Mobile Localisation

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Outline

- Generalities.
- Mobile localisation using time of arrival.
- Mobile localisation using angle of arrival.
- Conclusion.

Generalities

Introduction

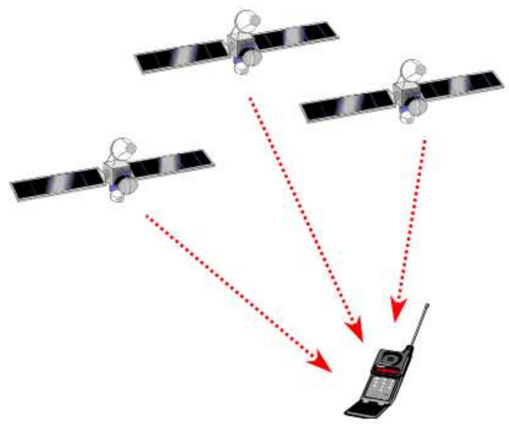
- **Objective:** Find the mobile position (x, y) in a cellular network.
- **Interest:**
  - Localisation services: Emergency, hotels, close restaurants, ...
  - Trafic Localisation, navigation, ...
- **Possible approaches:**
  - Use of GPS (satellite) system.
  - Terrestrial base station (BS) based localization: (Focus on the mobile localization in UMTS-FDD).
  - Hybrid solutions (GPS + BS).

Introduction: Some history...

- GPS is the first localization system (operational since 1991). Developped by US army mainly for military applications and navigation aid.

- New requirement by the FCC (federal communications commission) for all mobile operators to provide a localisation service for emergencies (911 service):
  - *Phase 1*: Localization with a precision ≤ 125m in 67% of the cases.
  - *Phase 2*: Localization with a precision ≤ 300m in 99% of the cases.

GPS

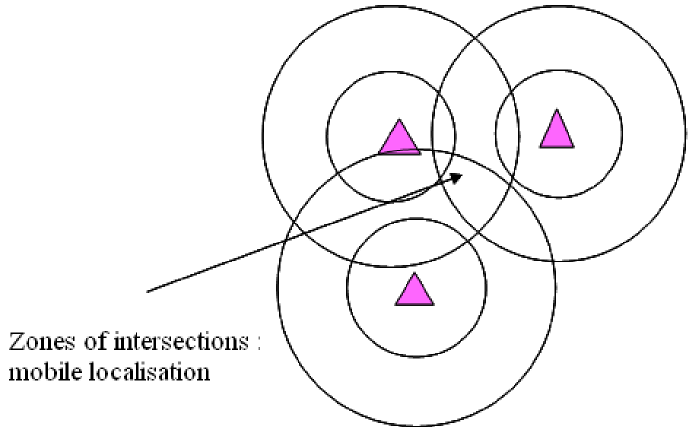


- Advantage : high precision.
- Drawbacks :
  - Requires the visibility by at least 3 satellites.
  - Generation of new mobiles : extra cost for the mobile operators.
  - Heavy initialization system.

Localisation techniques

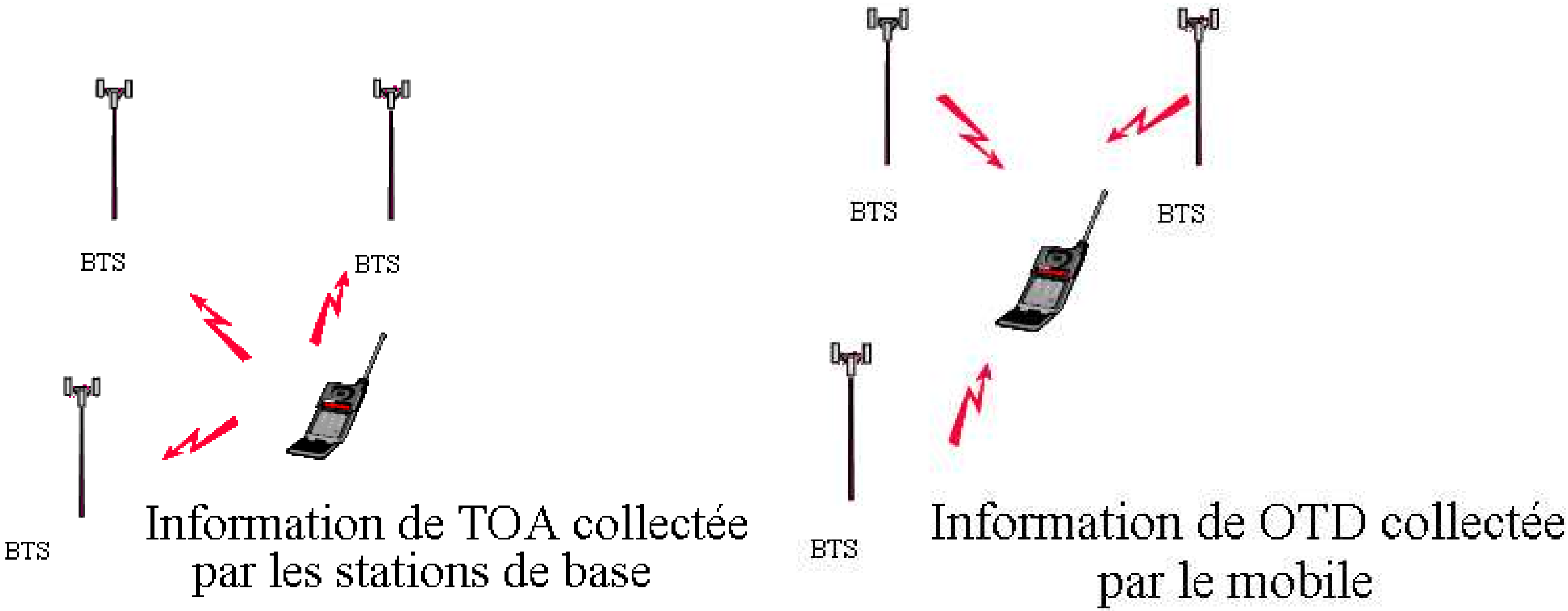
1. **Distance measures** $\Rightarrow$ **at least 3 base stations (BSs)** :
   - Power measure : exists in the standard.
   - Time of arrival (TOA) : Synchronisation of the BSs.
2. **Angle of arrival (AOA)** $\Rightarrow$ **at least 2 BSs** :
   - Installation of multi-sensor antennae : up-link.
3. **Angle-distance measure** $\Rightarrow$ **1 BS** :
   - AOA + distance measure (in the near field case).

Timing Advance (TA)

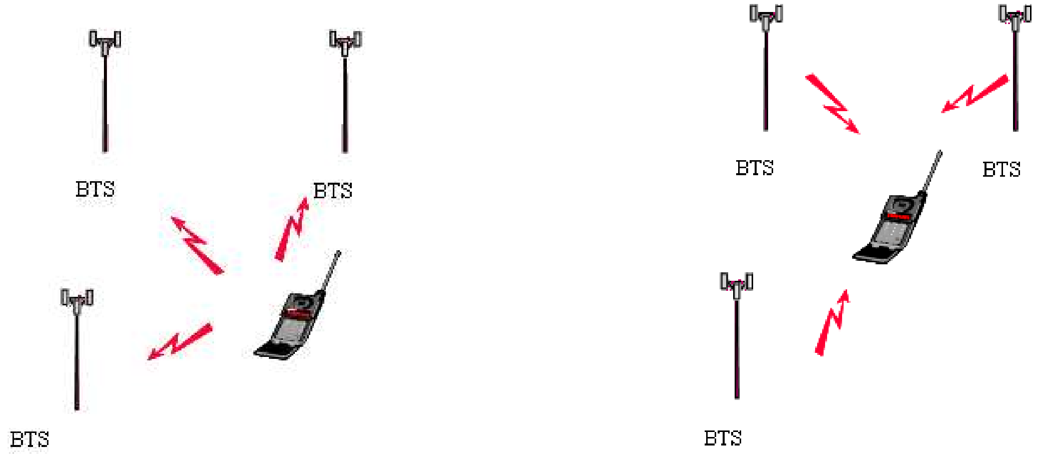


- TA: Proportional to the propagation time between the BS and the mobile.
- Quantification with 6 bits of the TA $\Rightarrow$ precision error of about 500m!!!
- Triangulation with the TA and al least 3 BS.

TOA / OTD (1)


BTS
BTS
BTS
Information de TOA collectée
par les stations de base
BTS
BTS
BTS
Information de OTD collectée
par le mobile

Localisation via time delays



- Necessitates the synchronisation of the BSs.
- Necessitates the use of at least 3 BSs.

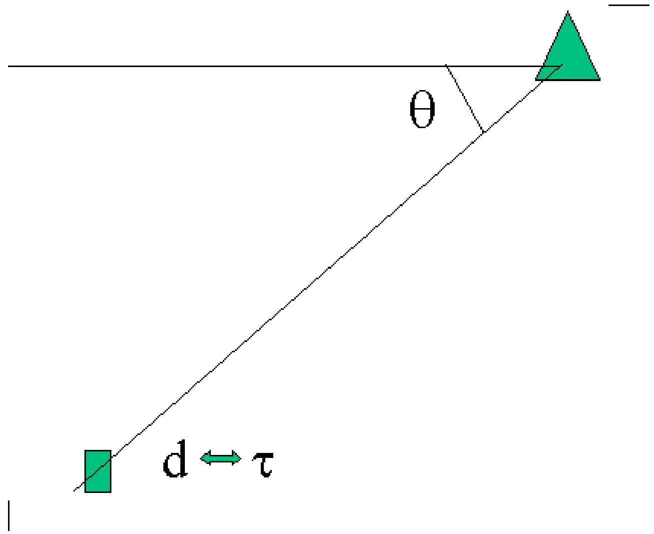
TOA / OTD (2)

- Time Of Arrival
  - Installation of heavy and expensive equipments at the BSs.
  - sensitive to multi-paths.
- Observed Time Difference
  - Certain improvement over the previous technique (signals are synchronized in the down-link).
  - Drawbacks:
    - Generation of new mobiles.
    - sensitive to multi-paths.

Power measures

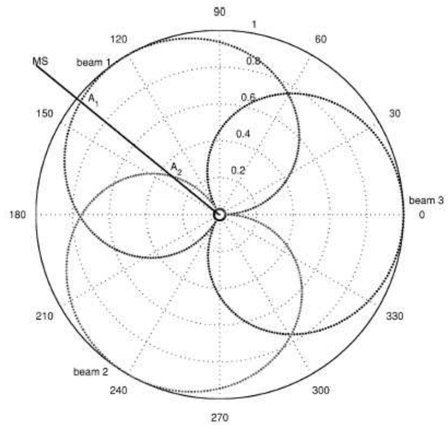
- $P_r = P_e(\frac{\lambda}{4\pi d})^\alpha$.
- Advantages:
  - Exists already in the standard.
  - Triangulation possible with more than 3 BSs. de base.
- Drawbacks:
  - Very sensitive to the received power model (tough modelization problem!!).

Angles and delays



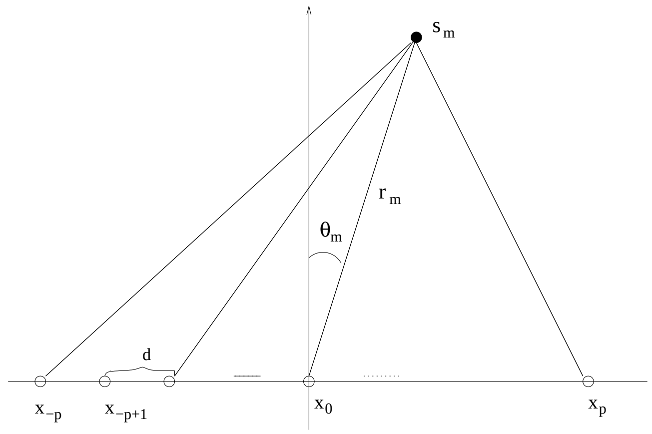
- Avantages :
  - Localisation is possible with only 1 BS (if synchronization).
- Drawbacks :
  - Requires multiple receivers (antenna array) at the BS.
  - Even more sensitive to multipaths effect.

