrier," *Proc. Globecom*, vol 1, San Antonio, TX, 2001.

[6] M. Morelli and Umberto Mengali, "A comparison of pilot-aided channel estimation methods for OFDM systems," *IEEE Trans. Signal Processing*, vol. 49, Dec. 2001.

[7] B. Muquet *et al*, "A subspace based blind and semi-blind channel identification method for OFDM systems," *IEEE Trans. Signal Processing*, vol. 50, July 2002.

[8] S. Adireddy, L. Tong and H. Viswanathan, "Optimal placement of training for frequency-slective block-fading channels," *IEEE Trans. Information Theory*, vol. 48, Aug 2002.

[9] How much training is needed in multiple-antenna wireless links?," *IEEE Trans. Information Theory*, vol. 49, April 2003.

[10] S. Ohno, and G. B. Giannakis, "Optimal training and redundant precoding for block transmissions with application to wirelesss OFDM," *IEEE Trans. Communications*, vol. 50, Dec 2002.

[11] Z. Liu *et al*, "Linear constellation precoding for OFDM with multipath diversity and coding gains," *IEEE Trans. Communications*, vol. 51, March 2003.

[12] Z. Wang and G. B. Giannakis, "Complex-field coding for OFDM over fading wireless channels," *IEEE Trans. Information Theory*, vol. 49, Mar. 2003.

[13] S. Ohno, and G. B. Giannakis, "Capacity optimizing MMSE-optimal pilots for wireless OFDM over frequency-selective block Rayleigh-fading channels," *IEEE Trans. Information Theory*, vol. 50, Sep 2004.

[14] G.T. Zhou *et al*, "First-order statistical method for channel estimation," *IEEE Signal Processing Lett.*, 10, Mar 2003.

[15] A. G. Orozco-Lugo *et al*, "Channel estimation using implicit training," *IEEE Trans. Signal Processing*, vol. 52, Jan 2004.

[16] M. Ghogho, *et al*, "Channel estimation and symbol detection for block transmission using data-dependent superimposed training", *IEEE Signal Processing Lett.*, 12, Mar 2005.

[17] M. Ghogho *et al*, "SISO and MIMO Channel Estimation and Symbol Detection using Data-Dependent Superimposed Training," *IEEE ICASSP'05*, Philadelphia, March 2005.

[18] M. Ghogho and A. Swami, "Optimal Training for Affine-Precoded and Cyclic-Prefixed Block Transmissions," *Proc SSP'05*, Bordeaux, France, July 2005.

[19] M. Ghogho and A. Swami, "Training Design for Multipath Channel and Frequency-Offset Estimation in MIMO Systems," to be published in *IEEE Trans. Signal Processing*, vol. .