Powers and delays



- Avantages :
  - Localisation is possible with the existing BSs that use 'sectorial' sensors.
- Drawbacks :
  - Very sensitive to the received model power.
  - Requires time synchronization of the BS with the mobiles.

Angles and ranges (Near Field)



- Avantages:
  - Localisation is possible with only 1 BS (without synchronization).
- Drawbacks:
  - Applicable only in the near-field!!

Differences between GSM & UMTS

- Advantages in favor of UMTS:
  - Better time resolution due to the oversampling w.r.t symbol duration.
  - Frequency re-use factor equal to 1: Mobile seen by neighboring cells.
- Advantages en faveur du GSM :
  - Relatively reduced multi-paths effect.

Preliminary results for the GSM

Power measure	140 meters (experiment realised in Paris)
Timing Advance	550 meters
OTA/TOA	110 meters
GPS	5 to 10 meters
Angle of arrival	$\approx$ 100 meters

Limiting factors in UMTS-FDD

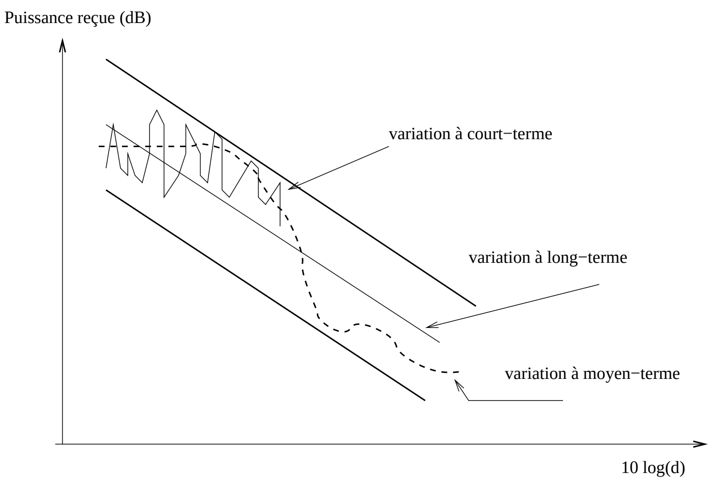
- **Estimation accuracy:** An error of one chip period $T_c \Rightarrow$ an error of 73m.
- **Hearing problem (particular to l'UMTS-FDD):** communication between the mobile and the far-located BSs.
  - First considered solutions:
    - Down-link: use of ldle periods.
    - Up-link: $\nearrow$ mobile power.

      $\Longrightarrow$ Reduces the system capacity and the mobile autonomy.
- **Non-line of sight (NLOS) problem:**
  - Considered solutions: Use redundant measures and perform selection.

Mobile Localization in UMTS-FDD

Using OTD (Down-link)

Signal Model



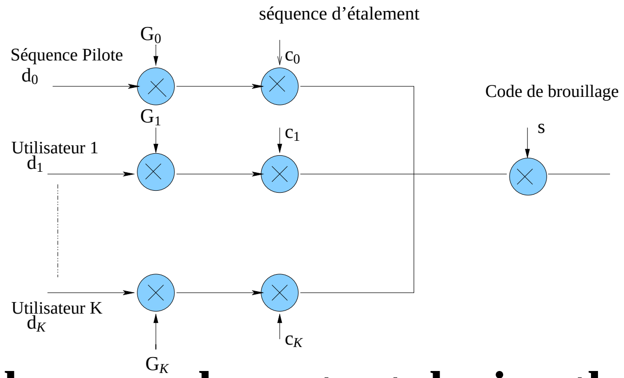
Tap	Retard relatif (ns)	Puissance relative σ_i (dB)	Spectr de Doppler $S(f)$
1	100	-3.2	CLASS
2	200	-5	CLASS
3	500	-4.5	CLASS
4	600	-3.6	CLASS
5	850	-3.9	CLASS
6	900	0.0	CLASS
7	1050	-3.0	CLASS
8	1350	-1.2	CLASS
9	1450	-5.0	CLASS
10	1500	-3.5	CLASS

$$h(t,\tau) = \sum_{i=1}^{M} h_i(t)\delta(t - \tau_i(t))$$

Channel model in a macro-cellular environment proposed by CODIT

The down link

- **Why chosing the down-link:**
  - A high power pilot existing during all the transmission period.
  - Transmitted signals are synchronized.
- **Signal transmitted by the BS:**



- **Propagation channel assumed constant during the slot period l:**

$$h^l(t) = \sum_{r=1}^{R} \beta_{r,l} g(t - \tau_r)$$

Estimation of TOAs

- **Principle (RAKE estimator):**
  - Estimation of $\hat{h}_l(k)$: Correlation between the l-th slot received signal and the shifted version of the pilot signal.
  - TOAs Estimation: Averaging over L slots.

$$\hat{h}(k) = \frac{1}{L}\sum_{l=1}^{L}|\hat{h}_l(k)|$$

- **Estimation accuracy:** $T_c/2$
  Refining the accuracy:
  - By oversampling.
  - By using high resolution methods.
- **Floor effect:** RAKE estimator is not robust against interferences.

Hearing problem

- **Objective:** Improve the robustness of channel estimate against interferences especially for far-located BSs.
- **Difficulty:** The mobile does not know the other user's signatures.
- **Proposed solutions:**
  - Projection of the channel estimate onto the principal subspace of its covariance matrix $\mathbf{\Gamma}$ (RAKE-SP).

$$\mathbf{h}_l = \mathbf{U}\mathbf{g}_l$$

    where $\mathbf{U}$ *represents the matrix of principal eigenvectors of* $\mathbf{\Gamma}$.

  - Remove (substract) the pilot signal of the serving BS to estimate the channels of far-located BSs.

$$\tilde{x}_l(i) = x_l(i) - \hat{p}_l^1(i)$$

High resolution (MUSIC) algorithm

- : Estimation of the channel covariance matrix

$$\hat{\mathbf{\Gamma}} = \frac{1}{J} \sum_{j=1}^{J} \hat{\mathbf{h}}_j \hat{\mathbf{h}}_j^H \longrightarrow_{J\to\infty} \mathbf{A}(\tau)\mathbf{G}\mathbf{A}(\tau)^H + \sigma_0 \mathbf{R}_0$$

- Estimation of the generalized eigenvectors of $\hat{\mathbf{\Gamma}}$:

$$\hat{\mathbf{\Lambda}}\mathbf{e}_i = \lambda_i \mathbf{R}_0 \mathbf{e}_i$$

- Delay estimation by minimising:

$$v(\tau) = \frac{\mathbf{r}_\tau \mathbf{r}_\tau^H}{\mathbf{r}_\tau \mathbf{E}\mathbf{E}^H \mathbf{r}_\tau^H}$$

where $\mathbf{E}$ represents the matrix of noise eigenvectors of $\mathbf{\Lambda}$ and $\mathbf{r}_\tau$ is the pilot signal autocorrelation vector evaluated for a time lag τ.

Discussion

- MUSIC allows a better estimation of the time delay (see simulation results).
- However, MUSIC is relatively expensive $\Rightarrow$ especially for the down-link (limited mobile power).
- One should reduce its complexity (size of vector $\mathbf{h}$) by using a windowing around the first peak of the RAKE $\Rightarrow$ Two step procedure where MUSIC represents the 'refinement' step.

Triangulation with more than 3 BSs

- Relation between the TOAs and the mobile position (x, y) :

$$\hat{t}_i = \frac{\sqrt{(x - x_i)^2 + (y - y_i)^2}}{c} + t_0 + w_i$$

t_0= temps de référence et w_i= bruit d'estimation.

- System resolution:
  - Solving the system in the least squares sence (non-linear equations).
  - Explicit solution (after linearization):

$$\begin{pmatrix} c^2(t_2^2 - t_1^2) \\ \vdots \\ c^2(t_I^2 - t_1^2) \end{pmatrix} = -2 \begin{pmatrix} x_{2,1} & y_{2,1} & c(t_2 - t_1) \\ \vdots & \vdots & \vdots \\ x_{I,1} & y_{I,1} & c(t_I - t_1) \end{pmatrix} \begin{pmatrix} x \\ y \\ t_0 \end{pmatrix} + \begin{pmatrix} K_2 - K_1 \\ \vdots \\ K_C - K_1 \end{pmatrix}$$

Triangulation: Chan's method

- If the number of BS is 3:
  - One solves w.r.t. r_1 :

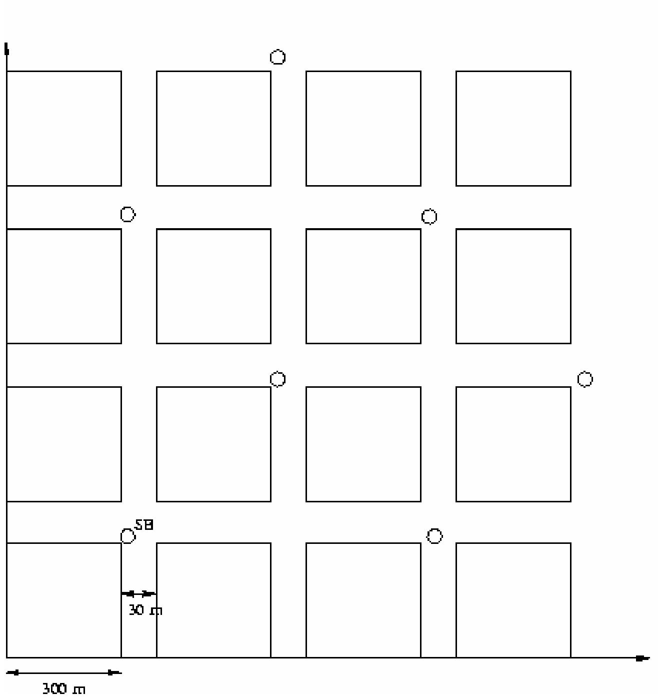
$$\begin{pmatrix} x \\ y \end{pmatrix} = - \begin{pmatrix} x_{2,1} & y_{2,1} \\ x_{3,1} & y_{3,1} \end{pmatrix} \left[\begin{pmatrix} r_{2,1} \\ r_{3,1} \end{pmatrix} r_1 + \frac{1}{2} \begin{pmatrix} r_{2,1}^2 - K_2 + K_1 \\ r_{3,1}^2 - K_3 + K_1 \end{pmatrix} \right]$$

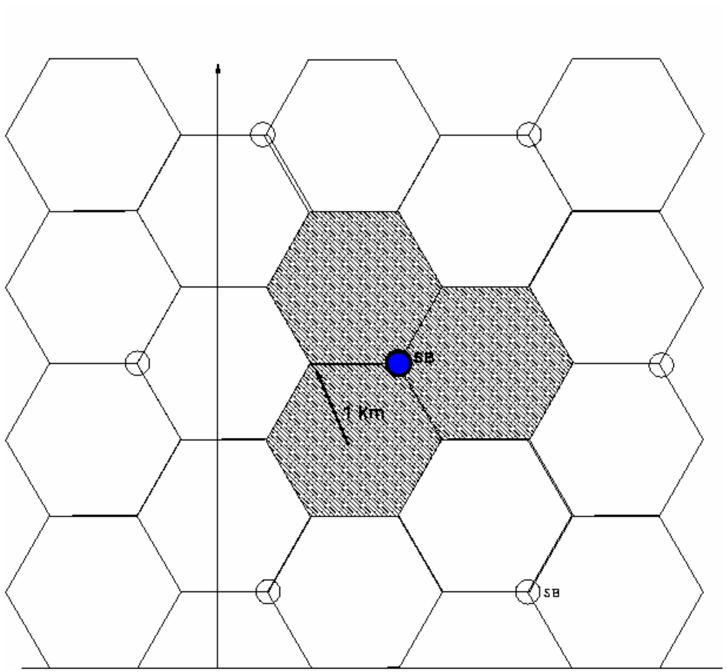
  - Then, we solve a second order polynomial equation in r_1 :

$$r_1^2 = \begin{pmatrix} x & y \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} - \begin{pmatrix} 2x_1 & 2y_1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} x_1 & y_1 \end{pmatrix} \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}$$

- Among the 2 possible solutions, one choses the one withing the area covered by the serving BS.

MICRO & MACRO cells)



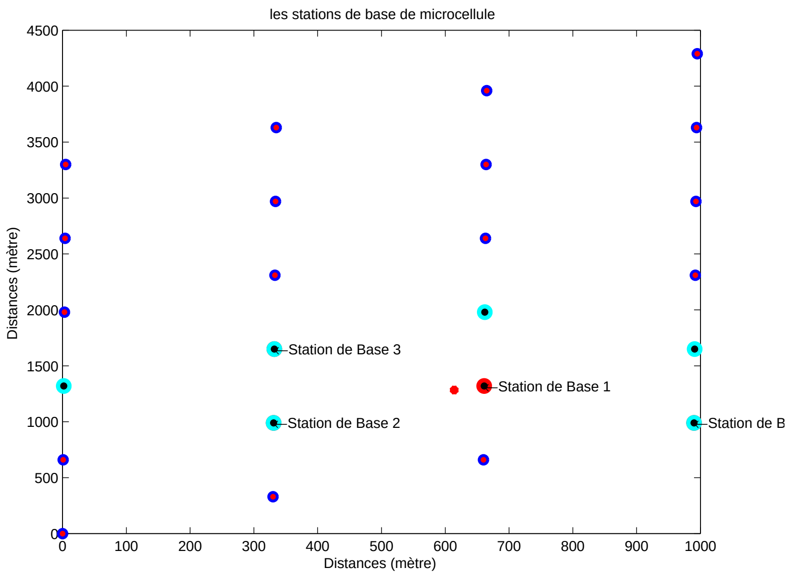


(g) Micro-cell (Manhattan)

(h) Macro-cell

Simulation

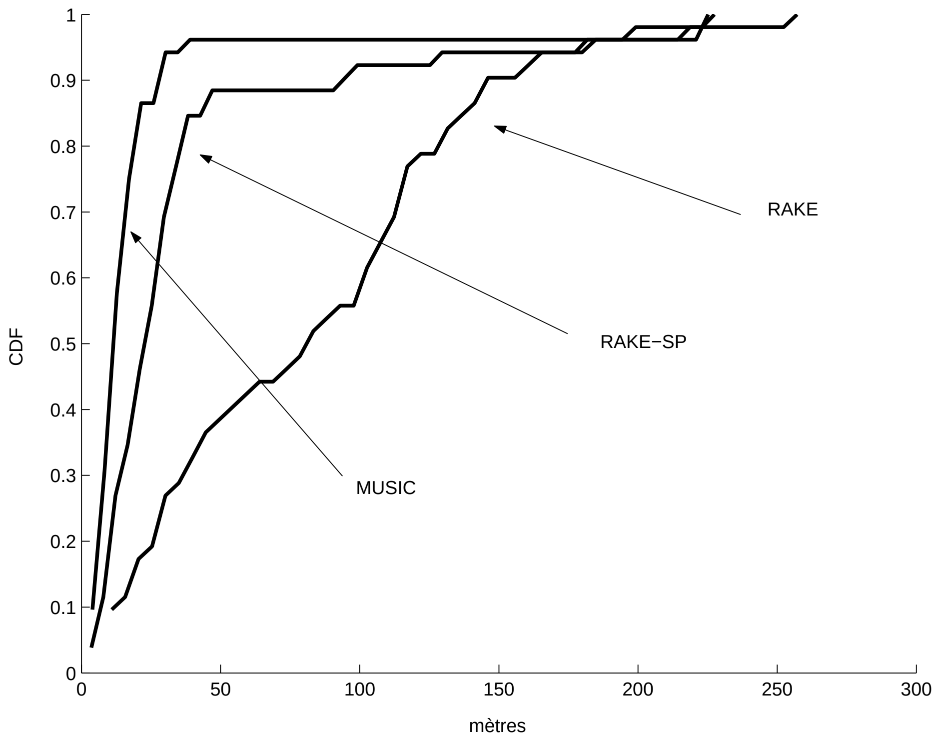
- Simulation in a micro-cell environment (Manhattan).
- Three paths per channel, triangulation with 4 BSs.
- Additif noise representing 10% of the total received power of the furthest BS.
- Loose power control (the ratio between the maximal and minimal powers is ≤ 10).



Micro-cell environment

RAKE-SP

- Comparison of the performance obtained by MUSIC, RAKE-SP and RAKE.



Random position of the mobile, 30 users, $L = 240$ slots.

Dealing with NLOS

- **Proposed solution:** Selection of the 3 'most coherent' TOA measures (we assume mobile hearing by more than 3 BSs).
- **Coherence criterion:**
  - Coherence of the estimated position $M_{i,j,l}(t_i, t_j, t_l)$ (using BSs i, j and l) with TOA t_k assuming a time reference t_0 known:

$$\xi_{i,j,l}^k(t_0) = \| \sqrt{(x_{i,j,l} - x_k)^2 + (y_{i,j,l} - y_k)^2} - c(t_k - t_0) \|^2$$

    Minimisation of $\xi_{i,j,l}^k$ over all possible choices of i, j, l

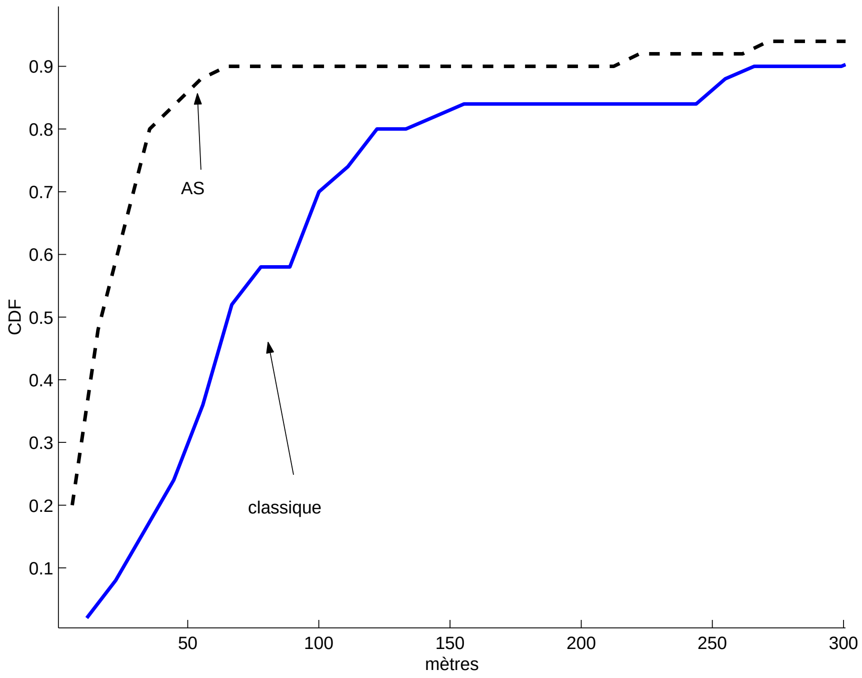
$$\hat{i}, \hat{j}, \hat{l} = \arg \min_{i,j,l,k} \xi_{i,j,l}^k(t_0)$$

  - The time reference t_0 being unknown (one minimizes numerically):

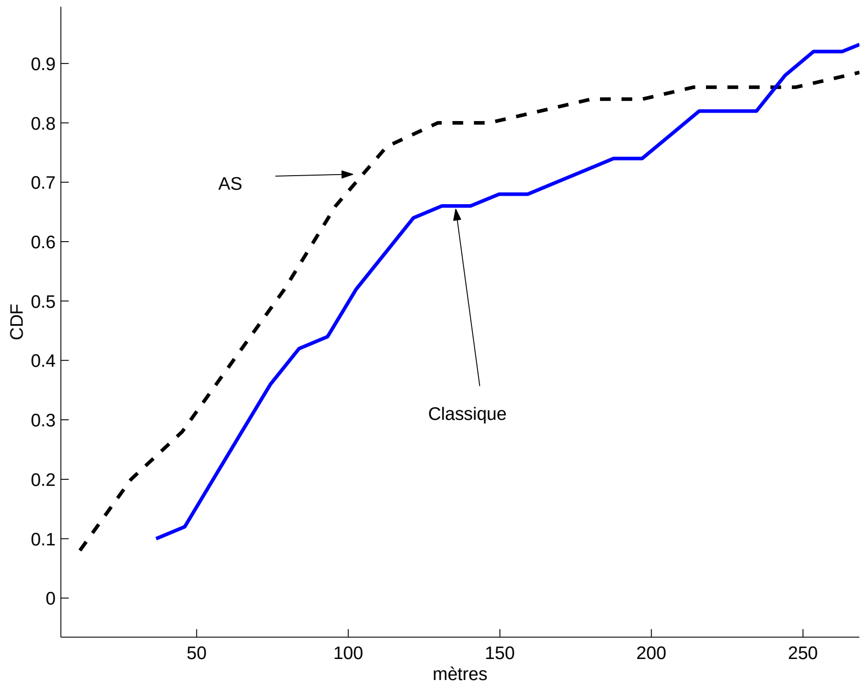
$$\hat{i}, \hat{j}, \hat{l}, \hat{t_0} = \arg \min_{i,j,l,k,t_0} \xi_{i,j,l}^k(t_0)$$

Algorithm of selection (AS)

random position of the mobile at each run, $L = 120$ slots, 8 BSs, K=15



(i) TOAs estimated by MUSIC, NLOS on the 2nd BS



(j) TOAs estimated by RAKE-SP, NLOS on the 2nd BS

Mobile Localisation Using Angle of Arrival (Up-Link)

Estimation of the AOA

- Requires at least two sensors $\Rightarrow$ Applicable in the uplink.
- Possible with existing BSs but poor estimation accuracy.
- Estimation using ‘smart antennae’ $\Rightarrow$ array processing for source localization.

Array Processing: Basic Concepts

Objectives

- Signal processing extracts information from measured signals.
- Array signal processing uses a group of sensors:
  - Signal enhancement / noise reduction.
    - Coherence adding.
    - Spatial filtering.
  - Source / channel characterizations :
    - number of sources.
    - location 'direction finding'.
    - waveforms 'information from the sources'.

Applications

- Wireless communications.
- Interference mitigation.
- Radar / Sonar.
- Biomedical.
- Speech.
- Seismic.
- ....

Coherent adding

- Let us have an array of M sensors $(m = 1, \cdots, M)$:

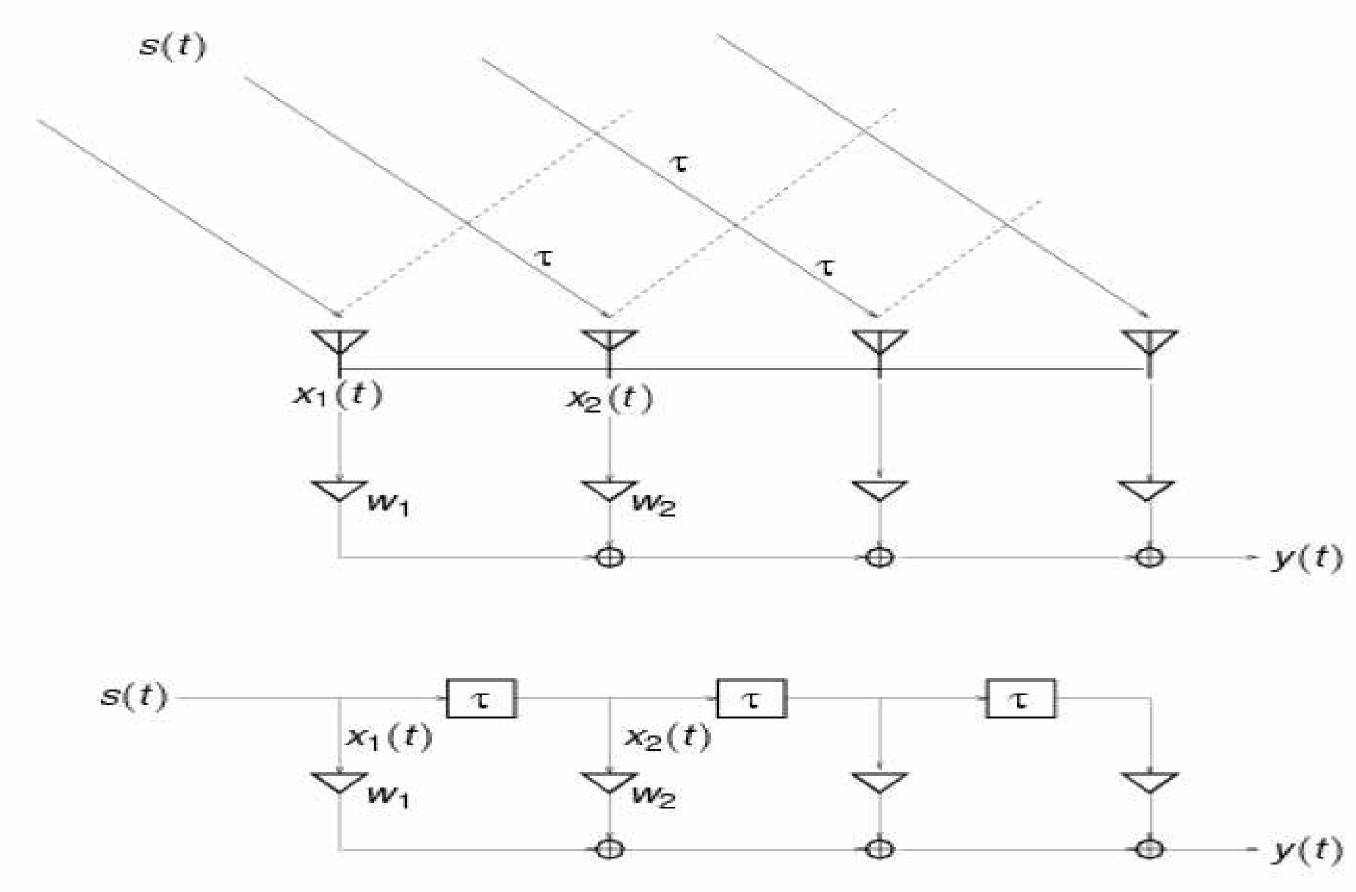
$$x_m(t) = s(t) + n_m(t), \qquad \text{noise variance } \sigma^2$$

- If the noise on the antennas is uncorrelated, then

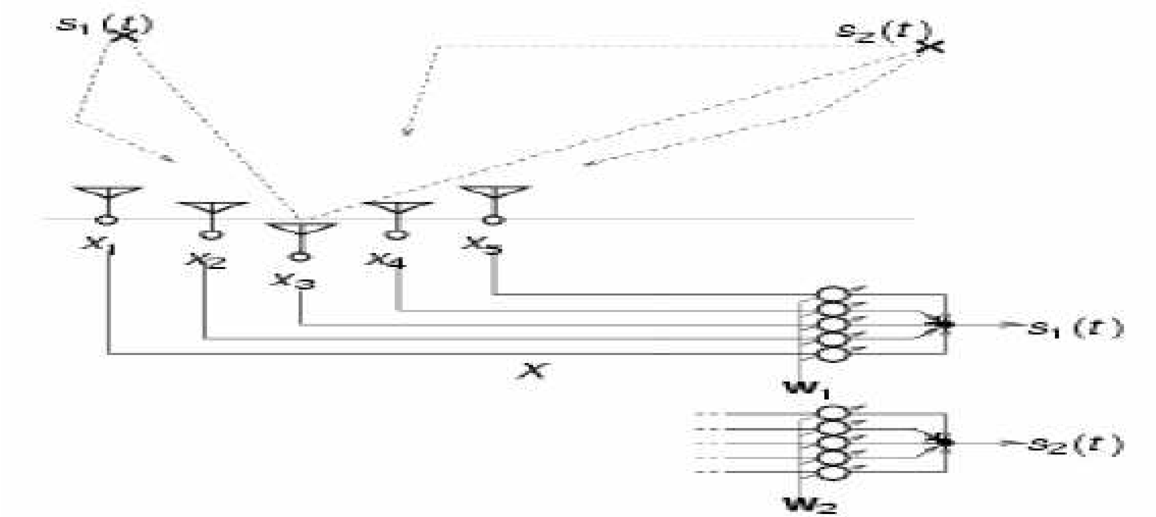
$$y(t) = \frac{1}{M} \sum_{m=1}^{M} x_m(t) = s(t) + \frac{1}{M} \sum_{m=1}^{M} n_m(t), \quad \text{noise variance } \frac{1}{M}\sigma^2$$

Hence the noise power is reduced by a factor M.

Spatial filtering



Spatial filtering



- Cancelling out interferers : Source separation
  - Classical beamforming requires known 'look directions', or a reference signal.
  - Blind beamforming : no a priori direction information. Relies on structural properties.

Data model

Baseband signal

- An antenna receives a real valued bandpass signal with center frequency f_c,

$$z(t) = \Re\{s(t)e^{j2\pi f_c t}\} = x(t)\cos(2\pi f_c t) - y(t)\sin(2\pi f_c t)$$

- The baseband signal is

$$s(t) = x(t) + jy(t)$$

  It is the complex envelope of $z(t)$

- $s(t)$ is recovered from $z(t)$ by demodulation : multiplying the received signal with $\cos(2\pi f_c t)$ and $\sin(2\pi f_c t)$ followed by low pass filtering.

Data model

Small delays of narrow band signals

- Recall $z(t) = \Re\{s(t)e^{j2\pi f_c t}\}$. We investigate the effect of small delays of $z(t)$ on the baseband signal $s(t)$

$$z_\tau(t) \triangleq z(t-\tau) = \Re\{s(t-\tau)e^{-j2\pi f_c \tau} e^{j2\pi f_c t}\}$$

- The complex envelope of the delayed signal is

$$s_\tau(t) = s(t-\tau)e^{-j2\pi f_c \tau}$$

Data model

Small delays of narrow band signals

- Let W be the bandwidth of $s(t)$. If $e^{-j2\pi f\tau} \approx 1$ for all frequencies $|f| \leq \frac{W}{2}$, then

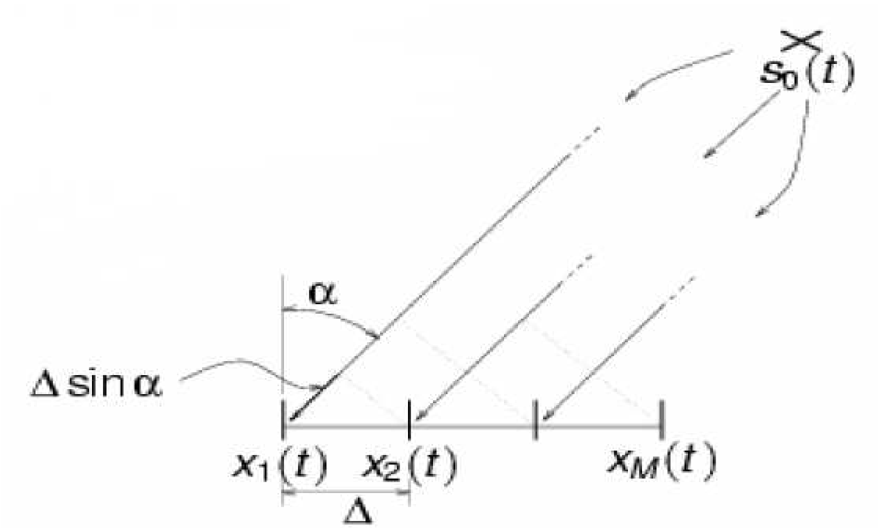
$$s(t-\tau) = \int_{-\frac{W}{2}}^{\frac{W}{2}} S(f) e^{j2\pi f(t-\tau)} df \approx \int_{-\frac{W}{2}}^{\frac{W}{2}} S(f) e^{j2\pi f t} df = s(t)$$

For narrowband signals, time delays shorter than the inverse bandwidth amount to phase shifts of the complex envelope.

Data model

Antenna array response

$$\tau = \frac{L}{c} = \frac{d\sin(\alpha)}{c} = \frac{\lambda\Delta\sin(\alpha)}{c} = \frac{\Delta\sin(\alpha)}{f_c}$$



- Far field assumption : planar waves.
- $a(\alpha)$: Antenna gain pattern.

Data model

Antenna array response

- Let $s(t)$ be the baseband signal at the first antenna : $x_1(t) = a(\alpha)s(t)$
- The signal received by x_2 at a distance of Δ wavelengths experiences an addition delay τ.
- If τ is small compared to the inverse bandwidth of $s(t)$, then

$$s_\tau(t) = s(t)e^{-j2\pi\Delta\sin(\alpha)}$$

- Collect the received signals into a vector $\mathbf{x}(t)$:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ \vdots \\ x_M(t) \end{bmatrix} = \begin{bmatrix} e^{-j2\pi\Delta_1\sin(\alpha)} \\ \vdots \\ e^{-j2\pi\Delta_M\sin(\alpha)} \end{bmatrix} a(\alpha)s(t) = \mathbf{a}(\alpha)s(t)$$

$\mathbf{a}(\alpha)$ is the array response vector. For uniform linear array $\Delta_k = (k-1)\Delta$.

Data model

Array manifold

$$\mathbf{x}(t) = \mathbf{a}(\alpha)s(t)$$

- The array manifold :

$$\mathbf{\Omega} = \{\mathbf{a}(\alpha) : -\pi \leq \alpha \leq \pi\}$$

- The knowledge of $\mathbf{\Omega}$ allows direction finding (i.e. determine α from $\mathbf{x}$).

Spatial Localisation

- Find the number and positions of the sources.
- Sweep all space directions using beamforming
  - Matched filter $\Rightarrow$ Bartelett's method.
  - MVDR $\Rightarrow$ Capon's method.
- Exploit the data model & covariance matrix structure
  - MUSIC (subspace) algorithm
  - ESPRIT algorithm.

Bartlett's method

- Estimate the covariance and sweep all angles

$$\varphi(\theta) = E(|y(t)|^2) = \mathbf{w}^H \mathbf{R} \mathbf{w}$$

- Sum-delay (matched filter) beamforming

$$\mathbf{w} = \frac{\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H \mathbf{a}(\theta)} \Rightarrow \varphi(\theta) = \frac{\mathbf{a}(\theta)^H \mathbf{R} \mathbf{a}(\theta)}{(\mathbf{a}(\theta)^H \mathbf{a}(\theta))^2}$$

- For a uniform linear array (ULA)

$$\varphi(\theta) = \frac{1}{N^2} \mathbf{a}(\theta)^H \mathbf{R} \mathbf{a}(\theta)$$

Computation using Fourier transform

- Development of the quadratic transform

$$\varphi(\theta) = \mathbf{a}(\theta)^H \mathbf{R}\mathbf{a}(\theta) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} \alpha_n^* R_{nm} \alpha_m$$

- For ULA

$$\alpha_n = (e^{-j2\pi\nu_\theta})^n \Rightarrow \varphi(\theta) = \sum_{n=0}^{N-1} \sum_{m=0}^{N-1} R_{nm} (e^{-j2\pi\nu_\theta})^{n-m}$$

- Fourier transform

$$\varphi(\theta) = \sum_{q=-N+1}^{N-1} (e^{-j2\pi\nu_\theta})^q \sum_{n=\max(0,q)}^{N-1+\min(0,q)} R_{n,n-q}$$

Capon's method (MVDR)

- Sweep all angle positions with the MVDR spatial filter

$$\mathbf{w} = \frac{\mathbf{R}^{-1}\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta)}$$

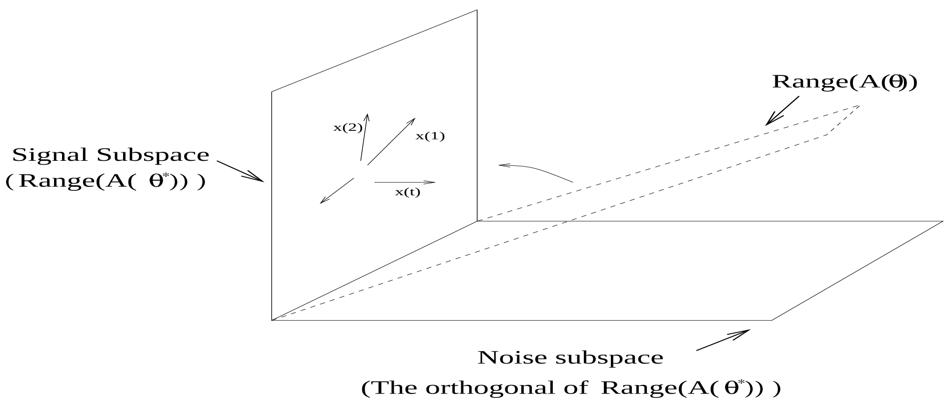
- The localisation function becomes

$$\varphi(\theta) = \frac{1}{\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta)}$$

car $\varphi(\theta) = \mathbf{w}^H\mathbf{R}\mathbf{w} = \frac{\mathbf{a}(\theta)\mathbf{R}^{-1}}{\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta)}\mathbf{R}\frac{\mathbf{R}^{-1}\mathbf{a}(\theta)}{\mathbf{a}(\theta)^H\mathbf{R}^{-1}\mathbf{a}(\theta)}$

- Can be computed using Fourier transform but with $\mathbf{R}^{-1}$ instead of $\mathbf{R}$.

MUSIC (subspace) method



Principle: Assume the following model: $\mathbf{x}(n) = \mathbf{A}(\theta)\mathbf{s}(n)$ with

$$\mathrm{Range}(\mathbf{A}(\theta)) = \mathrm{Range}(\mathbf{A}(\theta')) \Longleftrightarrow \theta = \theta'$$

Thus, θ can be estimated as:

$$\hat{\theta} = \arg\min_{\theta} \mathrm{d}(\mathrm{Range}\{\mathbf{x}(n)\},\ \mathrm{Range}(\mathbf{A}(\theta)))$$

MUSIC

- Estimate the signal (resp. noise) subspace as the principal (resp. minor) eigen-subspace of the data covariance matrix $\mathbf{R}_x$:

$$\mathbf{R}_x = \sum_n \mathbf{x}(n)\mathbf{x}^H(n) = [\mathbf{E}_s \; \mathbf{E}_n] \begin{bmatrix} \mathbf{\Lambda}_s & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \mathbf{E}_s^H \\ \mathbf{E}_n^H \end{bmatrix}$$

where $\quad \mathrm{Range}(\mathbf{E}_s) = \mathrm{Range}(A(\theta)) \perp \mathrm{Range}(\mathbf{E}_n).$

- Orthogonal relation still valid if additive white noise.

MUSIC

- The source angle locations are estimated by minimizing:

$$\min_{\theta} \mathbf{a}(\theta)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta)$$

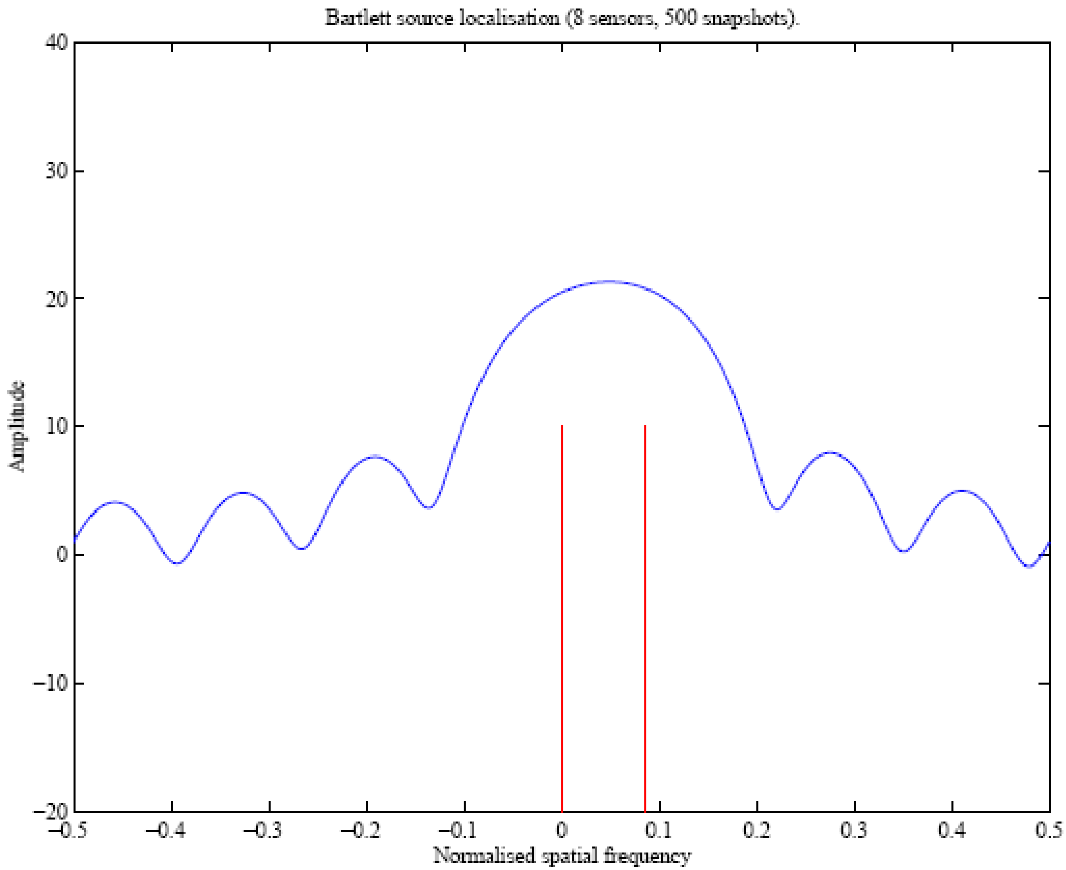
- Or equivalently by maximizing the MUSIC localisation function

$$\varphi(\theta) = \frac{1}{\mathbf{a}(\theta)^H \mathbf{E}_n \mathbf{E}_n^H \mathbf{a}(\theta)}$$

  The P sources locations correspond to the P maximas of the above function.

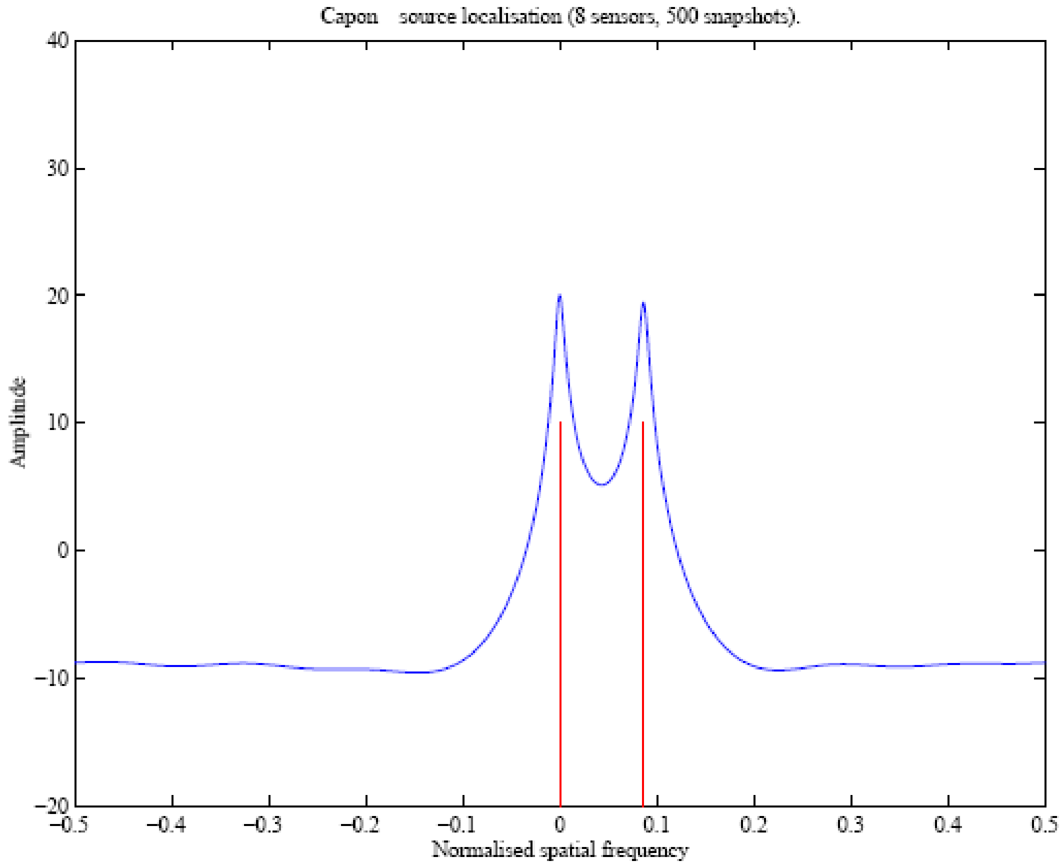
Example (1)

- Bartlett's method (SNR = 20dB)



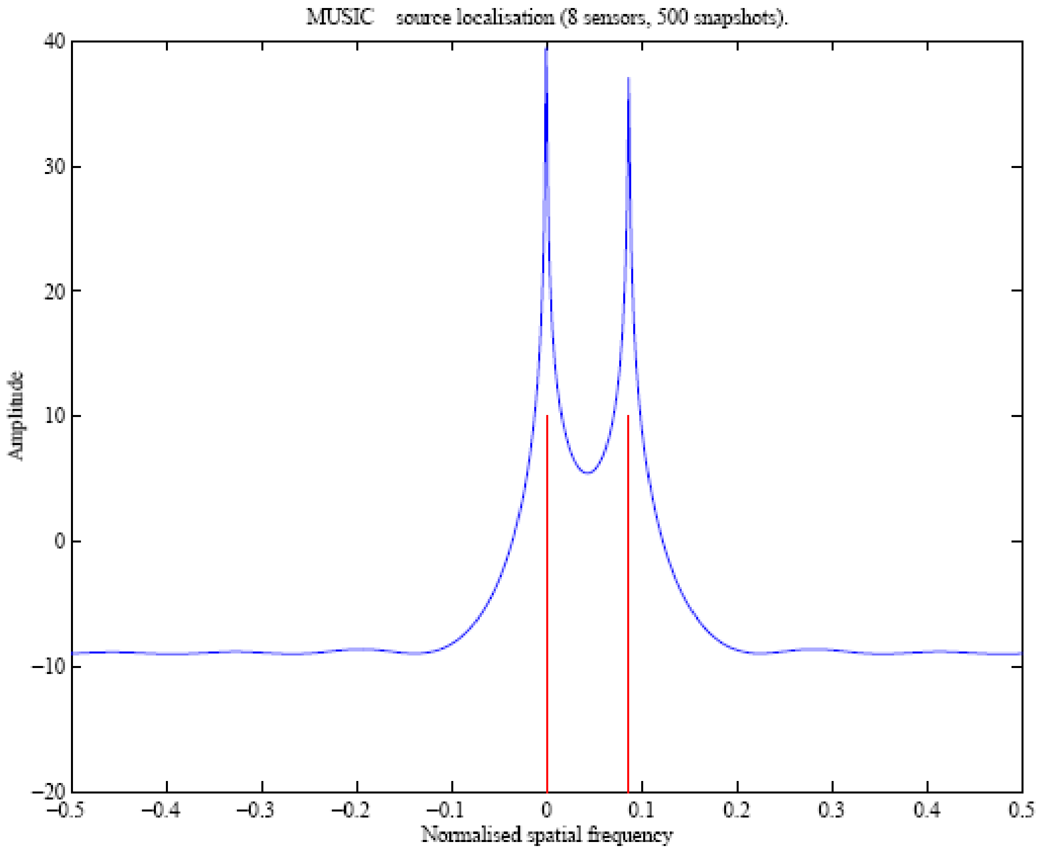
Example (2)

- Capon's method (SNR = 20dB)



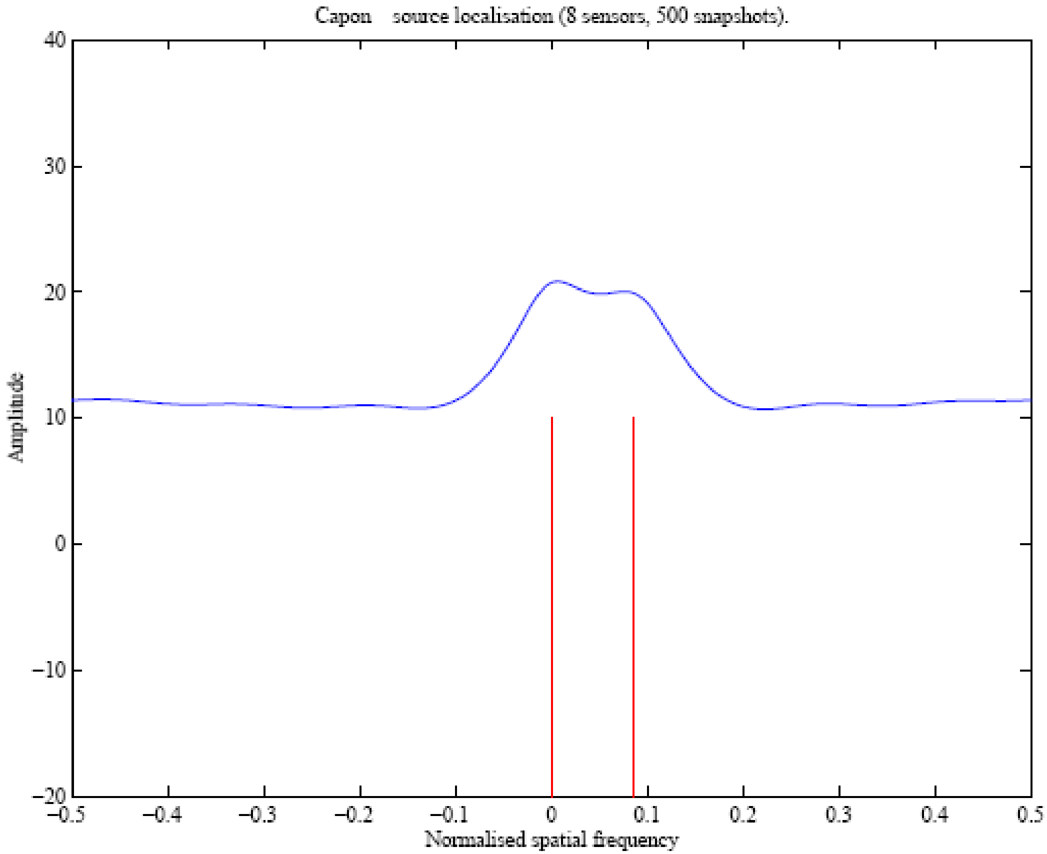
Example (3)

- MUSIC method (SNR = 20dB)



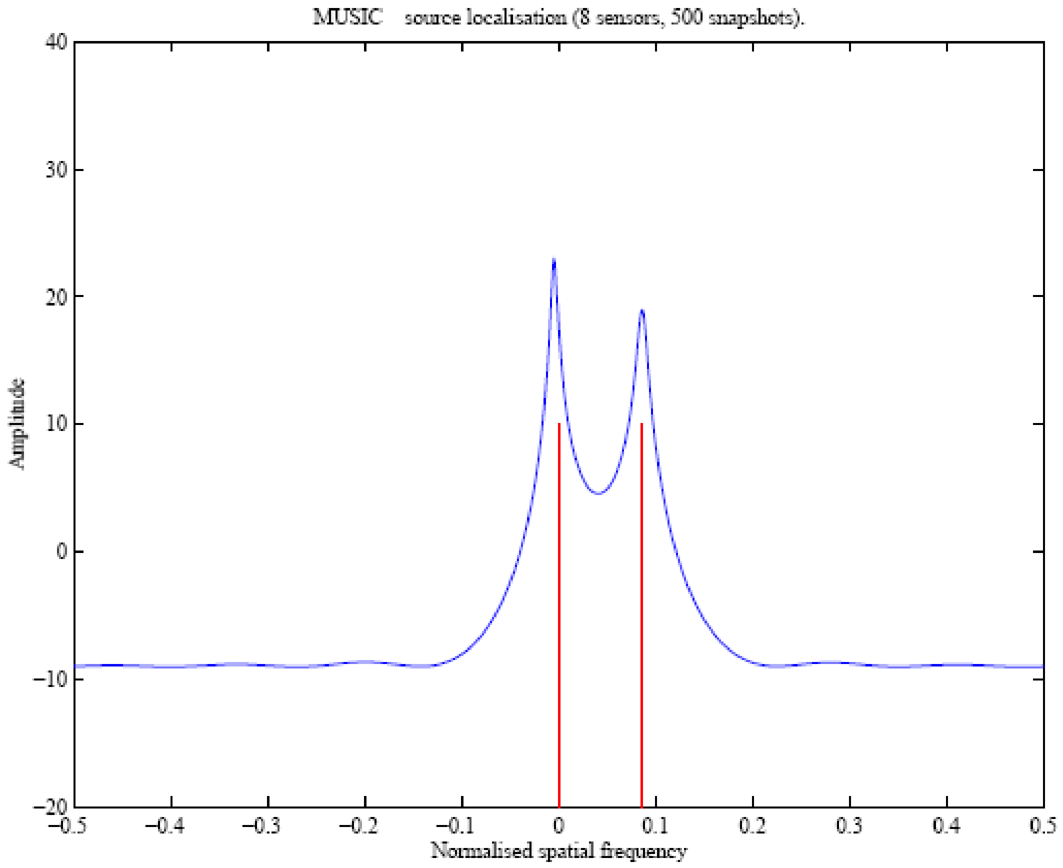
Example (4)

- Capon's method (SNR = 0dB)



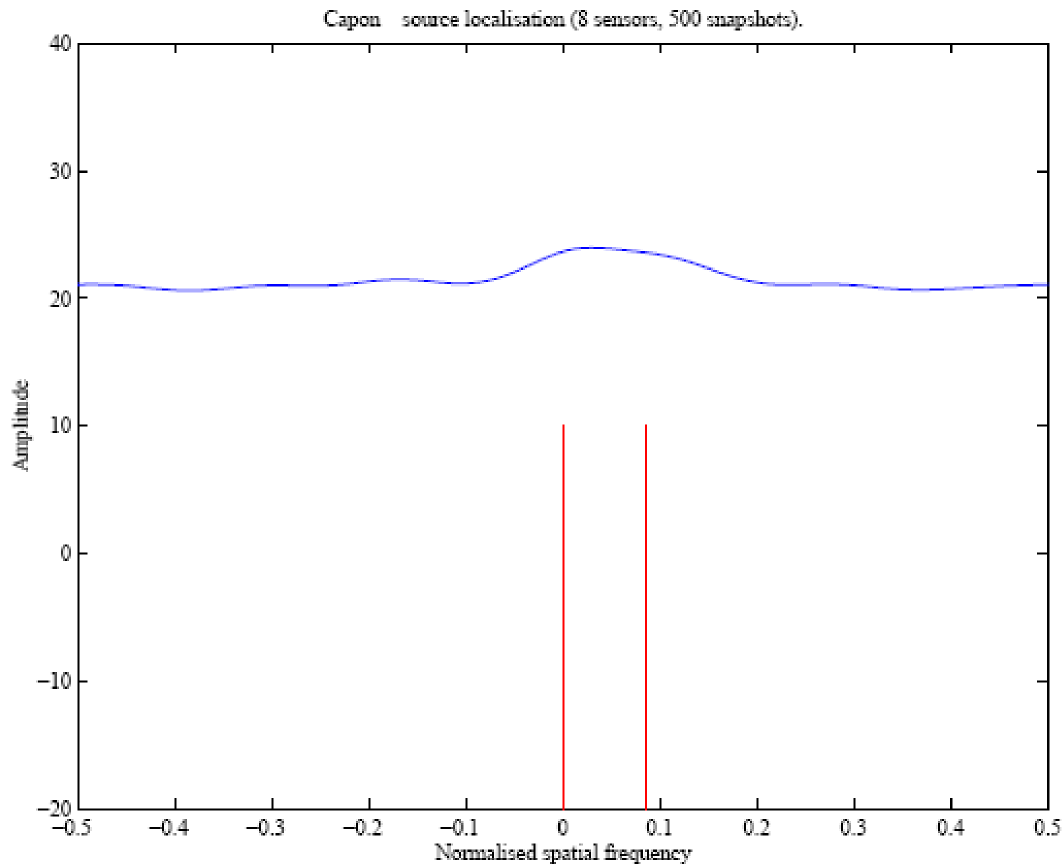
Example (5)

- MUSIC method (SNR = 0dB)



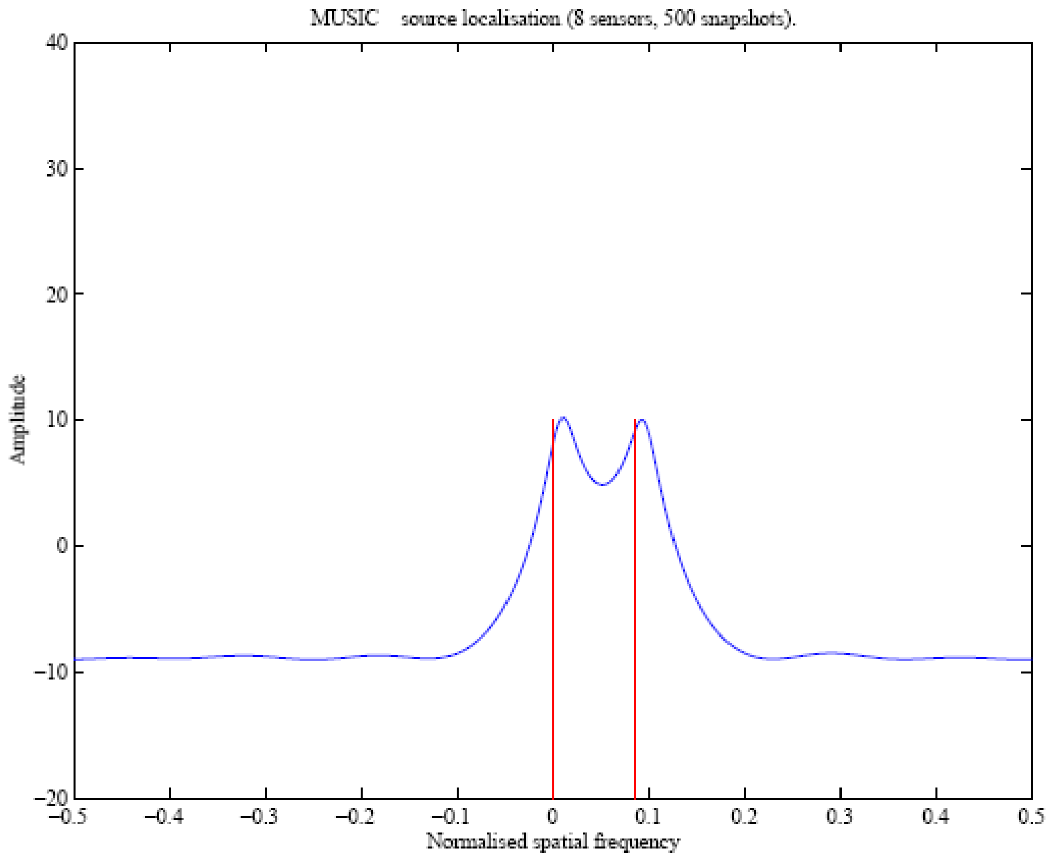
Example (6)

- Capon's method (SNR = -10dB)



Example (7)

- MUSIC method (SNR = -10dB)



ESPRIT Method

- Consider a ULA
- Structure of the directional vector

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{-j2\pi f \frac{d}{C} \sin\theta} \\ \vdots \\ e^{-j2\pi f (N-1) \frac{d}{C} \sin\theta} \end{bmatrix} = \begin{bmatrix} 1 \\ e^{-j2\pi\nu_\theta} \\ \vdots \\ \left(e^{-j2\pi\nu_\theta}\right)^{N-1} \end{bmatrix}$$

- By removing the first or the last entry of this vector, one obtains two linearly dependent subvectors of $\mathbf{a}(\theta)$.

Rotational invariance

- Oo the directional vector

$$\mathbf{a}(\theta) = \begin{bmatrix} \mathbf{a}_1(\theta) \\ \text{row } N \end{bmatrix} = \begin{bmatrix} \text{row } 1 \\ \mathbf{a}_2(\theta) \end{bmatrix} \Rightarrow \mathbf{a}_2(\theta) = \mathbf{a}_1(\theta) e^{-j2\pi\nu_\theta}$$

- On matrix $\mathbf{A}$

$$\mathbf{A} = [\mathbf{a}(\theta_1), \mathbf{a}(\theta_2), \cdots, \mathbf{a}(\theta_P)]$$

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_1 \\ \text{row } N \end{bmatrix} = \begin{bmatrix} \text{row } 1 \\ \mathbf{A}_2 \end{bmatrix} \Rightarrow \mathbf{A}_2(\theta) = \mathbf{A}_1 \Phi$$

$$\Phi = \text{diag}(e^{-j2\pi\nu_{\theta_p}})$$

- Matrix Φ provides directly the desired angles.

ESPRIT method

- The same transform on the eigenvectors of the signal subspace leads to

$$\mathbf{U}_1 = \mathbf{A}_1\mathbf{T}, \;\; \mathbf{U}_2 = \mathbf{A}_2\mathbf{T}$$

$$\mathbf{U}_2 = \mathbf{A}_1\Phi\mathbf{T} = \mathbf{U}_1\mathbf{T}^{-1}\Phi\mathbf{T}$$

- Il suffit de trouver Ψ tel que $\mathbf{U}_2 = \mathbf{U}_1\Psi$ By least squares estimation:

$$\Psi = (\mathbf{U}_1^H\mathbf{U}_1)^{-1}\mathbf{U}_1^H\mathbf{U}_2$$

- Φ and Ψ have the same eigenvalues

$$\text{Eig}(\Psi) = \text{diag}(e^{j2\pi\nu_{\theta_p}})$$

‘Generalized’ ESPRIT method

- ESPRIT can be used, not only for ULA but for any array containing 2 sub-arrays such that the 2nd is the translated version of the first one. Hence, for a source located at θ

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{a}_1(\theta) \\ \mathbf{a}_2(\theta) \end{bmatrix} s(t) \text{ with } \mathbf{a}_2(\theta) = \mathbf{a}_1(\theta) e^{-j2\pi\nu_\theta}$$

- Space shift: plays the role of the inter-sensors distance in ULA.

$$\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 \\ \mathbf{A}_2 \end{bmatrix} \mathbf{s}(t) \Rightarrow \mathbf{A}_1 = \mathbf{A}_2 \Phi$$

ESPRIT method: algorithm

- Eigen-decomposition of the covariance algorithm (noiselesss case)

$$\mathbf{R} = \mathbf{E}\left(\begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix} \begin{bmatrix} \mathbf{x}_1(t) \\ \mathbf{x}_2(t) \end{bmatrix}^H \right) = \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix} \Lambda \begin{bmatrix} \mathbf{U}_1 \\ \mathbf{U}_2 \end{bmatrix}^H$$

- Rotational invariance property for $\mathbf{A}$ and $\mathbf{U}_s$

$$\begin{aligned} \mathbf{U}_1 &= \mathbf{A}_1\mathbf{T} \\ \mathbf{U}_2 &= \mathbf{A}_2\mathbf{T} \end{aligned}$$

$$\exists\Psi \;\text{ such that }\; \mathbf{U}_2 = \mathbf{U}_1\Psi$$

- Matrix Φ is the matrix of eigenvalues of Ψ

$$\text{Eig}(\Psi) = \text{diag}(e^{j2\pi\nu_{\theta_p}})$$

Other localization methods: ML

- In the AWGN and deterministic inputs case, the likelihood function can be expressed as:

$$L(\theta, s(t), \sigma^2) = \prod_{t=1}^{T} (\pi\sigma^2)^{-N} e^{-\frac{\|x(t)-As(t)\|^2}{\sigma^2}}$$

- Let Π_A be the orthogonal projection matrix on Range$(\mathbf{A})$

$$\Pi_A = \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H \Rightarrow \mathbf{A}\hat{\mathbf{s}}(t) = \Pi_A \mathbf{x}(t)$$

$$\overline{\Pi}_A = \mathbf{I} - \Pi_A \Rightarrow \theta = \arg\min_{\theta} \mathrm{Tr}(\overline{\Pi}_A \hat{\mathbf{R}})$$

Other methods: Weighted subspace fitting

- Exploits the relation between $\mathbf{U}_s$ and $\mathbf{A}$

$$\exists \mathbf{T} \text{ such that } \mathbf{U}_s = \mathbf{A}\mathbf{T}$$

- Minimise the LS distance

$$\{\hat{\theta}, \hat{\mathbf{T}}\} = \arg\min_{\theta, \mathbf{T}} \|\hat{\mathbf{U}}_s - \mathbf{A}\mathbf{T}\|_W^2$$

- Solving in $\mathbf{T}$ first followed by an estimation of θ

$$\hat{\mathbf{T}} = \mathbf{A}^{\#}\mathbf{U}_s \text{ with } \mathbf{A}^{\#} = (\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$$

$$\text{then } \hat{\theta} = \arg\min_{\theta} \mathrm{Tr}(\overline{\Pi}_A \mathbf{U}_s \mathbf{W} \mathbf{U}_s)$$

Asymptotic optimal weighting $\mathbf{W} = (\Lambda_s - \sigma^2\mathbf{I})^2\Lambda_s^{-1}$

Discussion

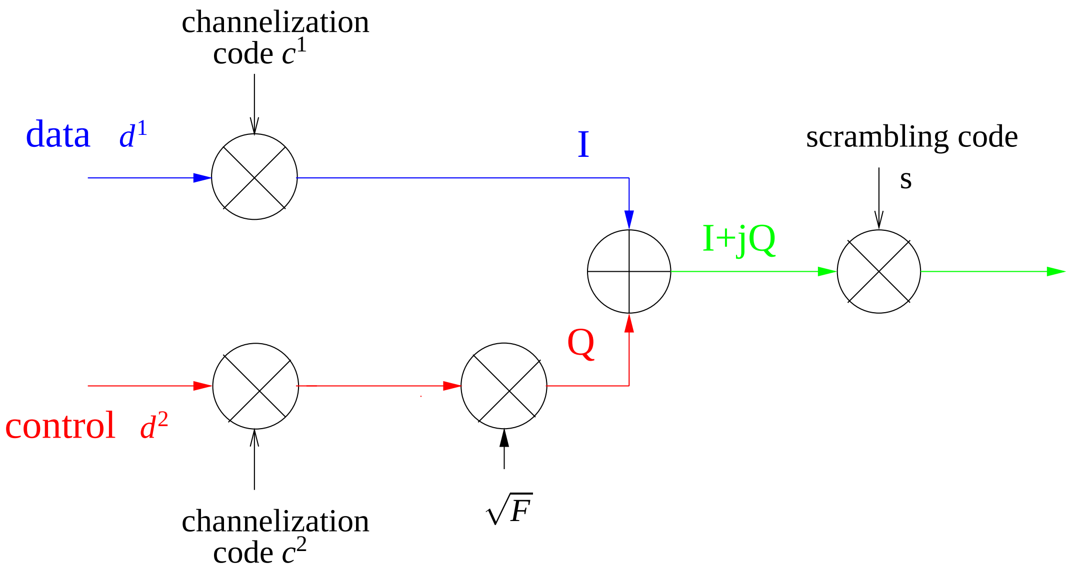
- Many existing localizaton methods.
- Compromise between resolution (MUSIC, ESPRIT, ..) and robustness and computational complexity (Beamforming).

- Many existing extentions:
  - Joint estimation of angles and delays (JADE algorithm)
  - Generalisation to wide-band sources,
  - Tracking and adaptive processing, ...

Application to Mobile Localisation in UMTS-FDD

Up-Link

- Transmitted signal by k-th user:



- Propagation channel

$$\mathbf{h}_k(t) = \sum_{i=1}^{R_k} \mathbf{a}(\theta_{k,i})\beta_{k,i}\mathbf{g}(t - \tau_{k,i})$$

Joint AOA and TOA estimation

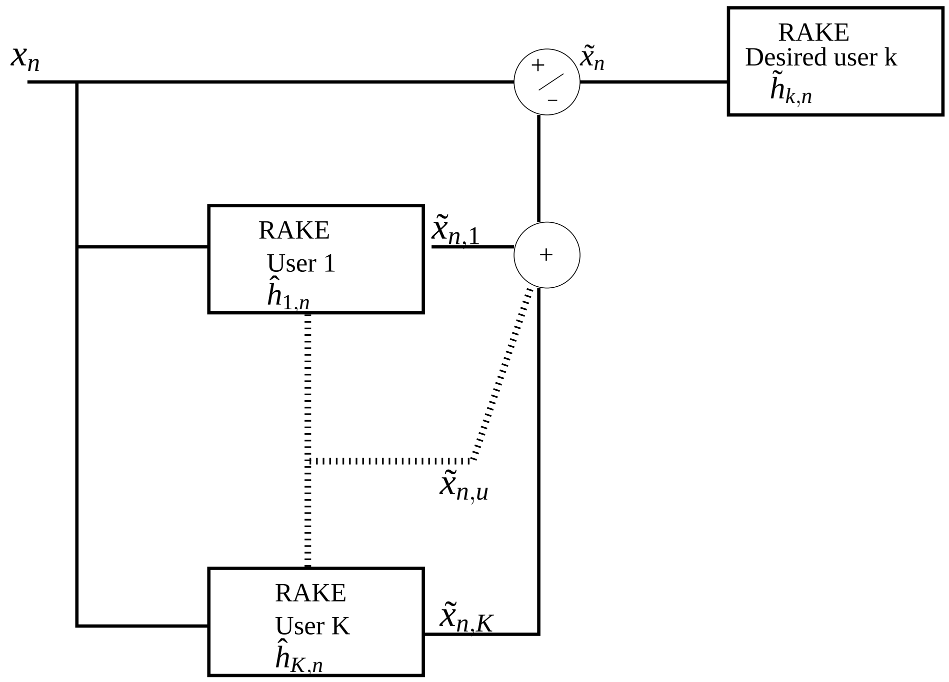
- **Raison:** Correspondance between the AOAs and TOAs of the multi-paths $\Rightarrow$ for joint angle-delay localization. Also, the direct path is chosen as the one associated with the smallest TOA.

- **State of the art**
  - Maximum likelihood approach [Wax & al. 1997].
  - Subspace methods: Time Space Time-MUSIC [Wax & al. 2001].
  - ESPRIT-like methods [Vanderveen & al. 1998].

- **Proposed method:**
  - Delay estimation using the channel FT matrix by ESPRIT.
  - Estimation of only the desired angle (i.e. the one corresponding to the smallest delay).

Hearing problem

- AOA-TOA estimation algorithms require a first channel estimation:
  - RAKE-type estimator: non-robust to near-far effect (interferences).
  - RAKE-estimator with interference cancellation (PIC):



Localization with antenna array:

- Channel model: $\mathbf{h}(t) = \begin{bmatrix} h_1(t) \\ \vdots \\ h_N(t) \end{bmatrix} = \sum_{i=1}^{d} \mathbf{a}(\theta_i)\beta_i \mathbf{g}(t-\tau_i)$
- For a uniform circular array : $\mathbf{a}(\theta_i) = \begin{bmatrix} e^{j\xi\cos(\theta_i-\gamma_1)} \\ \vdots \\ e^{j\xi\cos(\theta_i-\gamma_N)} \end{bmatrix}$
- Channel matrix :

$$\begin{aligned}
\mathbf{H} &\triangleq \left[\mathbf{h}(0)\ \mathbf{h}(\frac{T}{P}) \cdots \mathbf{h}(LT-\frac{T}{P})\right] \\
&= [\mathbf{a}(\theta_1)\cdots\mathbf{a}(\theta_d)] \begin{bmatrix} \beta_1 & & 0 \\ & \ddots & \\ 0 & & \beta_d \end{bmatrix} \begin{bmatrix} \mathbf{g}_{\tau_1} \\ \vdots \\ \mathbf{g}_{\tau_d} \end{bmatrix} \\
&= \mathbf{A}(\theta)\mathbf{B}\mathbf{G}(\tau)
\end{aligned}$$

Delays Estimation

- The FT of $\mathbf{H}$ transforms $\mathbf{G}(\tau)$ (up to a diagonal matrix) into:

$$\mathbf{V}(\tau) = \begin{bmatrix} 1 & \chi_1 & \chi_1^2 & \cdots & \chi_1^{LP-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & \chi_d & \chi_d^2 & \cdots & \chi_d^{LP-1} \end{bmatrix}$$

where $\chi_i = e^{\frac{-j2\pi\tau_i}{L}},\ 1 \leq i \leq d$.

- Matrix $\mathbf{H}_F$ has the rotational invariance property that allows for the estimation of τ_i using ESPRIT algorithm.

Angle estimation

- Once the delays are estimated we estimate the angle of the LOS path according to:
  - Inversion of the delays matrix:

$$\mathbf{H}' = \mathbf{H}\mathbf{G}(\tau)^{-1}$$

  - Selection of the first column $\mathbf{h}_1$ de $\mathbf{H}'$ and estimation of the AOA of the first path by maximizing :

$$\|\mathbf{a}(\theta)^H \mathbf{h}_1\|$$

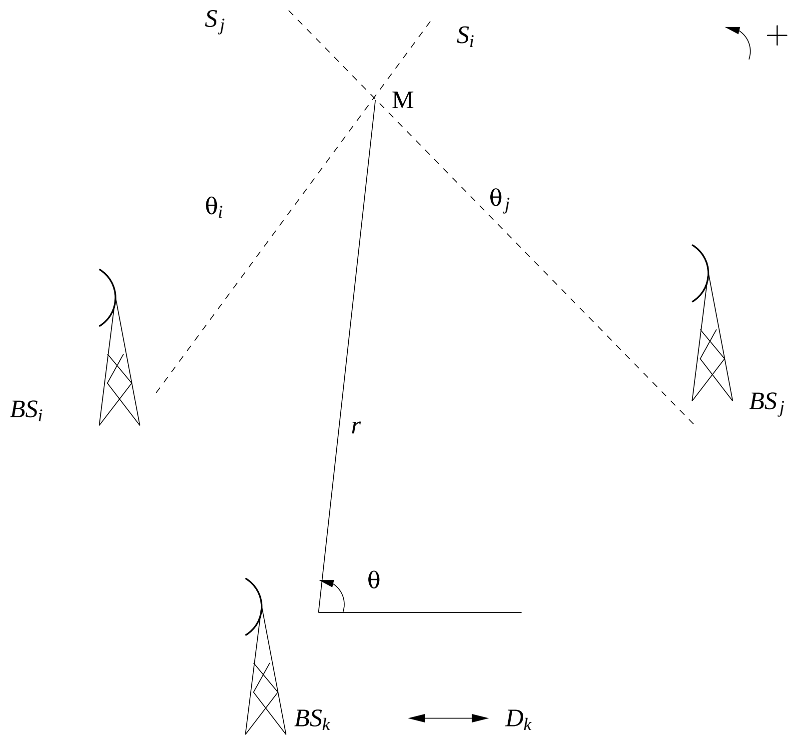
Proposed method for the NLOS

- **Idea:** Selection of the 2 'most reliable' measures: Coherence criterion of 2 given AOAs.
- **Coherence measure:**
  - If we know the distribution of the mobile position D_k w.r.t. a BS k:

$$P(\theta_i, \theta_j / D_k) = D_k(M)$$

    $\Rightarrow$ We would select the pair (θ_i, θ_j) that maximizes $P(\theta_i, \theta_j / D_k)$.
  - To give equal opprtunity to all BSs, we chose:

$$\hat{i}, \hat{j} = \arg \max_{i,j,k} P(\theta_i, \theta_j / D_k)$$

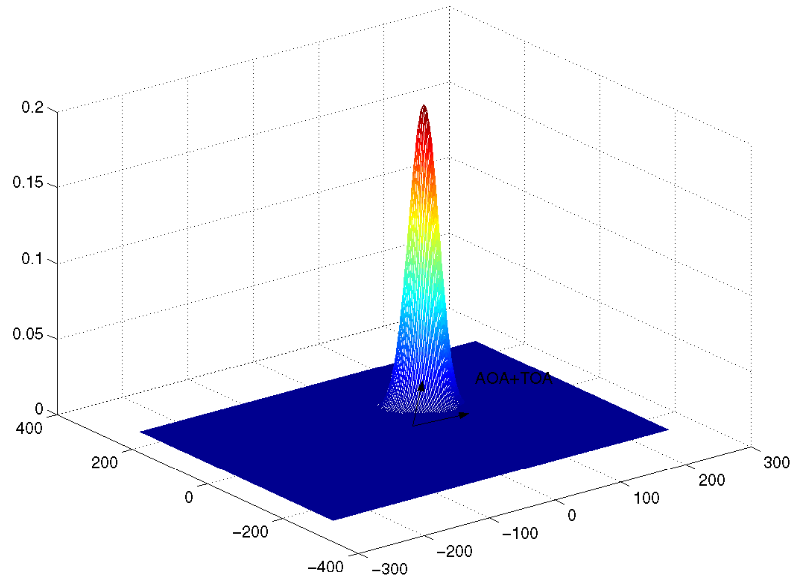


‘A priori’ mobile position distribution

Many possible distributions: We have chosen the Gaussian distribution.

- σ_r et σ_θ are ad-hoc.
- $\mu_\theta = \theta_k$.
- $\mu_r = d_k$: t_k is linked to d_k via the relation:

$$d_k = c(t_k - t_{0_k})$$



$$D(r,\theta) = \frac{1}{2\pi\sigma_r\sigma_\theta} e^{-\left(\frac{r-\mu_r}{\sqrt{2}\sigma_r}\right)^2} e^{-\left(\frac{\theta-\mu_\theta}{\sqrt{2}\sigma_\theta}\right)^2}$$

Synchronisation constraint

- **Problem:** Necessitates between the mobile and the BSs. Too constraining!!

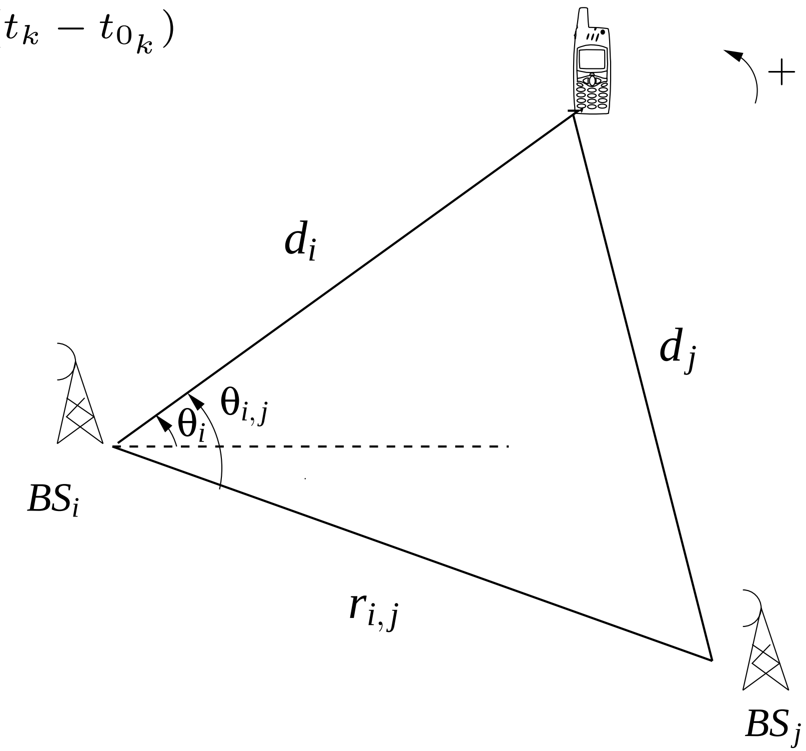
$$d_k = c(t_k - t_{0_k})$$

- **Alternative solution:**
  - Use a similar technique to the Timing Advance in GSM.
  - Estimate the time references by minimizing:

$$\hat{t}_{0_1}, \ldots, \hat{t}_{0_I} =$$

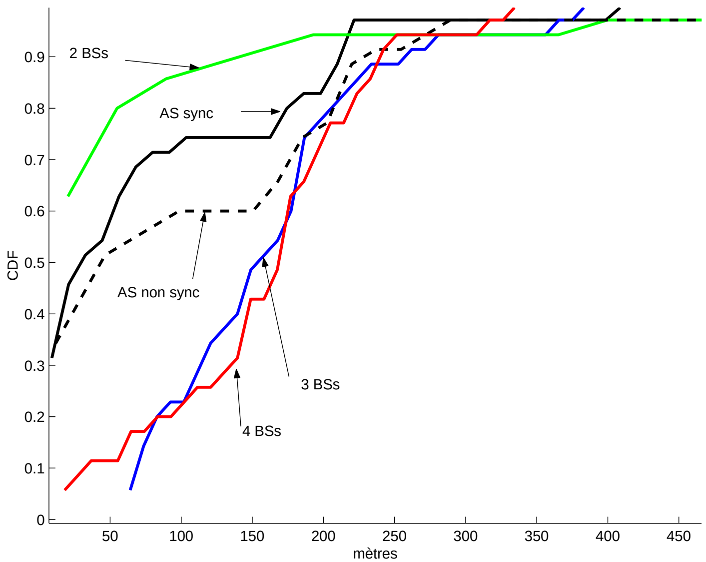
$$\arg\min \sum_{i=1}^{I} \sum_{j=1}^{I} \| d_j(t_{0_j}) - d_j(t_{0_i}) \|^2$$

where $d_{i,j}$ is given by: $\sqrt{r_{i,j}^2 + d_i^2 - 2\cos(\theta_{i,j}) r_{i,j} d_i}$

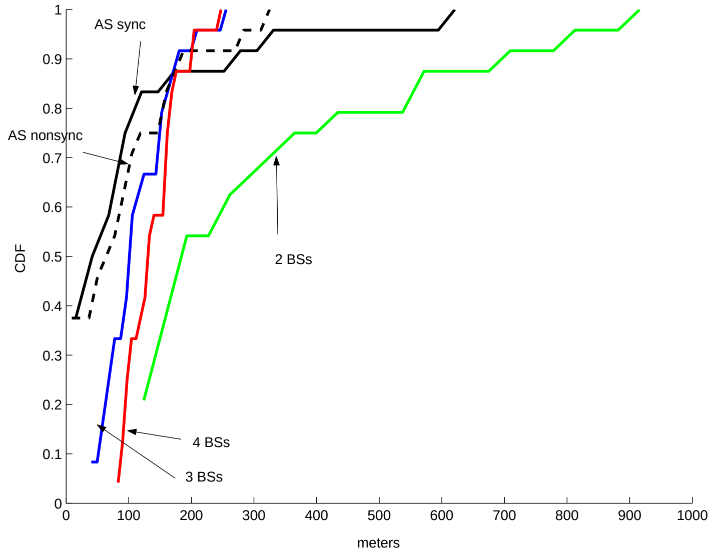


Localization results

- Comparison with standard triangulation techniques.



(k) NLOS sur les BS 3 et 4



(l) NLOS sur les BS 2 et 4

Random mobile position at each run, $L = 80$ slots, 4 BSs, $K = 20$.

Concluding Remarks

Conclusion

- **Main difficulties (Hearing + NLOS)**: No fully satisfactory solution (i.e. still an open problem). We have presented certain solutions using, when possible, partial interference cancellation and selection of the 'best' AOA/TOA estimates. Other solutions exist, e.g.
  - Using 'a priori' learning of the dependence of the channel impulse response on the mobile position (too expensive and requires regular up-dating),
  - Using a 'super calculator' which captures both the transmitted and received signals to extract the desired information,
  - Using Idle periods: reduces significantly the system capacity.

Conclusion

- **Estimation accuracy**: The best estimates are computationally demanding and the power in the downlink is 'restricted'. Good 'intermediate' solutions especially in adaptive schemes.
- **Tracking**: Many tracking algorithms exist using subspace tracking, Kalman filtering, particular filtering, gradient techniques, etc. Tracking might improve the estimation accuracy (at least for slowly moving mobiles) due to memory effect.
- **Hybrid solution**: Use both GPS and terrestrial BS signals for mobile location.